

Title	Black holes at future colliders
Author(s)	Ida, Daisuke; Oda, Kinya; Park, Seong Chan
Citation	
Version Type	A0
URL	https://hdl.handle.net/11094/78778
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Note	

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Black Holes at future colliders *

Daisuke Ida[†]

Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

Kin-ya Oda[‡]

Physik Dept. T30e, TU München, James Franck Str., D-85748 Garching, Germany

Seong Chan Park[§]

Korea Institute for Advanced Study (KIAS), Seoul 130-012, Korea

We consider the production and decay of TeV sized black holes. After discussing evaluation of the production cross section of higher dimensional rotating black holes and black rings, the master equation for general spin- s fields confined on brane world is derived. For five-dimensional (Randall-Sundrum) black holes, we obtain analytic formulae for the greybody factors in low frequency expansion.

I. INTRODUCTION

The scattering process of two particles at CM energies in the trans-Planck domain, is well calculable using known laws of physics, because gravitational interaction dominates over all other interactions. Non-trivial quantum gravitational (or string/M theoretical) phenomena are well behind the horizon [1]. If the impact parameter is less than the black hole radius corresponding to the CM energy then one naturally expects a black hole to form. When nature realizes TeV scale gravity scenario [2, 3, 4], one of the most intriguing prediction would be copious production of TeV sized black holes at near future particle colliders and in ultra high energy cosmic rays [5, 6, 7, 8, 9]. (See also some recent papers [10] for particle accelerator signals and [11, 12] for cosmic ray signals.) The production cross section of black hole in the higher dimensional case was obtained in ref. [13] under the assumptions of “Hoop conjecture” [14] by taking angular momenta into account and the result has been numerically proved in refs. [15, 16] (See Ref. [17] and also [12] where similar analysis were made to estimate the cross-section by taking angular momenta into account.). Once produced, black holes lose its masses and angular momenta through the Hawking radiation [18]. The

Hawking radiation is determined for each mode by the greybody factor, i.e. the absorption probability of an incoming wave of the corresponding mode. For four dimensional case, it is first calculated for spin 0 field by A. A. Starobinsky [19], then for spin 1, 2 and 1/2 fields by S. A. Teukolsky and Don N. Page [20, 21, 22, 23, 24, 25, 26]. The absorption cross section of a non-rotating BH for all frequencies and with an analytic expression was computed by N. Sanchez [27].

The master equation for general brane-fields with arbitrary spin- s was obtained in ref. [13] for rotating black holes in higher dimensional spacetime and its non-rotating limit was confirmed in ref. [28]. Analytic expressions of greybody factors for rotating black holes were obtained in five dimensional (Randall-Sundrum) case in ref. [13] and also for non-rotating limit in the series of papers [29, 30].

II. PRODUCTION OF ROTATING BLACK HOLES

First we briefly review the properties of the rotating $(4+n)$ - dimensional black hole [31]. In general, higher dimensional black hole may have $\lfloor (n+3)/2 \rfloor$ angular momenta. When the black hole is produced in the collision of two particles on the brane, where the initial state has only single angular momentum, it is sufficient to consider that the only single angular momentum is non-zero.

In the Boyer-Lindquist coordinate, the metric for the black hole with single angular momentum takes the following form [31]

$$g = \left(1 - \frac{\mu r^{-n+1}}{\Sigma(r, \vartheta)}\right) dt^2$$

*This paper is based on talks by SC in several conferences. They include DESY theory workshop in DESY, Hamburg, Germany Sep.23-26, 2003, International conference on Gravitation and Astrophysics of APCTP(ICGA-VI) in Ewha Woman’s University, Seoul, Korea, Oct. 6-9, 2003 and International conference on Flavour physics (ICFP-II) in KIAS, Seoul, Korea, Oct. 6-11, 2003.

