## Title
Prediction of Welding Deformation and Residual Stress by Elastic FEM Based on Inherent Strain (Report I): Mechanism of Inherent Strain Production (Mechanics, Strength & Structure Design)

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## Note

Prediction of Welding Deformation and Residual Stress by Elastic FEM Based on Inherent Strain (Report 1)†

- Mechanism of Inherent Strain Production -

Yu LUO*, Hidekazu MURAKAWA**, Yukio UEDA***

Abstract

The welding residual stress and the distortion are considered to be produced by the inherent strain. The plastic strain in bead welding computed through the Thermal-Elastic-Plastic analysis is an ideal example of the inherent strain, since there is no groove which may introduce additional mismatch. Such inherent strain produced in the welding process is primarily determined by the highest temperature reached and the constraint at each point. Thus, the mechanical phenomena in the bead welding process are closely analyzed by the Thermal-Elastic-Plastic FEM and the mechanism in which the inherent strain is produced is investigated from the aspect of the highest temperature reached and the constraint. Further, based on the knowledge obtained through the study, simple formulae to calculate the inherent strain distribution are proposed. Using the inherent strain given by the formulae, the welding residual stress and the distortion can be predicted by an Elastic FEM analysis.

KEY WORDS: (Inherent strain) (Welding Deformation) (Welding stress) (FEM)

1. Introduction

Along with the development of computer and numerical analysis technology, it becomes possible to simulate the mechanical processes during welding by Thermal-Elastic-Plastic (T.E.P.) FEM1,2 when the welded structures is simple and small. However, in case of large and complicated welded structures, it is difficult to analyze such non-linear problems because long computing time is required. To ensure the integrity of welded structures, it is necessary to develop a method to investigate welding deformation and residual stress with reasonable computational effort. For this reason, the concept of inherent strain3-6 has been introduced. Generally, the inherent strain can be considered as a source of both deformations and residual stresses which are produced during thermal processes such as welding, cutting and bending. If the inherent strain for an individual thermal process is known, the deformation and residual stress can be estimated using Elastic FEM in stead of T.E.P. FEM. In this way the computing time can be greatly shortened. Thus, the determination of inherent strain is very important when investigating welding deformation and residual stress in an actual thermal process.

In this paper, the mechanical process in bead welding is simulated by T.E.P. FEM. The computed results clearly show that inherent strain produced in the welding process is primarily determined by the highest temperature reached and the constraint at each point. Thus, the mechanism in which inherent strain is produced is clarified from the aspect of the highest temperature reached and the constraint. Further, based on the knowledge obtained through the study, simple formulae to calculate the inherent strain distribution are proposed. Using the inherent strain given by the formulae, the welding residual stress and the distortion can be predicted by Elastic FEM analysis. The computed results agree well with the available experimental results.

2. Mechanism of inherent strain production

In order to study the mechanism of inherent strain production in welding process, the mechanical behavior of a simple model is investigated first.
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Fig. 1 History of stress under thermal cycle for a Bar model with elastic constraint.

2.1 A bar with elastic constraint

A simple model of a bar with elastic constraint is shown in Fig. 1. In order to represent the magnitude of the constraint, a parameter $\beta$ is introduced which is defined as follows,

$$\beta = k*/(k+k*)$$  \hspace{1cm} (1)

where,

- $k = aE/l$ stiffness of bar
- $k^*$ stiffness of spring
- $a$ cross-sectional area of bar
- $E$ Young's modulus of bar
- $l$ length of bar

The relation between the stress and the temperature of the bar during the thermal cycle is shown in Fig. 1. In the elastic stage, the relation among stress $\sigma$, constraint $\beta$ and temperature $T$ can be expressed as,

$$\sigma = \beta \alpha T E$$ \hspace{1cm} (2)

where, $\alpha$ is the coefficient of linear expansion.

