Dynamical electroweak symmetry breaking in $SO(5) \times U(1)$ gauge-Higgs unification with top and bottom quarks

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An $SO(5) \times U(1)$ gauge-Higgs unification model in the Randall-Sundrum warped space with top and bottom quarks is constructed. Additional fermions on the Planck brane make exotic particles heavy by effectively changing boundary conditions of bulk fermions from those determined by orbifold conditions. Gauge couplings of a top quark multiplet trigger electroweak symmetry breaking by the Hosotani mechanism, simultaneously giving a top quark the observed mass. The bottom quark mass is generated by the combination of brane interactions and the Hosotani mechanism, where only one ratio of brane masses is relevant when the scale of brane masses is much larger than the Kaluza-Klein scale ($\sim$ 1.5 TeV). The Higgs mass is predicted to be 49.9 (53.5) GeV for the warp factor 10^{15} (10^{17}). The Wilson line phase turns out $\frac{1}{2}\pi$ and the Higgs couplings to W and Z vanish so that the LEP2 bound for the Higgs mass is evaded. In the flat spacetime limit the electroweak symmetry is unbroken.

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I. INTRODUCTION

The Higgs particle is the only particle missing in the standard model of electroweak interactions. It is necessary to induce the electroweak symmetry breaking and is expected to be discovered at the LHC in the near future. However, it is not obvious if the Higgs particle appears as described in the standard model. Its mass and couplings to other particles may deviate from those in the standard model.

What we are going to learn from LHC experiments is, among others, the structure and origin of symmetry breaking. Both electroweak unified theory and grand unified theory are constructed with higher gauge symmetry, which, in turn, must break down at low energies. The mechanism of gauge symmetry breaking is the backbone of unification. For the first time in history we are going to directly see the mechanism of gauge symmetry breaking. In the current folklore this gauge symmetry breaking is supposed to be triggered by a Higgs scalar field. Unlike gauge interactions, however, there seems no guiding principle to regulate interactions of a Higgs field, including its self-interactions and Yukawa couplings to fermions. Further there arises the gauge hierarchy problem when the standard model is implemented in grand unified theory.

There are many scenarios proposed beyond the standard model. Besides supersymmetry, the Higgsless scenario, and the little Higgs scenario, there is another intriguing scenario called gauge-Higgs unification in which the 4D Higgs field is identified with the extra-dimensional component of gauge potentials in higher dimensional gauge theory (see e.g. [1,2] and references therein). Symmetry breaking is caused, at the quantum level, by dynamics of Wilson line phases in extra dimensions through the Hosotani mechanism [3–5]. Higgs couplings are controlled by gauge principle with a given spacetime background.

Significant progress has been achieved in the gauge-Higgs unification scenario in recent years. Chiral fermions are naturally accommodated on orbifolds [6]. At low energies the standard model is reproduced in models based on such groups as $SU(3)$ and $SO(5) \times U(1)$ [7–15]. Besides the naturalness of the small Higgs mass may follow from gauge invariance associated with Wilson line phases [16].

In models in flat space the Higgs mass is predicted too small, typically 1 order of magnitude smaller than m_W, and the WWZ coupling may significantly deviate from that in the standard model. One way out is to construct a model such that the Wilson line phase takes a sufficiently small value by tuning matter content [12,14]. In Ref. [14] an $SU(3)$ model has been proposed by incorporating fermions in **3**, **6**, and **10** representations of the group. Scrucca *et al.* explored a model with localized kinetic terms for gauge fields to a heavy Higgs field [9]. It has been also discussed that models in six or more dimensions may help Higgs fields acquire large masses from self-couplings. Antoniadis *et al.* discussed a model on $M^4 \times (T^2/Z_2)$. The very existence of Wilson line phases, however, implies flat directions in the Higgs potential at the tree level, resulting in a light Higgs field [7,13].

The alternative way to resolve the difficulties is to consider the gauge-Higgs unification in the Randall-Sundrum (RS) space [17–39]. It has been argued that the Higgs mass picks up an enhancement factor given by the logarithm of the warp factor in the RS space so that the Higgs mass can fall in the LHC range for generic values of the Wilson line phase θ_H [21]. Despite the fact that wave functions of W and Z get rotated in the group space at finite θ_H, their profile in the extra dimension remains almost flat in the RS space, whereas substantial deformation results in flat space. As a consequence the WWZ coupling remains universal to high accuracy irrespective of the value of θ_H in the RS space unlike the case in flat space [27,28]. However, the WWH and ZZH couplings, where H stands for the Higgs field, are suppressed by a factor $\cos\theta_H$ compared with those in the standard model [27,28,40]. This is a distinctive prediction and can be tested at the LHC. There are also predictions for Yukawa couplings [23], electroweak precision tests [15,33,41], anomalous magnetic moments [42], etc.

There remains a challenging task to incorporate quarks and leptons with dynamical electroweak (EW) symmetry breaking in the scheme. At low energies the quark-lepton content in the standard model must be reproduced with correct gauge couplings and mass spectrum. As a generic feature, fermion multiplets in gauge-Higgs unification tend to give rise to unwanted exotic light particles, which must be made heavy by some means such as brane mass terms [10]. More importantly the presence of fermions is vital to have EW symmetry breaking. The fermion content must be such that it triggers EW symmetry breaking by quantum effects.

To summarize, to include quarks and leptons with dynamical EW symmetry breaking with correct gauge couplings and mass spectra is a highly nontrivial task. An important step has been put forward by Medina, Shar, and Wagner who proposed an $SO(5) \times U(1)$ model in the RS space with fermions in **5** and **10** representations of $SO(5)$ [32]. It has been shown that the presence of a top quark induces electroweak symmetry breaking. The fermion structure of this model is elaborated in order to pass the electroweak precision tests, especially to obtain appropriate S and T parameters as well as the consistent $Zb\bar{b}$ coupling. Each generation follows from two **5** multiplets and one **10** multiplet in the bulk, where unwanted fields are made heavy by assigning a specific choice of boundary conditions. These boundary conditions can be achieved, even if one starts from the boundary conditions consistent with orbifolding, by introducing numbers of additional brane fermions in order to effectively modify the original orbifold boundary conditions to the desired ones for low-lying modes in the associated Kaluza-Klein towers. Note that the orbifolding by Z_2 parity plays an important role to remove the gravitational instability due to having the negative tension brane on the orbifold fixed point in the Randall-Sundrum spacetime in addition to solving the Einstein equations. It can be checked that the boundary conditions in Ref. [32] can be obtained from the orbifold ones by introducing $O(10)$ brane fermion multiplets for each generation. Although the success of the model is striking, there are many free parameters, which make the relation between the parameters in the Lagrangian and the low energy observables obscure. Therefore it is difficult to transparently see the theoretical structure when one further seeks simpler models.

In the present paper we propose a model with simpler fermion content written in the form of a Lagrangian with natural boundary conditions dictated by the orbifold structure in the RS space as a prototype to clarify the above-mentioned relation so that the electroweak symmetry breaking structure is transparent and the low energy fields can be explicitly written in terms of the bulk and brane fields, postponing the full analysis of the S and T parameters. The model is constructed to fit in with the criteria of Ref. [43] for suppressing radiative corrections to the ρ (T) parameter and $Zb\bar{b}$ coupling. Everything should follow from the equations of motion, or the action principle. Brane fermions introduced on the Planck brane, through couplings with bulk fermions, effectively change boundary conditions of some of the fermion components to push all unwanted exotic particles to the Kaluza-Klein mass scale. Quite interestingly the low energy physics does not depend on the values of various parameters introduced on the Planck brane except for one ratio of mass parameters which is fixed by m_b/m_t. Furthermore, it is found that the effective potential for the Wilson line phase θ_H is minimized at $\pm\frac{1}{2}\pi$ as a consequence of gauge dynamics of heavy top quarks. At $\theta_H = \pm\frac{1}{2}\pi$ the EW symmetry is dynamically broken to $U(1)_{\rm EM}$ and the WWH and ZZH couplings vanish at the tree level. With m_W and m_t given, the Higgs mass is predicted around 50 GeV. The LEP2 bound for the Higgs mass is evaded due to the vanishing ZZH coupling. The model predicts a light Higgs particle with a narrow width.

The paper is organized as follows. The model is specified in Sec. II. The spectrum in the gauge-Higgs sector, which is known in the literature, is briefly summarized in Sec. III in the form suited for subsequent applications. In Sec. IV the spectrum of fermions is analyzed by solving coupled equations of bulk and brane fermions. It will be shown how brane mass terms give rise to discontinuities in bulk fermions at the Planck brane, effectively changing boundary conditions there. Equations determining the spectrum take complicated forms in the presence of brane mass terms. The expressions are tremendously simplified when the scale of brane masses is much larger than the Kaluza-Klein mass scale $m_{\rm KK}$. It is found that the top mass m_t is generated by the Hosotani mechanism almost independent of brane masses, whereas the bottom mass is generated by the combination of brane masses and the Hosotani mechanism. With the knowledge of the mass spectrum the effective potential for the Wilson line phase $V_{\rm eff}(\theta_H)$ is evaluated to examine electroweak symmetry breaking in Sec. V. It will be found that the contribution from a top quark dominates over those from the gauge fields such that the potential is minimized at $\theta_H = \pm\frac{1}{2}\pi$ and the electroweak symmetry is dynamically broken. Contributions from light quarks and leptons are negligible.

In Sec. VI the Higgs mass is evaluated. Section VII is devoted to summary and discussions.

II. $SO(5) \times U(1)$ MODEL

The metric in the Randall-Sundrum (RS) warped spacetime [44] is given by

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (2.1)$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$, $\sigma(y) = \sigma(y + 2L)$, and $\sigma(y) \equiv k|y|$ for $|y| \le L$. The fundamental region in the fifth dimension is given by $0 \le y \le L$, which is sandwiched by the Planck brane at $y = 0$ and the TeV brane at $y = L$, respectively. The bulk region $0 < y < L$ is a sliced anti-de Sitter (AdS) spacetime with a negative cosmological constant $\Lambda = -6k^2$. The metric is specified with two parameters, k and kL. In the gauge-Higgs unification scenario discussed below the W boson mass is predicted as $m_W(k, kL, \theta_H)$ where θ_H is the Wilson line phase of gauge fields determined dynamically once the matter content is given. With m_W given, therefore, there remains only one parameter specifying the spacetime. In field theory in the Randall-Sundrum spacetime there appear Kaluza-Klein (KK) excitations in a tower for each particle, with the KK mass scale given by

$$m_{\mathrm{KK}} = \frac{\pi k}{e^{kL} - 1} \sim \pi k e^{-kL}. \quad (2.2)$$

We shall find that for $e^{kL} = 10^{15}$ (10^{17}), $k = 4.72 \times 10^{17}$ GeV $(5.03 \times 10^{19}$ GeV) and $m_{\mathrm{KK}} = 1.48$ TeV (1.58 TeV). The results in the present paper are insensitive to the value of k in the above range.

We consider an $SO(5) \times U(1)_X$ gauge theory in the Randall-Sundrum warped spacetime. The $SO(5) \times U(1)_X$ symmetry is reduced to $SO(4) \times U(1)_X$ by orbifold boundary conditions. The symmetry is further reduced to $SU(2)_L \times U(1)_Y$ on the Planck brane. In the present paper we address neither a question of how the orbifold structure of spacetime appears with orbifold conditions, nor a question of how the symmetry further reduces to the standard model symmetry $SU(2)_L \times U(1)_Y$ on the Planck brane. We imagine these happen at a high energy scale of $O(M_{\mathrm{GUT}})$ to $O(M_{\mathrm{Planck}})$ as described below.