[†]Electronic address: d.ida@th.phys.titech.ac.jp

[‡]Electronic address: odakin@ph.tum.de

[§]Electronic address: spark@kias.re.kr

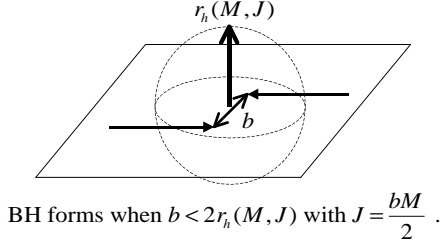


FIG. 1: Schematic picture for the condition of the black hole formation.

$$\begin{aligned}
& -\sin^2 \vartheta \left(r^2 + a^2 + a^2 \sin^2 \vartheta \frac{\mu r^{-n+1}}{\Sigma(r, \vartheta)} \right) d\varphi^2 \\
& + 2a \sin^2 \vartheta \frac{\mu r^{-n+1}}{\Sigma(r, \vartheta)} dt d\varphi - \frac{\Sigma(r, \vartheta)}{\Delta(r)} dr^2 - \Sigma(r, \vartheta) d\vartheta^2 \\
& - r^2 \cos^2 \vartheta d\Omega^n,
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
\Sigma(r, \vartheta) &= r^2 + a^2 \cos^2 \vartheta, \\
\Delta(r) &= r^2 + a^2 - \mu r^{-n+1}.
\end{aligned}$$

We can see that the horizon occurs when $\Delta(r) = 0$, i.e. when $r = r_h$ with

$$r_h = \left[\frac{\mu}{1 + a_*^2} \right]^{1/(n+1)} = (1 + a_*^2)^{-1/(n+1)} r_S, \tag{2}$$

where $a_* = a/r_h$ and r_S is the Schwarzschild radius for given mass. Note that there is only single horizon when $n \geq 1$ (contrary to the four-dimensional Kerr black hole which has inner and outer horizons) and its radius is independent of the angular coordinates. We can obtain the total mass M and angular momentum J of the black hole from the metric (1)

$$M = \frac{(n+2)A_{n+2}}{16\pi G} \mu, \quad J = \frac{2}{n+2} Ma, \tag{3}$$

where $A_{n+2} = 2\pi^{(n+3)/2}/\Gamma(\frac{n+3}{2})$ is the area of unit sphere S^{n+2} and G is the $(4+n)$ -dimensional Newton constant. Therefore we may consider μ and a (or r_h^{-1} and a_*) as the normalized mass and angular momentum parameters, respectively. We note that there are no upper bound on a when $n \geq 2$ nor on a_* when $n \geq 1$, contrary to the four-dimensional case where both a and a_* are bounded from above. We concentrate on the brane field equations and hence only the induced metric on the brane is relevant, where the last term in eq. (1) vanishes and the angular variables ϑ and φ are redefined to take the values $0 \leq \vartheta \leq \pi$ and $0 \leq \varphi < 2\pi$.

A. Production cross section

We estimate the production cross section of rotating black holes within the classical picture. Let us consider a collision of two massless particles with finite impact parameter b and CM energy $\sqrt{s} = M_i$ so that each particle has energy $M_i/2$ in the CM frame. (see Fig. 1 for schematic picture) The initial angular momentum before collision is $J_i = bM_i/2$ in the CM frame. Suppose that a black hole forms whenever the initial two particles can be wrapped inside the event horizon of the black hole with the mass $M = M_i$ and angular momentum $J = J_i$, i.e., when

$$b < 2r_h(M, J) = 2r_h(M_i, bM_i/2), \tag{4}$$

where $r_h(M, J)$ is defined through eqs. (2) and (3). Since the right hand side is monotonically decreasing function of b , there is maximum value b_{\max} which saturates the inequality (4)

$$b_{\max}(M) = 2 \left[1 + \left(\frac{n+2}{2} \right)^2 \right]^{-\frac{1}{n+1}} r_S(M), \tag{5}$$

where $r_S(M)$ is defined by $r_S(M) = \mu(M)^{1/(n+1)}$ and eq. (3). When $b = b_{\max}$, the rotation parameter a_* takes the maximal value $(a_*)_{\max} = (n+2)/2$.

The formula (5) fits the numerical result of b_{\max} with full consideration of the general relativity by Yoshino and Nambu [16] within the accuracy less than 1.5% for $n \geq 2$ and 6.5% for $n = 1$:

n	1	2	3	4	5	6	7
$R_{\text{Numerical}}$	1.04	1.16	1.23	1.28	1.32	1.35	1.37
R_{Analytic}	1.11	1.17	1.22	1.26	1.30	1.33	1.36

where R denotes $R = b_{\max}/r_S(M)$.