The temperature at which the stress just reaches the yield stress in the heating process is defined as $T_1$, such that,

$$T_1 = \sigma_Y / \beta \alpha E$$ \hspace{1cm} (3)

When the bar is constrained completely ($\beta = 1$), $T_1$ is dependent on only material properties and defined as $T_Y$,

$$T_Y = \sigma_Y / \alpha E$$ \hspace{1cm} (4)

If material properties ($\sigma_Y$, $\alpha$, $E$) are not dependent on temperature, $T_Y$ becomes constant.

Similarly, the heating temperature which is necessary to attain yield stress in the cooling process is defined as $T_2$, such that,

$$T_2 = 2T_1$$ \hspace{1cm} (5)

From Fig. 1, it can be clearly seen that the mechanical behaviors of the bar can be divided into three types according to the highest temperature:

1. $T_{max} < T_1$
   - When the highest temperature is lower than $T_1$ as O-A-O shown in Fig. 1, the thermal stress at the end of the heating process is smaller than the yield stress. So no inherent strain, deformation nor residual stress is produced.

2. $T_1 < T_{max} < T_2$
   - When the highest temperature is greater than $T_1$ but less than $T_2$, the thermal stress reaches the yield stress in the heating process, and plastic strain is produced. But in the cooling process, additional plastic strain is not produced.

3. $T_2 < T_{max}$
   - When $T_{max}$ is greater than $T_2$ as O-B-G-H-F in Fig. 1, inherent strain is produced in both the heating and the cooling processes.

2.2 A cube with three dimensional elastic constraint

2.2.1 Model of a cube with elastic constraint

In the preceding section, the production mechanism of inherent strain in a thermal cycle for a one dimension model (a bar with elastic constraint) is discussed conceptually. However, the mechanical phenomena in welding processes are not one dimensional. Therefore a cube model under three dimensional constraint as shown in Fig. 2 is studied. In this model a cube of size $s$ is constrained by springs which have stiffnesses $k^*_{x}$, $k^*_{y}$ and $k^*_{z}$ in $x$, $y$, $z$ directions, respectively. Similar to the bar model, constraint parameters $\beta_x$, $\beta_y$ and $\beta_z$ are defined as follows,

$$\beta_i = k*_{i}/(k+k^*_{i}) \hspace{1cm} (i=x, y, z)$$ \hspace{1cm} (6)

where,

- $k = Es$ stiffness of cube
- $k^*_{i}$ stiffness of spring in $i$-th direction
- $s$ size of cube

Fig. 2 Three dimensional constraint model.
Fig. 3 shows the relation between the temperature and the yield stress and Young's modulus used in this study. Three kinds of materials are considered. Material-3 is close to actual mild steel. Both yield stress and Young's modulus are dependent on temperature. In the case of material-2, only yield stress is dependent on temperature. In the case of material-1, neither yield stress nor Young's modulus is dependent on temperature. The thermal and physical constants used are shown in Fig. 4. The constraint parameter $\beta_X$, $\beta_Y$, $\beta_Z$ was determined with reference to material properties at room temperature.

To clarify the mechanism of inherent strain production under three dimensional constraint, a series of computations are made using Thermal Elastic Plastic (T.E.P.) FEM.

2.2.2 Relation between highest temperature and residual stress

Corresponding to the welding model discussed in this report, x, y and z directions refer to the welding, the transverse and the thickness directions, respectively. In general, the constraint is the strongest in the welding direction and that in the thickness direction is the weakest. Fig. 5 shows the relation between the highest temperature and the residual stress which is obtained as computed results for material-1 when $\beta_Y=\beta_Z=0$. From this figure, it is clearly seen that the mechanical behavior of the cube is the same as that of the bar model since the constraints in y and z directions $\beta_Y$, $\beta_Z$ are zero. Thus, when $T<T_{1x}$, no residual stress is produced. When $T_{1x}<T<T_{2x}$, the residual stress increases with the temperature. When $T>T_{2x}$, the residual stress becomes equal to the yield stress independent of temperature, where,

$$T_{1x}=\alpha Y/\beta X \alpha E, \quad T_{2x}=2T_{1x}$$

On the other hand, $T_{1x}$ increases with the decrease of $\beta_X$. It is only when $T_{1x}<T_{max}<T_{2x}$ that $\beta_X$ influences the value of the residual stress.

