Trans. JWRI, Vol. 23 (1994), No. 1

Restraint Stress and Strain in Circular Patch Welds†

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Abstract

Patch welding is mainly performed for repair, reinforcement and so on.

In this study, the characteristics of the restraint stress and strain produced in circular patch welding and their production mechanism are elucidated from the results of the thermal elastic-plastic analysis. Then, the analytical calculation procedure of the restraint stress and strain produced by circular patch welding is shown based on the production mechanism of the restraint stress and strain. As the result, the formula to provide the restraint stress and strain is proposed.

Moreover, the applicability of the restraint intensity as a mechanical measure for cold crack initiation in circular patch welding is investigated. Severity of the mechanical restraint condition in the elastic range can be evaluated by using the restraint intensity in circular patch welding as a parameter. Meanwhile, severity of the mechanical restraint condition in the plastic range can not be directly evaluated by using the restraint intensity. However, the relative severity of the mechanical restraint may be evaluated by using the restraint intensity only in the limited range.

KEY WORDS: (Repair) (Patch Welding) (Circular Patch Welding) (Residual Stress) (Plastic Strain) (Restraint Stress) (Restraint Strain) (Restraint Intensity) (Inherent Shrinkage) (Analytical Calculation Method)

1. Introduction

Patch welding is mainly performed for repair, reinforcement and so on. As the mechanical restraint condition of patch welding is severe, a weld cold crack sometimes occurs. So it is important to know the characteristics of the restraint stress and strain produced by patch welding.

One way to elucidate the mechanical behavior during welding accurately is a numerical simulation by the thermal elastic-plastic analysis based on FEM (Finite Element Method). However, the thermal elastic-plastic analysis should be carried out whenever the heat input and the boundary condition change. If the purpose is to know the various basic mechanical behaviors during welding, the thermal elastic-plastic analysis is not an economical option because it needs much labor and cost.

In this paper, circular patch welding is considered. First, the thermal elastic-plastic analysis is carried out. Then the characteristics of the restraint stress and strain produced in circular patch welding and their production mechanism are elucidated from the analysis. Next, the procedure to obtain the restraint stress and strain by analytical means is shown to elucidate the various basic mechanical phenomena in circular patch welding systematically. Moreover, the applicability of the restraint intensity as a mechanical measure for cold crack initiation is investigated.

2. Finite Element Modeling

2.1 Model

Figure 1 shows the finite element model. Thickness h and the ratio of the diameter of the inner plate, 2a, to that of the outer plate, 2b: 2a/2b=1/5 are fixed. The diameter of the inner plate, 2a, varies as 2a=40, 80, 120, 160(mm).

The first pass welding is performed in a counterclockwise direction from point P_S. The heat input is 1.7(kJ/mm), travel speed is 2.5(mm/s) and thermal efficient is 0.75.

First, unstationary thermal conduction analysis (moving heat source) is carried out considering the temperature dependency of physical constants[1]. Next, using the temperature history obtained by the unstationary thermal conduction analysis, the thermal elastic-plastic stress analysis (considering the temperature

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Transactions of JWRI is published by Welding Research Institute, Osaka University, Ibaraki, Osaka 567, Japan

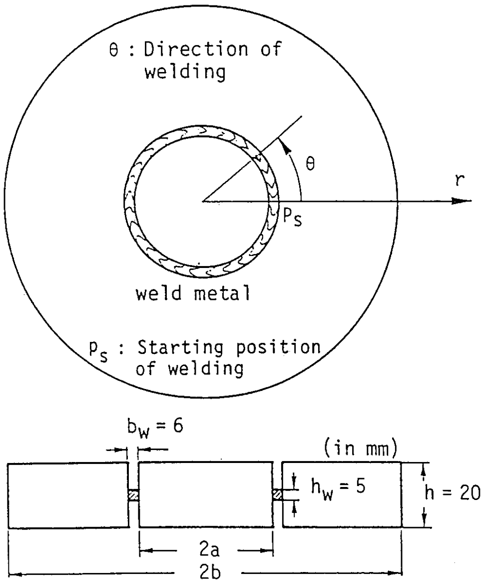

Fig.1 Circular patch weld joint

dependency of the mechanical properties[1]) is performed as the plane stress problem. The analysis in Section 2 is done by FEM.

