Rotating Black Holes at Future Colliders

Daisuke Ida
Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan
Kin-ya Oda
Physik Dept. T30e, TU München, James Franck Str., D-85748 Garching, Germany
Seong Chan Park
Korea Institute for Advanced Study (KIAS), Seoul 130-012, Korea

Abstract

We consider the production and decay of TeV-scale black holes. Evaluation of the production cross section of higher dimensional rotating black holes is made. The master field equation for general spin-s fields confined on brane world is derived. For five-dimensional (Randall-Sundrum) black holes, we obtain analytic formulae for the greybody factors in low frequency expansion.

1. Introduction

The scattering process of two particles at CM energies in the trans-Planck domain, is well calculable using known laws of physics, because gravitational interaction dominates over all other interactions. Non-trivial quantum gravitational (or string/M theoretical) phenomena are well behind the horizon. If the impact parameter is less than the black hole radius corresponding to the CM energy then one naturally expects a black hole to form. When nature realizes TeV scale gravity scenario [1], one of the most intriguing prediction would be copious production of TeV sized black holes at near future particle colliders [2, 3]. The production cross section of black hole in the higher dimensional case was obtained in ref. [4] by taking angular momenta into account and the result has been numerically proved in refs. [5](See also Ref. [7] and [8].). Once produced, black holes lose its masses and angular momenta through the Hawking radiation [9]. The Hawking radiation is determined for each mode by the greybody factor, i.e. the absorption probability of an incoming wave of the corresponding mode. The master equation for general brane-fields with arbitrary spin-s was obtained in ref. [4] for rotating black holes in higher dimensional spacetime and its non-rotating limit was confirmed in ref. [10]. Analytic expressions of greybody factors for rotating black holes were obtained in five dimensional (Randall-Sundrum) case in ref. [4] and also for non-rotating limit in the series of papers [11].

2. Production of rotation black holes

Let us imagine a collision of two massless particles with finite impact parameter b and the center of mass (CM) energy $\sqrt{s} = M_i$ so that each particle has energy $M_i/2$ in the CM frame. The initial angular momentum before collision is $J_i = bM_i/2$ in the CM frame. Suppose that a black hole forms whenever the initial two particles can be wrapped inside the event horizon of the black hole with the mass $M = M_i$ and angular momentum $J = J_i$, i.e., when

$$b \ < \ 2r_h(M, J) = 2r_h(M_i, bM_i/2), \tag{1}$$

where $r_h(M, J)$ is the size of event-horizon for given energy M and angular momentum J[12]. Since the right hand side is monotonically decreasing function of b, there is maximum value b_{max} which saturates the inequality (1). The production cross section is now given by

$$\begin{aligned} \sigma(M) &= \pi b_{max}^2 \\ &= 4\left[1 + \left(\frac{n+2}{2}\right)^2\right]^{-2/(n+1)} \pi r_S(M)^2 \\ &= F\,\pi r_S(M)^2. \end{aligned} \tag{2}$$

The form factor F is summarized as

n	1	2	3	4	5	6	7
$F_{\rm NY}$	1.084	1.341	1.515	1.642	1.741	1.819	1.883
$F_{\rm Our}$	1.231	1.368	1.486	1.592	1.690	1.780	1.863

and the result fits the numerical result of $\sigma(M)$ with full consideration of the general relativity by Yoshino and Nambu [6] within the accuracy less than 1.5% for $n \geq 2$ and 6.5% for $n = 1$. This result implies that we would underestimate the production cross section of black holes if we did not take the angular momentum into account and that it becomes more significant for higher dimensions.

3. Decay of rotating black holes

Black hole decays through the Hawking radiation. The Hawking radiation is not exactly thermal but modified by the so-called greybody factor which could be obtained by solving wave equations under the black hole background metric. For the higher dimensional rotating black holes, the brane field equations for massless spin s field was derived utilizing the Newman-Penrose formalism [13]. By the standard decomposition $\Phi_s = R_s(r)S(\vartheta)e^{-i\omega t+im\varphi}$, the master wave equation is given as:

$$\frac{1}{\sin\vartheta}\frac{d}{d\vartheta}\left(\sin\vartheta\frac{dS}{d\vartheta}\right)+[(s-a\omega\cos\vartheta)^2-(s\cot\vartheta+m\csc\vartheta)^2+\hat{A}]S=0, \tag{3}$$

