PHYSICAL REVIEW D **71,** 124039 (2005)

Rotating black holes at future colliders. II. Anisotropic scalar field emission

Daisuke Ida*

Department of Physics, Gakushuin University, Tokyo 171-8588, Japan

Kin-ya Oda†

Physikalisches Institut der Universität Bonn, Nussallee 12, Bonn 53115, Germany

Seong Chan Park‡

Institute for High Energy Phenomenology, Floyd R. Newman Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York 14853, USA

(Received 7 March 2005; published 29 June 2005)

This is the sequel to the first paper of the series, where we have discussed the Hawking radiation from five-dimensional rotating black holes for spin 0, 1/2, and 1 brane fields in the low frequency regime. We consider the emission of a brane-localized scalar field from rotating black holes in general space-time dimensions without relying on the low frequency expansions.

DOI: 10.1103/PhysRevD.71.124039 PACS numbers: 04.50.+h, 04.70.Dy, 11.25.Mj, 11.25.Wx

I. INTRODUCTION

The black hole is one of the most important key objects in theoretical physics. Though its quantum behavior and thermodynamic property have played great roles in the path to understanding yet an unknown quantum theory of gravity (see e.g. Refs. [1–3]), a direct experimental test had been believed almost impossible. Recently, the scenarios of large [4] and warped [5] extra dimension(s) have led to an amazing possibility of producing black holes at future colliders with distinct signals [6,7] (see also Refs. [8,9] for earlier studies).

When the center-of-mass (c.m.) energy of a collision exceeds the Planck scale, which is of the order of TeV here, the cross section is dominated by a black hole production [10], which is predicted to be of the order of the geometrical one [11–16], increasing with the c.m. energy. In this trans-Planckian energy domain, the larger the c.m. energy, the larger the mass of the resulting black hole, and hence the better its decay is treated semiclassically via Hawking radiation [17]. The main purpose of this series of work is to discuss such decay signals in the hope that these will serve as the basis to pursue stringy or quantum gravitational corrections to them.

In previous publications, we have pointed out that the production cross section of a black hole increases with its angular momentum, so that the produced black holes are highly rotating [18,19] (see also Ref. [20] for an earlier attempt). The form factor for the production cross section, taking this rotation into account [18], is larger than unity and increases with the number of dimensions $D = 4 + n$. The result qualitatively agrees with an independent numerical simulation of a classical gravitational collision of two massless point particles [13,14]. We note that this form factor is hardly interpretative without considering the angular momentum. It is indispensable to take into account the angular momentum of the black hole when we perform a realistic calculation of its production and evaporation.

The black hole radiates mainly into the SM fields that are localized on the brane [21] unless it is highly rotating at which case the so-called superradiant bulk graviton emission is expected to be sizable [22] based on the numerical study in four dimensions [23]. (We note that there is an upper bound on the angular momentum of the black hole when it is produced by a particle collision [18].) In the previous paper [18], we have shown that the massless brane field equations with spin 0, 1/2, and 1, i.e. for all the standard model fields, are of a variables separable type in the rotating black hole background. Then we have obtained the analytic expressions for the grey-body factor of $D = 4 + n = 5$ dimensional (Randall-Sundrum 1) black hole by solving these equations under the low energy approximation of the radiating field. In this paper we present a generalized result to the higher dimensional black hole in $D > 5$ for the brane-localized scalar field without relying on the low energy approximation and discuss its physical implications.

This paper is the longer version of the brief report [19]. We also note that the related works by Harris [24] and by Harris and Kanti [25] appeared more recently.