Our result is obtained in the approximation that we neglect all the effects by the junk emissions in the balding phase and hence that the initial CM energy M_i and angular momentum J_i become directly the resultant black hole mass $M = M_i$ and angular momentum $J = J_i$. The coincidence of our result with the numerical study [16] suggests that this approximation would be actually viable for higher dimensional black hole formation at least unless b is very close to b_{\max} .

Once we neglect the balding phase, the initial impact parameter b directly leads to the resultant angular momentum of the black hole $J = bM/2$. Since the impact parameter $[b, b+db]$ contributes to the cross section $2\pi b db$, this relation between b and J tells us the (differential) production cross section of the black hole with its mass M and its angular momentum in $[J, J+dJ]$

$$d\sigma(M, J) = \begin{cases} 8\pi J dJ / M^2 & (J < J_{\max}) \\ 0 & (J > J_{\max}) \end{cases}, \tag{6}$$

where

$$J_{\max} = \frac{b_{\max} M}{2} = j_n \left(\frac{M}{M_P} \right)^{\frac{n+2}{n+1}} \quad (7)$$

with

$$j_n = \left[\frac{2^n \pi^{\frac{n-3}{2}} \Gamma\left(\frac{n+3}{2}\right)}{(n+2) \left[1 + \left(\frac{n+2}{2}\right)^2\right]} \right]^{1/(n+1)},$$

$$M_P = \left(\frac{(2\pi)^n}{8\pi G} \right)^{1/(n+2)}. \quad (8)$$

It is observed that the differential cross section (6) linearly increases with the angular momentum. We expect that this behavior is correct as the first approximation, so that the black holes tend to be produced with larger angular momenta. At the typical LHC energy $M/M_P = 5$, the value of J_{\max} is $J_{\max} = 2.9, 4.5, \dots, 10, 12$ for $n = 1, 2, \dots, 6, 7$, respectively. This means that the semi-classical treatment of the angular momentum becomes increasingly valid for large n .

Integrating the expression (6) simply gives

$$\begin{aligned} \sigma(M) &= \pi b_{\max}^2 \\ &= 4 \left[1 + \left(\frac{n+2}{2} \right)^2 \right]^{-2/(n+1)} \pi r_S(M)^2 \\ &= F \pi r_S(M)^2. \end{aligned} \quad (9)$$

The form factor F [35] is summarized as

n	1	2	3	4	5	6	7
F_{NY}	1.084	1.341	1.515	1.642	1.741	1.819	1.883
F_{Our}	1.231	1.368	1.486	1.592	1.690	1.780	1.863

This result implies that we would underestimate the production cross section of black holes if we did not take the angular momentum into account and that it becomes more significant for higher dimensions. We point out that this effect has been often overlooked in the literature.

B. Rotating black ring

A higher-dimensional black hole can have various non-trivial topology, and the uniqueness property of stationary black holes fails in five (and probably in higher) dimensions. The typical example in five dimensions has been recently given by Emparan and Reall [32]. They have explicitly provided a solution of the five-dimensional vacuum Einstein equation, which represents the stationary rotating black ring (homeomorphic to $S^1 \times S^2$). In this case, the centrifugal force prevents the black ring from collapsing. When the angular momentum is not large enough, the black ring will collapse to the Kerr black hole due to the gravitational attraction and some

effective tension of the ring source. In fact, this five-dimensional black ring solution has the minimum possible value of the angular momentum given by

$$J_{\min} = k_{\text{BR}} \left(\frac{M}{M_P} \right)^{3/2}, \quad (10)$$

where $k_{\text{BR}} = 0.282$. On the other hand, we have the upper bound for the angular momentum of the black holes produced by particle collisions:

$$J_{\max} = j_1 \left(\frac{M}{M_P} \right)^{3/2}, \quad (11)$$

where $j_1 = 0.256$. Since these numerical values are of the same order, we cannot conclude the possibility of black ring productions at colliders.

