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Addenda to "On Multiple Distributions."

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By Tadashige ISHIHARA

In the present addenda we note that a "multiple distribution" can be considered in a more general case than the one considered in the paper mentioned in the title, which has wider ranges of applications. We give also here some corrections of the errata found in the paper.

In the original paper we defined the modified topology of $\mathfrak{D}^{\pi}(x, t)$ by the introduction of a collection of bounded sets (Arnold's family) in $\mathfrak{D}^{\pi}(t)$, and denoted the obtained space by $\mathfrak{D}_{Q}^{\pi}(x, t)$ or $\tilde{\mathfrak{D}}_{Q}^{\pi}(x, t)$. We considered further the space $'\mathfrak{D}_{Q}^{\pi}(x, t)$ or $'\tilde{\mathfrak{D}}_{Q}^{\pi}(x, t)$ which is a closure of a $\mathfrak{D}^{\nu}(x, t)$ by the dual topology $\tilde{\mathfrak{D}}'_{Q}{}^{\pi}(x, t)$ or $\mathfrak{D}'_{Q}{}^{(\pi)}(x, t)$.

Now we can modify the topology of $\mathfrak{D}^{\pi}(x) \otimes \mathfrak{D}^{\nu}(t)$ also by the similar Arnold's family. We consider further a closure of a $\mathfrak{D}^{\nu}(x, t)$ by the dual topology of the obtained topological vector space and denote the obtained space similarly by $'\mathfrak{D}_{Q}^{\pi}(x, t)$ or $'\tilde{\mathfrak{D}}_{Q}^{\pi}(x, t)$. The new $'\mathfrak{D}_{Q}^{\pi}(x, t)$ or $'\tilde{\mathfrak{D}}_{Q}^{\pi}(x, t)$ in a special case concides with the old $'\mathfrak{D}(x, t)$ or $'\tilde{\mathfrak{D}}_{Q}^{\pi}(x, t)$ and any element $T \in {}'\mathfrak{D}_{Q}^{\pi}(x, t)$ or $\in {}'\tilde{\mathfrak{D}}_{Q}^{\pi}(x, t)$ defines a multiple distribution T_{Q} which has the same (and often refined) properties as the old ones.

This extension is attained without essential alteration of proofs. That is to say, the following modifications of the notations, phrases and clauses in the original paper give us the extended theory.

(i) Replacemet of $\mathfrak{D}^{\pi}(t)$ or $\mathfrak{B}_{\pi}(t)$ by $\mathfrak{D}^{\nu}(t)$ or $\mathfrak{B}_{\nu}(t)$, and similar modification of suffix. (pp. 191, 192, 193, 197, 198 and $\mathfrak{D}^{(\pi)'}(t) \rightarrow \mathfrak{D}^{(\mu)'}(t)$ in p. 194).

(ii) Deletion of superfluous clauses and phrases about $\mathfrak{D}^{\pi}(x, t)$, or replacement of $\mathfrak{D}^{\pi}(x, t)$ by $\mathfrak{D}^{\pi}(x) \otimes \mathfrak{D}^{\nu}(t)$. (p. 192).

(iii) Deletion of superfluous conditions about the order π, ν, μ which originate in the equality of the order π of $\mathfrak{D}^{\pi}(x)$ and of $\mathfrak{D}^{\pi}(t)$, or additions of the necessary conditions of the contrary case. (pp. 196, 197, 199, 200 or pp. 193, 197).

After these three sorts of modifications, all the lemmas and theorems in the paper still hold if we give the new meanings to the notations

in the original paper, (for example $\mathfrak{D}_Q^{\pi}(x, t)$ mentioned above). Some theorems (Theorem 5, 5′, Lemma 10, 12 etc.) which have properties concerning the order π of $\mathfrak{D}^{\pi}(t)$ can be refined. For example Lemma 12 which insists that $T \in {}'\mathfrak{D}_P^{\pi} \to f(t)g(x)T \in {}'\mathfrak{D}_P$ is true for $f \in \mathfrak{D}^{\nu}(t)$ and $g(x) \in \mathfrak{E}^{\pi}(x)$, which are necessary conditions for $f(t)g(x)T$ and $f(t)Q$ to be a well-defined distribution for $T \in D_P^{(\pi)\prime}$, $\mathfrak{Q} \in D^{(\nu)\prime}$.

By virtue of these refinements we can see Theorems 12 and 13 still hold for the equation (i)

$$\partial U(x, t)/\partial t + \sum_{|\rho| \leq \sigma} A_{\rho}(t) E_{\rho}(x) D_x^{\beta} U(x, t) = B(x, t)$$

where $A_{\rho}(t) \in \mathfrak{D}^{0}(t)$ and $E_{\rho}(x) \in \mathfrak{D}^{\pi+|\rho|}(x)$.

Obviously these conditions are the nesessary conditions for this equation to have a meaning for a continuous (as $\mathfrak{V} \to \mathfrak{D}^{(\pi)\prime}(x)$) solution $U(x, t)$.

The proofs of the theorems can also be done by the insertions of $E_{\rho}(x)$ and deletion of the superfluous conditions for $A_{\rho}(t)$. The conditions "for $\pi \geq 1$" and "for $\pi \geq 2$" which appeared in Theorem 11 and 12 are also superfluous.

Errata.

Superfluous condition and errata (several obvious misprints are omitted):

(a) We often considered $(\nu, \rho Q)$-sequence or family for $\rho=1$, 0 but the consideration for $\rho=1$ suffices. (pp. 195, 196, 197 and 198.)

(b) p. 188, line 1 from the bottom. For "$\lambda B \subset \lambda B^* \subset \mu B^* \cap \nu B^*$" read "$g+\lambda B \subset g+\lambda B^* \subset g+\mu B^* \cap \nu B^*$".

p. 190, line 20. For "Lemma 9" read "Lemma 7".

line 4 from the bottom. For "π-times" read "π-times continuously".

p. 191, line 7. For suffix "μ'" read "$(\mu)'$". For "$\mu < \nu$" read "$\mu \leq \nu$".

p. 197, line 4. For "π'" and "π" read "$(\mu')'$" and "$(\mu)'$".

line 6. For "$\nu \geq \mu$, $\nu' \geq \mu'$" read "$\nu, \nu' \geq \mu, \mu'$".

p. 199, line 24. Add "($\mathfrak{B}^0$; $\mathfrak{B}^0$ for (ν, fQ)-sequence or $\mathfrak{B}^0$ for $(\nu, f\tilde{Q})$-family)" to the end of the line.

p. 200, line 8. For "$T\delta_{t_0}^{(\mu)}$" read "$T_{(-1)^{\mu}}\delta_{t_0}^{(\mu)}$".

line 10. For "$T_{t_0}^{(\sigma)}$" read "$T_{(-1)^{\sigma}}\delta_{t_0}^{(\sigma)}$".

p. 202, line 17. Insert "[(*)]" nex to "say".

p. 203, line 2. For "(1)" read "(*)".

line 4. For "$u\delta^{(\mu)}$" read "read" "$(-)^{\mu} u\delta^{(\mu)}$".

p. 204, line 19. For "7′" read "5′".

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