II. BRANE SCALAR FIELD EMISSION

A brane-localized scalar field Φ in the higher $(4 + n)$-dimensional rotating black hole background [26] can be decomposed into the radial and angular parts $R(r)$ and $S_{\ell m}(\vartheta)$, respectively [18]

$$\Phi = R(r)S_{\ell m}(\vartheta)e^{-i\omega t+im\varphi}, \tag{1}$$

where the Boyer-Lindquist coordinate $(t, r, \vartheta, \varphi)$ reduces

*Electronic address: daisuke.ida@gakushuin.ac.jp
†Electronic address: odakin@th.physik.uni-bonn.de
‡Electronic address: spark@mail.lns.cornell.edu

to the spherical coordinate at spatial infinity and ℓ, m are the angular quantum numbers. The resultant equations are shown to be separable [18]. The angular part $S_{\ell m}$ obeys the equation for the spheroidal harmonics while the radial equation becomes

$$\left[\frac{d}{dr}\Delta\frac{d}{dr}+\frac{[(r^2+a^2)\omega-ma]^2}{\Delta}+2ma\omega - a^2\omega^2-A\right]R=0, \quad (2)$$

where

$$\Delta(r)=(r^2+a^2)-(r_h^2+a^2)\left(\frac{r}{r_h}\right)^{1-n}, \quad (3)$$

with r_h and a being the horizon radius and the rotation parameter of the black hole, respectively. Note that $\Delta(r_h)=0$.

The power spectrum of the Hawking radiation is governed by, for each scalar mode,

$$\frac{dE}{dtd\omega d\cos\vartheta}=\frac{\omega\Gamma_{\ell m}}{e^{(\omega-m\Omega)/T}-1}|S_{\ell m}(\vartheta)|^2, \quad (4)$$

where Ω and T are the angular velocity and the Hawking temperature of the black hole

$$\Omega=\frac{a_*}{(1+a_*^2)r_h}, \qquad T=\frac{(n+1)+(n-1)a_*^2}{4\pi(1+a_*^2)r_h}, \quad (5)$$

with $a_*=a/r_h$. The $\Gamma_{\ell m}$ is the grey-body factor that determines the departure from the black body spectrum, which is the main object of this paper. One immediate observation is that the contribution from $m>0$ modes dominates over that from $m<0$ modes in the rapidly rotating case $\Omega\gg\omega$.

III. NUMERICAL EVALUATION OF THE GREY-BODY FACTOR

The asymptotic forms of the radial wave function at the near horizon (NH) and far field (FF) limits, $r\to r_h$ and $r\to\infty$ respectively, are [18]

$$R_{\rm NH}=Y_{\rm in}e^{-ikr_*}+Y_{\rm out}e^{ikr_*}, \quad (6)$$

$$R_{\rm FF}=Z_{\rm in}\frac{e^{-i\omega r_*}}{r}+Z_{\rm out}\frac{e^{i\omega r_*}}{r}, \quad (7)$$

where $k=\omega-ma/(r_h^2+a^2)$ and the tortoise coordinate r_* is defined by $r_*(r)\to r$ for $r\to\infty$ and

$$\frac{dr_*}{dr}=\frac{r^2+a^2}{\Delta(r)}. \quad (8)$$

We obtain the grey-body factors in the following steps.

(1) Put the purely ingoing boundary condition $Y_{\rm out}=0$ by imposing

$$R(r_0)\to e^{-ikr_*(r_0)},$$
$$R'(r_0)\to -ik\frac{r_0^2+a^2}{\Delta(r_0)}e^{-ikr_*(r_0)}, \quad (9)$$

at the NH region $r_0=r_h(1+\epsilon)$.

(2) Numerically integrate the master equation (2) from the NH region to the FF regime $r=r_{\rm max}$.

(3) Perform a least squares fit to the obtained data by the function (7) around $r=r_{\rm max}$ to get $Z_{\rm in}$ and $Z_{\rm out}$.

(4) Finally the grey-body factor for the (ℓ,m) mode is given by the absorption rate

$$\Gamma_{\ell m}=1-\left|\frac{Z_{\rm out}}{Z_{\rm in}}\right|^2. \quad (10)$$

For the angular eigenvalue A, we employed the small $a\omega$ expansions up to 6th order in [27]. [The last 6th order term in the expansion is less than a few percent of the leading order term at $a\omega\lesssim 3$ for the $(\ell,m)=(0,0)$ and $(2,0)$ modes and at $a\omega\lesssim 4$ for all other modes.] We have also performed the above procedure in the ingoing Kerr-Newman coordinate as a cross-check.