2.2 Temperature distribution

As one example of the result of the unstationary thermal conduction analysis (moving heat source), **Fig.2** shows the transient of the isothermal contour for a diameter 2a=80(mm) inner plate.

As the neighbor of the heat source is always at a high temperature during movement (t<100.5(s)) of the heat source (P_f is the present position of welding), the isothermal contour is a biased distribution. However, when welding finishes and as time goes on (t=167.2(s)), a bias of temperature due to the moving heat source is vanished. So the isothermal contour shows a tendency toward axisymmetric distribution.

On the other hand, the temperature of the inner plate is higher than that of the outer one. That is, heat is conducted in the inner plate. This is the characteristic

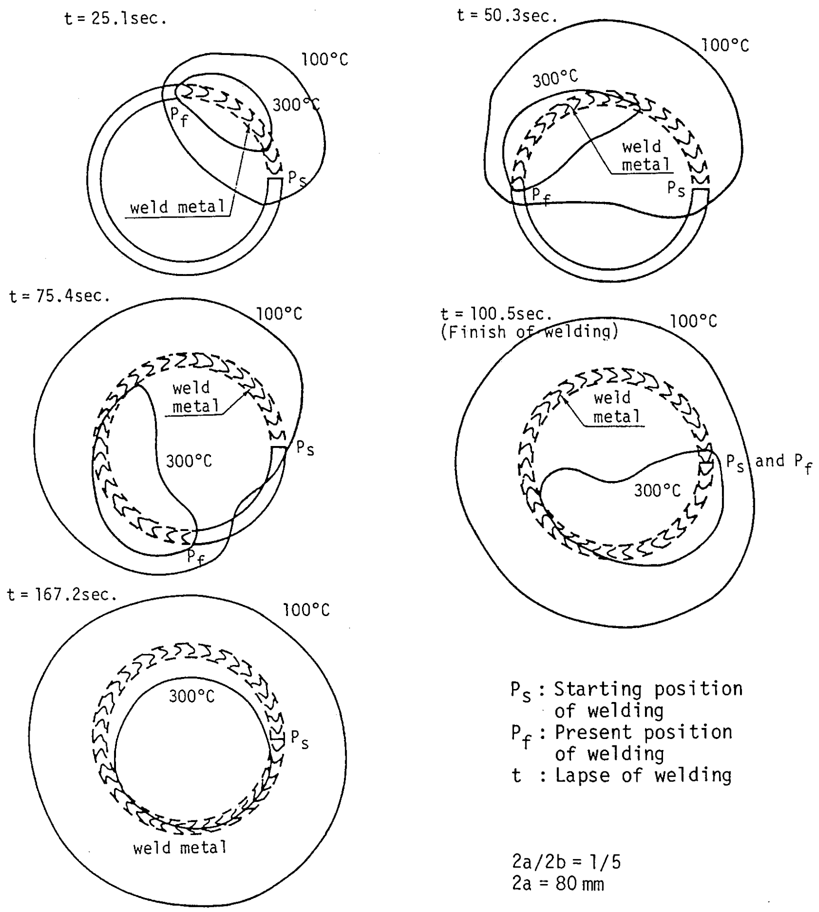

Fig.2 Transient isothermal contours

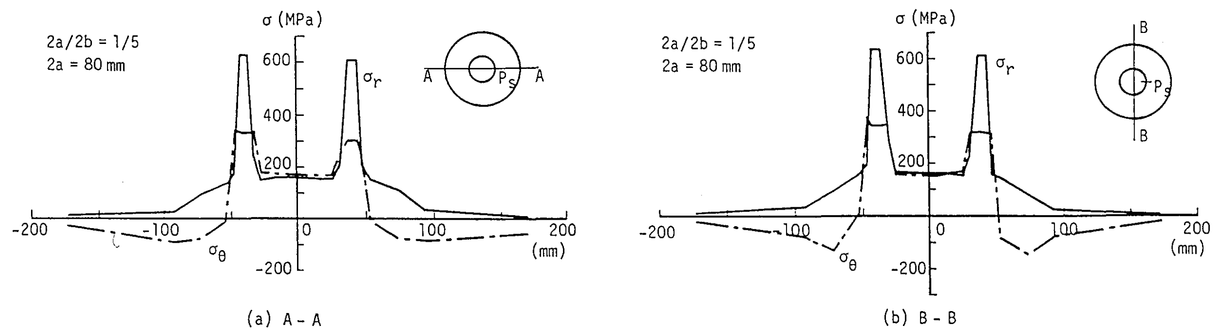

Fig.3 Residual stress distributions

phenomenon of patch welding. So far as a diameter of the inner plate does not become extremely large, the temperature has the same tendency.

2.3 Characteristics of the restraint stress and strain

According to the results of the thermal elastic-plastic stress analysis, the restraint stress (residual stress) has the same tendency regardless of the size of the inner plate. **Figure 3** shows the restraint stress distribution obtained for the model, whose diameter of the inner plate, 2a, is 80(mm).

The distributions of σ_r (the residual stress component in a radial direction) and σ_θ (the residual stress component in a circumferential direction) show almost axisymmetric distributions. In the first pass welding, σ_r produced in weld metal is a larger tensile compared to σ_θ. So concerning the plastic strain, only the component in a radial direction, εp_r, is noted here.