The Lagrangian density consists of four parts:

$$\mathcal{L} = \mathcal{L}^{\mathrm{gauge}}_{\mathrm{bulk}} + \mathcal{L}^{\mathrm{scalar}}_{\mathrm{Pl.\ brane}} + \mathcal{L}^{\mathrm{fermion}}_{\mathrm{bulk}} + \mathcal{L}^{\mathrm{fermion}}_{\mathrm{Pl.\ brane}}. \quad (2.3)$$

The bulk parts respect $SO(5) \times U(1)_X$ gauge symmetry. There are $SO(5)$ gauge fields A_M and $U(1)_X$ gauge field B_M. The former are decomposed as $A_M = \sum_{a_L=1}^3 A_M^{a_L} T^{a_L} + \sum_{a_R=1}^3 A_M^{a_R} T^{a_R} + \sum_{\hat{a}=1}^4 A_M^{\hat{a}} T^{\hat{a}}$ where T^{a_L, a_R} $(a_L,\ a_R = 1, 2, 3)$ and $T^{\hat{a}}$ $(\hat{a} = 1, 2, 3, 4)$ are the generators of $SO(4) \sim SU(2)_L \times SU(2)_R$ and $SO(5)/SO(4)$, respectively. $\mathcal{L}^{\mathrm{gauge}}_{\mathrm{bulk}}$ is given by

$$\mathcal{L}^{\mathrm{gauge}}_{\mathrm{bulk}} = -\mathrm{tr}\,\tfrac{1}{2}F^{(A)MN}F^{(A)}_{MN} - \tfrac{1}{4}F^{(B)MN}F^{(B)}_{MN} \quad (2.4)$$

with the associated gauge fixing and ghost terms, where $F^{(A)}_{MN} = \partial_M A_N - \partial_N A_M - ig_A[A_M, A_N]$ and $F^{(B)}_{MN} = \partial_M B_N - \partial_N B_M$.

The orbifold boundary conditions at $y_0 = 0$ and $y_1 = L$ for gauge fields are given by

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1},$$
$$\begin{pmatrix} B_\mu \\ B_y \end{pmatrix}(x, y_j - y) = \begin{pmatrix} B_\mu \\ -B_y \end{pmatrix}(x, y_j + y),$$
$$P_j = \mathrm{diag}(-1, -1, -1, -1, +1) \qquad (j = 0, 1), \quad (2.5)$$

which reduce the $SO(5) \times U(1)_X$ symmetry to $SO(4) \times U(1)_X$. We introduce a scalar field $\Phi(x)$ on the Planck brane which belongs to $(0, \frac{1}{2})$ representation of $SO(4) = SU(2)_L \times SU(2)_R$ and has a charge of $U(1)_X$. With

$$\mathcal{L}^{\mathrm{scalar}}_{\mathrm{Pl.\ brane}} = \delta(y)\{-(D_\mu\Phi)^\dagger D^\mu\Phi - \lambda_\Phi(\Phi^\dagger\Phi - w^2)^2\},$$
$$D_\mu\Phi = \partial_\mu\Phi - i\left(g_A \sum_{a_R=1}^{3} A_\mu^{a_R} T^{a_R} + \frac{g_B}{2} B_\mu\right)\Phi \quad (2.6)$$

the $SU(2)_R \times U(1)_X$ symmetry breaks down to $U(1)_Y$, the massless modes of $A_\mu^{1_R}$, $A_\mu^{2_R}$, and $A_\mu^{\prime 3_R}$ acquiring large masses. Here

$$\begin{pmatrix} A_M^{\prime 3_R} \\ B_M^Y \end{pmatrix} = \begin{pmatrix} c_\phi & -s_\phi \\ s_\phi & c_\phi \end{pmatrix} \begin{pmatrix} A_M^{3_R} \\ B_M \end{pmatrix},$$
$$c_\phi \equiv \frac{g_A}{\sqrt{g_A^2 + g_B^2}}, \qquad s_\phi \equiv \frac{g_B}{\sqrt{g_A^2 + g_B^2}}. \quad (2.7)$$

We suppose that w is much bigger than the Kaluza-Klein mass scale, being of $O(M_{\mathrm{GUT}})$ to $O(M_{\mathrm{Planck}})$. The net effect for low-lying modes of the Kaluza-Klein towers of $A_\mu^{1_R}$, $A_\mu^{2_R}$, and $A_\mu^{\prime 3_R}$ is that they effectively obey Dirichlet boundary conditions at the Planck brane. We note that the effective change of boundary conditions occurs for A_μ components, but not for A_y components as seen from (2.6). This also conforms with the invariance under large gauge transformations which shift the Wilson line phase by a multiple of 2π. We see in the subsequent sections, in a concrete manner, that a similar effective change of boundary conditions takes place for fermions as well.

We remark that massive modes in the Kaluza-Klein towers of the A_y components are unphysical. Their spectrum, in general, depends on gauge-fixing conditions imposed. In the bulk we adopt the gauge fixing in Refs. [22,23] so that the kinetic terms of the A_μ and A_y of $SO(5) \times U(1)_X$ become diagonal.

On the Planck brane at $y = 0$ one further adds a gauge-fixing condition ($\propto \delta(y)$) suitable for the gauge symmetry breaking $SU(2)_R \times U(1)_X \to U(1)_Y$ induced by the scalar field $\Phi(x)$. The most convenient choice is the standard R_ξ gauge, in which the ghost fields in the bulk have the same boundary conditions and spectrum as the corresponding A_μ components for $w \gg M_{\rm KK}$.

In our scheme the A_y components of $SU(2)_R \times U(1)_X$ are odd under parity around $y = 0$ and obey the Dirichlet condition so that they do not enter into the gauge-fixing conditions at the Planck brane. It is possible to allow discontinuities in A_y at $y = 0$ by enlarging the configuration space for A_y. In this case one may include discontinuities in A_y in the gauge-fixing condition at the Planck brane. This possibility is examined in Ref. [2]. It is shown there that the boundary conditions for A_y at $y = 0$ and the resulting spectrum depend on the gauge parameters introduced. In one limit (the ξ parameter in the bulk approaching 0) A_y obeys the Dirichlet boundary conditions at $y = 0$, whereas in another limit ($\xi \to \infty$) corresponding to the unitary gauge A_y obeys the Neumann boundary conditions which were adopted in Ref. [32]. In this paper all A_y components are supposed to be continuous at the Planck and TeV branes.

The resultant spectrum at low energies in the gauge sector is that of the standard model as discussed in detail in Ref. [28]. There appear W, Z bosons and photons as massless gauge fields, and an $SU(2)_L$ doublet Higgs boson ϕ from the A_y component. The Higgs boson, which is nothing but a fluctuation mode of the Wilson line phase θ_H, is massless at the tree level, but becomes massive at the quantum level. We shall see below that the effective potential $V_{\rm eff}(\theta_H)$ is minimized at $\theta_H = \pm\frac{1}{2}\pi$ due to a contribution from the top quark multiplet so that the electroweak symmetry breaks down to $U(1)_{\rm EM}$. At the same time the Higgs field acquires a mass $\sim$50 GeV.

To describe top and bottom quarks we introduce two fermion multiplets in the vector representation of $SO(5)$. In the bulk one has

$$\mathcal{L}_{\rm bulk}^{\rm fermion} = \sum_{a=1}^{2} \bar{\Psi}_a \mathcal{D}(c_a)\Psi_a,$$

$$\mathcal{D}(c_a) = \Gamma^A e_A{}^M\Big(\partial_M - \frac{1}{8}\omega_{MBC}[\Gamma^B,\Gamma^C] - ig_A A_M - iq_a\frac{g_B}{2}B_M\Big) - c_a\sigma'(y). \tag{2.8}$$

Here $e_A{}^M$'s are tetrads and Γ^A's are Dirac matrices in the orthonormal frame. We take in the present paper

$$\Gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & \end{pmatrix} \qquad (\mu = 0,1,2,3),$$
$$\Gamma^5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \qquad \sigma^\mu = (1,\vec\sigma), \qquad \bar\sigma^\mu = (-1,\vec\sigma). \tag{2.9}$$

The c_a term in (2.8) gives a bulk kink mass [45], where $\sigma'(y) = k\epsilon(y)$ is a periodic step function with a magnitude k. The dimensionless parameter c_a plays an important role in the Randall-Sundrum warped spacetime. The $U(1)_X$ charges are $q_1 = \frac{2}{3}$ and $q_2 = -\frac{1}{3}$. The notation $\bar\Psi = i\Psi^\dagger\Gamma^0$ has been adopted. The orbifold boundary conditions are given by

$$\Psi_a(x, y_j - y) = -P_j\Gamma^5\Psi_a(x, y_j + y). \tag{2.10}$$

With P_j in (2.5) the first four components of Ψ_a are even under parity for the 4D left-handed ($\Gamma^5 = -1$) components.

To facilitate discussions below, it is useful to express the $SU(2)_L \times SU(2)_R$ content of an $SO(5)$ vector Ψ. The first four components ψ_k $(k = 1,\cdots,4)$ belong to $(\frac{1}{2},\frac{1}{2})$, whereas the fifth component ψ_5 to (0, 0). We define $\hat\psi$ by

$$\hat\psi = \begin{pmatrix} \hat\psi_{11} & \hat\psi_{12} \\ \hat\psi_{21} & \hat\psi_{22}\end{pmatrix} \equiv \frac{1}{\sqrt2}(\psi_4 + i\vec\psi\cdot\vec\tau)i\tau_2 = \frac{1}{\sqrt2}\begin{pmatrix} -i\psi_1 - \psi_2 & i\psi_3 + \psi_4 \\ i\psi_3 - \psi_4 & i\psi_1 - \psi_2 \end{pmatrix}, \tag{2.11}$$

which transforms, under an $SU(2)_L \times SU(2)_R$ transformation $\Omega_L \otimes \Omega_R$, as $\hat\psi' = \Omega_L\hat\psi\Omega_R{}^t$. $(\hat\psi_{11},\hat\psi_{21})^t$ and $(\hat\psi_{12},\hat\psi_{22})^t$ form $SU(2)_L$ doublets. In the following we express components of Ψ as $\Psi = (\hat\psi_{11},\hat\psi_{21},\hat\psi_{12},\hat\psi_{22},\psi_5)^t$. With this notation, the two fermion multiplets contain

$$\Psi_1 = \begin{pmatrix} T \\ B \\ t \\ b \\ t'\end{pmatrix}\begin{matrix}\frac{5}{3}\\ \frac{2}{3}\\ \frac{2}{3}\\ -\frac{1}{3}\\ \frac{2}{3}\end{matrix}, \qquad \Psi_2 = \begin{pmatrix} U \\ D \\ X \\ Y \\ b'\end{pmatrix}\begin{matrix}\frac{2}{3}\\ -\frac{1}{3}\\ -\frac{1}{3}\\ -\frac{4}{3}\\ -\frac{1}{3}\end{matrix}. \tag{2.12}$$

The numbers written on the side are values of the electric charge $Q_E = T^{3_L} + T^{3_R} + Q_X$. (T, B), (t, b), (U, D), and (X, Y) form $SU(2)_L$ doublets. ψ_4, which couples to ψ_5 through the Wilson line phase θ_H, is given by $(\hat\psi_{12} - \hat\psi_{21})/\sqrt2$.

If there were no additional interactions on the branes, there would appear, before the EW symmetry breaking, massless modes in 4D in $SU(2)_L$ multiplets

$$Q_{1L} = \begin{pmatrix} T_L \\ B_L\end{pmatrix}, \qquad q_L = \begin{pmatrix} t_L \\ b_L\end{pmatrix}, \qquad t'_R,$$
$$Q_{2L} = \begin{pmatrix} U_L \\ D_L\end{pmatrix}, \qquad Q_{3L} = \begin{pmatrix} X_L \\ Y_L\end{pmatrix}, \qquad b'_R. \tag{2.13}$$

After the EW symmetry breaking by the Hosotani mechanism the top quark and extra b' would acquire finite masses, but other left-handed modes would remain massless. We want the top and bottom to acquire the observed masses, whereas other light modes to gain large masses of the Kaluza-Klein scale.