IV. RESULTS

A. Comparison with analytic expansions for $D=5$

The numerical results of the grey-body factors for the $D=4+n=5$ dimensional black hole are shown in Figs. 1–3. They are in good agreement with the previous analytic expression in [18] in the region $r_h\omega\lesssim 0.3$. At $\omega=\omega_0$ with

$$\omega_0=m\Omega=\frac{ma}{r_h^2+a^2}, \quad (11)$$

the Bose statistics factor diverges, but this divergence is regularized to give a finite emission rate due to the zero absorption rate at this point, where grey-body factors cross

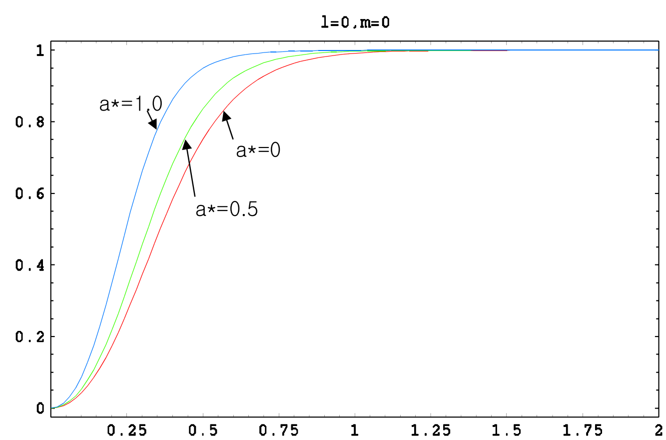

FIG. 1 (color online). The grey-body factor for the brane scalar emission into the $\ell=0$ mode from the $D=5$ dimensional black hole.

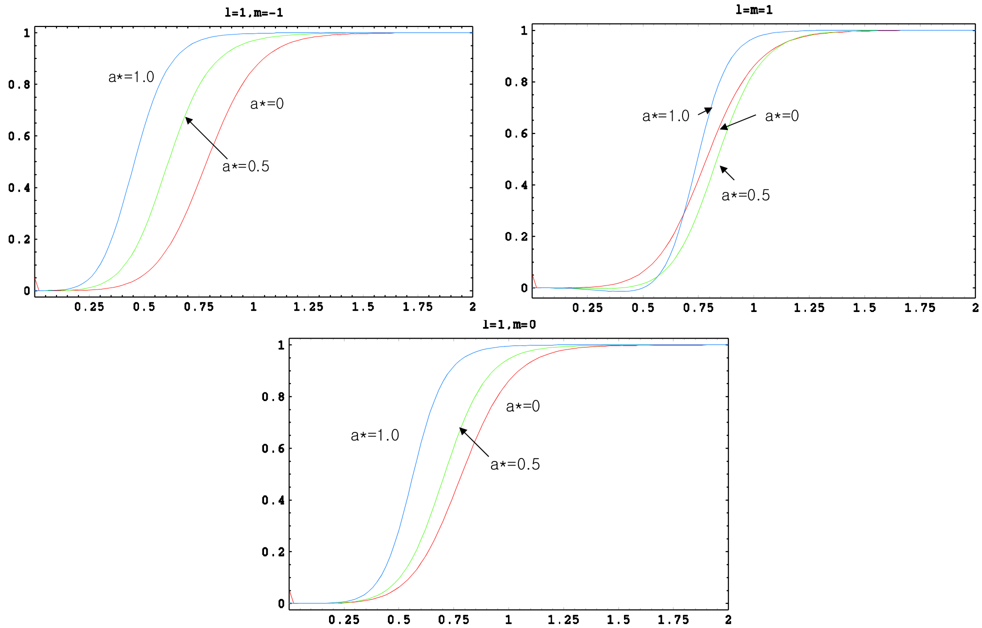

FIG. 2 (color online). The grey-body factors for the brane scalar emission into the $\ell = 1$ modes from the $D = 5$ dimensional black hole.

