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Wave-particle power transfer in a steady-state driven system

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The general expression of the power transfer from a high-energy ion beam to a background electrostatic plasma wave is obtained for arbitrary wave amplitude. It is verified that phase space gradients produced by a finite amplitude wave enhance the power transfer significantly.

In an earlier work,[1] the expression for the power transfer from an ion beam (injected at high energy and undergoing particle annihilation) to an electrostatic plasma wave, was derived in the limits of small and large amplitudes. It was not possible to completely describe the power transfer rate in the transition region of the linear to nonlinear theory. Here, we construct a solution to this problem that holds for intermediate amplitudes and we demonstrate an enhanced maximum of the power transfer at finite amplitude. This demonstrates that the response of a nonlinear wave[2,3] can differ dramatically from simple Landau damping predictions, as was also discussed in Ref. 1. Thus, the saturation level of the nonlinear wave may be much larger than the expectation of simple dimensional arguments. This arises here because the source of particles establishing the steady state cannot feed particles in the trapping region, which produces a discontinuity at the separatrix of passing and trapped particles, which causes the enhancement of the power transfer. Similar enhancements were observed in a drift wave calculation even without a particle annihilation mechanism present.[4]

We assume a high-energy ion beam is injected into a background plasma where a single mode of an electrostatic wave is present. The injected ions slow down and annihilate as a result of classical drag and charge exchange to form a weakly destabilizing distribution function. We also assume that the distribution of the initial velocity v_0 of the injected ions is given by $I_b(v_0) = QS_I(v_0)$, with the normalization $\int_{-\infty}^{\infty} S_I(v_0)dv_0 = 1$. We write the distribution function of the fast ions as $F(t,x,v) = \int_{-\infty}^{\infty} fS_I(v_0)dv_0$, where $f = f(t,x,v;v_0)$ satisfies:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q\mathscr{E}}{m}\frac{\partial f}{\partial v} = -\nu_a f + a\frac{\partial f}{\partial v} + Q\delta(v - v_0). \tag{1}$$

Here each fast ion with the mass m and the charge q is assumed to slow down with a drag coefficient a and to annihilate at a rate ν_a. The electrostatic electric field $\mathscr{E}$ is $\mathscr{E} = -\partial\varphi/\partial x$, with $\varphi = \varphi_0\cos(kx - \omega t)$, where φ_0 is constant under the assumption that the growth rate of the wave is small. Transforming the independent variables to $\psi = kx - \omega t$ ($k > 0$) and $E = \tilde{u}^2/2 + \Phi\cos\psi + \alpha\psi$, where $\Phi = q\varphi_0/m$, $\alpha = a/k$, and $\tilde{u} = v - \omega/k$, we rewrite Eq. (1) as

$$\frac{\nu}{u}f^{\pm} \pm \frac{\partial f^{\pm}}{\partial\psi} = \frac{gQ}{k}\,\delta\!\left(E - \Phi\cos\psi - \alpha\psi - \frac{u_0^2}{2}\right), \tag{2}$$

where $u(\psi,E) = |\tilde{u}| = \sqrt{2(E - \Phi\cos\psi - \alpha\psi)}$, $\nu = \nu_a/k$, $u_0 = v_0 - \omega/k$,

$$g = \begin{cases} 1, & \text{if } \tilde{u} > 0, \\ 0, & \text{if } \tilde{u} < 0, \end{cases} \quad \text{and} \quad f = \begin{cases} f^{+}(\psi,E), & \text{if } \tilde{u} > 0 \\ f^{-}(\psi,E), & \text{if } \tilde{u} < 0. \end{cases}$$

