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New Measuring Method of Axisymmetric Three Dimensional Residual Stresses Using Inherent Strains as Parameters†

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Abstract

For Measurement of axisymmetric three dimensional residual stresses, Sachs' method is often used. The accuracy of this method is not high when even small errors are contained in observed strains.

Several years ago, the authors presented a new measuring principle of residual stresses in which inherent strains (source of residual stresses) are dealt as parameters and formulated a general measuring theory based on the finite element method.

In this paper, based on the general measuring theory, a new measuring theory of axisymmetric three dimensional residual stresses is developed and the practical procedure of the measurement is presented. The accuracy of measured residual stresses by this method is much improved. This method is applied to an actual measurement of residual stresses produced in a quenched shaft and its reliability and practicability are also demonstrated.

KEY WORDS: (Inherent Strain) (Residual Stress) (Axisymmetric Stress) (Measurement of Residual Stress)

1. Introduction

In recent years, large size industrial machines are constructed. This tendency is generally evident among other steel structures. Following this, it is very important to have accurate information on three dimensional residual stresses caused by thermal processing of steel plates such as quenching, welding, etc., in order to investigate the security of steel structures.

The authors introduced "the measuring principle of residual stresses in which inherent strains (the source of residual stresses) are dealt as parameters" as a general measuring principle of residual stresses and formulated a general measuring theory based on the finite element method. They also developed it by the statistical method so that the accuracy of measured three dimensional residual stresses is investigated[1),2)]. In this consequence, measurement of three dimensional residual stresses including internal ones became theoretically possible, and as actual application three dimensional residual stresses in welded joints were observed[2)−4)].

The drilling method or Sachs' method[5)] is practically used for three dimensional residual stresses in an axisymmetric body where residual stresses are axially uniform. Nevertheless, if errors are contained in the observed strains at measurement, they propagate and the reliability of the measured residual stresses is lowered.

Based on the above-mentioned general measuring theory, the authors propose a measuring theory of axisymmetric three dimensional residual stresses in which inherent strains are dealt as parameters and show the concrete measuring method and procedure. Both this new method and Sachs' method are used to measure axisymmetric three dimensional residual stresses of a same model for numerical experiments. The measured results indicate that the new method is very accurate without the inevitable defect of Sachs' method since they are based on the different measuring principles. The applicability and practicability of this new method are also demonstrated at actual measurement of three dimensional residual stresses caused by quenching in a N_i-C_r steel shaft (axisymmetric body).

2. Measurement of Three Dimensional Residual Stresses by Sachs' Method and Error

Inevitable problems involved in measurement of residual stresses by Sachs' method (drilling method) are demonstrated by conducting a numerical experiment on an analytical model.

2.1 Modeling

The specimen used in the numerical experiment and its coordinate system are shown in **Fig. 1.** Isotropic inherent strains ϵ^* are uniformly given in the axial direction of the specimen. They distribute in the cross section as shown in the following equations.

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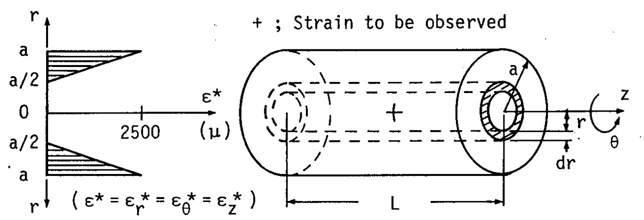