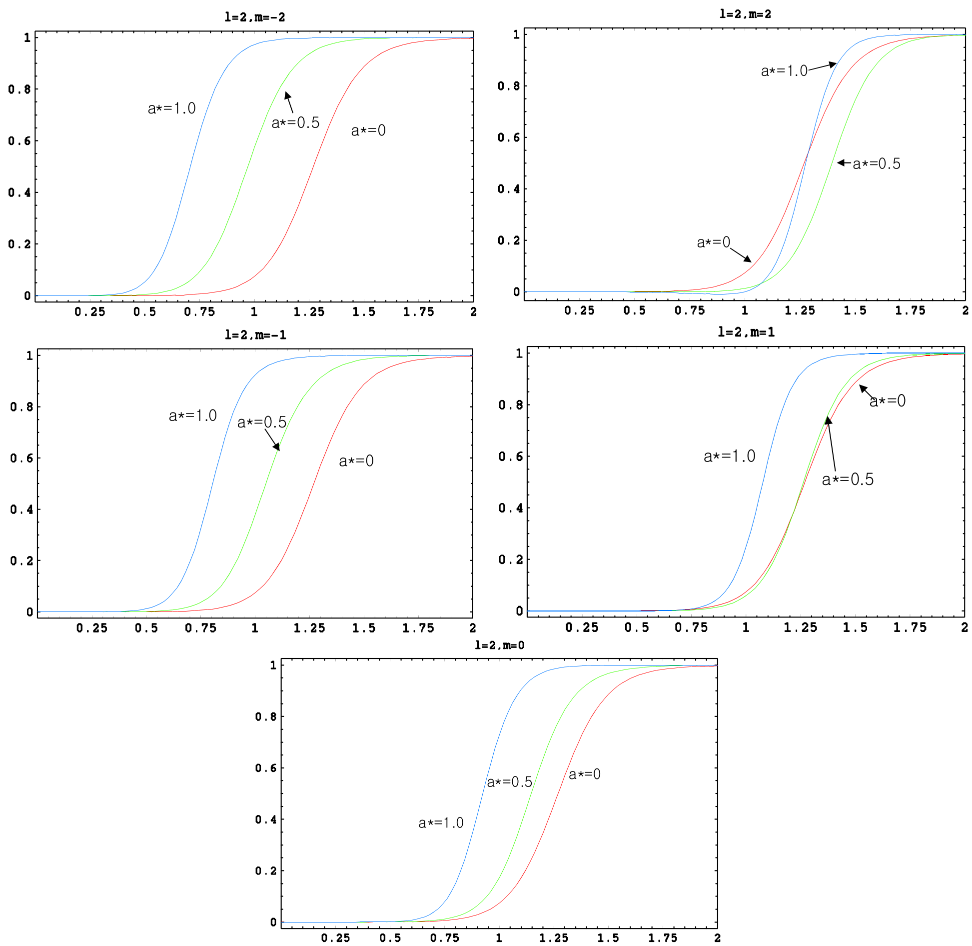

FIG. 3 (color online). The grey-body factors for the brane scalar emission into the $\ell = 2$ modes from the $D = 5$ dimensional black hole.