![Fig. 3 Temperature dependent mechanical properties.](image)

![Fig. 4 Temperature dependent thermal and physical constants.](image)

![Fig. 5 Relation between residual stress and maximum temperature.](image)
2.2.3 Influence of highest temperature and constraint on inherent strain

The relation among the constraint, the highest temperature and the inherent strain is given in Fig. 6. The conditions for the analysis in Fig. 6(a) are the same as those in the preceding section. For the computation in Fig. 6(b), \( \beta_y=1 \) is assumed to represent the effect of the constraint in the transverse direction. From Fig. 6 it is seen that the curve can be divided into three regions according to \( \beta_x \), such that,

\[
\begin{align*}
\varepsilon^*_{x} &= 0 & T_{1x} > T_{\text{max}} \\
\varepsilon^*_{x} &= \alpha T_{\text{max}} + \varepsilon_Y / \beta_x & T_{1x} < T_{\text{max}} < T_{2x} \\
\varepsilon^*_{x} &= -\varepsilon_Y / \beta_x & T_{\text{max}} > T_{2x}
\end{align*}
\]

where,

\[
\begin{align*}
\varepsilon_Y &= \gamma / E \\
T_{1x} &= \gamma / \beta_x \alpha E \\
T_{2x} &= 2 \gamma / \beta_x \alpha E
\end{align*}
\]

As described by the above formulas, the inherent strain \( \varepsilon^*_{x} \) is influenced by the highest temperature \( T_{\text{max}} \) only when \( T_{1x} < T_{\text{max}} < T_{2x} \).

Comparing Fig. 6(a) and Fig. 6(b), it is found that the curve is basically similar, especially when \( 0.2 < \beta_x < 0.8 \). In the welding process, the constraint parameter in the welding direction is about in this range. Thus, the inherent strain in the welding direction is mainly determined by the constraint parameter, \( \beta_x \). Influence from the constraint in the transverse direction \( \beta_y \) is small.

It is also seen that, the inherent strain has a peak value at certain value of \( \beta_x \). If constraint is larger or smaller than this peak, the absolute value of inherent strain decreases. Since \( \beta_x = 1 \) in the welding direction, the inherent strain is nearly equal to the yield strain. On the other hand, constraint in the transverse direction \( \beta_y \) is very small. Thus, the inherent strain \( \varepsilon^*_{y} \) is large and dependent on both the constraint and the highest temperature.

Assuming that \( \beta_y = 0.01, \beta_x = 0 \) and considering material-3, which is closer to actual steel, serial computations are made. The relations between the inherent strain \( \varepsilon^*_{x} \), \( \varepsilon^*_{y} \) and the highest temperature are shown in Fig. 7 and Fig. 8, respectively. It is seen from Fig. 7 that \( \varepsilon^*_{x} \) is independent of temperature and determined by \( \beta_x \) when the temperature is higher than \( T_{2x} \). It is also clearly seen from Fig. 8 that \( \varepsilon^*_{y} \) is dependent on temperature and not influenced by \( \beta_x \). When the highest temperature is lower than the mechanical

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**Fig. 6** Effect of maximum temperature and constraint on inherent strain in welding direction.

**Fig. 7** Relation between maximum temperature and inherent strain in welding direction.

**Fig. 8** Relation between maximum temperature and inherent strain in transverse direction.
Fig. 9 Relation between $\beta_X$ and $e_{X*}$ for three materials at 800°C.

The melting point, $e_{Y*}$ is nearly equal to zero and it increases with the temperature when the temperature is high.

2.2.4 Influence of material properties

Fig. 9 shows the relation between $\beta_X$ and $e_{X*}$ for three different materials when the highest temperature is 800°C. From Fig. 9, it is concluded that the inherent strain is not influenced by material properties when the constraint is strong (in this case $\beta_X>0.25$). But when the constraint is very small, the inherent strain is strongly dependent on material properties.