Fig. 1 Analysis model for numerical experiment

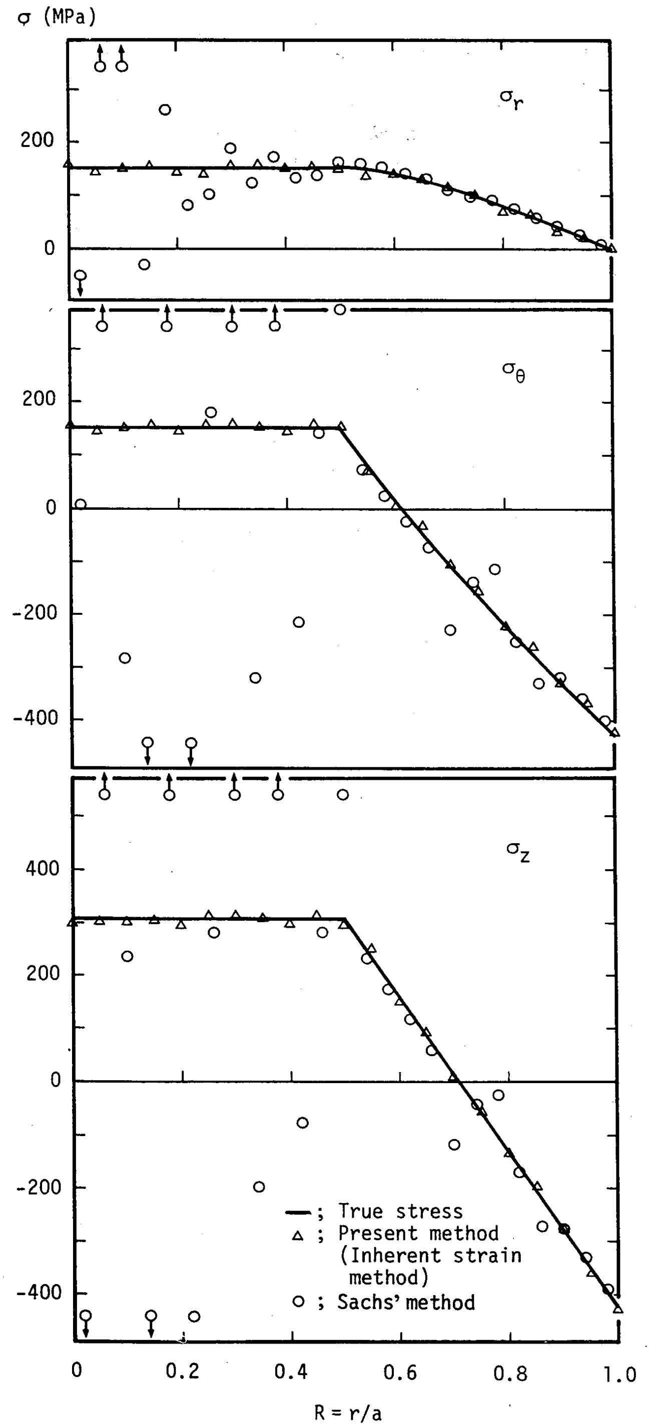


Fig. 2 Three dimensional residual stresses in the model

$$\epsilon^* = \epsilon_r^* = \epsilon_\theta^* = \epsilon_z^*$$
$$\epsilon^* = 0 \qquad [0 \leqslant R \leqslant 0.5] \qquad (1)$$
$$\epsilon^* = 5000\mu\,(R - 0.5) \qquad [0.5 < R \leqslant 1.0]$$

where, $R = r/a$, r and a: radial distance and outer radius of the model. The exact analytical solution gives stress components (radial stress σ_r, circumferential stress σ_θ and axial stress σ_z) produced by these inherent strains in the central cross section of the axisymmetric model. The distributions of these stress components along the radius are shown in **Fig. 2** by solid lines. These stresses are considered the true residual stresses and Sachs' method is applied to their measurement.

2.2 Standard application of Sachs' method

Strains relaxed by concentric drilling are observed on the outer surface of the specimen (Fig. 1). Using these strains, three dimensional residual stresses including internal ones are measured. Since $\epsilon_{\theta m}$ and ϵ_{zm} are theoretically equal, they are treated in Chapter 2 as $\epsilon_{\theta m} = \epsilon_{zm} = \epsilon_m$.

When the specimen is drilled in a concentric configuration with radius r, relaxed strains observed on the outer surface are ϵ_m. When the specimen is further drilled widening the radius for dr, the change of relaxed strains becomes $d\epsilon_m$. Using the observed value ϵ_m and $d\epsilon_m/dR$, residual stresses can be obtained as follows.

$$\sigma_r = E'\,[\frac{1-R^2}{2R^2}]\,\epsilon_m$$
$$\sigma_\theta = E'\,[\frac{1-R^2}{2R}\frac{d\epsilon_m}{dR} - \frac{1+R^2}{2R^2}\epsilon_m] \qquad (2)$$
$$\sigma_z = E'\,[\frac{1-R^2}{2R}\frac{d\epsilon_m}{dR} - \epsilon_m]$$
$$E' = E/(1-\nu)$$

E: elastic modulus, ν: Poisson's ratio

If no error is contained, the observed relaxed strain ϵ_m ought to coincide with the analytical solution of Eq.(3).

$$\epsilon_m = \frac{25000\mu}{24}\,\frac{R^2}{1-R^2} \qquad [0 \leqslant R \leqslant 0.5]$$
$$\epsilon_m = \frac{5000\mu}{24}\,\frac{R^2}{1-R^2}\,[-\frac{1}{R^2} - 16R + 17] \qquad (3)$$
$$[0.5 < R \leqslant 1.0]$$

Therefore, residual stresses estimated by substituting Eq.(3) into Eq.(2) agree with the true residual stresses (the solid lines in Fig. 2).

2.3 Problems involved in application of Sachs' method

Various problems involved in measurement of residual stresses by Sachs' method are as follows.

1) Stresses need by axially uniform. Therefore, as predicted from the Saint-Venant's principle, the axial length L of the axisymmetric body needs be at least twice as long as its diameter, $2a$, ($L \geqslant 4a$). If the diameter of the body is large, a large size drilling machine and highly skilled drilling technique are required because such a body should be drilled profoundly in a concentric configuration. Therefore, application of Sachs' method is unpractical.
2) Depending on the magnitude of residual stresses, the axisymmetric body may be newly plastified during drilling. Though such possibility cannot be avoided because the drilling procedure (drilling precedes from the center of axis toward the outer surface) is unchangeable.
3) Owing to various causes, errors may be contained in observed values of strains. Since relaxed strains are observed on the outer surface of the body, the errors contained in the observed values may propagate and have bad influence on the measuring result of residual stresses.

Especially, the propagation of errors contained in observed relaxed strains stated in 3) is the most significant problem which influences the reliability of the measured residual stresses by Sachs' method.

2.4 Propagation of errors contained in observed relaxed strains

Relaxed strains ϵ_m due to drilling are observed by electrical-resistance strain gages attached on the outer surface of the model. Errors which may be contained in the observed values are considered to be originally independent random variables and obey a normal distribution. In order to simplify the problem, it is assumed that the random errors of $\pm 25\mu$ are contained in observed values of relaxed strains as shown in **Fig. 3** by 「●」 (the solid line indicates the true relaxed strains). It is further assumed that 「●」 corresponds to the relaxed strains observed on the outer surface of the newly drilled model. Using these relaxed strains containing the error (「●」), three dimensional residual stresses are estimated by Sachs' method as shown by 「○」 in Fig. 2 (「△」 in the figure indicates three dimensional residual stresses estimated by "the inherent strain method" which will be discussed later). The residual stresses estimated by this method may differ from the true one estimated on the assumption that no error is contained.

The propagation of errors in Sach's method can be easily estimated by Eq.(2). Attention is paid to the radial stress σ_r for example.