We show in the present paper that the presence of brane fermions [10] on the Planck brane (at $y = 0$) cures these problems naturally. We introduce three right-handed multiplets belonging to $(\frac{1}{2}, 0)$ representation of $SU(2)_L \times SU(2)_R$:

$$\hat{\chi}_{1R} = \begin{pmatrix} \hat{T}_R \\ \hat{B}_R \end{pmatrix}_{7/6}, \quad \hat{\chi}_{2R} = \begin{pmatrix} \hat{U}_R \\ \hat{D}_R \end{pmatrix}_{1/6}, \quad \hat{\chi}_{3R} = \begin{pmatrix} \hat{X}_R \\ \hat{Y}_R \end{pmatrix}_{-5/6}. \tag{2.14}$$

Here the subscripts 7/6, etc. represent the $U(1)_X$ charges of the corresponding multiplets. We write down a general $SU(2)_L \times U(1)_Y$ invariant brane action:

$$\mathcal{L}^{\rm fermion}_{\rm Pl.\,brane} = i\delta(y)\bigg\{\sum_{\alpha=1}^{3} \hat{\chi}^\dagger_{\alpha R}\bar{\sigma}^\mu D_\mu \chi_{\alpha R} - \sum_{\alpha=1}^{3} \mu_\alpha(\hat{\chi}^\dagger_{\alpha R} Q_{\alpha L} - Q^\dagger_{\alpha L}\hat{\chi}_{\alpha R}) - \tilde{\mu}(\hat{\chi}^\dagger_{2R} q_L - q^\dagger_L \hat{\chi}_{2R})\bigg\}, \tag{2.15}$$

where D_μ in the kinetic term has the same form as in (2.6) with $A^{a_R}_\mu T^{a_R}$ replaced by $A^{a_L}_\mu T^{a_L}$. There are four brane mass parameters, μ_α and $\tilde{\mu}$, which have dimensions of $(\text{mass})^{1/2}$. In the subsequent discussions we suppose that μ_α^2 and $\tilde{\mu}^2$ are much larger than the Kaluza-Klein scale $m_{\rm KK} \sim 1.5$ TeV, possibly being of order $M_{\rm GUT}$ or $M_{\rm Planck}$. It will be shown below that the only relevant parameter for the spectrum at low energies is the ratio $\tilde{\mu}/\mu_2$ so long as $\mu_\alpha^2, \tilde{\mu}^2 \gg m_{\rm KK}$.

In Ref. [32] a model with top and bottom quarks residing in two **5** multiplets and one **10** multiplet has been considered. It is assumed that the fermions satisfy boundary conditions differing from those obtained by simple orbifolding. It has been stated there that this change of boundary conditions can follow from brane mass interactions at the TeV brane. We show below by solving equations of motion that desired change of boundary conditions for low-lying modes in Kaluza-Klein towers takes place as a result of brane mass interactions at the Planck brane (2.15), keeping the custodial $SO(4)$ symmetry at the TeV brane.

III. SPECTRUM IN THE GAUGE-HIGGS SECTOR

The spectrum in the gauge-Higgs sector described by $\mathcal{L}^{\rm gauge}_{\rm bulk} + \mathcal{L}^{\rm scalar}_{\rm Pl.\,brane}$ with (2.4) and (2.6) has been well spelled out in Ref. [28]. In this section we summarize the results obtained there, which makes it necessary to evaluate the effective potential for the Wilson line phase θ_H.

The y coordinate in the Randall-Sundrum spacetime in (2.1) is suited for seeing the orbifold structure. In finding the spectrum of particles and their wave functions in the fifth dimension the conformal coordinate $z \equiv e^{\sigma(y)}$ is useful, with which the metric becomes

$$ds^2 = \frac{1}{z^2}\bigg\{\eta_{\mu\nu}dx^\mu dx^\nu + \frac{dz^2}{k^2}\bigg\}. \tag{3.1}$$

The fundamental region $0 \le y \le L$ is mapped to $1 \le z \le z_L = e^{kL}$. z_L is called a warp factor, which we will find to be around 10^{15} to 10^{17}. In the bulk region $0 < y < L$, one has $\partial_y = kz\partial_z$, $A_y = kzA_z$, etc.

The fifth dimensional component of gauge potentials A_y or A_z has zero modes in the $SO(5)/SO(4)$ part with generators $T^{\hat{a}}$ ($\hat{a} = 1, \cdots, 4$),

$$\begin{aligned} A^{\hat{a}}_z(x,z) &= \phi^a(x)\sqrt{\frac{2}{k(z_L^2-1)}}\,z + \cdots, \\ \Phi_H(x) &= \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^4 - i\phi^3 \end{pmatrix}. \end{aligned} \tag{3.2}$$

Φ_H corresponds to the $SU(2)_L$ doublet Higgs field in the standard model. Without loss of generality one can assume that $\langle \phi^a \rangle = v\delta^{a4}$ when the EW symmetry is spontaneously broken. The Wilson line phase θ_H is given by $\exp\{\frac{i}{2}\theta_H(2\sqrt{2}T^{\hat{4}})\} = \exp\{ig_A\int_1^{z_L} dz\langle A_z\rangle\}$ so that [21]

$$\theta_H = \frac{1}{2}g_A v\sqrt{\frac{z_L^2-1}{k}} \sim \frac{g_4 v}{2}\frac{\pi\sqrt{kL}}{m_{\rm KK}}. \tag{3.3}$$

Here $g_4 = g_A/\sqrt{L}$ is the four-dimensional $SU(2)_L$ gauge coupling constant. We remark that θ_H is a phase variable so that physics is periodic in θ_H with a period 2π.

The spectrum is determined with $\theta_H \neq 0$. Various components in $SO(5)$ mix among each other. Following Falkowski [29], we define basis functions for mass eigenmodes in the gauge-Higgs sector by solutions of the Bessel equation

$$\begin{aligned} &\bigg(\frac{d^2}{dz^2} - \frac{1}{z}\frac{d}{dz} + \lambda^2\bigg)\begin{pmatrix} C(z;\lambda) \\ S(z;\lambda) \end{pmatrix} = 0, \qquad C(z_L;\lambda) = z_L, \\ &C'(z_L;\lambda) = 0, \qquad S(z_L;\lambda) = 0, \qquad S'(z_L;\lambda) = \lambda. \end{aligned} \tag{3.4}$$

Here $C' = dC/dz$ and a relation $CS' - SC' = \lambda z$ holds. The Neumann and Dirichlet boundary conditions for A_μ correspond to solutions $C(z_L;\lambda)$ and $S(z_L;\lambda)$, respectively. The dimensionless eigenvalue λ is related to a 4D mass by $m = k\lambda$. As the scalar interactions on the Planck brane at $z = 1$ effectively change the boundary conditions there, it is most convenient to use the base functions C and S defined with boundary conditions at $z = z_L$ as in (3.4). They generalize trigonometric functions in flat space, and are given in terms of Bessel functions by

$$\begin{aligned} C(z;\lambda) &= \frac{\pi}{2}\lambda z z_L F_{1,0}(\lambda z, \lambda z_L), \\ C'(z;\lambda) &= \frac{\pi}{2}\lambda^2 z z_L F_{0,0}(\lambda z, \lambda z_L), \\ S(z;\lambda) &= -\frac{\pi}{2}\lambda z F_{1,1}(\lambda z, \lambda z_L), \\ S'(z;\lambda) &= -\frac{\pi}{2}\lambda^2 z F_{0,1}(\lambda z, \lambda z_L). \end{aligned} \tag{3.5}$$

Here $F_{\alpha,\beta}(u,v)$ is defined as

$$F_{\alpha,\beta}(u,v) = J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v), \tag{3.6}$$

which is the same as in Ref. [28]. The relation $F_{\alpha,\alpha-1}(u,u) = 2/\pi u$ has been used in (3.5).

A. KK towers of 4D gauge fields

(i) $(1_L, 1_R, \hat{1})$, $(2_L, 2_R, \hat{2})$ components (W tower)

With $\theta_H \neq 0$, three components $A_\mu^{a_L}$, $A_\mu^{a_R}$, and $A_\mu^{\hat{a}}$ ($a = 1, 2$) mix among each other. The mass spectrum $m_n = k\lambda_n$ is determined by

$$C(1;\lambda_n) = 0, \tag{3.7}$$

$$2S(1;\lambda_n)C'(1;\lambda_n) + \lambda_n \sin^2\theta_H = 0 \qquad (W\,\text{tower}). \tag{3.8}$$

The spectrum (3.7) contains only massive modes. The spectrum (3.8), which depends on θ_H, contains a W boson as the lowest mode λ_0. When the warp factor $z_L = e^{kL} \gg 1$, one finds that $\lambda_0 z_L \ll 1$ for any value of θ_H. Employing approximate formulas for Bessel functions, one finds that the W boson mass is given by

$$m_W \sim \sqrt{\frac{k}{L}} e^{-kL} |\sin\theta_H| \sim \frac{m_{\rm KK}}{\pi\sqrt{kL}} |\sin\theta_H|. \tag{3.9}$$

Later it will be found that the effective potential is minimized at $\theta_H = \frac{1}{2}\pi$. Hence, we find that for $e^{kL} = 10^{15}$ (10^{17}), $k = 4.72 \times 10^{17}$ GeV (5.03×10^{19} GeV), and $m_{\rm KK} = 1.48$ TeV (1.58 TeV).

(ii) $(3_L, 3_R, \hat{3}, B)$ or $(3_L, 3'_R, \hat{3}, Y)$ components (γ and Z towers)

Four components $A_\mu^{3_L}$, $A_\mu^{3_R}$, $A_\mu^{\hat{3}}$, and B_μ mix among each other. The spectrum is given by

$$C'(1;\lambda_n) = 0 \qquad (\text{photon tower}), \tag{3.10}$$

$$C(1;\lambda_n) = 0, \tag{3.11}$$

$$2S(1;\lambda_n)C'(1;\lambda_n) + \lambda_n(1 + s_\phi^2)\sin^2\theta_H = 0 \qquad (Z\,\text{tower}). \tag{3.12}$$

Here s_ϕ is defined in (2.7). The spectrum (3.10) contains a zero mode $\lambda_0 = 0$, corresponding to a photon. The spectrum (3.12) contains a Z boson, whose mass is given by

$$m_Z \sim \frac{m_W}{\cos\theta_W}, \qquad \cos\theta_W = \frac{1}{\sqrt{1 + s_\phi^2}}. \tag{3.13}$$

The approximate equality is valid to the $O(0.1\%)$ accuracy for $m_{\rm KK} = O(\text{TeV})$ [28]. Notice that the Weinberg angle θ_W is almost independent of θ_H, which is not the case in the corresponding model in flat spacetime $M^4 \times (S^1/Z_2)$.

We would like to add a comment on wave functions. The profile of the photon wave function in the fifth dimension is exactly constant. The W and Z wave functions are almost constant in the fifth coordinate except for the vicinity of the TeV brane, though they have significant θ_H dependence in the weight of the $SO(5)$ group components. It has been known that the approximate flatness in the fifth dimension assures the universality in the WWZ, $WWWW$, and $WWZZ$ couplings, whereas the nontrivial θ_H dependence in the group space leads to the deviation of WWH and ZZH couplings from those in the standard model [27,28,31]. In the flat space the W and Z wave functions acquire significant dependence on the fifth coordinate when θ_H becomes $O(1)$, which leads to the deviation of WWZ coupling from that in the standard model [28].

(iii) $(\hat{4})$ component

The spectrum of $A_\mu^{\hat{4}}$ is given by $C(1;\lambda_n) = 0$. It contains only massive modes.