We note that E is a constant of motion of a slowing down particle and the particle is reflected by the effective potential $\Phi_{\text{eff}}(\psi) = \Phi\cos\psi + \alpha\psi$ at the turning point ψ_t, which is defined as the minimum value of ψ_t that satisfies $u(\psi_t,E) = 0$; other roots ψ_t of $u(\psi_t,E) = 0$ are inaccessible to particles born to the left of the turning points (Fig. 1). The boundary conditions of Eq. (2) are, for a fixed E, $f^{+} \to 0$ as $\psi \to -\infty$ and $f^{+} = f^{-}$ at the turning point $\psi = \psi_t$. There are a finite number $[N(E)]$ of "birth" points $\{\psi_b^{(i)}\}$ $[1 < i < N(E)]$, that satisfy $E = u_0^2/2 + \Phi\cos\psi_b^{(i)} + \alpha\psi_b^{(i)}$. Then the solution to Eq. (2) is

$$f^{\pm}(\psi,E) = \sum_{i=1}^{N(E)} \frac{Q}{ku|\partial u/\partial\psi|_{\psi=\psi_b^{(i)}}} \times \exp\left(-\nu\int_{\psi_b^{(i)}}^{\psi_t}\frac{d\psi'}{u} \mp \nu\int_{\psi_t}^{\psi}\frac{d\psi'}{u}\right). \tag{3}$$

The wave to particle power transfer P_{total} is

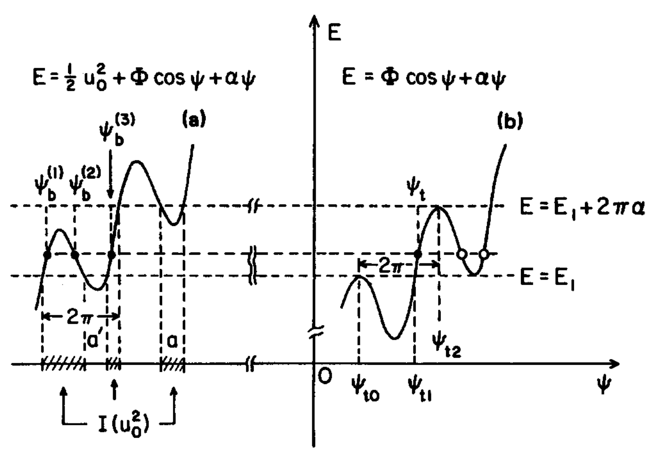

FIG. 1. (a) The birth-point curve $E = \frac{1}{2}u_0^2 + \Phi\cos\psi + \alpha\psi$ and (b) the effective potential curve $E = \Phi_{\text{eff}} = \Phi\cos\psi + \alpha\psi$. For a given E, the birth points $\psi_b^{(i)}$ ($i = 1,2,3$) and the turning point ψ_t are also shown. Here $\psi_{t2} = \psi_{t0} + 2\pi$. We note that O indicates inaccessible points ψ that satisfy $u(\psi,E) = 0$. The total length of the integral region $I(u_0^2)$ is, in general, 2π, which is easily seen in the special case of this figure by moving the section a to a'.

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$$P_{\text{total}} = \frac{kq}{2\pi}\int_0^{2\pi/k} dx \int_{-\infty}^{\infty} dv\, \mathscr{E} vF$$
$$= \int_{-\infty}^{\infty} du_0\, P(u_0) S_I\Big(u_0 + \frac{\omega}{k}\Big), \tag{4}$$

where

$$P(u_0) = \frac{\omega q \varphi_0}{2\pi}\int_0^{2\pi} d\psi \int_{\Phi\cos\psi + \alpha\psi}^{\infty} dE\, \frac{\sin\psi}{u(\psi,E)}$$
$$\times (f^+ + f^-)\Big[1 + \mathscr{O}\Big(\frac{ku}{\omega}\Big)\Big]. \tag{5}$$