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR}{dr}\right)+\Bigg[\frac{K^2}{\Delta}+s\left(4i\omega r-i\frac{\Delta_{,rr}K}{\Delta}+\Delta_{,rr}-2\right) +2ma\omega-a^2\omega^2-A\Bigg]R=0, \tag{4}$$

where $K=(r^2+a^2)\omega-ma$ and $\hat{A}=-s(s-1)+A$. The solution of the angular equation is known as spin-weighted spheroidal harmonics ${}_sS_{lm}$. We solved the radial equation for $n=1$ Randall-Sundrum black hole with the low frequency expansion. Here we outline our procedure: First we obtain the "near horizon" and "far field" solutions in the corresponding limits; Then we match these two solutions at the "overlapping region" in which both limits are consistently satisfied; Finally we impose the "purely ingoing" boundary condition at the near horizon side and then read the coefficients of "outgoing" and "ingoing" modes at the far field side. The ratio of these two coefficients can be translated into the absorption probability of the mode, which is nothing but the greybody factor itself. Finally, the greybody factor Γ (=the absorption probability) could be written as follows.

$$\Gamma = 1-\left|\frac{Y_{\rm out}Z_{\rm out}}{Y_{\rm in}Z_{\rm in}}\right| = 1-\left|\frac{1-C}{1+C}\right|^2, \tag{5}$$

where

$$C=\frac{(4i\tilde{\omega})^{2l+1}}{4}\left(\frac{(l+s)!(l-s)!}{(2l)!(2l+1)!}\right)^2(-iQ-l)_{2l+1}, \tag{6}$$

with $(\alpha)_n=\prod_{n'=1}^{n}(\alpha+n'-1)$ being the Pochhammer's symbol and some useful dimensionless quantities $\xi=\frac{r-r_h}{r_h}$, $\tilde{\omega}=r_h\omega$ and $Q=\frac{\omega-m\Omega}{2\pi T}$. We note that the so-called s-wave dominance is maximally violated for fermion and vector fields since there are no $l=0$ modes for them.

4. Summary

We have studied theoretical aspects of the rotating black hole production and evaporation. For production, we present an estimation of the geometrical cross section up to unknown mass and angular momentum loss in the balding phase. Our result of the maximum impact parameter b_{max} is in good agreement with the numerical result by Yoshino and Nambu when the number of extra dimensions is $n \geq 1$ (i.e. within 6.5% when $n = 1$ and 1.5% when $n \geq 2$). Relying on this agreement, we obtain the (differential) cross section for a given mass and an angular momentum. The result shows that black holes tend to be produced with large angular momenta. For evaporation, we first derive the master equation for brane fields for general spin and for an arbitrary number of extra dimensions. We show that the equations are separable into radial and angular parts. From these equations, we obtain the greybody factors for brane fields with general spin for the five-dimensions ($n = 1$) Kerr black hole within the low-frequency expansion.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998); L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[2] T. Banks and W. Fischler, arXiv:hep-th/9906038; P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B **441**, 96 (1998).

[3] S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002); S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001).

[4] D. Ida, K. y. Oda and S. C. Park, Phys. Rev. D **67**, 064025 (2003) [arXiv:hep-th/0212108].

[5] D. M. Eardley and S. B. Giddings, Phys. Rev. D **66**, 044011 (2002) [arXiv:gr-qc/0201034].

[6] H. Yoshino and Y. Nambu, Phys. Rev. D **67**, 024009 (2003) [arXiv:gr-qc/0209003].

[7] S. C. Park and H. S. Song, J. Korean Phys. Soc. **43**, 30 (2003) [arXiv:hep-ph/0111069].

[8] L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, Phys. Rev. D **65**, 124027 (2002) [arXiv:hep-ph/0112247].

[9] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[10] C. M. Harris and P. Kanti, JHEP **0310**, 014 (2003) [arXiv:hep-ph/0309054].

[11] P. Kanti and J. March-Russell, Phys. Rev. D **67**, 104019 (2003) [arXiv:hep-ph/0212199]; Phys. Rev. D **66**, 024023 (2002) [arXiv:hep-ph/0203223].

[12] R. C. Myers and M. J. Perry, Annals Phys. **172**, 304 (1986).

[13] E. Newman and R. Penrose, J. Math. Phys. **3**, 566 (1962).