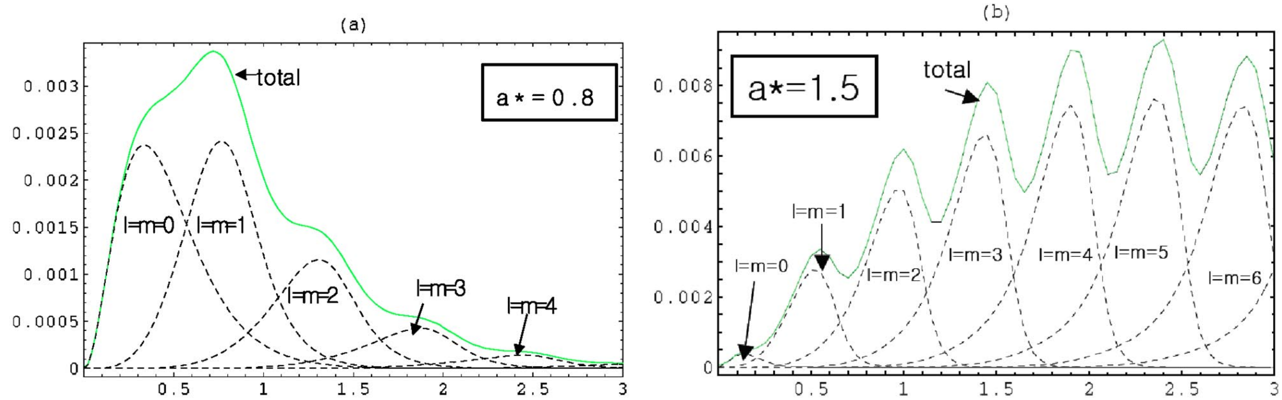

FIG. 4 (color online). Total power spectrum of the $D = 5$ dimensional black hole. The covering curves correspond to the total power spectrum and the dotted ones to the contributions from the $\ell = m$ modes.

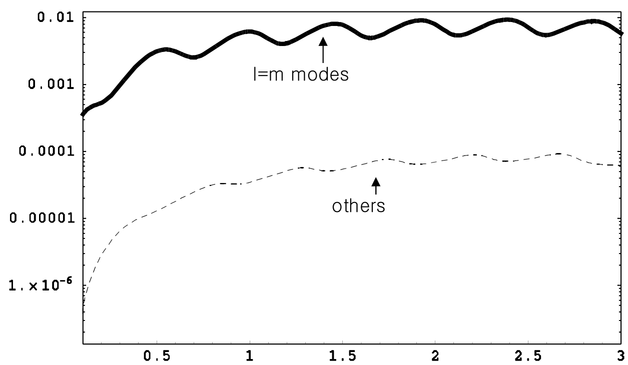

FIG. 5. Total power spectrum of the $D = 5$ dimensional black hole with $a_* = 1.5$. The small concave curve denotes the contribution from the sum of other than $\ell = m$ modes.

the zero [28]. As we claimed in Ref. [19], our analytic expression in Ref. [18] correctly shows for which ω there emerges the superradiance with the negative grey-body factor $\propto \tilde{Q} = (1 + a_*^2)\omega - ma_* < 0$.

B. Power spectrum

In Fig. 4 we show the power spectrum for the $D = 4 + n = 5$ dimensional rotating black hole. We can see that when the hole is rotating the total power spectrum is essentially determined by $\ell = m$ modes, with each peak corresponding to each angular mode. We can safely neglect the modes other than the $\ell = m$ ones. As a check we plot the $\ell = m$ contribution and that from the other modes in Fig. 5 for $a_* = 1.5$.