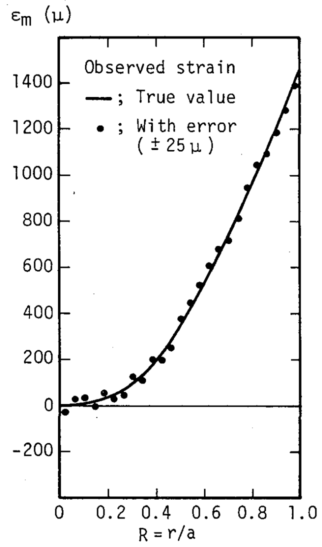


Fig. 3 Observed strains on the outer surface

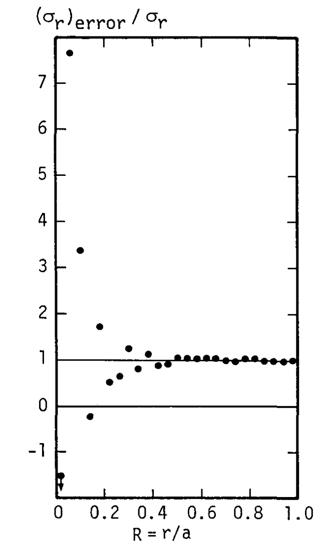


Fig. 4 Relative errors of residual stress σ_r

The residual stress σ_r indicated in Eq.(2) can be determined in proportion to the relaxed strain observed on the outer surface, ϵ_m, and $(1-R^2)/2R^2$. It is evident that if $R = r/a$ approaches zero, the accuracy of σ_r estimated by Eq.(2) is rapidly lowered. Therefore, the proportion between residual stress containing the error of $\pm 25\mu$, $(\sigma_r)_{error}$, and the true residual stress σ_r can be shown as in **Fig. 4** : $(\sigma_r)_{error}/\sigma_r$. The relative error is enlarged in the region where $R = r/a$ is small ($R < 0.5$).

As is mentioned above, application of Sachs' method to measurement of internal stresses may lead to an incorrect result because of the propagation of errors. This is an unavoidable drawback of Sachs' method.

3. Measuring Theory of Axisymmetric Three Dimensional Residual Stresses Using Inherent Strains as Parameters

The measuring principle of residual stresses in which inherent strains are dealt as parameters indicates that 1) inherent strains (invariable value) which is the source of residual stresses can be estimated and 2) using these inherent strains, residual stresses can be measured. Residual stresses of an axisymmetric body which are axially uniform except at the ends and in their vicinities are to be measured here. In this case, inherent strains or the source of residual stresses are considered to be axially uniform, too. In consideration of such characteristic of inherent strain distributions, the measuring theory of axisymmetric three dimensional residual stresses in which inherent strains are dealt as parameters is proposed. Based on this theory, a numerical experiment is conducted on the model presented in the previous chapter and the applicability and practicability of this method are demonstrated. It is also shown that the new method does not contain such defect as in Sachs' method.

3.1 Basic measuring theory using inherent strain as parameters

The basic measuring theory of three dimensional residual stresses using inherent strains as parameters is briefly explained (see Refs. 1) and 2) for details).

There are two kinds of inherent strain components; one is effective to produce residual stresses and the other is ineffective. Only the effective one is taken into consideration here. Accordingly, the relation of inherent strains $\{\epsilon^*\}$ or the source of residual stresses with elastic strains $\{\epsilon\}$ and stress $\{\sigma\}$ produced by $\{\epsilon^*\}$ at an arbitrary point of a three dimensional body can be expressed by the following elastic response equations.

$$\{\epsilon\} = [H^*]\{\epsilon^*\}$$
$$\{\sigma\} = [D]\{\epsilon\} = [D]\,[H^*]\{\epsilon^*\} \tag{4}$$

where, $[H^*]$: elastic response matrix.
$[D]$: elasticity matrix.

It is now shown that in the measuring theory of residual stresses using inherent strains as parameters, three dimensional residual stresses can be accurately measured using the elastic response relation equations if $\{\epsilon^*\}$ is accurately estimated.

On the other hand, the most remarkable feature of this theory is to take advantage of the fact that inherent strains (the source of stresses) are invariants. That is to say, when the measuring object, or R-specimen, is cut,

(1) residual stress distributions change, but
(2) inherent strain distributions stay the same as in the original R-specimen.

Utilizing these features, inherent strains $\{\epsilon^*\}$ are estimated as follows.

Elastic strains produced in a three dimensional body are observed as much as possible such as by cutting and slicing. Observed strains are denoted by $\{{}_m\epsilon\}$. Since $\{{}_m\epsilon\}$ may contain various observation errors, the following measuring equation is derived from the above-mentioned elastic response equations.

$$\{{}_m\epsilon\} - [H^*]\{\hat{\epsilon}^*\} = \{V\} \tag{5}$$

where, $\{\hat{\epsilon}^*\}$: the most probable value of inherent strain
$\{V\}$: residual

The most probable value of inherent strains $\{\hat{\epsilon}^*\}$ can be obtained under the condition that the sum of square of residual becomes the minimum, that is,

$$\{\hat{\epsilon}^*\} = ([H^*]^T[H^*])^{-1}[H^*]^T\{{}_m\epsilon\} \tag{6}$$

Substituting this $\{\hat{\epsilon}^*\}$ into $\{\epsilon^*\}$ in Eq.(4), elastic strains and residual stresses produced at an arbitrary point of a three dimensional body can be measured. That is,

$$\{\hat{\epsilon}\} = [H^*]\{\hat{\epsilon}^*\}$$
$$\{\hat{\sigma}\} = [D]\{\hat{\epsilon}\} = [D]\,[H^*]\{\hat{\epsilon}^*\} \tag{7}$$

3.2 Measuring theory of axisymmetric residual stresses using inherent strains as parameters

Based on the basic theory mentioned in the previous section, the measuring theory of axisymmetric three dimensional residual stresses which is on the assumption that inherent strain distributions are axially uniform is described here.

3.2.1 Assumptions used in the measuring theory

If inherent strain distributions are axially uniform, the inherent strain component γ^*_{zr} which produces unsymmetric stresses except at the end portions of a body can be neglected. Therefore, residual stresses can be considered to be produced only by the inherent strain components ϵ^*_r, ϵ^*_θ and ϵ^*_z which are the functions of r alone. For concrete measurement, the following assumptions are made.

(1) Cutting is accompanied by only elastic change of strains and does not produce any new inherent strain (special attention should be paid to control the raise of temperature caused by cutting).
(2) Stresses remain in thinly sliced plates are in the plane stress state.