We assume that $S_I(v_0)$ is a peaked function about $v_0 = V_0$ with a relatively narrow width ΔV, where ΔV satisfies $(\Phi + \alpha)/U_0 \ll \Delta V \ll U_0 \equiv V_0 - \omega/k$. It is assumed that the contribution to P is from a narrow region in velocity space where the speed of particles is near zero in the wave frame, so that $\mathscr{O}(ku/\omega)$ are ignorable. Since $f^\pm$ satisfies $f^\pm(\psi + 2\pi, E + 2\pi\alpha) = f^\pm(\psi,E)$, we transform $E \to E - 2\pi n\alpha$, $\psi \to \psi - 2\pi n$ (where n are positive integer values) and then we invert the order of integration of Eq. (5)

$$\int_0^{2\pi} d\psi \int_{\Phi\cos\psi + \alpha\psi}^{\infty} dE \cdots = \int_{E_1}^{E_1 + 2\pi\alpha} dE \int_{-\infty}^{\psi_t(E)} d\psi \cdots, \tag{6}$$

where E_1 is a local maximum of $E = \Phi \times \cos\psi + \alpha\psi$ at $\psi = \psi_{t0}$ (as shown in Fig. 1). Using the relation

$$\int_{y_1}^{y_2} \frac{d\psi}{u} = -\frac{u}{\alpha}\Big|_{y_1}^{y_2} + \frac{\Phi}{\alpha}\int_{y_1}^{y_2} \frac{\sin\psi}{u}\, d\psi,$$

we readily obtain from Eq. (3), if $\nu\sqrt{\Phi}/\alpha \ll 1$,

$$f^+ + f^- \simeq \sum_{(i)} \frac{2Q}{ku_0 |\partial u/\partial\psi|_{\psi = \psi_b^{(i)}}} \exp\Big(-\frac{\nu u_0}{\alpha}\Big)$$
$$\times \Big(1 - \frac{\nu\Phi}{\alpha}\int_{\psi_b^{(i)}}^{\psi_t} \frac{\sin\psi}{u}\, d\psi\Big), \tag{7}$$

With the use of Eqs. (5)–(7), $P(u_0)$ is

$$P(u_0) = \frac{\omega\varphi_0 qQ}{\pi k}\exp\Big(-\frac{\nu u_0}{\alpha}\Big)\Big[\int_{I(u_0^2)} d\psi_b$$
$$\times \int_{-\infty}^{\psi_t[E(u_0^2,\psi_b)]} \frac{\sin\psi}{u[\psi, E(u_0^2,\psi_b)]}\, d\psi$$
$$- \frac{\nu\Phi}{\alpha}\int_{I(u_0^2)} d\psi_b$$
$$\times \Big(\int_{-\infty}^{\psi_t[E(u_0^2,\psi_b)]} \frac{\sin\psi}{u[\psi,E(u_0^2,\psi_b)]}\, d\psi\Big)^2\Big], \tag{8}$$

where we used $E(u_0^2,\psi_b) = u_0^2/2 + \Phi\cos\psi_b + \alpha\psi_b$,

$$\int_{E_1}^{E_1 + 2\pi\alpha} dE \sum_{i=1}^{N(E)} \frac{1}{u_0|\partial u/\partial\psi|_{\psi=\psi_b^{(i)}}} = \int_{I(u_0^2)} d\psi_b$$

and $I(u_0^2)$ is the domain of ψ on which the energy $E(u_0^2,\psi) = u_0^2/2 + \Phi\cos\psi + \alpha\psi$; it lies between E_1 and $E_1 + 2\pi\alpha$ for a given u_0. As explained in Fig. 1, the total length of $I(u_0^2)$ is 2π. We have used the relations $dE = (\alpha - \Phi\sin\psi_b^{(i)})d\psi_b^{(i)}$ at each birth point $\psi_b^{(i)}$ for a fixed u_0 and $u\,\partial u(E,\psi)/\partial\psi = \Phi\sin\psi - \alpha$ for a fixed E. The lower limit $\psi_b^{(i)}$ in Eq. (7) is replaced by $-\infty$ in Eq. (8) as $u_0 \gg \Phi^{1/2}$.