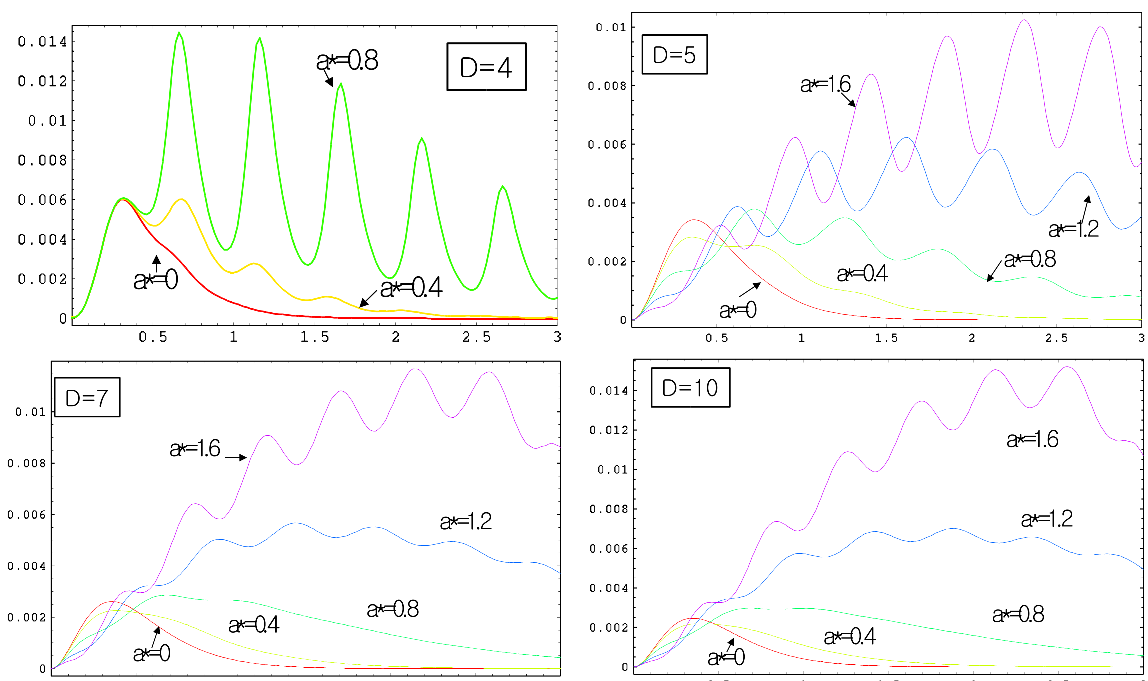

FIG. 6 (color online). Total power spectrum for a fixed number of space-time dimensions D and varying rotation parameter a_*.

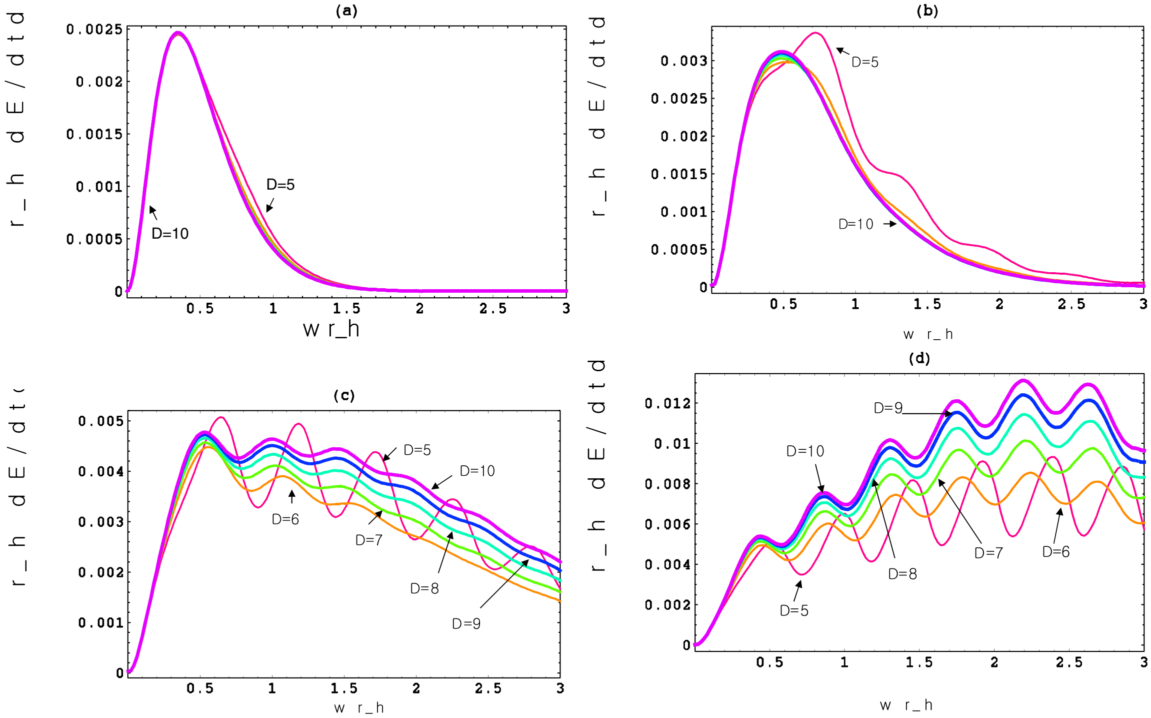

FIG. 7 (color online). Total power spectrum for fixed rotation parameter a_* and varying space-time dimensions D. (a)–(d) correspond to $a_* = 0$, 0.5, 1.0, and 1.5, respectively.