Now we consider the possibility of the higher dimensional black rings, which is homeomorphic to, say, $S^1 \times S^{n+1}$. Corresponding Newtonian situation will be the system of a rotating massive circle. For simplicity, we just consider the gravitational attraction and the centrifugal force of the massive circle and neglect the effect of tension. Let ℓ , M and J be the radius, the mass and the angular momentum of the massive circle. The minimum value of the angular momentum for exploding black ring is estimated:

$$J \gtrsim J_{\min} = k_n \left(\frac{M}{M_P} \right)^{(n+2)/(n+1)}, \quad (12)$$

where

$$k_n = 2^{-\frac{2n^2+3n+7}{2(n+1)}} \pi^{\frac{(n+6)(n-1)}{4(n+1)}} \left[\frac{\Gamma\left(\frac{n+2}{2}\right)}{n+1} \right]^{-\frac{n-1}{2(n+1)}}. \quad (13)$$

J_{\min} for exploding black rings is one or two order(s) of magnitude smaller than J_{\max} for collision limit when n is large. Therefore we expect that the exploding black rings are possibly produced at colliders if there are many extra dimensions, though they will suffer from the black string instability when they become sufficiently large thin rings.

III. RADIATIONS FROM ROTATING BLACK HOLE

In this section, we study the Hawking radiation [18] from the higher dimensional Kerr black hole [31]. The Hawking radiation is thermal but not strictly black body due to the frequency dependent greybody factor Γ , which is identical to the absorption probability (by the hole) of the corresponding mode [18, 25]. The quantity $1 - \Gamma$ for each mode can be computed from the solution (to the wave equation of that mode) which has no outgoing flux at the horizon as the ratio of the incoming and outgoing flux at infinity.

A. Brane field equations

We derive the wave equations of the brane modes using the induced four dimensional metric of the $(4+n)$ -dimensional rotating black hole [31]. The wave equations can be understood as generalization of the Teukolsky equation [20, 22, 23, 24] to the higher dimensional Kerr geometry. The derivation is shown in Appendix.

We present the brane field equations for massless spin s field which are obtained from the metric (1) with the standard decomposition

$$\Phi_s = R_s(r)S(\vartheta)e^{-i\omega t + im\varphi}, \quad (14)$$

utilizing the Newman-Penrose formalism [33]

$$\begin{aligned} & \frac{1}{\sin \vartheta} \frac{d}{d\vartheta} \left(\sin \vartheta \frac{dS}{d\vartheta} \right) + [(s - a\omega \cos \vartheta)^2 \\ & - (s \cot \vartheta + m \csc \vartheta)^2 - s(s-1) + A]S = 0, \end{aligned} \quad (15)$$

$$\begin{aligned} & \Delta^{-s} \frac{d}{dr} \left(\Delta^{s+1} \frac{dR}{dr} \right) \\ & + \left[\frac{K^2}{\Delta} + s \left(4i\omega r - i \frac{\Delta_{,rr} K}{\Delta} + \Delta_{,rr} - 2 \right) \right. \\ & \left. + 2ma\omega - a^2\omega^2 - A \right] R = 0, \end{aligned} \quad (16)$$

where

$$K = (r^2 + a^2)\omega - ma. \quad (17)$$

The solution of eq. (15) is called spin-weighted spheroidal harmonics ${}_sS_{lm}$ (see e.g. ref. [23, 34]).

B. Hawking radiation and greybody factor

Since we have shown that massless brane field equations are separable into radial and angular parts, we may write down the power spectrum of the Hawking radiation [18] for each massless brane mode

$$\frac{dE_{s,l,m}}{dt d\omega d\varphi d\cos \vartheta} = \frac{\omega}{2\pi} \frac{{}_s\Gamma_{l,m}}{e^{\frac{\omega - m\Omega}{T}} \mp 1} |{}_sS_{lm}|^2, \quad (18)$$

where T and Ω are the Hawking temperature and the angular velocity at the horizon, respectively given by

$$T = \frac{(n+1) + (n-1)a_*^2}{4\pi(1+a_*^2)r_h}, \quad \Omega = \frac{a_*}{(1+a_*^2)r_h}, \quad (19)$$

and ${}_s\Gamma_{l,m}(r_h, a; \omega)$ is the greybody factor [18, 25] which is identical to the absorption probability of the incoming wave of the corresponding mode.