3.2.2 Separation of three dimensional inherent strain components

Because inherent strains are invariable, it is considered to separate three dimensional inherent strain components in order to simplify measurement of residual stresses.

A cross section perpendicular to the z-axis is cut out from R-specimen (**Fig. 5 (a)**). This is called T-specimen (Fig. 5 (b)). Similarly, a rectangular plate containing and parallel to the z-axis is cut out. This is called L-specimen (Fig. 5 (c)). Cutting procedure is exampled in Fig. 5. No special device but an electric sawing machine is used. Each specimen is cut out from an arbitrary position in an arbitrary cutting order. If it is predicted that the portions of R-specimen from which T- and L-specimens are to be taken out are restricted so severely as to be newly plastified by cutting, each specimen can be cut out by relaxing the surrounding restraints. This implies the flexibility of the method in the cutting order and no disturbance by plastification as described problem 2) involved in Sachs' method. It is natural that the same inherent strain

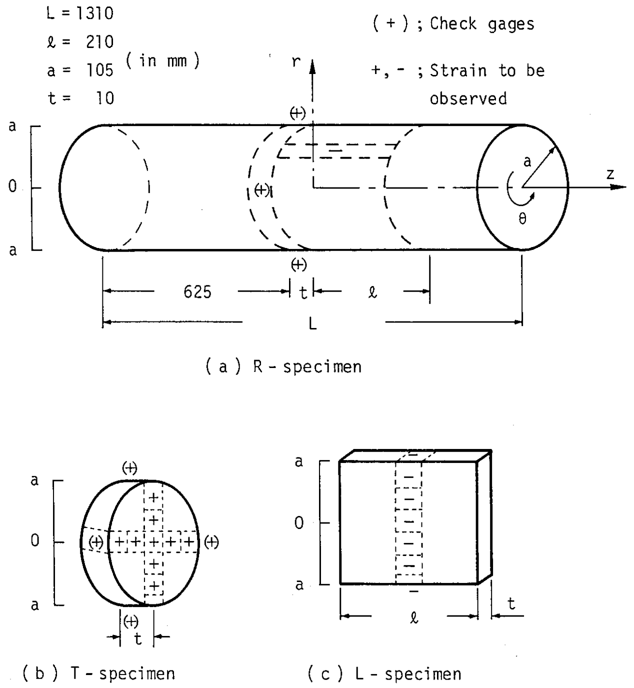


Fig. 5 Analysis model and location of T- and L-specimens

distributions in R-specimen exist in the cut out T- and L-specimens.

On the other hand, inherent strain components perpendicular to the plate surfaces do not contribute to stresses remain in thinly sliced T- and L-specimens. That is to say, stresses remain in T-specimen are produced by the cross sectional inherent strains (ϵ_r^*, ϵ_θ^*) and stresses in L-specimen are only by the axial inherent strain (ϵ_z^*). Three dimensional inherent strain components (ϵ_r^*, ϵ_θ^*, ϵ_z^*) are thus divided into cross sectional ones (ϵ_r^*, ϵ_θ^*) and axial one (ϵ_z^*).

For the above reason, measurement of three dimensional residual stresses is decomposed into that of two dimensional problem. This is an important simplification of the measuring theory of three dimensional residual stresses taking into account of the special feature of inherent strain distribution which is uniform in one direction. Being treated as a two dimensional problem, stresses remain in each specimen can be directly observed. As a result, errors contained in observed strains do not propagate, since residual stresses are measured by giving the most probable values of the inherent strains estimated by Eq.(6) to the non-stressed R-specimen. This will be detailed in 3.3.

Consequently, three dimensional residual stresses $\{ \sigma \}$ of R-specimen can be obtained as the sum of stresses $\{ \sigma^A \}$ produced by only cross sectional inherent strains and stresses $\{ \sigma^B \}$ by only axial inherent strains as,

$$\{ \sigma \} = \{ \sigma^A \} + \{ \sigma^B \} \tag{8}$$

The concrete measuring method and procedure of three dimensional residual stress components $\{ \sigma^A \}$ and $\{ \sigma^B \}$ are described below.

3.2.3 Three dimensional residual stresses by cross sectional inherent strains, $\{ \sigma^A \}$

If inherent strains are axially uniform, three dimensional residual stresses $\{ \sigma^A \}$ produced by cross sectional inherent strains can be directly measured using two dimensional stresses (plane stresses) remaining in T-specimen.

Let us assume that only cross sectional inherent stains are given to stress-free R-specimen uniformly in the z-direction. Even after R-specimen is deformed by these inherent strains, its cross section at the position inside from the ends by the diameter or more keeps plane as before (hereinafter called plane deformation). On the other hand, stresses produced by giving these inherent strains to the plane strain state are in a state of equilibrium in the cross section since stresses in the plane is self-equilibrating. Therefore in the cross section free from the effect of the free ends, the above-mentioned plane deformation coincides with the plane strain state[2)].

Residual stresses remaining in the thinly sliced T-specimen are in the plane stress state and they, $\{ \sigma^{AO} \} = \{ \sigma_r^{AO}, \sigma_\theta^{AO} \}$, can be directly observed by the conventional sectioning method using strain gages. Substituting directly observed $\{ \sigma^{AO} \}$ into the following equations which express the relation of stresses between plane stress and plane strain, three dimensional residual stresses $\{ \sigma^A \} = \{ \sigma_r^A, \sigma_\theta^A, \sigma_z^A \}$ can be obtained.

$$\begin{aligned} \sigma_r^A &= \sigma_r^{AO} / (1 - \nu^2) \\ \sigma_\theta^A &= \sigma_\theta^{AO} / (1 - \nu^2) \\ \sigma_z^A &= \nu (\sigma_r^{AO} + \sigma_\theta^{AO}) / (1 - \nu^2) \end{aligned} \tag{9}$$

3.2.4 Three dimensional residual stresses by axial inherent strains, $\{ \sigma^B \}$

When three dimensional residual stresses produced by axially uniform inherent strains, $\{ \sigma^B \}$, are measured, different measuring methods are applied, depending on the relative proportion between the length l of L-specimen and the diameter $2a$ of the original R-specimen, that is $l/2a$.