B. KK towers of 4D scalar fields

Mode functions of the extra-dimensional component A_z satisfy, in place of (3.4),

$$\left(\frac{d^2}{dz^2} - \frac{1}{z}\frac{d}{dz} + \frac{1}{z^2} + \lambda^2\right)\begin{pmatrix} C'(z;\lambda) \\ S'(z;\lambda) \end{pmatrix} = 0. \tag{3.14}$$

The Neumann and Dirichlet boundary conditions for A_z correspond to solutions $S'(z_L;\lambda)$ and $C'(z_L;\lambda)$, respectively. In classifying the spectra for A_z it is convenient to introduce

$$\begin{pmatrix} A_z^{a_V} \\ A_z^{a_A} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} A_z^{a_L} + A_z^{a_R} \\ A_z^{a_L} - A_z^{a_R} \end{pmatrix} \qquad (a = 1, 2, 3). \tag{3.15}$$

(i) $(1_V, 2_V, 3_V, B)$ components

The spectrum is determined by $C(1;\lambda_n) = 0$, which contains no zero mode.

(ii) $(a_A, \hat{a})$ $(a = 1, 2, 3)$ components

The spectrum is given by

$$S(1;\lambda_n)C'(1;\lambda_n) + \lambda_n \sin^2\theta_H = 0. \tag{3.16}$$

We note that this spectrum is obtained for A_z satisfying the orbifold boundary conditions which are not modified by the additional dynamics on the Planck brane described by (2.6). As described in Sec. II and Ref. [28], it is related to the large gauge invariance. The spectrum for this part is different from that used in Ref. [32].

(iii) $(\hat{4})$ component (Higgs tower)

The spectrum is given by $\lambda_n S(1;\lambda_n) = 0$. There is a zero mode $\lambda_0 = 0$, corresponding to the physical neutral Higgs field ϕ^4 in (3.2). It acquires a finite mass quantum mechanically by the Hosotani mechanism.

C. KK towers of ghost fields

The free part of the equations obeyed by the ghost fields in the bulk are the same as for the A_μ part. The boundary conditions obeyed by the ghost fields for the group components outside $SU(2)_R \times U(1)_X/U(1)_Y$ are obviously the same as for the A_μ. Even for the group components in $SU(2)_R \times U(1)_X/U(1)_Y$, as explained in Sec. II, the ghost fields obey the same boundary conditions at the Planck brane as A_μ, once the R_ξ gauge is adopted on the Planck brane. Hence in this gauge all components of the ghost fields have the same spectrum as the corresponding A_μ.

IV. SPECTRUM OF FERMIONS

The fermion spectrum is found in a similar manner. The presence of boundary interactions on the Planck brane (2.15) among bulk fermions Ψ_a and brane fermions $\hat{\chi}_{jR}$ induces discontinuities in a part of the bulk fermion fields. It also effectively changes boundary conditions at the Planck brane, yielding a desired mass spectrum. The role of brane mass terms for making exotic fermions heavy was discussed by Burdman and Nomura several years ago [10]. We shall see below how this is achieved by solving equations of motion for both bulk and brane fermions.

A. Basis functions

Before writing down full equations in the presence of a nonvanishing Wilson line phase θ_H, let us recall the basic structure of Dirac equations in the absence of gauge interactions. In the Randall-Sundrum spacetime the rescaled spinor field in the bulk, $\check{\Psi} = e^{-2\sigma}\Psi = z^{-2}\Psi$, satisfies a simple equation. If there were no brane interactions, it would obey

$$\left\{\begin{pmatrix} & \sigma\partial \\ \bar{\sigma}\partial & \end{pmatrix} - k\begin{pmatrix} D_-(c) & \\ & D_+(c)\end{pmatrix}\right\}\begin{pmatrix}\check{\Psi}_R \\ \check{\Psi}_L\end{pmatrix} = 0, \quad (4.1)$$

$$D_\pm(c) = \pm\frac{d}{dz} + \frac{c}{z}. \quad (4.2)$$

Here c is the bulk mass parameter and Ψ_R (Ψ_L) represents the right-handed component with $\Gamma^5 = +1$ (left-handed component with $\Gamma^5 = -1$). The parity of Ψ_R at the brane is opposite to that of Ψ_L. Without brane interactions the component with even (odd) parity satisfies a Neumann (Dirichlet) condition there. Neumann conditions for $\check{\Psi}_R$ and $\check{\Psi}_L$ are given by $D_-(c)\check{\Psi}_R = 0$ and $D_+(c)\check{\Psi}_L = 0$, respectively.

In the rest of the present paper we always discuss fermions in terms of rescaled fields $\check{\Psi}$. To simplify expressions we henceforth drop a symbol check ($\check{\ }$). Repeated use of Eq. (4.1) gives

$$\begin{aligned}&\{\partial^2 - k^2 D_+(c)D_-(c)\}\Psi_R = 0,\\ &\{\partial^2 - k^2 D_-(c)D_+(c)\}\Psi_L = 0.\end{aligned} \quad (4.3)$$

Taking into account the fact that there are brane interactions at the Planck brane at $z = 1$, we define basis functions for fermions as

$$\begin{aligned}&\{D_+(c)D_-(c) - \lambda^2\}\begin{pmatrix} C_R(z;\lambda,c) \\ S_R(z;\lambda,c)\end{pmatrix} = 0,\\ &\{D_-(c)D_+(c) - \lambda^2\}\begin{pmatrix} C_L(z;\lambda,c) \\ S_L(z;\lambda,c)\end{pmatrix} = 0,\\ &C_R = C_L = 1,\\ &D_-(c)C_R = D_+(c)C_L = 0,\\ &S_R = S_L = 0,\\ &D_-(c)S_R = D_+(c)S_L = \lambda \quad \text{at } z = z_L.\end{aligned} \quad (4.4)$$

Explicit forms of these functions are given by

$$\begin{aligned}C_L(z;\lambda,c) &= +\frac{\pi}{2}\lambda\sqrt{zz_L}F_{c+(1/2),c-(1/2)}(\lambda z,\lambda z_L),\\ S_L(z;\lambda,c) &= -\frac{\pi}{2}\lambda\sqrt{zz_L}F_{c+(1/2),c+(1/2)}(\lambda z,\lambda z_L),\\ C_R(z;\lambda,c) &= -\frac{\pi}{2}\lambda\sqrt{zz_L}F_{c-(1/2),c+(1/2)}(\lambda z,\lambda z_L),\\ S_R(z;\lambda,c) &= +\frac{\pi}{2}\lambda\sqrt{zz_L}F_{c-(1/2),c-(1/2)}(\lambda z,\lambda z_L),\end{aligned} \quad (4.5)$$

where $F_{\alpha,\beta}$ is defined in (3.6). These functions are related to each other by

$$\begin{aligned}&D_+(c)\begin{pmatrix} C_L \\ S_L\end{pmatrix} = \lambda\begin{pmatrix} S_R \\ C_R\end{pmatrix}, \quad D_-(c)\begin{pmatrix} C_R \\ S_R\end{pmatrix} = \lambda\begin{pmatrix} S_L \\ C_L\end{pmatrix},\\ &\begin{pmatrix} C_L \\ S_L\end{pmatrix}(z;\lambda,-c) = -\begin{pmatrix} C_R \\ S_R\end{pmatrix}(z;\lambda,c), \quad C_LC_R - S_LS_R = 1.\end{aligned} \quad (4.6)$$

B. Equations and the spectrum

To find the spectrum resulting in the theory with (2.8) and (2.15), we start with writing full equations. Recall that the Wilson line phase θ_H mixes (B, t) with t' in the $Q_{\rm EM} = \frac{2}{3}$ sector and (D, X) with b' in the $Q_{\rm EM} = -\frac{1}{3}$ sector, respectively. The brane mass interactions connect B to $\hat{B}_R$, U and t to $\hat{U}_R$, in the $Q_{\rm EM} = \frac{2}{3}$ sector, whereas they connect D and b to $\hat{D}_R$, and X to $\hat{X}_R$, in the $Q_{\rm EM} = -\frac{1}{3}$ sector.

Strategy for solving equations in the presence of $\theta_H \neq 0$ is to first move to a new twisted gauge in which the background field vanishes, $\tilde{A}^c_z = 0$, as described in Refs. [28,29]. This is achieved by

$$\begin{aligned}&\Omega(z) = \exp\{i\theta(z)\sqrt{2}T^{\hat{4}}\}, \qquad \theta(z) = \frac{z_L^2 - z^2}{z_L^2 - 1}\theta_H,\\ &gA^c_z = \dot{\theta}\sqrt{2}T^{\hat{4}}, \qquad \dot{\theta} = \frac{d\theta}{dz}, \qquad \tilde{\Psi} = \Omega(z)\Psi.\end{aligned} \quad (4.7)$$

Note that $\theta(1) = \theta_H$ and $\Omega(z_L) = 1$. In the standard vectorial representation $\Psi = (\psi_1, \cdots, \psi_5)^t$, $\Omega(z)$ takes the form

$$\Omega(z) = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & c & s \\ & & & -s & c \end{pmatrix}, \qquad \begin{cases} c = \cos\theta(z), \\ s = \sin\theta(z). \end{cases} \tag{4.8}$$

In the twisted gauge the equations in the bulk are the same as in free theory, whereas the boundary conditions at the Planck brane at $z = 1$ become more involved. In the basis (B, t, t') employed in (2.11) and (2.12)

$$\begin{pmatrix} \tilde{B} \\ \tilde{t} \\ \tilde{t}' \end{pmatrix} = \tilde{\Omega} \begin{pmatrix} B \\ t \\ t' \end{pmatrix}, \qquad \tilde{\Omega} = \begin{pmatrix} \frac{1}{2}(1+c) & \frac{1}{2}(1-c) & -\frac{1}{\sqrt{2}}s \\ \frac{1}{2}(1-c) & \frac{1}{2}(1+c) & \frac{1}{\sqrt{2}}s \\ \frac{1}{\sqrt{2}}s & -\frac{1}{\sqrt{2}}s & c \end{pmatrix}. \tag{4.9}$$

A similar relation applies to (D, X, b').

(i) $Q_{\rm EM} = \frac{5}{3}$ sector

In the two component basis there are T_L, T_R, and $\hat{T}_R$ in this sector. As there is no coupling to θ_H, $\tilde{T} = T$. The parity assignments of the bulk fields are $T_L(+,+)$, $T_R(-,-)$. With (2.8) and (2.15) equations of motion are given by

$$\sigma\partial T_L - kD_-(c_1)T_R - \mu_1\delta(y)\hat{T}_R = 0,$$
$$\bar{\sigma}\partial T_R - kD_+(c_1)T_L = 0, \qquad \bar{\sigma}\partial\hat{T}_R - \mu_1 T_L = 0. \tag{4.10}$$

Recall that $D_\pm = (e^{-\sigma}/k)(\pm d/dy + c\,d\sigma/dy)$ in the y coordinate. Integrating the first equation from $y = -\epsilon$ to $y = +\epsilon$ and making use of $T_R(x, -y) = -T_R(x, y)$, one finds

$$T_R|_{y=\epsilon} = -T_R|_{y=-\epsilon} = \frac{\mu_1}{2}\hat{T}_R(x), \tag{4.11}$$

that is, parity-odd T_R develops a discontinuity at the Planck brane. Inserting (4.11) into the second equation in (4.10) and making use of the third equation in (4.10), one finds $(kD_+(c_1) - \mu_1^2)T_L = 0$ at $y = \epsilon$. The boundary conditions for the bulk field T_L are thus given by

$$\begin{cases} (D_+(c_1) - \frac{\mu_1^2}{2k})T_L = 0 & \text{at } z = 1, \\ D_+(c_1)T_L = 0 & \text{at } z = z_L. \end{cases} \tag{4.12}$$

Boundary conditions for T_R are given by (4.11) and $T_R|_{z=z_L} = 0$, which follow from (4.10) and (4.12).