Figure 4 shows the transient and residual plastic strain along the weld line, εp_r, produced in the weld metal of the model, whose diameter 2a is 80(mm). εp_r shows that the constant value produced in the cooling process from the axisymmetric distribution to room temperature is uniformly added to the local value, which

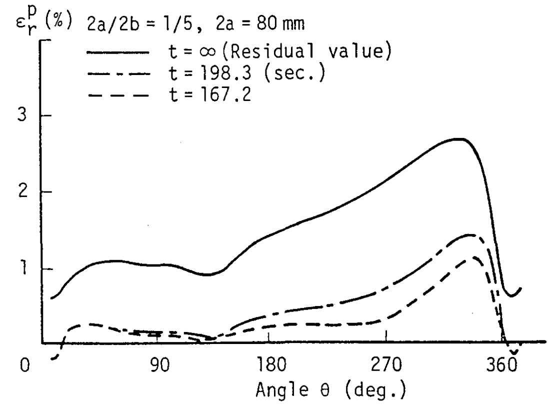

Fig.4 Transient plastic strain distribution in the weld metal

is produced till the isothermal contour becomes an axisymmetric distribution (Fig.2). εp_r is mainly produced in the cooling process from axisymmetric distribution to room temperature.

2.4 Production mechanism of the restraint stress and strain

Figure 5 shows the dislocation (displacement) in a radial direction, S, at the groove produced at the position, θ=90, 270°, and the plastic strain component, εp_r, obtained for the model whose diameter of the inner plate 2a is 80(mm).

The position where is θ=90° is noted (Fig.5(a)). Before the heat source passes ($t<t_p$) at the position θ=90°, the groove dislocates to the closing direction (in the positive direction) due to the expansion of the already welded part. When the heat source passes ($t=t_p$), the groove expands ($t_p<t<t_r$), because of the welding heat, until the weld metal cools to temperature T_m ($t=t_r$), at which it recovers rigidity and the groove dislocates to the further closing direction. Next, noting the position where is θ=270° (Fig.5(b)), the difference in behavior at the position θ=90° is only that the groove opens before the heat source passes ($t<<t_p$). After it passes, the groove ($t>t_p$) shows the same behavior at the position θ=90°. The reason why the groove at the position θ=270° opens before the heat source passes ($t<<t_p$) is because the groove at the position where is θ=90° closes.