(1) When the length of L-specimen, l, is smaller than $4a$: $l < 4a$

As stated in 3.1, measurement of the residual stresses should be based on the measuring theory in which inherent strains are dealt as parameters. The procedure is as follows.

1) Elastic strains remaining in L-specimen are observed by strain gages, etc..
2) Using these observed values, the most probable value of axial inherent strains, $\hat{\epsilon}_z^*$, is estimated by Eq.(6).
3) Giving only $\hat{\epsilon}_z^*$ to stress-free R-specimen, three dimensional elastic analysis is conducted to obtain $\{\sigma^B\} = \{\sigma_r^B, \sigma_\theta^B, \sigma_z^B\}$.

If axial inherent strains are estimated in this procedure, there is no special limitation but for the case the length of L-specimen, l, is extremely short. In general, a specific method such as the finite element method should be applied to the three dimensional elastic analysis. However, if the residual stresses $\{\sigma^B\}$ of R-specimen are in the portion inside from the ends by more than the diameter, that is, free from the effects of the free ends, the residual stress components can be analytically obtained by the following equations (analytical solutions) without applying the finite element method.

$$\begin{aligned}\sigma_r^B &= \frac{\nu E}{1-\nu^2}\left[\frac{1}{a^2}\int_0^a \hat{\epsilon}_z^* r dr - \frac{1}{r^2}\int_0^r \hat{\epsilon}_z^* r dr\right]\\ \sigma_\theta^B &= \frac{\nu E}{1-\nu^2}\left[-\hat{\epsilon}_z^* + \frac{1}{a^2}\int_0^a \hat{\epsilon}_z^* r dr + \frac{1}{r^2}\int_0^r \hat{\epsilon}_z^* r dr\right]\\ \sigma_z^B &= \frac{E}{1-\nu^2}\left[-\hat{\epsilon}_z^* + \frac{2}{a^2}\int_0^a \hat{\epsilon}_z^* r dr\right]\\ &[\nu\sigma_z^B = \sigma_r^B + \sigma_\theta^B]\end{aligned} \qquad (10)$$

(2) When l is larger than $4a$: $l \geqslant 4a$

In this case, it is unnecessary to estimate inherent strains because three dimensional residual stress $\{\sigma^B\}$ can be obtained directly from observed strains.

If R-specimen is long enough, its portion inside from the ends by more than its diameter is in the state of plane deformation. As L-specimen can be cut out from it in order to satisfy the size ratio of $L \geqslant 4a$, the central portion of this L-specimen keeps the state of plane deformation. In this case, when L-specimen is cut out from R-specimen, relaxed strain ${}_m\epsilon_{zl}$ is observed and elastic strain ${}_m\epsilon_{zn}$ remains in the sliced L-specimen (these can be directly observed such as by strain gages)[6]. Then, elastic strain ϵ_z produced in L-specimen before cutting is given by,

$$\epsilon_z = {}_m\epsilon_{zl} + {}_m\epsilon_{zn} \qquad (11)$$

where,

${}_m\epsilon_{zl}$: linear component of elastic strains which can be observed by strain gages attached on the top and bottom surfaces of L-specimen before cutting out from R-specimen,

${}_m\epsilon_{zn}$: elastic strains which remain in L-specimen and can be obtained by slicing L-specimen into thin layers.

On the other hand, the whole axial strains e_z of R-specimen produced only by axial inherent strains can be given as the sum of inherent strains ϵ_z^* and elastic strains ϵ_z.

$$e_z = \epsilon_z^* + \epsilon_z \qquad (12)$$

or

$$\epsilon_z^* = e_z - \epsilon_z \qquad (12)'$$

The above equation is substituted into Eq.(10) and the following equilibrium equation;

$$\int_0^a \sigma_z^B 2\pi r dr = 0$$

leads to

$$\int_0^a \epsilon_z r dr = 0$$

As a result, three dimensional residual stresses $\{\sigma^B\}$ produced by axial inherent strains can be analytically obtained as follows.

$$\begin{aligned}\sigma_r^B &= \frac{\nu E}{1-\nu^2}\left[\frac{1}{r^2}\int_0^r \epsilon_z r dr\right]\\ \sigma_\theta^B &= \frac{\nu E}{1-\nu^2}\left[\epsilon_z - \frac{1}{r^2}\int_0^r \epsilon_z r dr\right]\\ \sigma_z^B &= \frac{E}{1-\nu^2}\epsilon_z\end{aligned} \qquad (13)$$

When the length l of L-specimen cut out from R-specimen in which inherent strains are axially uniform is shorter than $4a$ ($l < 4a$), three dimensional residual stresses $\{\sigma^B\}$ can be obtained by substituting estimated $\hat{\epsilon}_z^*$ into Eq.(10). If l is long enough to satisfy the condition, $l \geqslant 4a$, axial inherent strains need not be estimated because $\{\sigma^B\}$ can be analytically obtained by Eq.(13). In the above two cases, if inherent strains are axially uniform, three dimensional residual stresses can be measured without limitation of the length of R-specimen, L. This means that large the diameter of the axisymmetric specimen may be, the three dimensional residual stresses can be measured without any technical difficulty. That is, this method is not concerned with the problem 1) of Sachs' method.

3.3 Propagation of errors contained in observed strains

The same numerical experimental model as used for Sachs' method is now employed for measurement of residual stresses by the present measuring method. In this experiment, the propagation of errors contained in observed strains is concretely characterized.

Inherent strains are invariable and do not vary if no plastic deformation is added by cutting. Admitting this characteristic of inherent strains, a numerical experiment is to be conducted. Accordingly, elastic strains produced by giving isotropic inherent strains ϵ^* ($= \epsilon_r^* = \epsilon_\theta^* = \epsilon_z^*$) shown in Eq.(1) respectively to *T*-specimen and *L*-specimen are considered to be the true residual strains. They are shown in **Fig. 6** by solid lines. Similarly to the case of Sachs' method, random errors of $\pm 25\mu$ are added to these strains as shown in the same figure by「○」. Strains containing these errors are considered newly observed residual strains such as by strain gages.