In the bulk region $1 < z < z_L$, T_L and T_R satisfy free equations. Mode functions are obtained with an ansatz $T_{L,R} = e^{ipx}u_{L,R}(p)f_{L,R}(z)$ for each mass eigenstate. $f_{L,R}(z)$ satisfy $D_+f_L = \lambda f_R$ and $D_-f_R = \lambda f_L$ so that T_L and T_R satisfy (4.3). The boundary condition (4.12) at $z = z_L$ implies that $f_L(z) \propto C_L(z;\lambda,c_1)$. The boundary condition at $z = 1$ is then satisfied if

$$\lambda S_R^{(1)} - \frac{\mu_1^2}{2k}C_L^{(1)} = 0, \tag{4.13}$$

where $S_R^{(j)} = S_R(1;\lambda,c_j)$, etc. If there were no boundary interaction ($\mu_1 = 0$), then the spectrum contains a zero mode ($\lambda_0 = 0$). For $\mu_1^2/2k \gg \lambda$ the second term dominates over the first term. The lowest mass $m_0 = k\lambda_0$ determined by $C_L(1;\lambda_0,c) = 0$ is at the Kaluza-Klein mass scale for $c > 0$. In other words, as long as $\mu_1^2 \gg m_{\rm KK}$, the mass of the lowest mode is $O(m_{\rm KK})$ for $c_1 > 0$.

Here we have been observing an effective change of boundary conditions. The Neumann condition corresponding to $\lambda S_R(1;\lambda,c) = 0$ changes to the Dirichlet condition corresponding to $C_L(1;\lambda,c) = 0$ for low-lying modes in the Kaluza-Klein tower. We note that for $c < -\frac{1}{2}$ the lowest mass value determined by $C_L(1;\lambda,c) = -C_R(1;\lambda,|c|) = 0$ becomes nonvanishing but remains small.

(ii) $Q_{\rm EM} = \frac{2}{3}$ sector

In this sector six fields U, B, t, t', $\hat{U}_R$, and $\hat{B}_R$ mix with each other. In the basis (B, t, t') the Wilson line phase θ_H gives a background field

$$-igA_z^c = \frac{\dot{\theta}}{\sqrt{2}}\begin{pmatrix} & & -1 \\ & & 1 \\ 1 & -1 & \end{pmatrix}. \tag{4.14}$$

We note that $-igA_z^c = (d\tilde{\Omega}^\dagger/dz)\tilde{\Omega}$. With (4.14) equations of motion in the original gauge are given by

$$\sigma\partial\begin{pmatrix} U_L \\ B_L \\ t_L \\ t'_L \end{pmatrix} - k\begin{pmatrix} D_-^{(2)} & & & \\ & D_-^{(1)} & & \frac{1}{\sqrt{2}}\dot{\theta} \\ & & D_-^{(1)} & -\frac{1}{\sqrt{2}}\dot{\theta} \\ & -\frac{1}{\sqrt{2}}\dot{\theta} & \frac{1}{\sqrt{2}}\dot{\theta} & D_-^{(1)} \end{pmatrix}\begin{pmatrix} U_R \\ B_R \\ t_R \\ t'_R \end{pmatrix} - \delta(y)\begin{pmatrix} \mu_2\hat{U}_R \\ \mu_1\hat{B}_R \\ \tilde{\mu}\hat{U}_R \\ 0 \end{pmatrix} = 0,$$
$$\bar{\sigma}\partial\begin{pmatrix} U_R \\ B_R \\ t_R \\ t'_R \end{pmatrix} - k\begin{pmatrix} D_+^{(2)} & & & \\ & D_+^{(1)} & & -\frac{1}{\sqrt{2}}\dot{\theta} \\ & & D_+^{(1)} & \frac{1}{\sqrt{2}}\dot{\theta} \\ & \frac{1}{\sqrt{2}}\dot{\theta} & -\frac{1}{\sqrt{2}}\dot{\theta} & D_+^{(1)} \end{pmatrix}\begin{pmatrix} U_L \\ B_L \\ t_L \\ t'_L \end{pmatrix} = 0, \qquad \bar{\sigma}\partial\begin{pmatrix} \hat{U}_R \\ \hat{B}_R \end{pmatrix} - \begin{pmatrix} \mu_2 & 0 & \tilde{\mu} \\ 0 & \mu_1 & 0 \end{pmatrix}\begin{pmatrix} U_L \\ B_L \\ t_L \end{pmatrix} = 0. \tag{4.15}$$

Here $D_{\pm}^{(j)} = D_{\pm}(c_j)$. Recall that U_L, B_L, t_L, and t'_R have parity $(+,+)$ whereas U_R, B_R, t_R, and t'_L have parity $(-,-)$. Integrating the first equation above from $y = -\epsilon$ to $y = +\epsilon$ and making use of the odd nature of U_R, B_R, and t_R under parity, one finds

$$U_R|_{y=\epsilon} = \frac{\mu_2}{2}\hat{U}_R,\quad B_R|_{y=\epsilon} = \frac{\mu_1}{2}\hat{B}_R,\quad t_R|_{y=\epsilon} = \frac{\tilde{\mu}}{2}\hat{U}_R. \tag{4.16}$$

Another parity-odd field t'_L satisfies $t'_L|_{y=\epsilon} = 0$ as it follows from the second equation in (4.15). U_R, B_R, and t_R develop discontinuities at the Planck brane, but t'_L does not.

Now to find boundary conditions for U_L, B_L, t_L at $y = 0$ ($z = 1$) we insert (4.16) into the second equation in (4.15) and use the third equation in (4.15). Noting that t'_L vanishes there, one obtains

$$D_+^{(2)}U_L - \frac{\mu_2}{2k}(\mu_2 U_L + \tilde{\mu}t_L) = 0,\quad D_+^{(1)}B_L - \frac{\mu_1}{2k}\cdot\mu_1 B_L = 0,\quad D_+^{(1)}t_L - \frac{\tilde{\mu}}{2k}(\mu_2 U_L + \tilde{\mu}t_L) = 0,\qquad t'_L = 0, \tag{4.17}$$

at $z = 1$, and

$$D_+^{(2)}U_L = D_+^{(1)}B_L = D_+^{(1)}t_L = t'_L = 0 \tag{4.18}$$

at $z = z_L$.

At this stage we move to the twisted gauge defined in (4.7) and (4.9) in which the bulk fields satisfy free equations. Taking into account the fact that $\tilde{t}'_L = t'_L = 0$ at $z = z_L$, we find that

$$D_+^{(2)}\tilde{U}_L = D_+^{(1)}\tilde{B}_L = D_+^{(1)}\tilde{t}_L = \tilde{t}'_L = 0 \tag{4.19}$$

at $z = z_L$. Making use of (4.9) and (4.17), we find

$$\begin{aligned} &s_H(\tilde{B}_L - \tilde{t}_L) - \sqrt{2}c_H\tilde{t}'_L = 0,\qquad \left(D_+^{(2)} - \frac{\mu_2^2}{2k}\right)\tilde{U}_L - \frac{\tilde{\mu}\mu_2}{4k}(\tilde{B}_L + \tilde{t}_L) + \frac{\tilde{\mu}\mu_2}{4c_H k}(\tilde{B}_L - \tilde{t}_L) = 0,\\ &-\frac{\tilde{\mu}\mu_2}{2k}\tilde{U}_L + \left(D_+^{(1)} - \frac{\mu_1^2 + \tilde{\mu}^2}{4k}\right)(\tilde{B}_L + \tilde{t}_L) - \frac{\mu_1^2 - \tilde{\mu}^2}{4c_H k}(\tilde{B}_L - \tilde{t}_L) = 0,\\ &\frac{\tilde{\mu}\mu_2}{2k}\tilde{U}_L - \frac{\mu_1^2 - \tilde{\mu}^2}{4k}(\tilde{B}_L + \tilde{t}_L) + \left(c_H D_+^{(1)} - \frac{\mu_1^2 + \tilde{\mu}^2}{4c_H k}\right)(\tilde{B}_L - \tilde{t}_L) + \sqrt{2}s_H D_+^{(1)}\tilde{t}'_L = 0 \end{aligned} \tag{4.20}$$

at $z = 1$ where $c_H = \cos\theta_H$ and $s_H = \sin\theta_H$. All the fields satisfy free equations. With the boundary conditions (4.19) at $z = z_L$, mode functions can be expressed as

$$\tilde{U}_L = a_U C_L(z;\lambda,c_2),\qquad \tilde{B}_L \pm \tilde{t}_L = a_{B\pm t}C_L(z;\lambda,c_1),\qquad \tilde{t}'_L = a_{t'}S_L(z;\lambda,c_1). \tag{4.21}$$

Eigenvalues for λ are determined by the boundary conditions (4.20) at $z = 1$. Inserting (4.21) into (4.20), one finds after lengthy but straightforward manipulation that

$$\begin{aligned} &\lambda A_1(\lambda) + \lambda B_1(\lambda)\sin^2\theta_H = 0,\\ &A_1(\lambda) = \left(\lambda S_R^{(1)} - \frac{\mu_1^2}{2k}C_L^{(1)}\right) \times\left(\lambda S_R^{(1)}S_R^{(2)} - \frac{\tilde{\mu}^2}{2k}C_L^{(1)}S_R^{(2)} - \frac{\mu_2^2}{2k}S_R^{(1)}C_L^{(2)}\right),\\ &B_1(\lambda) = \frac{1}{S_L^{(1)}}\left(\lambda^2 S_R^{(1)}S_R^{(2)} - \lambda\frac{\mu_2^2}{2k}S_R^{(1)}C_L^{(2)} - \lambda\frac{\mu_1^2 + \tilde{\mu}^2}{4k}C_L^{(1)}S_R^{(2)} + \frac{\mu_1^2\mu_2^2}{8k^2}C_L^{(1)}C_L^{(2)}\right), \end{aligned} \tag{4.22}$$

where $S_R^{(j)} = S_R(1;\lambda,c_j)$, etc.

When boundary masses μ_j and $\tilde{\mu}$ vanish, (4.22) reduces to

$$\lambda S_R^{(1)}\cdot\lambda\left(S_R^{(1)} + \frac{\sin^2\theta_H}{S_L^{(1)}}\right)\cdot\lambda S_R^{(2)} = 0. \tag{4.23}$$

The first and third factors correspond to the KK towers of $t + B$ and U, respectively. The second factor, yielding $S_L^{(1)}S_R^{(1)} + \sin^2\theta_H = 0$ for $\theta_H \neq 0$, gives a spectrum of the KK towers of $t - B$ and t'. The zero mode of $t - B$ acquires a mass by $\theta_H \neq 0$, but the zero modes of $t + B$ and U remain massless. With nonvanishing boundary masses these unwanted light modes acquire masses of $O(m_{\rm KK})$ for $c_1, c_2 > 0$.