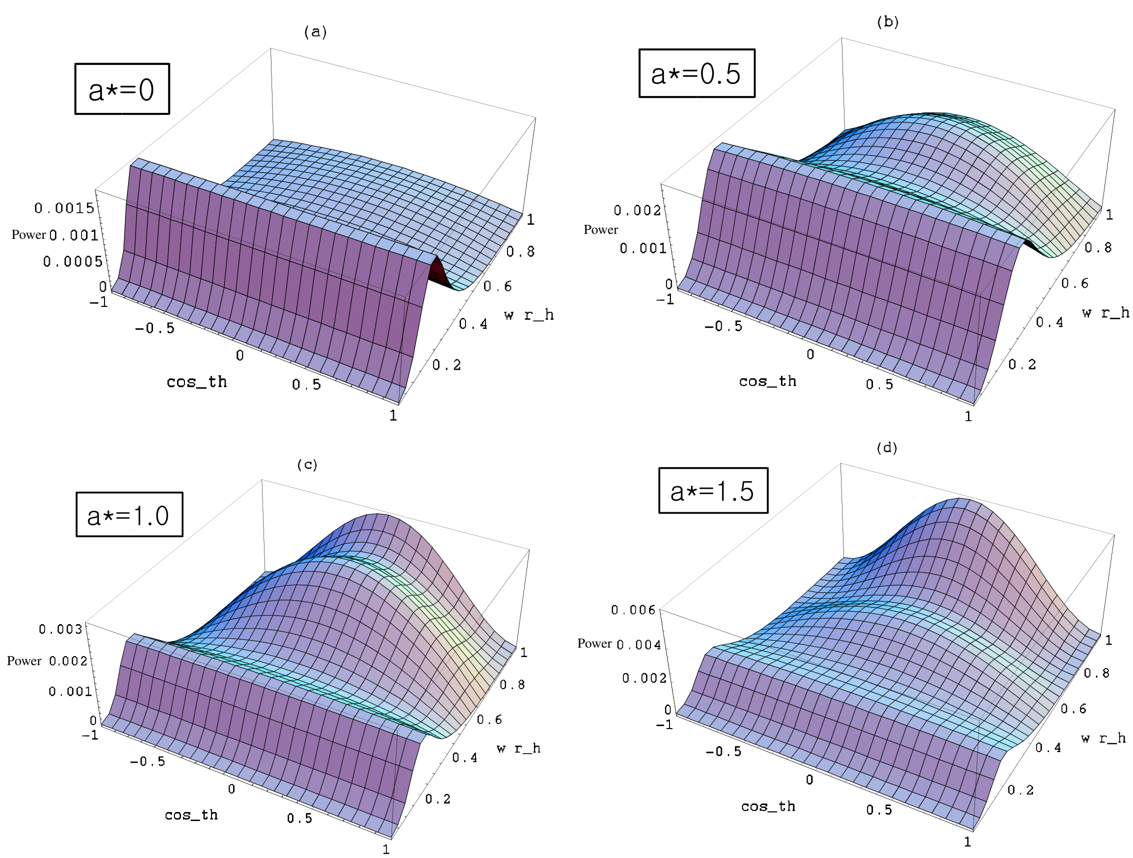

FIG. 8 (color online). Angular distribution for $D = 5$ dimensional black hole.