Using these observed strains containing errors, $\{ \sigma^A \}$ is obtained from Eq.(9) in the above-mentioned procedure. On the other hand, $\{ \sigma^B \}$ is obtained by estimating the most probable value of axial inherent strain $\hat{\epsilon}_z^*$ and substituting it into $\hat{\epsilon}_z^*$ of Eq.(10). Finally, three dimensional residual stresses $\{ \sigma \} = \{ \sigma_r, \sigma_\theta, \sigma_z \}$ are obtained from Eq.(8) as the sum of $\{ \sigma^A \}$ and $\{ \sigma^B \}$. The measured results are shown in Fig. 2 by「△」.

In this new measuring method, errors contained in observed strains do not propagate so that three dimensional residual stresses $\{ \sigma \}$ can be fairly accurately measured. It is now understood that this method is far more reliable than Sachs' method. Because inherent strains are invariable as far as plastic deformation does not newly occur, inherent strains or the source of residual stresses can be estimated using strains directly observed on the sliced specimens, and residual stresses can be measured from them. This is the most advantageous characteristic of this measuring method and the theoretical background for high accuracy of the measured results. Thus, the problem 3) of Sachs' method is not involved in the new method.

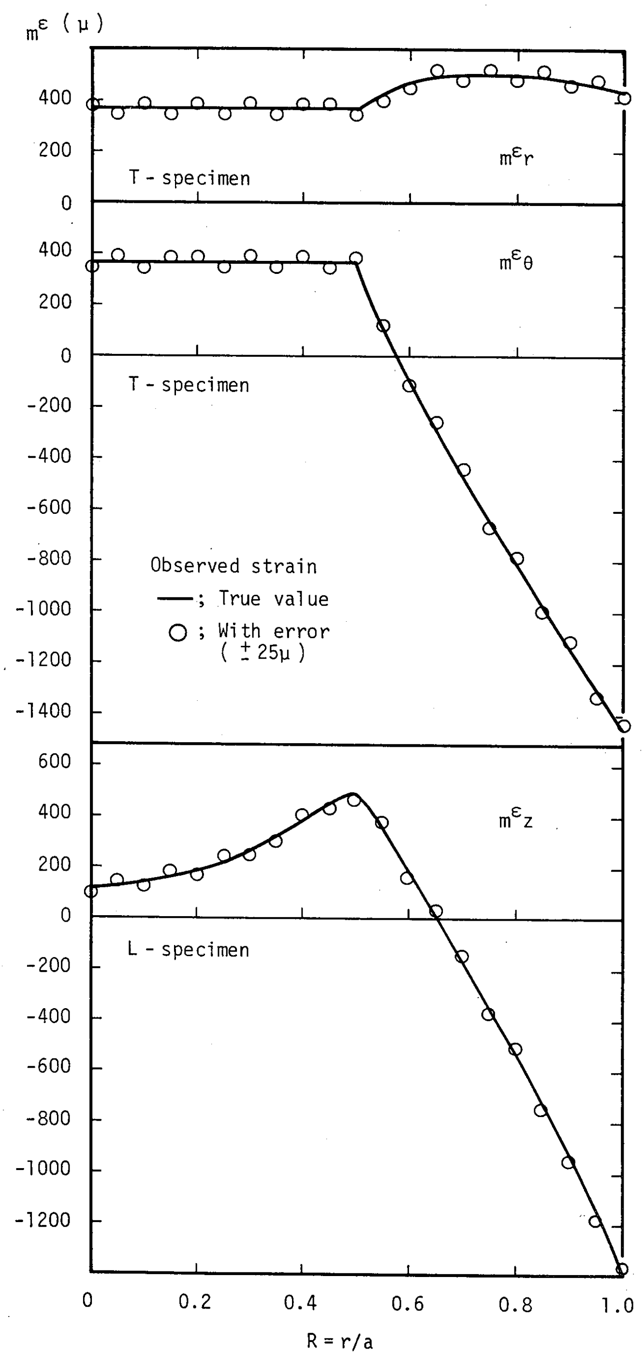


Fig. 6 Residual strains (remaining elastic strain) distributions in *T*- and *L*-specimen of analysis model

4. Actual Measurement of Three Dimensional Residual Stress Distributions in Axisymmetric Quenched Shaft

Based on the newly developed measuring theory of axisymmetric three dimensional residual stresses in which inherent strains are dealt as parameters, residual stresses due to quenching of N_i-C_r steel shaft (axisymmetric body) are measured.

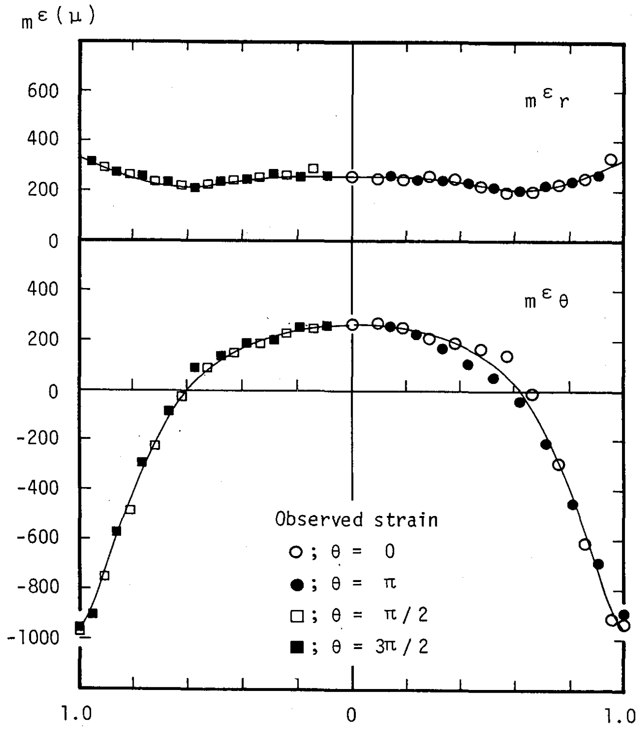
4.1 Specimen and observing points of strains