(iii) $Q_{\rm EM} = -\frac{1}{3}$ sector

This sector has a similar structure to that in the $Q_{\rm EM} = \frac{2}{3}$ sector. Six fields b, D, X, b', $\hat{D}_R$, and $\hat{X}_R$ mix with each other. b_L, D_L, X_L, and b'_R have parity $(+,+)$ whereas b_R, D_R, X_R, and b'_L have parity $(-,-)$. Equations of motion are given by

$$
\sigma\partial\begin{pmatrix} b_L \\ D_L \\ X_L \\ b_L' \end{pmatrix} - k\begin{pmatrix} D_-^{(1)} & & & \\ & D_-^{(2)} & & \frac{1}{\sqrt{2}}\dot{\theta} \\ & & D_-^{(2)} & -\frac{1}{\sqrt{2}}\dot{\theta} \\ & -\frac{1}{\sqrt{2}}\dot{\theta} & \frac{1}{\sqrt{2}}\dot{\theta} & D_-^{(2)} \end{pmatrix}\begin{pmatrix} b_R \\ D_R \\ X_R \\ b_R' \end{pmatrix} - \delta(y)\begin{pmatrix} \tilde{\mu}\hat{D}_R \\ \mu_2\hat{D}_R \\ \mu_3\hat{X}_R \\ 0 \end{pmatrix} = 0,
$$

$$
\bar{\sigma}\partial\begin{pmatrix} b_R \\ D_R \\ X_R \\ b_R' \end{pmatrix} - k\begin{pmatrix} D_+^{(1)} & & & \\ & D_+^{(2)} & & -\frac{1}{\sqrt{2}}\dot{\theta} \\ & & D_+^{(2)} & \frac{1}{\sqrt{2}}\dot{\theta} \\ & \frac{1}{\sqrt{2}}\dot{\theta} & -\frac{1}{\sqrt{2}}\dot{\theta} & D_+^{(2)} \end{pmatrix}\begin{pmatrix} b_L \\ D_L \\ X_L \\ b_L' \end{pmatrix} = 0, \qquad \bar{\sigma}\partial\begin{pmatrix} \hat{D}_R \\ \hat{X}_R \end{pmatrix} - \begin{pmatrix} \tilde{\mu} & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}\begin{pmatrix} b_L \\ D_L \\ X_L \end{pmatrix} = 0. \tag{4.24}
$$

This time b_R, D_R, and X_R develop discontinuities at the Planck brane:

$$
b_R|_{y=\epsilon} = \frac{\tilde{\mu}}{2}\hat{D}_R, \quad D_R|_{y=\epsilon} = \frac{\mu_2}{2}\hat{D}_R, \quad X_R|_{y=\epsilon} = \frac{\mu_3}{2}\hat{X}_R. \tag{4.25}
$$

With (4.25) boundary conditions for the left-handed bulk fields are found to be

$$
\begin{aligned}
&D_+^{(1)} b_L - \frac{\tilde{\mu}}{2k}(\tilde{\mu} b_L + \mu_2 D_L) = 0, \\
&D_+^{(2)} D_L - \frac{\mu_2}{2k}(\tilde{\mu} b_L + \mu_2 D_L) = 0, \\
&D_+^{(2)} X_L - \frac{\mu_3}{2k}\cdot \mu_3 X_L = 0, \qquad b_L' = 0,
\end{aligned} \tag{4.26}
$$

at $z = 1$, and $D_+^{(1)} b_L = D_+^{(2)} D_L = D_+^{(2)} X_L = b_L' = 0$ at $z = z_L$. In the twisted gauge, mode functions are expressed, as in (4.21), as

$$
\begin{aligned}
\tilde{b}_L &= a_b C_L(z; \lambda, c_1), \\
\tilde{D}_L \pm \tilde{X}_L &= a_{D\pm X} C_L(z; \lambda, c_2), \\
\tilde{b}_L' &= a_{b'} S_L(z; \lambda, c_2).
\end{aligned} \tag{4.27}
$$

The boundary conditions at $z = 1$, (4.26), are satisfied if

$$
\begin{aligned}
&\lambda A_2(\lambda) + \lambda B_2(\lambda)\sin^2\theta_H = 0, \\
&A_2(\lambda) = \left(\lambda S_R^{(2)} - \frac{\mu_3^2}{2k} C_L^{(2)}\right) \\
&\qquad \times \left(\lambda S_R^{(1)} S_R^{(2)} - \frac{\tilde{\mu}^2}{2k} C_L^{(1)} S_R^{(2)} - \frac{\mu_2^2}{2k} S_R^{(1)} C_L^{(2)}\right), \\
&B_2(\lambda) = \frac{1}{S_L^{(2)}}\left(\lambda^2 S_R^{(1)} S_R^{(2)} - \lambda\frac{\mu_2^2 + \mu_3^2}{4k} S_R^{(1)} C_L^{(2)} \right. \\
&\qquad \left. - \lambda\frac{\tilde{\mu}^2}{2k} C_L^{(1)} S_R^{(2)} + \frac{\tilde{\mu}^2\mu_3^2}{8k^2} C_L^{(1)} C_L^{(2)}\right).
\end{aligned} \tag{4.28}
$$

There is a subtle difference between the $Q_{\rm EM} = \frac{2}{3}$ and $-\frac{1}{3}$ sectors. The expression (4.28) can be obtained from (4.22) by formally interchanging (c_1, c_2), (μ_1, μ_3), and $(\mu_2, \tilde{\mu})$. $\hat{\chi}_{2R}$, which was introduced to lift the lowest mode of $Q_{2L} = (U_L, D_L)$ of Ψ_2 to the KK scale with the mass term μ_2, also couples to $q_L = (t_L, b_L)$ of Ψ_1 with the mass term $\tilde{\mu}$. It is, therefore, natural to suppose that $\tilde{\mu}^2 \ll \mu_j^2$. We will see below that this leads to $m_b \ll m_t$ as desired.

(iv) $Q_{\rm EM} = -\frac{4}{3}$ sector

Y and $\hat{Y}_R$ belong to this sector. Equations of motion are obtained from (4.10) by replacing $(T, \hat{T}_R)$ by $(Y, \hat{Y}_R)$ and (c_1, μ_1) by (c_2, μ_3). The spectrum is determined by

$$
\lambda S_R^{(2)} - \frac{\mu_3^2}{2k} C_L^{(2)} = 0. \tag{4.29}
$$

C. Top and bottom masses

The fermion mass spectrum is determined by the relations (4.13), (4.22), (4.28), and (4.29). The brane mass terms are expected to emerge when the RS warped spacetime is generated at high energy scale. Even though we do not know how they emerge, it is natural to imagine that all μ_j^2 and $\tilde{\mu}^2$ are at that high scale, namely, of $O(M_{\rm GUT})$ or $O(M_{\rm Planck})$. What we need and assume in the present paper is much more modest. We only suppose that μ_j^2, $\tilde{\mu}^2 \gg m_{\rm KK}$. It will be seen below that the only relevant parameter for low energy physics is $\tilde{\mu}/\mu_2$ in this case.

For low-lying modes in the Kaluza-Klein towers $m = \lambda k \ll \mu_j^2$ so that (4.13) and (4.29) in the $Q_{\rm EM} = \frac{5}{3}, -\frac{4}{3}$ sectors are approximated by $C_L^{(1)} = 0$ and $C_L^{(2)} = 0$, respectively. The lowest mode in each sector has a mass of $O(m_{\rm KK})$ for $c_j > 0$.

In a similar manner the relation (4.22) in the $Q_{\rm EM} = \frac{2}{3}$ sector is approximated by

$$
C_L^{(1)} = 0, \qquad \mu_2^2 C_L^{(2)}\left\{S_R^{(1)} + \frac{\sin^2\theta_H}{2S_L^{(1)}}\right\} + \tilde{\mu}^2 C_L^{(1)} S_R^{(2)} = 0. \tag{4.30}
$$

The first one gives a KK tower of $(B, \hat{B}_R)$. The second one contains towers of (t, t') and $(U, \hat{U}_R)$. It is found below that m_t and m_b can be reproduced if $c_1 \sim c_2$. In the limiting case $c_1 = c_2$ the second one splits into $C_L^{(1)} = 0$ for $(U, \hat{U}_R)$ and

$$2\left(1+\frac{\tilde{\mu}^2}{\mu_2^2}\right)S_L^{(1)}S_R^{(1)}+\sin^2\theta_H=0 \tag{4.31}$$

for (t, t'). Notice the appearance of a factor 2 in (4.31) compared with the similar expression in the middle of (4.23) due to the brane interactions.

The spectrum determined by (4.30) contains one light mode, namely, a top quark. For $\lambda z_L \ll 1$ and $0 < c < \frac{1}{2}$, $C_L(1;\lambda,c) \sim z_L^c$ and $S_L(1;\lambda,c) \sim -\lambda z_L^{1+c}/(1+2c)$. Making use of these relations, one finds the top quark mass, $m_t = \lambda k$, to be given, for $0 < c_1, c_2 < \frac{1}{2}$, by

$$\begin{aligned} m_t &\sim \frac{m_{\rm KK}}{\sqrt{2}\pi}\frac{\sqrt{1-4c_1^2}|\sin\theta_H|}{(1+\frac{\tilde{\mu}^2}{\mu_2^2}\frac{1-2c_1}{1-2c_2}z_L^{2(c_1-c_2)})^{1/2}} \\ &\sim \frac{m_{\rm KK}}{\sqrt{2}\pi}\sqrt{1-4c_1^2}|\sin\theta_H| \quad \text{for } \frac{\tilde{\mu}^2}{\mu_2^2}z_L^{2(c_1-c_2)} \ll 1. \end{aligned} \tag{4.32}$$

With $\theta_H = \pm\frac{1}{2}\pi$, we find $c_1 \sim 0.43$ for $m_t = 172$ GeV.

The spectrum in the $Q_{\rm EM} = -\frac{1}{3}$ sector is obtained in a similar manner. Equation (4.29) is approximated by

$$C_L^{(2)}=0, \qquad \tilde{\mu}^2C_L^{(1)}\left\{S_R^{(2)}+\frac{\sin^2\theta_H}{2S_L^{(2)}}\right\}+\mu_2^2C_L^{(2)}S_R^{(1)}=0. \tag{4.33}$$

There exists one light mode, which is identified with the bottom quark. Its mass is given, for $0 < c_1, c_2 < \frac{1}{2}$, by

$$\begin{aligned} m_b &\sim \frac{m_{\rm KK}}{\sqrt{2}\pi}\frac{\sqrt{1-4c_2^2}|\sin\theta_H|}{(1+\frac{\mu_2^2}{\tilde{\mu}^2}\frac{1-2c_2}{1-2c_1}z_L^{2(c_2-c_1)})^{1/2}} \\ &= \sqrt{\frac{1+2c_2}{1+2c_1}}\left|\frac{\tilde{\mu}}{\mu_2}\right|z_L^{c_1-c_2}\cdot m_t, \end{aligned} \tag{4.34}$$

which justifies the approximation employed in (4.32). $|c_1 - c_2|$ must be small to get a reasonable value for $\tilde{\mu}/\mu_2$. In the most attractive scenario $c_1 = c_2$, which results if all t, t', b, and b' belong to one single multiplet in a larger unified theory, one finds that

$$\left|\frac{\tilde{\mu}}{\mu_2}\right| = \frac{m_b}{m_t} \ll 1. \tag{4.35}$$

We stress that only the ratio $\tilde{\mu}/\mu_2$ among the brane masses is relevant for m_b. Individual values of the brane mass parameters μ_j^2, $\tilde{\mu}^2$ are irrelevant so long as they are much bigger than $m_{\rm KK}$. To have nonvanishing m_b we need both $\theta_H \neq 0$ and $\tilde{\mu} \neq 0$. b_L in Ψ_1 must be connected with b_R' in Ψ_2.

One may wonder if there are other values for c_1 and c_2 to reproduce m_t and m_b. In the cases $0 < c_1 < \frac{1}{2} < c_2$ and $-\frac{1}{2} < c_2 < 0 < c_1 < \frac{1}{2}$, one obtains the same relation for m_b/m_t as the second relation in (4.34), which demands unnaturally large $\tilde{\mu}/\mu_2$ as $z_L \sim 10^{15}$. In the current scheme, the observed m_t and m_b are realized only for $0 < c_1, c_2 < \frac{1}{2}$.

It is straightforward to incorporate light quarks in the first and second generations. For each generation two **5** multiplets and associated brane fermions are introduced. The bulk mass parameter $c_1 = c_2 \equiv c$ and brane masses μ_1, μ_2, μ_3, and $\tilde{\mu}$ take values depending on the generation. As their masses are much smaller than m_W, it will be found that $c > \frac{1}{2}$. The spectrum is determined by Eqs. (4.30) and (4.33). For up- and down-type quarks we find, for $c > \frac{1}{2}$, that

$$m_{\rm up} \sim \frac{m_{\rm KK}}{\sqrt{2}\pi}\frac{\sqrt{4c^2-1}|\sin\theta_H|}{(1+\frac{\tilde{\mu}^2}{\mu_2^2})^{1/2}z_L^{c-(1/2)}}, \qquad m_{\rm down} \sim \left|\frac{\tilde{\mu}}{\mu_2}\right|m_{\rm up}. \tag{4.36}$$

With $\theta_H = \pm\frac{1}{2}\pi$ and $z_L = 10^{15}$, we find $c \sim 0.653$ and 0.853 for $m_c = 1.4$ GeV and $m_u = 4$ MeV, respectively. A similar construction is done for leptons by putting (e_R^c, ν_R^c, e_L^c) and ν_L^c in Ψ_1 and Ψ_2, respectively. Large hierarchy in fermion masses can be naturally explained by modest distribution in the bulk mass parameter c, as was pointed out in Ref. [46] in general context and in Ref. [23] in the gauge-Higgs unification scenario.