3. Computation of welding deformation and residual stress

3.1 Model for analysis

As shown in Fig. 10, a bead weld was applied on a plate of width: 400mm, length: 200 mm and thickness: 16 mm. Four cases of heat input are assumed as follows, case(1): $Q=533J/mm$, ($Q*=5.0$)

$$Q*=Q/hr^2c_0T_Y$$

where $h$, $c$ and $q$ are the thickness, the thermal capacity and the density, respectively. $T_Y$ is given by Eq. (4). The welding speed is 4.3mm/sec. The mechanical properties are given in Fig. 3 (material-3) and the physical properties defined in Fig. 4 are used. The bead welding is simulated using both the plane strain model and the three dimensional model.

3.2 Plane strain model

3.2.1 Examination of inherent strain in plane strain model

To examine the inherent strain in bead welding, the T.E.P. FEM is used. The distribution of the highest temperature and the inherent strains on the surface for case(2) are shown in Figs. 11(a) and (b). From these figures, the features of inherent strain can be described as follows.

1. for $e_{X*}$

$$T_{max}<T_{1x}$$

$$e_{X*}=0$$

(no plastic strain)

(a) Distribution of temperature

(b) Distribution of inherent strain

Fig. 11 Distribution of temperature and inherent strains in transverse direction.
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\[ T_{1x} < T_{max} < T_{2x} \]
\[ e_x = T_{max} \]
\[ T_{2x} < T_{max} \]
\[ e_x = -\sigma_y / \beta_x \]

(where, \( T_{1x} = T_{max} \) for plane strain \( \beta_x = 1 \))
\[ \sigma_y = -0.00114 \]
\[ \theta = \alpha_0 / E = 0.00225 \]
\[ T_{1x} = 1450 \, ^\circ C, \quad T_{max} = 1500 \, ^\circ C \]

(2) for \( e_y \)
\[ T_{max} < T_{1x} \]
\[ e_y = 0 \]

(no plastic strain)
\[ T_{1x} < T_{max} < T_{m} \]
\[ e_y = e_z \]

(because \( \beta_y = \beta_z < \beta_x \))
\[ T_{m} < T_{max} \]
\[ e_y \]

(increases with \( T_{max} \))

where, \( T_{m} \) is the mechanical melting point.

(3) for \( e_z \)
\[ e_z = -(e_x + e_y) \]

(incompressibility)

3.2.2 Formulae of inherent strain for plane strain model

As explained in the preceding section, the main factors which influence the inherent strain are the highest temperature and the constraint. Simple formulae to calculate the inherent strain, which is a function of the highest temperature and the constraint, are proposed as follows for the plane strain model.

For inherent strain in welding direction
\[ e_x = 0 \]
\[ e_x = -\alpha (T_{max} - T_{1x}) \]
\[ e_x = e_y \]
\[ e_x = e_z \]

(8)

For inherent strain in transverse direction
\[ e_y = 0 \]
\[ e_y = e_z = -e_y / 2 \]
\[ T_{1x} < T_{max} < T_{m} \]

(incompressibility)
\[ e_y = -\alpha (T_{max} - T_{m}) + e_y / 2 \]
\[ T_{m} < T_{max} \]

(9)

To derive Eq.(9), it is assumed that \( e_y \) is a linear function of \( T_{max} \) in this region as observed in Fig.7. Also, the continuity condition of the inherent strain \( e_y \) at \( T_{max} = T_{m} \) is assumed, i.e.,
\[ e_y = e_y / 2 \]

For inherent strain in thickness direction
\[ e_z = -(e_x + e_y) \]

(incompressibility)

For shear inherent strain

According to the symmetry, the shear inherent strain is assumed to be an odd function of \( y \). Considering the results of T.E.P. FEM, the distribution of the shear inherent strain is represented by the following equation.
\[ \gamma_{yz} = B \alpha (T_{max} - T_{2x}) (y / h) \]

(11)

where, the material properties at room temperature are given as,
\[ \sigma_y = 240 \, MPa \]
\[ \alpha = 1.14 \times 10^{-5} \, ^\circ C^{-1} \]
\[ E = 210 \, GPa \]
\[ T_{1x} = 100 \, ^\circ C \]

\[ T_{2x} = 200 \, ^\circ C \]
\[ T_{m} = 750 \, ^\circ C \]