In Eq. (8) $\mathscr{P}(u_0^2) \equiv P(u_0)\exp(\nu u_0/\alpha)$ is a periodic function of u_0^2 with period $4\pi\alpha$. Thus the power transfer is

$$P_{\text{total}} = \frac{1}{2}\int_0^{\infty} \frac{dz}{\sqrt{z}}\, e^{-\nu\sqrt{z}/\alpha}\mathscr{P}(z) S_I\Big(\sqrt{z} + \frac{\omega}{k}\Big)$$
$$\simeq e^{-\nu U_0/\alpha}\frac{1}{2}\sum_{n=0}^{\infty} S_I\Big(\sqrt{z_n} + \frac{\omega}{k}\Big)\frac{\Delta z}{\sqrt{z_n}}$$
$$\cdot \frac{1}{\Delta z}\int_{z_n}^{z_n + \Delta z} dz\, \mathscr{P}(z), \tag{9}$$

where $z = u_0^2$, $z_n = n\,\Delta z\ (n \gg 0)$, and $\Delta z = 4\pi\alpha$. We used that $\exp(-\nu\sqrt{z}/\alpha)$ and $S_I(\sqrt{z} + \omega/k)/\sqrt{z}$ are slowly varying functions of z. The average $\overline{P}$ of $\mathscr{P}(z)$ is independent of z_n. Then as

$$\frac{1}{2}\sum_{n=0}^{\infty} S_I\Big(\sqrt{z_n} + \frac{\omega}{z}\Big)\frac{\Delta z}{\sqrt{z_n}} \simeq \int_0^{\infty} S_I\Big(u_0 + \frac{\omega}{k}\Big)du_0 = 1, \tag{10}$$

$$P_{\text{total}} = \frac{\exp(-\nu U_0/\alpha)}{2\pi\alpha}\int_{z_n}^{z_n+\Delta z} dz\, \mathscr{P}(z)$$
$$= \frac{\omega\varphi_0 qQ}{\pi k}\,\frac{\exp(-\nu U_0/\alpha)}{4\pi\alpha}\int_{z_n}^{z_n+\Delta z} du_0^2$$
$$\times \int_{I(u_0^2)} d\psi_b\, g[E(y_0^2,\psi_b)], \tag{11}$$

where

$$g[E(u_0^2,\psi_b)] = \int_{-\infty}^{\psi_t} \frac{\sin\psi}{u}\, d\psi - \frac{\nu\Phi}{\alpha}\Big(\int_{-\infty}^{\psi_t} \frac{\sin\psi}{u}\, d\psi\Big)^2. \tag{12}$$

Now u_0^2 is transformed to $E = E(u_0^2,\psi_b)$ to give

$$\frac{1}{2}\int_{z_n}^{z_n+\Delta z} du_0^2 \int_{I(u_0^2)} d\psi_b = \int\int_{\Omega} dE\, d\psi_b$$
$$= 2\pi\int_{E_1}^{E_1+2\pi\alpha} dE, \tag{13}$$

where the domain Ω of the integration is transformed to a rectangular region, as illustrated in Fig. 2.

We consider the contribution to P_{total} from the first term of the right-hand side of Eq. (12). Averaging Eq. (11) over u_0^2 with the use of Eq. (13) leads us to consider

$$G_2' = -\frac{\Phi}{\alpha}\int_{E_1}^{E_1+2\pi\alpha} dE \int_{-\infty}^{\psi_t(E)} \frac{\sin\psi}{u(\psi,E)}\, d\psi. \tag{14}$$

We define ψ_{t1} as the coordinate satisfying $E_1 \equiv \Phi\cos\psi_{t0} + \alpha\psi_{t0} = \Phi\cos\psi_{t1} + \alpha\psi_{t1}$ as in Fig. 1. Note that we take $\psi_{t0} < \psi_{t1} < \psi_{t2} + 2\pi$ (see Fig. 1) and $\psi_{t1} = \psi_{t0}$ only if $\alpha/\Phi > 1$. We split the integration over ψ in Eq. (14) into the two