V. DYNAMICAL EW SYMMETRY BREAKING

The value for θ_H is determined by the location of the global minimum of the effective potential $V_{\rm eff}(\theta_H)$, which becomes nontrivial at the quantum level [3]. When θ_H takes a nontrivial value, the standard model symmetry $SU(2)_L \times U(1)_Y$ dynamically breaks down to $U(1)_{\rm EM}$. In pure gauge theory without fermions the symmetry remains unbroken. We shall show below that the presence of a top quark induces the symmetry breaking.

The evaluation of the effective potential $V_{\rm eff}(\theta_H)$ in the RS warped spacetime was initiated by Oda and Weiler [22]. Since then a powerful method for the evaluation has been developed by Falkowski [29]. Concrete evaluation in the gauge-Higgs unification models of electroweak interactions in the RS spacetime has been given in Refs. [32,36].

The effective potential $V_{\rm eff}(\theta_H)$ at the one-loop level is determined by the dependence of the mass spectrum on θ_H. We have seen in the preceding sections that spectra in both gauge-Higgs and fermion sectors are determined by zeros of equations of the type $A(\lambda) + B(\lambda)f(\theta_H) = 0$. For gauge fields and fermions in the vector representation, we have seen $f(\theta_H) = \sin^2\theta_H(\equiv f_1(\theta_H))$. For matter fields in the spinor representation one finds $f(\theta_H) = \sin^2\frac{1}{2}\theta_H$. (See, for example, Refs. [27,28].)

We rewrite the equation in the form $1 + \tilde{Q}(\lambda)f(\theta_H) = 0$ ($\tilde{Q} = B/A$), which yields a spectrum $\{\lambda_n(\theta_H)\}$. Then one-loop contribution to $V_{\rm eff}(\theta_H)$ coming from particles with masses $m_n(\theta_H) = k\lambda_n$ is given by

$$V_{\rm eff}(\theta_H) = \pm\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\sum\ln(p^2+m_n(\theta_H)^2)$$
$$= \pm I[Q(q); f(\theta_H)],$$
$$I[Q(q); f(\theta_H)] = \frac{(kz_L^{-1})^4}{(4\pi)^2}\int_0^\infty dq\, q^3 \times \ln\{1+Q(q)f(\theta_H)\},$$
$$Q(q) = \tilde{Q}(iqz_L^{-1}). \quad (5.1)$$

Here $\pm$ corresponds to bosons or fermions, and θ_H-independent constant terms have been ignored. It will be seen below that the integral is dominated by the integrand in a range $0 < q < 10$.

A. Contributions from the gauge field sector

The W tower (3.8), the Z tower (3.12), and the scalar tower (3.16), with associated ghost contributions, contribute to $V_{\rm eff}(\theta_H)$. Let us define

$$\hat{F}_{\alpha,\beta}(u,v) = I_\alpha(u)K_\beta(v) - e^{-i(\alpha-\beta)\pi}K_\alpha(u)I_\beta(v),$$
$$F_{\alpha,\beta}(iu,iv) = -\frac{2}{\pi}e^{i(\alpha-\beta)\pi/2}\hat{F}_{\alpha,\beta}(u,v), \quad (5.2)$$

where I_α and K_α are modified Bessel functions and $F_{\alpha,\beta}$ is defined in (3.6). The effective potential is given by

$$V_{\rm eff}(\theta_H)^{\rm gauge} = 2\cdot 2\cdot I\left[\frac{1}{2}Q_0\left(q,\frac{1}{2}\right); f_1(\theta_H)\right] + 2\cdot I\left[\frac{1}{2}(1+s_\phi^2)Q_0\left(q,\frac{1}{2}\right); f_1(\theta_H)\right] + 3\cdot I\left[Q_0\left(q,\frac{1}{2}\right); f_1(\theta_H)\right],$$
$$Q_0(q,c) = \frac{z_L}{q^2}\frac{1}{\hat{F}_{c-(1/2),c-(1/2)}(qz_L^{-1},q)\hat{F}_{c+(1/2),c+(1/2)}(qz_L^{-1},q)},$$
$$f_1(\theta_H) = \sin^2\theta_H. \quad (5.3)$$

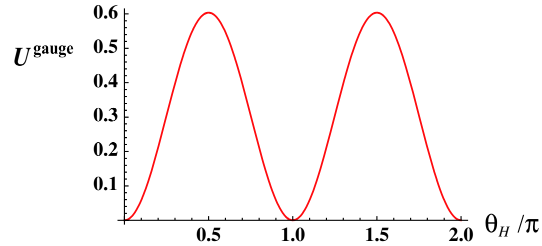

FIG. 1 (color online). The effective potential $V_{\rm eff}(\theta_H)^{\rm gauge}$ in pure gauge theory without fermions. The plot is for $U^{\rm gauge}(\theta_H/\pi) = (4\pi)^2(kz_L^{-1})^{-4}V_{\rm eff}^{\rm gauge}$ at $z_L = 10^{15}$.

The behavior of $V_{\rm eff}(\theta_H)^{\rm gauge}$ is depicted in Fig. 1. It has global minima at $\theta_H = 0$ and π. $SU(2)_L\times U(1)_Y$ symmetry remains unbroken in pure gauge theory.

The flat spacetime limit $k\to 0$ of $V_{\rm eff}(\theta_H)^{\rm gauge}$ is obtained by replacing $kz_L^{-1}\sim m_{\rm KK}/\pi$ by $1/L$, and $Q_0(q,\frac{1}{2})$ by $Q_{\rm flat}(q) = 1/\sinh^2 q$. The shape of $V_{\rm eff}^{\rm gauge}$ in the RS space is similar to that in flat space. The magnitude of $U^{\rm gauge} = (4\pi)^2(kz_L^{-1})^{-4}V_{\rm eff}^{\rm gauge}$ in RS is reduced compared to that in flat space by a factor $2/kL$.

B. Contributions from the fermion sector

In the fermion sector the spectra in the $Q=\frac{2}{3}$ and $-\frac{1}{3}$ sectors, (4.22) and (4.28), yield nontrivial contributions to $V_{\rm eff}(\theta_H)$. $\tilde{Q}$ is given by B_1/A_1 or B_2/A_2 in each sector. When there were no boundary masses, $\mu_j, \tilde{\mu} = 0$, then $\tilde{Q}$ would take the form $1/S_L^{(j)}S_R^{(j)}$ ($j = 1, 2$). It immediately follows that

$$V_{\rm eff}(\theta_H)^{\rm fermion}|_{\mu_j,\tilde{\mu}=0} = -4\{I[Q_0(q,c_1); f_1(\theta_H)] + I[Q_0(q,c_2); f_1(\theta_H)]\}. \quad (5.4)$$

Here the factor 4 accounts for the number of degrees of freedom. We have seen that $c_1\sim 0.43$ for the top quark multiplet. With this value the magnitude of the contribution from the top quark multiplet ($-4I[Q_0; f_1]$) is three times larger than that of $V_{\rm eff}(\theta_H)^{\rm gauge}$ in (5.3). The global minima are found at $\theta_H = \pm\frac{1}{2}\pi$, which implies the EW symmetry breaking, although with vanishing μ_j, $\tilde{\mu}$ there appear unwanted massless particles. We remark that a contribution $I[Q_0(q,c); f_1(\theta_H)]$ becomes negligible for $c > 0.6$ compared with the gauge field contributions. As a consequence contributions from light quarks and leptons become negligibly small in the RS space.

To get $Q_j(q)$ for μ_j, $\tilde{\mu}\neq 0$ from $\tilde{Q}_j(\lambda) = B_j/A_j$, it is sufficient to make the replacement

$$\lambda\to iqz_L^{-1},$$
$$\begin{pmatrix}S_L\\S_R\end{pmatrix}\to\pm iqz_L^{-1/2}\hat{F}_{c\pm(1/2),c\pm(1/2)}(qz_L^{-1},q), \quad (5.5)$$
$$\begin{pmatrix}C_L\\C_R\end{pmatrix}\to qz_L^{-1/2}\hat{F}_{c\pm(1/2),c\mp(1/2)}(qz_L^{-1},q).$$

The resultant expressions for $Q_j(q)$'s are not illuminating. When μ_j^2, $\tilde{\mu}^2\gg m_{\rm KK}$, they tremendously simplify. In particular for $c_1 = c_2 = c$ they become

$$Q_1(q)\simeq\frac{\mu_2^2}{2(\mu_2^2+\tilde{\mu}^2)}Q_0(q,c),$$
$$Q_2(q)\simeq\frac{\tilde{\mu}^2}{2(\mu_2^2+\tilde{\mu}^2)}Q_0(q,c). \quad (5.6)$$

The approximation is valid for $q\ll\mu_j^2/m_{\rm KK}$, $\tilde{\mu}^2/m_{\rm KK}$. As

$Q_j(q)$ becomes negligibly small for $q > 10$, the expression (5.6) can be safely used in the integral $I[Q_j(q), f_1(\theta_H)]$ for numerical evaluation. One finds

$$\begin{aligned} V_{\rm eff}(\theta_H)^{\rm fermion} \simeq -4\bigg\{ & I\bigg[\frac{1}{2(1+r)} Q_0(q,c); f_1(\theta_H)\bigg] \\ & + I\bigg[\frac{r}{2(1+r)} Q_0(q,c); f_1(\theta_H)\bigg]\bigg\}, \\ r = \frac{\tilde{\mu}^2}{\mu_2^2} = \bigg(\frac{m_b}{m_t}\bigg)^2. & \end{aligned} \tag{5.7}$$

As $r \ll 1$, the first term in (5.7) coming from Ψ_1 dominates. The factor $\frac{1}{2}$ in the argument of I is due to the fact that t' couples through θ_H to $\psi_4 = (t - B)/\sqrt{2}$, and the B component becomes heavy. As for the Ψ_2 contribution, both D and X, the partners of b', become heavy so that no component except for a small mixture of b characterized by a factor r remains light. This accounts for the difference between (5.4) and (5.7).

C. Symmetry breaking

The total effective potential $V_{\rm eff}(\theta_H)$ is the sum of $V_{\rm eff}(\theta_H)^{\rm gauge}$ in (5.3) and $V_{\rm eff}(\theta_H)^{\rm fermion}$ in (5.7). It is displayed in Fig. 2. With $c \sim 0.43$ the top contribution dominates over others. $V_{\rm eff}(\theta_H)$ has global minima at $\theta_H = \pm\frac{1}{2}\pi$, where the EW symmetry dynamically breaks down to $U(1)_{\rm EM}$. The contributions from other quarks and leptons are negligible, as the corresponding bulk mass parameters c range from 0.6 to 0.9 [23]. We conclude that the presence of a heavy top quark triggers EW symmetry breaking.

The effective potential $V_{\rm eff}(\theta_H)$ depends on the parameter z_L. There are two critical values for z_L. As z_L is decreased, the value of the bulk mass parameter c also decreases to reproduce the observed m_t. At $z_L = 9.4 \times 10^3 \equiv z_L^{c1}$, which corresponds to $k = 2.3 \times 10^6$ GeV, c becomes 0. For $z_L < z_L^{c1}$ there exists no solution with the observed m_t. One can set c to be 0 and examine the behavior of $V_{\rm eff}(\theta_H)$ for $z_L < z_L^{c1}$. It is found that for $z_L < z_L^{c2} = 905$, the global minima of $V_{\rm eff}(\theta_H)$ shift to $\theta_H = 0$

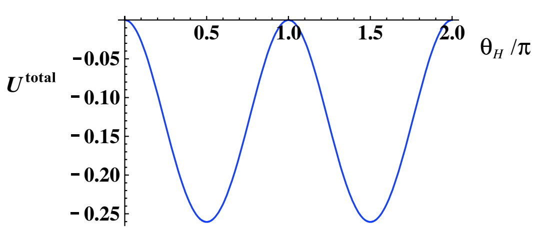

FIG. 2 (color online). The effective potential $V_{\rm eff}(\theta_H)$ in the model with top and bottom quarks. The plot is for $U^{\rm total}(\theta_H/\pi) = (4\pi)^2(kz_L^{-1})^{-4}V_{\rm eff}$ at $z_L = 10^{15}$. Contributions from light quarks and leptons are negligible. The global minima are located at $\theta_H = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$, where the EW symmetry dynamically breaks down to $U(1)_{\rm EM}$.

and π so that the EW symmetry is unbroken. One may take the flat space limit ($k \to 0$) with the bulk mass ck kept fixed. In this case $c \to \infty$ as $k \to 0$ so that contributions of fermions to the effective potential are exponentially suppressed. We conclude that the EW symmetry is unbroken in flat space in our scheme.