Approximately, the time dependence of M and J can be determined by

$$\begin{aligned} & -\frac{d}{dt} \begin{pmatrix} M \\ J \end{pmatrix} \\ & = \frac{1}{2\pi} \sum_{s,l,m} g_s \int_0^\infty d\omega \frac{{}_s\Gamma_{l,m}(r_h, a; \omega)}{e^{(\omega - m\Omega)/T} \mp 1} \begin{pmatrix} \omega \\ m \end{pmatrix}, \end{aligned} \quad (20)$$

where g_s is the number of ‘massless’ degrees of freedom at temperature T , namely the number of degrees of freedom whose masses are smaller than T , with spin s . Therefore, once we obtain the greybody factors, we completely determine the Hawking radiation and the subsequent evolution of the black hole up to the Planck phase, at which the semi-classical description by the Hawking radiation breaks down and a few quanta radiated is not predictable.

C. Greybody factors for Randall-Sundrum black hole

We find analytic expression of the greybody factors for $n = 1$ Randall-Sundrum black hole within the low frequency expansion. Here we outline our procedure: First we obtain the ‘near horizon’ and ‘far field’ solutions in the corresponding limits; Then we match these two solutions at the ‘overlapping region’ in which both limits are consistently satisfied; Finally we impose the ‘purely ingoing’ boundary condition at the near horizon side and then read the coefficients of ‘outgoing’ and ‘ingoing’ modes at the far field side. The ratio of these two coefficients can be translated into the absorption probability of the mode, which is nothing but the greybody factor itself.

First for convenience, we define dimensionless quantities

$$\xi = \frac{r - r_h}{r_h}, \quad \tilde{\omega} = r_h \omega, \quad \tilde{Q} = \frac{\omega - m\Omega}{2\pi T}. \quad (21)$$

Matching the NH and FF solutions in the overlapping region $1 + |\tilde{Q}| \ll \xi \ll 1/\tilde{\omega}$, we obtain

$$R_\infty = Y_{\text{in}} e^{-i\tilde{\omega}\xi} \left(\frac{\xi}{2} \right)^{-1} + Y_{\text{out}} e^{i\tilde{\omega}\xi} \left(\frac{\xi}{2} \right)^{-2s-1}, \quad (22)$$

where

$$\begin{aligned} & Y_{\text{in}} \\ & = \frac{\Gamma(2l+1)\Gamma(2l+2)}{\Gamma(l-s+1)\Gamma(l+s+1)} \frac{\Gamma(1-s-i\tilde{Q})}{\Gamma(l+1-i\tilde{Q})} (-4i\tilde{\omega})^{-l+s-1} \\ & \quad + \frac{\Gamma(-2l)\Gamma(-2l-1)}{\Gamma(-l-s)\Gamma(-l+s)} \frac{\Gamma(1-s-i\tilde{Q})}{\Gamma(-l-i\tilde{Q})} (-4i\tilde{\omega})^{l+s}, \\ & Y_{\text{out}} \\ & = \frac{\Gamma(2l+1)\Gamma(2l+2)}{[\Gamma(l-s+1)]^2} \frac{\Gamma(1-s-i\tilde{Q})}{\Gamma(l+1-i\tilde{Q})} (4i\tilde{\omega})^{-l-s-1} \end{aligned}$$

$$+ \frac{\Gamma(-2l)\Gamma(-2l-1)}{[\Gamma(-l-s)]^2} \frac{\Gamma(1-s-i\tilde{Q})}{\Gamma(-l-i\tilde{Q})} (4i\tilde{\omega})^{l-s}. \quad (23)$$

Finally, the greybody factor Γ (=the absorption probability) could be written as follows.

$$\Gamma = 1 - \left| \frac{Y_{\text{out}} Z_{\text{out}}}{Y_{\text{in}} Z_{\text{in}}} \right| = 1 - \left| \frac{1-C}{1+C} \right|^2, \quad (24)$$

where

$$C = \frac{(4i\tilde{\omega})^{2l+1}}{4} \left(\frac{(l+s)!(l-s)!}{(2l)!(2l+1)!} \right)^2 (-i\tilde{Q}-l)_{2l+1}, \quad (25)$$

with $(\alpha)_n = \prod_{n'=1}^n (\alpha + n' - 1)$ being the Pochhammer's symbol.