From the above, noting the mechanical behavior of the groove, two steps are recognized until the weld metal cools down to temperature T_m, at which it recovers rigidity. In the first step, the groove expands freely due to the welding heat and in the second one, the heat deformation produced in the first step dislocates returning to the initial position while the temperature cools down to room temperature. When the groove returns in the second step, the weld metal has already recovered rigidity. So the weld metal itself restricts the return of the groove. Then, as the groove cannot return the initial position, the restraint stress and strain are produced in the weld

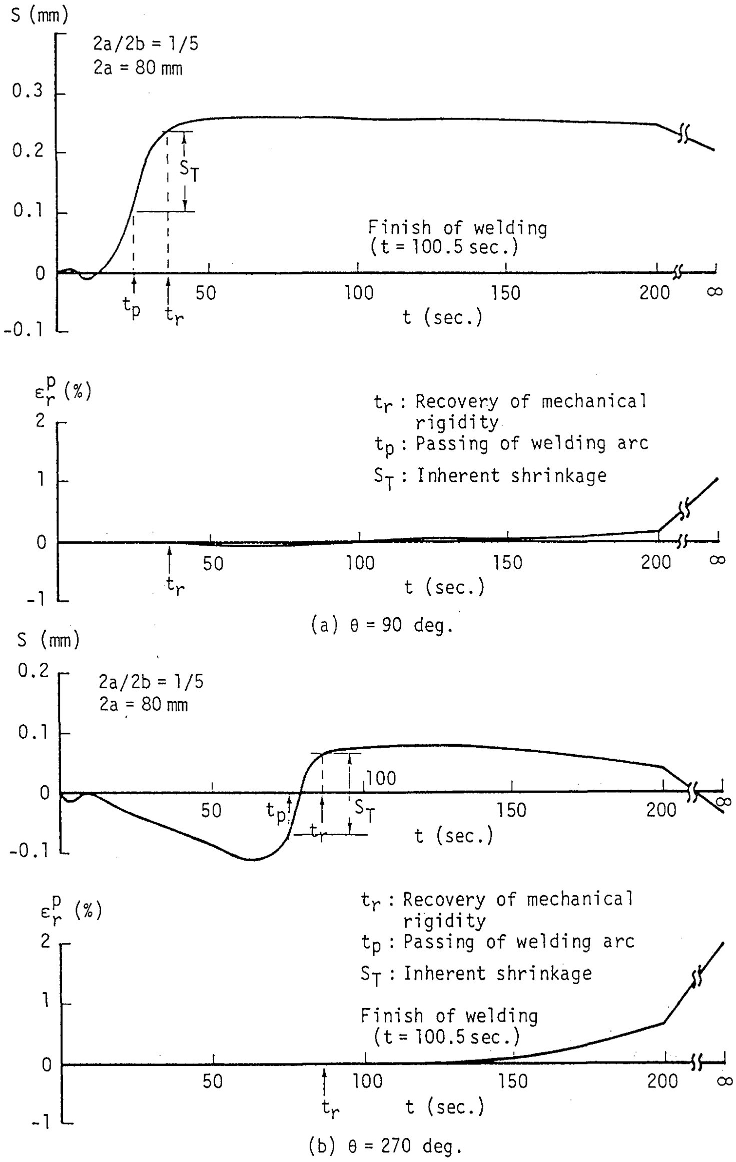

Fig.5 Transient of dislocation at the groove and of plastic strain in the weld metal

metal. Therefore, the dislocation S_T of the groove, which is produced from when the heat source passes ($t=t_p$) to when the weld metal cools to a temperature at which rigidity is mechanically recovered ($t_p<t<t_r$), can be considered to be the origin of restraint stress and strain generation[2]. S_T is called inherent shrinkage, hereinafter. The analytical calculation method of the restraint stress and strain is developed based on the inherent shrinkage.

3. Analytical calculation procedure of the restraint stress and strain

Here, the following assumptions were made during the analytical calculation:

i) The heat source is instantaneous one.

ii) Thermal expansion and shrinkage of the weld metal itself are ignored.

iii) Considering that the heat source is instantaneous plane one when the thickness of the base metal, h, is smaller than the critical thickness, h_{cr}, the analysis is treated as a two-dimensional problem. On the contrary, if h is larger than h_{cr}, the heat source is regarded as instantaneous linear one on the thickness center. However, obtaining S_T, the analysis is treated as a two-dimensional problem assuming that the temperature of the thickness center distributes uniformly to the thickness direction. If the analysis is treated like this, it is theoretically proved that an approximate value with high accuracy can be obtained[3].

iv) The base metal is completely elastic and the weld metal shows perfect elastic-plastic behavior. They follows Tresca's criterion of yielding*).