We would also like to remark that if fermions were introduced in the spinor representation of $SO(5)$, then there would be no EW symmetry breaking. In the effective potential fermions would give $f(\theta_H) = \sin^2\frac{1}{2}\theta_H$ in the expression (5.1) so that the global minimum would appear either at $\theta_H = 0$ or π.

VI. HIGGS MASS

The four-dimensional Higgs field (3.2) acquires a finite mass at the one-loop level. The physical neutral Higgs field $\phi^4 \equiv \phi_H$ is related to the Wilson line phase θ_H by (3.3). The effective potential $V_{\rm eff}(\theta_H)$ evaluated in the previous section translates to the effective potential for the Higgs field ϕ_H. By expanding $V_{\rm eff}$ around the minimum one obtains

$$\begin{aligned} V_{\rm eff} &= {\rm const} + \frac{1}{2}m_H^2(\phi_H - v)^2 + \cdots, \\ m_H^2 &= \frac{\pi^2 g_4^2 kL}{4m_{\rm KK}^2} \frac{d^2V_{\rm eff}}{d\theta_H^2}\bigg|_{\rm min}. \end{aligned} \tag{6.1}$$

We recall that

$$v = \langle\phi_H\rangle = \frac{2\theta_H}{\pi g_4}\frac{m_{\rm KK}}{\sqrt{kL}} = \frac{2}{g_4}\frac{\theta_H}{|\sin\theta_H|}m_W. \tag{6.2}$$

The relation between v and m_W deviates from that in the standard model by a factor $\frac{1}{2}\pi$ at the global minimum $\theta_H = \pm\frac{1}{2}\pi$.

Inserting (5.3) and (5.7) into (6.1), we find

$$\begin{aligned} m_H^2 \simeq \frac{g_4^2 kL m_{\rm KK}^2}{64\pi^4}\bigg\{ & -4G\bigg[\frac{1}{2}Q_0\bigg(q,\frac{1}{2}\bigg)\bigg] \\ & - 2G\bigg[\frac{1}{2}(1+s_\phi^2)Q_0\bigg(q,\frac{1}{2}\bigg)\bigg] - 3G\bigg[Q_0\bigg(q,\frac{1}{2}\bigg)\bigg] \\ & + 4G\bigg[\frac{1}{2(1+r)}Q_0(q,c)\bigg] \\ & + 4G\bigg[\frac{r}{2(1+r)}Q_0(q,c)\bigg]\bigg\}, \\ G[Q(q)] = \int_0^\infty dq\, q^3 \frac{2Q(q)}{1+Q(q)}. & \end{aligned} \tag{6.3}$$

The contribution from the bottom quark (the last term in the parenthesis) to m_H^2 is negligible. With numerical values m_W, m_t, $\alpha_W(m_Z) = g_4^2/4\pi = 0.0338$, and $z_L = 10^{15}$ ($kL = 34.5$) given, one finds that $k = 4.7 \times 10^{17}$ GeV, $m_{\rm KK} = 1.48$ TeV, $c = 0.429$, and $m_H = 49.9$ GeV. The numbers are tabulated for various values of z_L in Table. I.

TABLE I. The Higgs mass m_H, with the value of z_L given, k, $m_{\rm KK}$, c, and m_H are determined. Input parameters are $m_W = 80.4$ GeV, $\alpha_W = 0.0338$, and $m_t = 172$ GeV. For $z_L < 9.4 \times 10^3$ there is no value for c which reproduces m_t. For $z_L < 905$, $V_{\rm eff}(\theta_H)$ with $c = 0$ is minimized at $\theta_H = 0$, π so that the electroweak symmetry remains unbroken.

$z_L = e^{kL}$	k (GeV)	$m_{\rm KK}$ (TeV)	c	m_H
10^{17}	5.0×10^{19}	1.58	0.438	53.5
10^{15}	4.7×10^{17}	1.48	0.429	49.9
10^{13}	4.4×10^{15}	1.38	0.417	46.1
10^{10}	3.9×10^{12}	1.21	0.388	39.9
10^{5}	2.7×10^{7}	0.86	0.226	26.9
9.4×10^{3}	2.3×10^{6}	0.76	0	23.5

With m_W, m_t, m_b, α_W, and $z_L = e^{kL}$ given, all other relevant parameters at low energies are determined. The effective potential is minimized at $\theta_H = \pm\frac{1}{2}\pi$ where EW symmetry spontaneously breaks down. We stress that the Higgs mass m_H is mostly determined by m_W, α_W, and m_t.

It is seen that the Higgs mass is predicted around 50 GeV for $k = 10^{15} \sim 10^{19}$ GeV. One might wonder if this is in conflict with the LEP2 bound for m_H which states that $m_H < 114$ GeV is excluded. We contend that $m_H \sim 50$ GeV is in no conflict with the LEP2 bound in the current gauge-Higgs unification scenario.

The crucial observation is that the ZZH coupling vanishes at $\theta_H = \frac{1}{2}\pi$ as shown in Refs. [27,28]. The WWH and ZZH couplings in the $SO(5) \times U(1)_X$ model are suppressed, compared with those in the standard model, by a factor $\cos\theta_H$. The process $e^+e^- \to Z \to ZH$ cannot take place at $\theta_H = \pm\frac{1}{2}\pi$ so that the LEP2 bound is not applicable. The $ZZHH$ coupling, on the other hand, is multiplied by a factor $\cos 2\theta_H$ to the coupling in the standard model [31] so that $e^+e^- \to ZHH$ can proceed. Light Higgs particles might have been already produced. It is of great interest that a similar scenario emerges in a version of the minimal supersymmetric standard model (MSSM) where the lightest Higgs boson has a different coupling to Z from that of the Higgs boson in the standard model [47–50][1] and in the strongly interacting light Higgs scenario [40]. A distinctive feature in the gauge-Higgs unification scenario is that the light Higgs particle with vanishing WWH and ZZH couplings follows from the dynamics in the theory, but not by tuning parameters.

We would like to mention that the Higgs mass is expected to remain finite to all orders in perturbation theory. It is finite at the one-loop level as the θ_H-dependent part of the effective potential $V_{\rm eff}$ is finite as shown originally in Ref. [3], generally in Ref. [51] and also in the present paper. The finiteness has been shown at the two loop level in a toy model of five-dimensional QED [52].

A few comments are in order about the estimate of the Higgs mass given in Ref. [21]. It has been argued there, without either specifying the detailed fermion content or performing explicit computation of the effective potential $V_{\rm eff}(\theta_H)$, that in generic gauge-Higgs unification models in the RS space the Higgs mass should turn out in the range 140–280 GeV. In the present model we have found $m_H \sim 50$ GeV. The discrepancy stems from a couple of sources. First, in the evaluation of the effective potential we observed that the contribution from the top quark is halved due to the brane mass interactions. Second, we found that the effective potential takes the minimum at $\theta_H = \frac{1}{2}\pi$ whereas $\theta_H = (0.2 \sim 0.3)\pi$ was supposed in Ref. [21]. In the current model $m_H \propto m_W/|\sin\theta_H|$ so that smaller θ_H would give larger m_H. Third, the c dependence of $V_{\rm eff}(\theta_H)$ was not well appreciated in Ref. [21]. We have seen that for $c \sim 0.43$ there is partial cancellation between contributions from the top quark and gauge fields. If $c \sim 0.4$ ($m_t \sim 200$ GeV), then m_H would be increased by 40% to 73 GeV. The LEP2 bound $m_H \sim 114$ GeV would be achieved if one takes an unrealistic value $m_t \sim 262$ GeV ($c \sim 0.31$).[2] The appearance of the enhancement factor $kL/2$ in various physical quantities remains valid.

VII. SUMMARY AND DISCUSSIONS

In the present paper we constructed an $SO(5) \times U(1)_X$ gauge-Higgs unification model in the RS space with top and bottom quarks realized in two multiplets in the vector representation (**5**) of $SO(5)$. Additional brane fermions are introduced on the Planck brane to make all unwanted exotic particles heavy by brane mass terms, and at the same time to give a bottom quark a finite mass. Everything follows from equations of motion derived from the action principle with the orbifold boundary conditions. The effective change of boundary conditions results for low-lying modes of the Kaluza-Klein towers of exotic particles. The effective potential for the Wilson line phase and the Higgs mass are determined from the other observed quantities.

It was shown that the presence of a top quark triggers the electroweak symmetry breaking by the Hosotani mechanism. The effective potential was minimized at the Wilson line phase $\theta_H = \pm\frac{1}{2}\pi$. The Higgs mass m_H is predicted, once m_W, α_W, m_t, and z_L are given. It is found that $m_H \sim 50$ GeV for $z_L = 10^{15} \sim 10^{17}$. The WWH and ZZH couplings vanish at $\theta_H = \pm\frac{1}{2}\pi$ so that the LEP2 bound is evaded. We stress that the prediction is robust. It does not

[1]Note that, since the lightest Higgs is not standard model-like, the naive decoupling limit cannot be taken in this light Higgs MSSM scenario. In our case, the Kaluza-Klein scale $m_{\rm KK}$ is related to m_W by Eq. (3.9) so that one cannot arbitrarily take $m_{\rm KK} \to \infty$ limit for decoupling.

[2]Recall that our model does not need to give $m_H > 114$ GeV because of the vanishing ZZH coupling.

depend on the values of brane masses so long as the scale of the brane masses is much larger than m_{KK}. In short, the top mass determines the Higgs mass.

One may wonder if the vanishing, or suppression, of the WWH and ZZH couplings leads to the violation of the tree unitarity in the scattering of longitudinal components of W and Z. In Ref. [34] it has been shown that KK excited states of W and Z contribute to restore the unitarity at high energies through $WW^{(n)}H$ and $ZZ^{(n)}H$ couplings.

Phenomenology of the Higgs particle is of great interest. From the study of the $SU(3)$ model [23] it is expected that Yukawa couplings of the Higgs particle to quarks are suppressed compared with those in the standard model. The suppression would be milder for the top quark with $c \sim 0.4$ than that for lighter quarks with $c > 0.6$. The suppressed Yukawa coupling to the bottom quark implies that the Higgs particle has a rather narrow decay width.

When θ_H becomes large, generically large corrections are expected for the electroweak precision measurements, especially to the S and T parameters [18–20,25,26]. Our model, unlike the preceding ones, does not need any brane dynamics for the effective change of the boundary conditions at the TeV brane. It is manifest that our model fits into the criteria of Ref. [43] for suppressing radiative corrections to the ρ (T) parameter and $Zb\bar{b}$ coupling thanks to the custodial symmetry in the bulk and on the TeV brane and the extended $SO(5) \times Z_2 \simeq O(5)$ symmetry. In Ref. [32] it has been pointed out that sizable loop corrections to T may result when t_L and t'_R are placed in one multiplet. It is important to study such corrections in more detail in our framework.

The gauge-Higgs unification scenario predicts significant departure from the standard model, particularly in the Higgs sector. The forthcoming experiments at the LHC will give us clues in understanding the structure of the symmetry breaking and the origin of the Higgs particle.

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