\( h \): thickness of plate

The unknown coefficients in the above formulae are \( A \) and \( B \). These can be determined by T.E.P. FEM or by experiment. For example, from the results computed by T.E.P. FEM for the four cases, it is found that,
\[ T_{max} = 1450 \, ^\circ C, \quad e_y = 0.00225 \] at \( y = 0, \quad z = 0 \)

By substituting this condition into Eq.(9), it is found that,
\[ A = 0.35 \]

Also from the computed results by T.E.P. FEM, the constant \( B \) is determined to be,
\[ B = 1.7 \]

3.2.3 Comparison of results by T.E.P. FEM and Elastic FEM's

Using the inherent strain, the deformations and the residual stresses caused by a bead welding are computed by Elastic FEM. The inherent strain is calculated using the proposed formulae and the highest temperature to be used is computed by FEM. The highest temperature can be also estimated using appropriate analytical solutions. Figs. 12, 13 show the distribution of inherent strains for case(2) obtained from the T.E.P. FEM and that

![Fig.12 Distribution of inherent strain by thermal-elastic-plastic-analysis (case(2)).](image1)

![Fig.13 Distribution of inherent strain computed by proposed formula (case(2)).](image2)
calculated using the proposed formulae, respectively. The distributions of residual stresses computed by the T.E.P. and the Elastic FEM's are shown in Figs.14 and 15. Similarly, the comparison between angular distortions is shown in Fig.16. From these figures, it is seen that the deformation and the residual stresses can be predicted with reasonable accuracy by Elastic FEM using the inherent strain.

3.3 3-D model for bead welding with moving heating source

3.3.1 Feature of inherent strain in the 3-D model

The 3-D model is different from the plane strain model mainly in constraint. In case of plane the strain model, deformation in the welding direction is fully constrained, whereas that in the transverse direction is nearly free, i.e.,

\[ \beta_{x}=1, \quad \beta_{y} = \beta_{z} = 0.0 \]

In case of the 3-D model, the constraint in the welding direction is smaller than that in the plane strain model, but it is larger in the transverse direction.

The distribution of the highest temperature and the inherent strains on the surface at the middle transverse section in case(2) are shown in Figs.17(a) and (b). From these figures, the following features of the inherent strain are observed.

Fig.14 Stress distribution computed by thermal-elastic-plastic analysis (case(2)).

Fig.15 Stress distribution computed by elastic FEM using inherent strains (case(2)).

Fig.16 Comparison between angular distortion computed thermal-elastic analysis and proposed elastic analysis.

Fig.17 Distribution of temperature and inherent strain by thermal-elastic-plastic analysis and proposed formula (case(2)).
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(1) for $\varepsilon^*_{x}$
   
   $T_{\text{max}} < T_{1x}$
   
   $\varepsilon^*_{x} = 0$
   
   (no plastic strain)

   $T_{1x} < T_{\text{max}} < T_{2x}$
   
   $\varepsilon^*_{x}$ (increases with $T_{\text{max}}$)

   $T_{2x} < T_{\text{max}}$
   
   $\varepsilon^*_{x} = -\varepsilon_{y}/r_{x}$
   
   ($\beta_{x} < 1$)

   where, $T_{1x} = \sigma_{y}/E_{y}^x$, $T_{2x} = 2T_{1x}$

(2) for $\varepsilon^*_{y}$

   $T_{\text{max}} < T_{2x}$
   
   $\varepsilon^*_{y} = 0$
   
   (because $\beta_{y} < \beta_{x}$, $T_{1y} < T_{2y}$)

   $T_{2x} < T_{\text{max}}$
   
   $\varepsilon^*_{y}$ (increases with $T_{\text{max}}$)

(3) for $\varepsilon^*_{z}$

   $\varepsilon^*_{z} = -(\varepsilon^*_{x} + \varepsilon^*_{y})$
   
   (incompressibility)