3.1 Analytical calculation procedure of the inherent shrinkage

The inherent shrinkage S_T is the dislocation of the groove when the weld metal cools to temperature T_m at which mechanical rigidity is recovered. Therefore, when the weld metal cools down to T_m, temperature T(r) at the position, r, must be known. The axisymmetrical heat source is performed instantaneously on the radius a (radius of inner plate) from the origin. In this case, if the radius a satisfies the condition of the following equation (because a is larger than 12.2(mm) if the welding condition is the same as in Section 2.1, there is no problem for practical use), temperature T(r) at which the weld metal cools down to T_m (in the case mild steel is at 700°C) can be obtained as Eq.(2).

$$2\pi(a/h'_{cr})^2 > 3 \tag{1}$$

$$a > \sqrt{(3/2\pi)}\cdot h'_{cr} \geq 0.7h'_{cr} \tag{1'}$$

$$T(r)=2\pi(T_m-T_i)a/h'_{cr}\cdot \exp\{-\pi(r^2+a^2)(1/h'_{cr})^2\} \cdot I_o\{2\pi ar(1/h'_{cr})^2\}+Ti \tag{2}$$

where,

q: heat input (J/mm), c: specific heat (J/kg°C),
ρ: density (kg/mm^3),
T_i: initial temperature (°C), I_o: Bessel function
$h'_{cr} = h^2_{cr}/h$ in the case of $h \leq h_{cr}$
$h'_{cr} = h_{cr}$ in the case of $h > h_{cr}$
$h_{cr}=\sqrt{q/c\rho(T_m-T_i)}$:critical plate thickness (mm).

Temperature T(r) at which the weld metal cools down to T_m could be obtained. Therefore, the dislocation S_T of the groove when temperature is T(r) is calculated as follows. Thermal elastic stress σ^T_r, which is produced along the circle[4] whose radius r is a at T_m, can be calculated by using T(r) (Eq.(2)).

$$\sigma_r^T = \alpha E/a^2\{\int_0^a T(r)\cdot rdr - (a/b)^2 \int_0^b T(r)\cdot rdr\} \tag{3}$$

where,

α: thermal expansion coefficient (1/°C),
E: Young's modulus (MPa).

Here, it is difficult to obtain thermal elastic stress σ^T_r in Eq.(3) analytically. So in this study numerical integration is performed. Curve fitting is applied to the numerical results. Then, the σ^T_r is derived within ±2% error as the following equation:

$$\sigma_r^T = \pi\alpha E(Tm-Ti)h'_{cr}/a\cdot\{0.1592 - 0.3182(a/b)^2 - 0.0259(h'_{cr}/a)\} \tag{4}$$

$$[1 \leq a/h'_{cr} \leq 30,\ \ a/h'_{cr}+1.5 < b/h'_{cr}]$$

According to this, σ^T_r at T_m can be simply obtained as a function of a/b and of h'_{cr}/a.

As σ^T_r can be easily obtained, S_T can be simply obtained by using the restraint intensity, R_p, for the uniform load of the circular patch welded joint as mentioned in Section 4.

$$S_T\cdot R_p = \sigma_r^T\cdot h \tag{5}$$

$$\therefore\ S_T = \sigma_r^T h/R_p \tag{5'}$$

Using σ^T_r obtained from Eq.(4) and R_p calculated from Eq.(14) mentioned in Section 4, the inherent shrinkage S_T, which is the origin of restraint stress and strain generation, can be easily obtained.

3.2 The analytical calculation procedure of the restraint stress and strain

Here, distinguishing between the case in which the weld metal is elastic and the case in which it becomes plastic, the analytical calculation procedure of the restraint stress σ_w and strain ε_w based on the inherent shrinkage S_T is shown. The restraint strain ε_w in a radial direction is obtained from the following equation as the sum of elastic strain ε^e_r and plastic one ε^p_r.

$$\varepsilon_w = \varepsilon_r^e+\varepsilon_r^p \tag{6}$$

i) In the case when the weld metal is elastic ($\sigma_w < \sigma_Y$)

$$\sigma_w = \sigma_r^T h/h_w$$

$$\varepsilon_w = \varepsilon_r^e = \sigma_w/E \tag{7}$$

where,

ε_r^e : elastic strain
$\varepsilon_r^p = 0$: plastic strain
h_w: the throat thickness (mm).