R-specimen used in this experiment is made of N_i-C_r steel (SNC-21). Primarily, oil quenching is applied to the specimen. Secondly, after heating it at *870°C* for three hours, water quenching is applied. The mechanical properties and chemical compositions of *R*-specimen are shown in **Table 1**. Material constants used in the following calculations are assumed: modulus of elasticity *E = 206 GPa*, Poisson's ratio $\nu = 0.3$. The dimensions and the coordinate system of *R*-specimen are shown in Fig. 5. The portions where *T*- and *L*-specimens are to be cut out and the points where strain gages (check gages) (indicated by「(+)」) are to

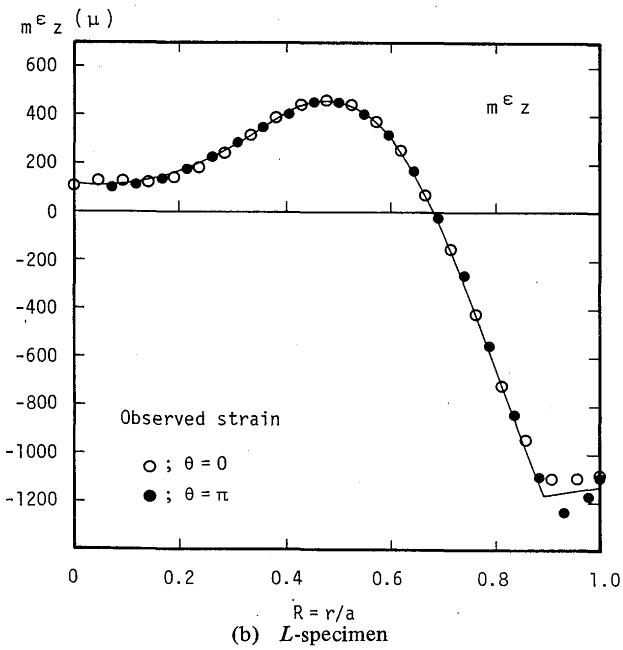
Table 1 Chemical compositions and mechanical properties of quenched shaft

Chemical composition (%)										Mechanical properties	
C	S_i	M_n	P	S	C_u	N_i	C_r	M_o	V	σ_Y (MPa)	σ_U (MPa)
0.17	0.23	0.56	0.007	0.005	0.03	2.20	0.31	0.01	0.08	< 588	< 784

σ_Y ; Yield strength , σ_U ; Tensile strength be attached so as to directly observe three dimensional residual stresses after cutting are also shown in Fig. 5. Three dimensional residual stresses of R-specimen are directly observed by the check gages at ⌈(+)⌋ and used for nothing but for investigation of the accuracy of residual stresses measured by this measuring method. The plate thickness is *10 mm* in both T- and L-specimens and the plate length $l = 210\ mm$ in L-specimen.



(a) *T*-specimen



(b) *L*-specimen

Fig. 7 Observed strain distributions in *T*- and *L*-specimens of quenched shaft

Observing points of strains remaining in each specimen are shown in Fig. 5. Strain gages with gage length of *2 mm* are used. After these gages are attached to the corresponding points on the front and back surfaces, specimens are cut into pieces. The average value of the strains observed on both surfaces is assumed to be the residual strains in each specimen. The results are shown in **Figs.** 7 (a) and (b) respectively for T- and L-specimens. The figures depict the strain distributions as axisymmetric.

4.2 Measurement of three dimensional stresses $\{\sigma^A\}$ by cross sectional inherent strains

Stresses $\{\sigma^{AO}\}$ in the plane stress state are obtained by using the observed strains in T-specimen (Fig. 7 (a)). Substituting this $\{\sigma^{AO}\}$ into Eq.(9), $\{\sigma^A\}$ is obtained. The results are shown in **Fig. 9**.

4.3 Measurement of three dimensional stresses $\{\sigma^B\}$ by axial inherent strains

The most probable value $\hat{\epsilon}_z^*$ and standard deviation $\hat{s}^*$

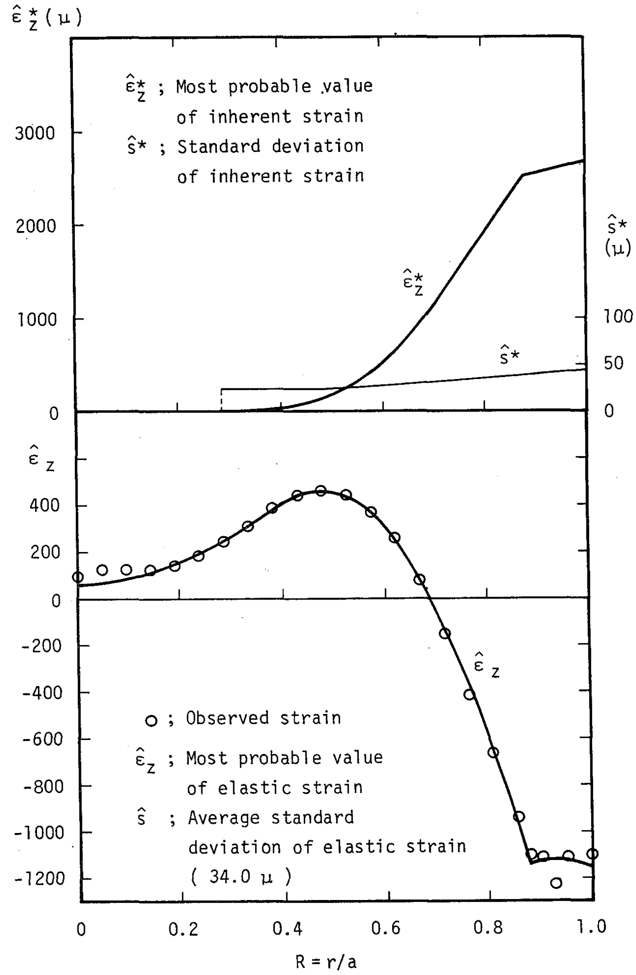


Fig. 8 Distributions of axial inherent strain and elastic strain

of inherent strains estimated by using the observed strains in L-specimen (Fig. 7 (b)) are shown in **Fig. 8**. Giving $\hat{\epsilon}_z^*$ to L-specimen, most probable residual strains $\hat{\epsilon}_z$ which correspond to residual stresses are reproduced. The deviation, $\hat{s}$, of these residual strains from the observed values is 34μ on the average (Fig. 8).