3.3.2 Formulae of inherent strain for the 3-D model

The shear components of the inherent strain $\gamma_{xy}, \gamma_{zx}$ are assumed to be negligible. Simple formulae which determine the inherent strain in the 3-D model as functions of the highest temperature and the constraint are given as,

When $T_{\text{max}} < T_{1x}$

$\varepsilon^*_{x} = \varepsilon^*_{y} = \varepsilon^*_{z} = \gamma_{xy} = \gamma_{zx} = \gamma_{yz} = 0$

(12)

When $T_{1x} < T_{\text{max}} < T_{2x}$

$\varepsilon^*_{x} = -\sigma_{x}/E_{y}^x$

$\varepsilon^*_{y} = \varepsilon^*_{z} = 0$

(13)

$\gamma_{yz} = 0$

When $T_{2x} < T_{\text{max}}$

$\varepsilon^*_{x} = -\varepsilon_{y}/r_{x}$

$\varepsilon^*_{y} = -\varepsilon_{x}/r_{y}$

$\gamma_{yz} = \Delta (T_{\text{max}} - T_{2x})/(y/h)$

(14)

The unknown coefficients in the above formulae are $A$, $B$, $C$ and $\beta_{x}$.

In case of the 3-D model, $\beta_{x}$ depends on heat input $Q^*$. From the definition, $\beta_{x}$ and $T_{1x}$ can be related through the following equation.

$T_{1x} = \sigma_{y}/E_{y}^x Q^*$

(15)

$T_{1x}$ can be considered as the temperature at which $\varepsilon^*_{x}$ just begins to be produced. From the results of T.E.P. FEM for the four cases, the relation between $T_{1x}$ and $Q^*$ is obtained as follows.

$T_{1x} = 100 + 5Q^*$

(16)

By comparing Eqs.(15) and (16), the following equation is derived.

$\beta_{x} = 1/(1 + 0.05Q^*)$

(17)

The constant $C$ can be decided by considering the continuity condition of $\varepsilon^*_{y}$ at $T_{\text{max}} = T_{2x}$ in Eq.(14), i.e.,

$\varepsilon^*_{y}(T_{2x}) = 0$

Fig. 18 Comparison between stresses computed by thermal-elastic-plastic and proposed elastic analysis (case(3)).

From the above condition, it is found that,

$C = T_{2x}$

According to the results of T.E.P. FEM, the relation between $Q^*$ and $\varepsilon^*_{y}$ in the middle of the welding line ($x = 100, y = 0, z = 0, T_{\text{max}} = 1450'C$) can be approximated as a linear function, i.e.,

$\varepsilon^*_{y} = -0.0035(1 + 0.4Q^*)$

(18)

By comparing Eqs.(18) and (14), the constant $A$ is determined to be,

$A = 0.26(1 + 0.4Q^*)$

Similarly, based on the results computed by T.E.P. FEM, $B$ can be determined, i.e.,

$B = 3.6 + 1.28Q^*$

(19)

3.3.3 Comparison between results by T.E.P. and Elastic FEM's

The distribution of inherent strains on the surface of the middle transverse section for case(2) calculated by the proposed formulae is shown in Fig.17(e). Fig. 18 shows the comparison between the residual stresses in case(3) computed by the T.E.P. and the Elastic FEM's. The transverse shrinkage and the angular distortion computed by the FEM's are compared with the values measured by Satoh et. al(?) in Figs.19 and 20, respectively. As seen from these comparisons, the elastic FEM using inherent strain determined by the proposed formulae can be used to predict welding residual stresses and distortions with an acceptable accuracy.
(2) The inherent strain produced in the bead welding process is primarily determined by the highest temperature reached and the constraint at each point. The mechanism by which the inherent strain is produced is examined from these aspects.

(3) Simple formulae to calculate the inherent strain distribution are proposed. Using the inherent strain given by the formulae, the welding residual stress and the distortion can be predict by elastic FEM analysis.

(4) The residual stresses and distortions predicted by Elastic FEM using proposed inherent strain formulae agree well with those computed by T.E.P. FEM, and also with reported experimental measurements.

References