ii) In the case when the weld metal is plastic ($\sigma_w \geq \sigma_Y$)

$$\sigma_w = \sigma_Y$$

$$\varepsilon_w = \varepsilon_r^e+\varepsilon_r^p \tag{8}$$

where,

$\varepsilon_r^e = \sigma_Y/E$
$\varepsilon_r^p = (S_T-S_e)/b_w$
σ_Y: yield stress of the weld metal (MPa)
b_w: the root gap (mm).

*) When Von Mises' criterion is applied, yield stress of the weld metal, σ_Y, can be obtained as follows:

$\sigma_Y = (2/\sqrt{3})\sigma_o$

where,

σ_o : yield stress of the weld metal obtained by the simple tensile test (MPa).

S_e in the above equation is the elastic deformation of the base metal. So it can be obtained from the following equation using the restraint intensity, R_p, in Eq.(14).

$$R_p \cdot S_e = \sigma_Y h_w \quad (9)$$

$$\therefore \; S_e = \sigma_Y h_w / R_p \quad (9)'$$

According to this, in the respective cases when the weld metal is elastic and plastic, the restraint stress and strain can be easily calculated.

3.3 Consideration

In order to investigate the applicability of the restraint stress and strain obtained by the analytical calculation, the thermal elastic-plastic stress analysis is carried out for the case in which the moving speed of the heat source is infinitely fast (the *assumption i)* used in the modeling), that is for the case when the instantaneous heat source is performed on the circular whose radius r is a. ε^p_r produced in the weld metal is shown by symbol •, and the result obtained by the analytical calculation is shown by a solid line in **Fig.6**. Regardless of the inner plate diameter size, 2a, both coincide well. Therefore, the validity of the analytical calculation is proved.

The result of the thermal elastic-plastic stress analysis with the moving effect of the heat source and that of the analytical calculation are compared in detail below.

Figure 7 shows the distribution of the plastic strain component in a radial direction produced in the weld metal of each model, ε^p_r, along the weld line. The feature ε^p_r does not depend on the size of the inner plate diameter, 2a. So ε^p_r becomes larger as it comes near the welding end, and the maximum value $(\varepsilon^p_r)_{max}$ is produced about 1.0~1.2h'_{cr} inside the welding end. The feature is made by adding the same quantity of the plastic strain (irrespective of the inner plate diameter) along the weld line from the welding end to the plastic strain

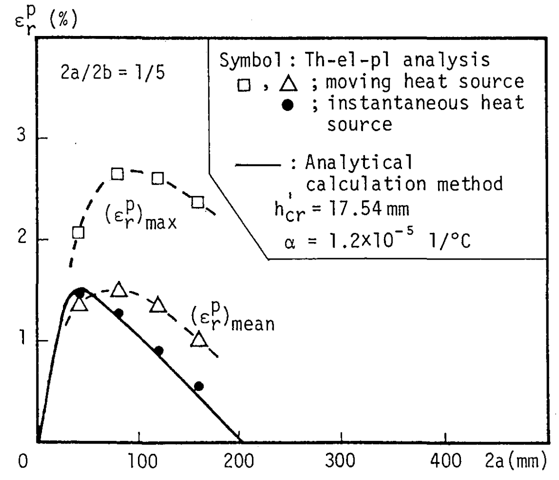

Fig.6 Validity of the analytical calculation results

obtained from the analytical calculation. The feature from the welding end is equal to ε^p_r produced during 167.2~198.3(s) after the starting of welding. The result of the thermal elastic-plastic stress analysis, assuming the case in which the welding speed was infinitely fast, coincided well with the one obtained from the analytical calculation. According to this fact, the feature of the added plastic strain from the welding end mentioned above is considered to be the characteristic of the welding phenomenon of the moving heat source. So the slower the welding speed is, the shorter the distance from the edge and the larger the absolute value becomes.