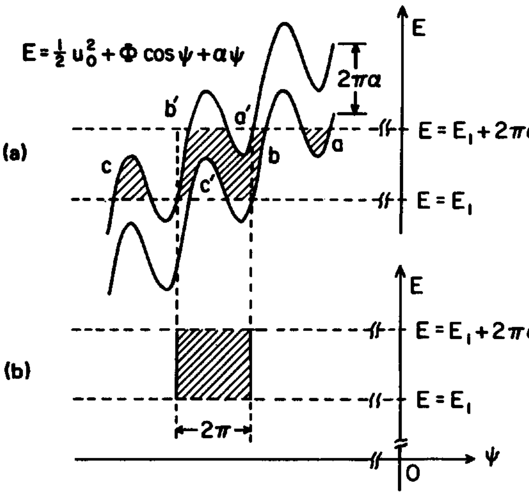

FIG. 2. The integral regions of Eq. (13). The hatched region of (a) indicates Ω, which may be transformed to the hatched region of (b) by transforming $a \to a'$, $b \to b'$, and $c \to c'$ in (a).

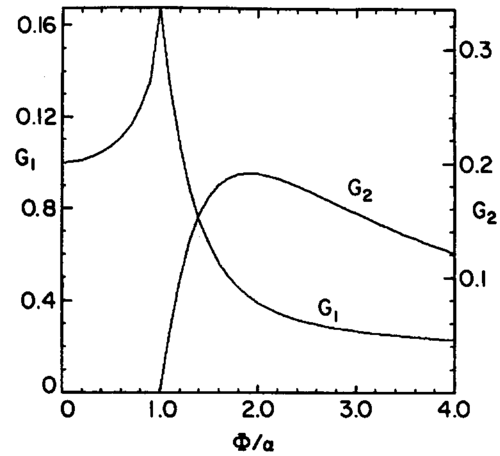

FIG. 3. Plots of the functions $G_1(y)$ and $G_2(y)$. For larger y, $G_1(5.0) = 0.20$, $G_2(5.0) = 0.096$; $G_1(10.0) = 0.14$, $G_2(10.0) = 0.041$; and $G_1(25.0) = 0.068$, $G_2(25.0) = 0.012$. For $y = 25$ the asymptotic form is about 10% larger than the numerical value for G_2 and 20% for G_1.

domains $(-\infty, \psi_{t1})$ and $[\psi_{t1}, \psi_t(E)]$ and exchange the order of the integrations over E and ψ. Changing the variable from E to $u = u(\psi, E)$ yields

$$G_2' = -\frac{\Phi}{\alpha}\left(\int_{-\infty}^{\psi_{t1}} d\psi \int_{u_1(\psi)}^{u_2(\psi)} du \sin\psi + \int_{\psi_{t1}}^{\psi_{t2}} d\psi \int_0^{u_2(\psi)} du \sin\psi\right), \tag{15}$$

where $u_1(\psi) \equiv u(\psi, E_1)$ and $u_2(\psi) \equiv u(\psi, E_1 + 2\pi\alpha)$. Performing the integration over u in Eq. (15) and using the relations $u_2(\psi - 2\pi) = u_1(\psi)$ we obtain

$$G_2' = \int_{\psi_{t0}}^{\psi_{t1}} d\psi\, u_1(\psi).$$

Clearly $G_2' = 0$ if $\Phi < \alpha$ (when $\psi_{t0} = \psi_{t1}$). Therefore the total power transfer is given by

$$P_{\text{total}} = \left[G_1\left(\frac{\Phi}{\alpha}\right) + \beta \frac{\omega}{k\alpha^{1/2}} G_2\left(\frac{\Phi}{\alpha}\right)\right] P_{\text{L}}, \tag{16}$$