Introducing the above $\hat{\epsilon}_z^*$ to Eq.(10), three dimensional stresses $\{ \sigma^B \}$ are obtained. The results are shown in Fig. 9.

4.4 Distributions of three dimensional residual stresses due to quenching

Three dimensional residual stresses $\{ \sigma \}=\{ \sigma_r, \sigma_\theta, \sigma_z \}$ are obtained as the sum of residual stress components $\{ \sigma^A \}$ and $\{ \sigma^B \}$ respectively obtained in 4.2 and 4.3. The results are shown in Fig. 9 in which ⌈○⌋ indicates three dimensional residual stresses directly observed on the surface of R-specimen. The measured values and directly observed values by the check gages show a good coincidence. These observed values (⌈○⌋) are used only to investigate the accuracy of the whole measured values. They are not used as observed values in the measurement of residual stresses by this method. That is to say, the measured value $\{ \sigma \}$ and the directly observed value on the surface (⌈○⌋) are quite independent. In this measuring method in which inherent strains are dealt as parameters, residual stresses inside of the specimen and those on the surface are treated the same. Therefore, the three dimensional residual stress distributions measured by this method are considered to be reliable enough in the entire body.

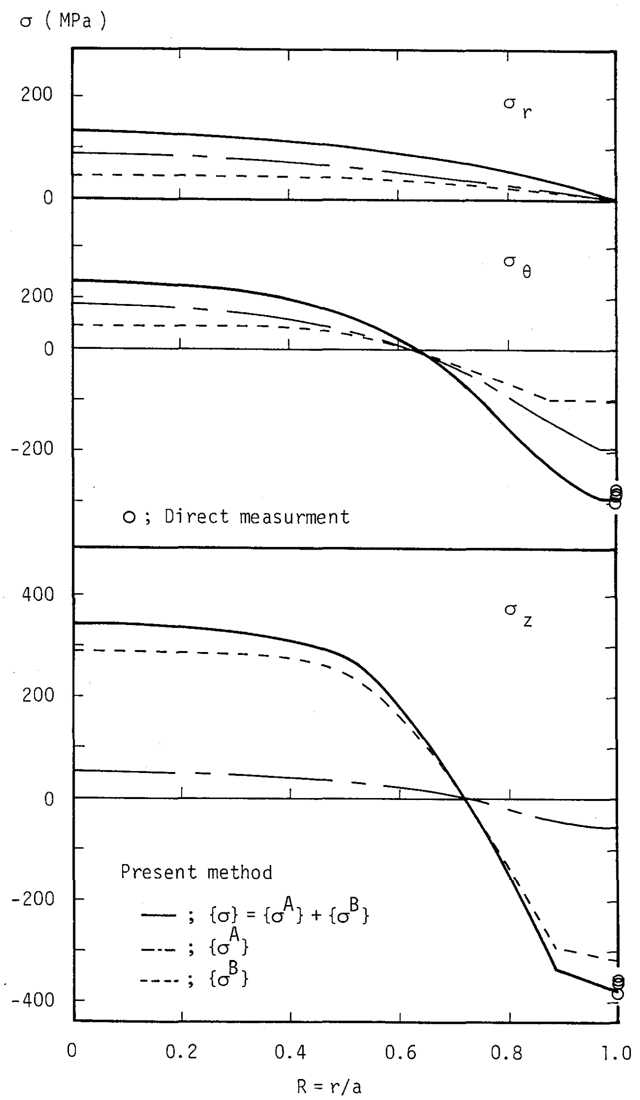


Fig. 9 Distributions of estimated three dimensional residual stresses in quenched shaft

5. Conclusion

In this study, the new measuring method of axisymmetric three dimensional residual stresses in which inherent strains are dealt as parameters is proposed and actual three dimensional residual stresses of a quenched axisymmetric shaft are measured by this method. The main results obtained are as follows.

(1) It is indicated that if Sachs' method which is very popular as a measuring method of residual stresses in axisymmetric bodies is applied, the measured residual stresses may be extremely inaccurate in some cases. This is because in Sachs' method, internal stresses are estimated by using relaxed strains observed on the outer surface of the body and if an error is contained in the observed value, the error may propagate. This is the inevitable problem involved in Sachs' method and should be taken into account on applying it.

(2) As a new measuring method of residual stresses which solves the problems involved in Sachs' method, a measuring theory of axisymmetric three dimensional residual stresses is proposed based on the measuring principle of residual stresses in which inherent strains are dealt as parameters. The new theory is developed without any approximation except the basic assumptions based on the theory of elasticity. It is shown that three dimensional residual stresses can be measured by this method in which two thin plates are cut out from R-specimen and two dimensional residual stresses and strains remaining in these sliced plates are directly observed. It is also shown that if the length of R-specimen is long enough, three dimensional residual stresses can be simply measured without estimating inherent strains. The applicability of this measuring method is demonstrated by using a concrete model for numerical experiments.

(3) Applying the measuring theory of three dimensional residual stresses in which inherent strains are dealt as

parameter, three dimensional residual stresses produced in a quenched axisymmetric shaft are actually measured. As a result, the practicability of the new method and the reliability of the measured three dimensional residual stresses are confirmed.

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