On the other hand, Δ and [] symbols in Fig.6 show the mean value $(\varepsilon^p_r)_{mean}$ and the maximum value $(\varepsilon^p_r)_{max}$ along the weld line, respectively. From Fig.6 (symbol []) it can be seen that, the restraint condition at the inner plate diameter 2a=80(mm) is the severest. The mean value along the weld line, $(\varepsilon^p_r)_{mean}$, of the thermal elastic-plastic stress analysis result corresponded well to ε^p_r obtained from the analytical calculation. The maximum value $(\varepsilon^p_r)_{max}$ may be made by adding 1.0~1.5(%) plastic strain to the value obtained from the analytical calculation irrespective of the size of the inner plate diameter, 2a. This 1.0~1.5(%) plastic strain value was almost equal to the maximum plastic strain value produced during 167.2~198.3(s) after the start of welding (see Fig.4).

In all cases, the severity of the relative mechanical condition can be systematically evaluated based on the restraint stress and strain simply obtained from the analytical calculation whether the weld metal is elastic or plastic.

4. Applicability of restraint intensity as a mechanical measure

Here, the restraint intensity as a mechanical measure for the uniform load is investigated based on the restraint

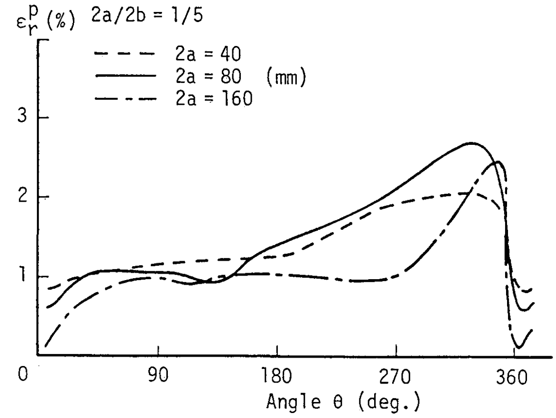

Fig.7 Plastic strain distributions in the weld metal

stress and strain obtained from the analytical calculation.

4.1 Restraint intensity for the uniform load

Restraint intensity is defined as "the force required to shrink unit the root gap of the joint per unit welding length" and it is expressed as the following equation.

$$R_p = p_o h/\delta \qquad (10)$$

where,

p_o: uniform loads (N/mm·mm),
h : thickness (mm)
δ : dislocation (mm).

When the uniform load p_o is applied to the outer edge of the circular plate with the hole at the origin **(Fig.8(a))**, the displacement u_1 at the loading edge is calculated as the following equation[5]:

$$u_1 = (p_o/E)\{a^2/(b^2-a^2)\}[(1-\nu)a+(1+\nu)b^2/a] \qquad (11)$$

where,

ν : Poisson's ratio

Similarly, in the case of Fig.8(b), the displacement u_2 at the loading edge is as follows[5]:

$$u_2 = (p_o/E)a(1-\nu) \qquad (12)$$

Therefore, the dislocation δ of the circular patch welded joint can be obtained as the following equation:

$$\delta = u_1+u_2 \qquad (13)$$

According to the above, obtaining the restraint intensity R_p for the uniform load of the circular patch welded joint following the defined equation (Eq.(10)), R_p can be calculated as follows:

$$R_p = (Eh/2a)\{1-(a/b)^2\} \qquad (14)$$

4.2 Applicability of restraint intensity as a mechanical measure

Figure 9 shows the relation between the restraint strain ε_w and restraint intensity R_p (Eq.(14)). In the elastic range, ε_w (same as in the restraint stress σ_w (Eq.(7)) and R_p are in a proportional relation. Therefore, the severity of the mechanical restraint condition can be evaluated using R_p instead of ε_w. On the other hand,

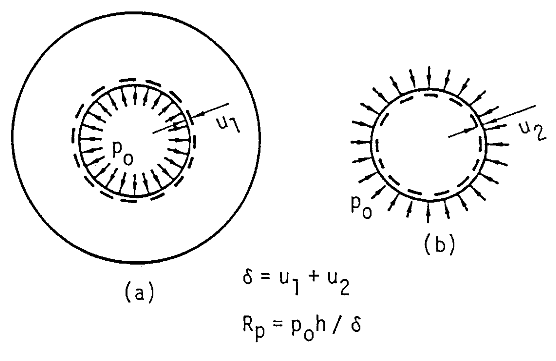

Fig.8 Definition of restraint intensity, R_p, for uniform loads in circular patch weld joint