where $\beta = ak/v_a\omega \approx \mathcal{O}(1)$, and

$$G_1\left(\frac{\Phi}{\alpha}\right) = \frac{1}{\pi^2}\int_0^{2\pi} d\xi \left(\int_{-\infty}^{\psi_t(\xi)} \frac{d\psi \sin\psi}{[\xi - (\Phi/\alpha)\cos\psi - \psi]^{1/2}}\right)^2, \tag{17}$$

$$\begin{aligned} G_2\left(\frac{\Phi}{\alpha}\right) &= \frac{2}{\pi^2}\frac{\alpha^{3/2}}{\Phi^2} G_2' \\ &= \frac{2\sqrt{2}}{\pi^2}\left(\frac{\alpha}{\Phi}\right)^2 \\ &\quad \times \int_{\psi_{t0}}^{\psi_{t1}} d\psi \left(\frac{\Phi}{\alpha}(\cos\psi_{t1} - \cos\psi) + \psi_{t1} - \psi\right)^{1/2}, \end{aligned} \tag{18}$$

$$\begin{aligned} P_{\text{L}} &= P_{\text{total}}(\Phi \to 0) \\ &= -\frac{\pi m\omega Q}{2}\Phi^2 \frac{v_a}{a^2} \exp\left[-\frac{v_a}{a}\left(V_0 - \frac{\omega}{k}\right)\right]. \end{aligned} \tag{19}$$

Note that $G_1(y \to 0) = 1$, $G_1(y \to \infty) \to 64/\pi^3 y$, $G_2(y) = 0$ if $y < 1$, and $G_2(y \to \infty) = 16/\pi^2 y^{3/2}$. The contribution to P_{total} from G_2 agrees with Ref. 1 for all y while the contribution from G_1 agrees with Ref. 1 in the asymptotic limit $\Phi/\alpha \gg 1$. The result for $\Phi/\alpha \approx 1$ is a new result. The numerical structure of $G_1(y)$ and $G_2(y)$ is given in Fig. 3.

Linear theory applies if $\Phi/\alpha \ll 1$ and the power transfer changes scale when $\Phi/\alpha > 1$. For $\Phi/\alpha < 1$, $G_2 = 0$, and the power transfer rate $G_1 P_{\text{L}}$ is comparable to the predicted linear rate. If the G_2 term were not present the power transfer rate, resulting from the nonlinearity of Φ in the G_1 term, would gradually change for finite Φ/α and for large Φ/α it would be reduced by a factor $64\alpha/\pi^3\Phi$. If in addition to the destabilizing drive a linear dissipative power transfer of the wave to the background plasma was present at a rate $G_d P_{\text{L}}$ (note $G_d < 1$ if linear instability is to occur), the level of saturation is determined by the zero power transfer condition $(G_1 - G_d)P_{\text{L}} = 0$ (if $G_d \ll 1$ the saturation level is $\Phi/\alpha = 64/\pi^3 G_d$). However, particle annihilation forces the presence of the G_2 term, which completely changes the scale of the saturation level. We note that the power transfer is amplified by a large factor $\omega/k\alpha^{1/2}$. The critical point, $\Phi/\alpha = 1$, just occurs when a separatrix arises and the particles slowing down from the source are unable to penetrate the trapping region. Saturation then occurs when $\Phi/\alpha \gg 1$. Then neglecting G_1, the zero power transfer condition, $(\beta\omega G_2/k\alpha^{1/2} - G_d)P_{\text{L}} = 0$, predicts that saturation occurs when $\Phi = (16\omega\beta\alpha/\pi^2 k G_d)^{2/3}$, roughly a factor $(\omega^2/k^2\alpha)^{1/3}$ larger than would be inferred from examining parameters arising in linear theory.

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[1] H. L. Berk and B. N. Breizman, Phys. Fluids B 2, 2226 (1990).
[2] T. O'Neil, Phys. Fluids 8, 2255 (1968).
[3] R. K. Mazitov, Zh. Prikl. Mekh. Fiz. 1, 27 (1965).
[4] H. L. Berk and B. Breizman, Phys. Fluids B 2, 2235 (1990).