In Fig. 6 we plot using only the $\ell = m$ modes. We take the angular modes from $\ell = m = 0$ to 7. In the low energy region ($r_h\omega \lesssim 0.3$) the spectrum of the highly rotating black hole is suppressed compared with the nonrotating power spectrum, confirming the analytic result in low energy expansions [18], while in the higher energy regime the spectrum is greatly enhanced. This tendency is stronger for larger dimensions. To see that more explicitly, we also plot the power spectrum varying the number of dimensions in Fig. 7.

C. Angular distribution

In Fig. 8 we plot the angular distribution for $D = 4 + n = 5$ dimensional black holes. We approximate the spheroidal harmonics by the spherical harmonics with the assumption $a\omega \ll 1$. We confirm the previous result in the low frequency approximation [18] that the anisotropy is greatly enhanced for a highly rotating black hole, due to the $\ell = m > 0$ mode.

V. DISCUSSION

We have explained the importance of the angular momentum when one considers TeV scale black hole production and evaporation. New numerical results are shown: the grey-body factor for the brane scalar emission from a general $D = 4 + n$ dimensional rotating black hole without relying on the low frequency expansions. The grey-body factors are obtained for general $D \geq 4$ dimensional cases and the various angular modes (ℓ, m). We confirmed the nontrivial angular dependence of the scalar emission at the middle energy region $r_h\omega \sim 0.5$ and found that it is even more enhanced at the higher energy region.

To understand the actual evolution of a black hole and to predict the collider signature, we need further investigations. It is important to determine the grey-body factors for spinor and vector fields. The evolution of the angular momentum and the mass can be determined once all the grey-body factors are determined [23,29]. In particular, the spin-down phase, whose time evolution has been impossible to determine so far, can be precisely described. One can in principle determine the angular momentum of the produced black hole from the nontrivial angular distribution of the signals.

[1] J. R. David, G. Mandal, and S. R. Wadia, Phys. Rep. **369**, 549 (2002).
[2] T. Padmanabhan, Phys. Rep. **406**, 49 (2005).
[3] G. Veneziano, J. High Energy Phys. 11 (2004) 001.
[4] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).
[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[6] S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002).
[7] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001).
[8] P. C. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett. B **441**, 96 (1998).
[9] T. Banks and W. Fischler, hep-th/9906038.
[10] G. 't Hooft, Phys. Lett. B **198**, 61 (1987).
[11] S. D. H. Hsu, Phys. Lett. B **555**, 92 (2003).
[12] D. M. Eardley and S. B. Giddings, Phys. Rev. D **66**, 044011 (2002).
[13] H. Yoshino and Y. Nambu, Phys. Rev. D **67**, 024009 (2003).
[14] H. Yoshino and V. S. Rychkov, hep-th/0503171 [Phys. Rev. D (to be published)].
[15] S. B. Giddings and V. S. Rychkov, Phys. Rev. D **70**, 104026 (2004).
[16] K. Kang and H. Nastase, hep-th/0410173.
[17] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[18] D. Ida, K. Oda, and S. C. Park, Phys. Rev. D **67**, 064025 (2003); **69**, 049901(E) (2004).
[19] D. Ida, K. Oda, and S. C. Park, hep-ph/0501210.
[20] S. C. Park and H. S. Song, J. Korean Phys. Soc. **43**, 30 (2003).
[21] R. Emparan, G. T. Horowitz, and R. C. Myers, Phys. Rev. Lett. **85**, 499 (2000).
[22] D. Stojkovic, Phys. Rev. Lett. **94**, 011603 (2005).
[23] D. N. Page, Phys. Rev. D **14**, 3260 (1976).
[24] C. M. Harris, hep-ph/0502005.
[25] C. M. Harris and P. Kanti, hep-th/0503010.
[26] R. C. Myers and M. J. Perry, Ann. Phys. (N.Y.) **172**, 304 (1986).
[27] E. Seidel, Classical Quantum Gravity **6**, 1057 (1989).
[28] The expression "Hawking absorption" [19] to describe this situation might be misleading.
[29] S. A. Teukolsky and W. H. Press, Astrophys. J. **193**, 443 (1974).