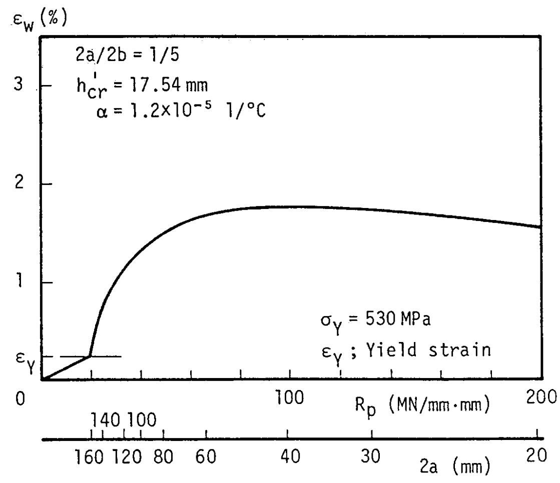

Fig.9 Relation between restraint intensity, R_p, and restraint strain, ε_w, in circular patch welding

noting the plastic range, ε_w shows the tendency of a simple increment not in proportion to R_p in the range where R_p is smaller than 100(MN/mm·mm). However, if R_p exceeds 100(MN/mm·mm), ε_w has a tendency of a decrease. According to this, in the plastic range, use of R_p as a measure which evaluates the severity of the relative mechanical condition has the applicable limits. If R_p exceeds the limit, it can not be used as a mechanical measure.

The applicability of the restraint intensity R_p of the circular patch welded joint is investigated. As a result, the severity of the mechanical condition can be evaluated by using R_p in the elastic range. However, R_p can not be a mechanical measure in the plastic range. But, considering the applicability of R_p as the relative measure of the severity of the mechanical condition, it is available if it is used as a simple measure within the applicable limits.

5. Conclusion

The results obtained are summarized as follows:

(1) According to the results of the unstationary thermal conduction analysis for circular patch welding, considering the moving effect of the heat source, heat is conducted in the inner plate. As time goes on, the influence of the moving effect of the heat source vanishes and the isothermal contour is the axisymmetric distribution.

(2) FEM modeling is treated as the moving heat source, the residual stress component in a radial direction becomes the almost an axisymmetric distribution. On the other hand, the plastic strain component in a radial direction $\varepsilon^p{}_r$ shows that the constant value produced in the cooling process from the

axisymmetric distribution to room temperature is uniformly added to the local value, which is obtained till the moving effect of the heat source is vanished (the isothermal contour becomes axisymmetric distribution). εp_r is almost produced in the cooling process from axisymmetric distribution to room temperature.

(3) The feature of εp_r does not depend on the size of the inner plate diameter, 2a. So the maximum value of εp_r is produced about $1.0 \sim 1.2h'_{cr}$ inside the welding end.

(4) Distinguishing between the case in which the weld metal is elastic and the case in which it becomes plastic, the analytical procedure obtained the restraint stress and strain based on the inherent shrinkage S_T was shown. Consequently, the severity of the relative mechanical condition can be systematically known for the various changes of the heat input, thickness of the base metal and the sizes of the inner and outer plates. Moreover, the validity of the analytical calculation procedure is proved.

(5) Based on the restraint stress and strain obtained from the analytical calculation procedure, the severity of the relative mechanical condition can be evaluated whether it is elastic or plastic.

On the other hand, the applicability of the restraint intensity, R_p, for the uniform load of the circular patch welded joint as a mechanical measure is investigated based on the restraint stress and strain obtained from the analytical calculation procedure.

(6) The relation between restraint strain ε_w (same as in restraint stress σ_w) and R_p is in proportion in the elastic case. Therefore, the severity of the mechanical restraint condition can be evaluated using R_p instead of ε_w in the elastic. By the way, the severity of the mechanical restraint condition can not be directly evaluated in the plastic case. However, R_p can be used as the relative measure of the severity of the mechanical condition within the limits where the restraint strain simply increases with the increments of R_p.

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