For concreteness, we write down the explicit expansion of eq. (24) up to $O(\tilde{\omega}^6)$ terms

$$\begin{aligned} {}_0\Gamma_{0,0} &= 4\tilde{\omega}^2 - 8\tilde{\omega}^4 + O(\tilde{\omega}^6), \\ {}_0\Gamma_{1,m} &= \frac{4\tilde{Q}\tilde{\omega}^3}{9} (1 + \tilde{Q}^2) + O(\tilde{\omega}^6), \\ {}_0\Gamma_{2,m} &= \frac{16\tilde{Q}\tilde{\omega}^5}{2025} \left(1 + \frac{5\tilde{Q}^2}{4} + \frac{\tilde{Q}^4}{4} \right) + O(\tilde{\omega}^{10}), \\ {}_{\frac{1}{2}}\Gamma_{\frac{1}{2},m} &= \tilde{\omega}^2 (1 + 4\tilde{Q}^2) - \frac{\tilde{\omega}^4}{2} (1 + 4\tilde{Q}^2)^2 + O(\tilde{\omega}^6), \\ {}_{\frac{1}{2}}\Gamma_{\frac{3}{2},m} &= \frac{\tilde{\omega}^4}{36} \left(1 + \frac{40\tilde{Q}^2}{9} + \frac{16\tilde{Q}^4}{9} \right) + O(\tilde{\omega}^8), \\ {}_1\Gamma_{1,m} &= \frac{16\tilde{Q}\tilde{\omega}^3}{9} (1 + \tilde{Q}^2) + O(\tilde{\omega}^6), \\ {}_1\Gamma_{2,m} &= \frac{4\tilde{Q}\tilde{\omega}^5}{225} \left(1 + \frac{5\tilde{Q}^2}{4} + \frac{\tilde{Q}^4}{4} \right) + O(\tilde{\omega}^{10}). \end{aligned} \quad (26)$$

Note that subleading terms in $\tilde{\omega}$ are already neglected when we obtain eq. (24) and the terms from these contributions are not written nor included in eqs. (24) and (26). We also note that the so-called s-wave dominance is maximally violated for spinor and vector fields since there are no $l=0$ modes for them.

IV. SUMMARY

We have studied theoretical aspects of the rotating black hole production and evaporation.

For production, we present an estimation of the geometrical cross section up to unknown mass and angular momentum loss in the balding phase. Our result of the maximum impact parameter b_{max} is in good agreement with the numerical result by Yoshino and Nambu when the number of extra dimensions is $n \geq 1$ (i.e. within 6.5% when $n=1$ and 1.5% when $n \geq 2$). Relying on this agreement, we obtain the (differential) cross section for a given mass and an angular momentum, which increases linearly with the angular momentum up to the cut-off value $J_{\text{max}} = b_{\text{max}} M/2$. This result shows that black holes tend to be produced with large angular momenta. We also studied the possibility of the black ring formation and find that it would possibly form when there are many extra dimensions. For evaporation, we first derive the master equation for brane fields for general spin and for an arbitrary number of extra dimensions. We show that the equations are separable into radial and angular parts as the four-dimensional Teukolsky equations. From these equations, we obtain the greybody factors for brane fields with general spin for the five-dimensions ($n=1$) Kerr black hole within the low-frequency expansion. We address several phenomenological implications of our results. The form factor of black hole production cross section is larger in the higher dimensional spacetime. The more precise determination of the radiation power is now available. We have shown that the black holes are produced with large angular momenta and that the resultant radiations will have strong angular dependence for $s=1/2$ and $s=1$ modes which points perpendicular to the beam axis while very small angular dependence is expected for scalar mode. More quantitative estimation will need the greybody factors for arbitrary frequency.

Acknowledgments

We would like to thanks to G. 't Hooft for nice comments on our work at DESY workshop and also to Robert Emparan, Yiota Kanti, V. Cardoso and M. Cavaglia for valuable discussions and comments. SC would like to thanks to Don Page, Valeri Frolov, Maurice van Putten and Eun-Joo Ahn for nice discussions at ICGA-VI. Y. Okada and Danny Marfatia gave constructive comments to SC at ICFP.

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$b_{\text{max}} = r_h(M, Mb_{\text{max}}/2)$ rather than Eq. (4), resulting in the form factor less than 1: $F \simeq 0.62\text{--}0.64$ in any dimensions $1 \leq n \leq 7$.