Characteristics of Stiffness and Strength of V-Joint in Tubular Offshore Structures†

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Abstract

In this study, simple(unstiffened) V-joint of tubular offshore structures for oil exploitation is investigated as one of three-dimensional joints, of which two braces exist in a plane perpendicular to the axis of the chord. The mechanical behaviors under various loads(normal force and bending moments) are analyzed by PNM(Plastic Node Method) which is one of very efficient and accurate methods for elastic-plastic analysis. Based on the results, the characteristics of the stiffness and strength of the V-joint are made clear, comparing with those of the T-joint which has the same dimensions. The elastic-plastic behavior is dependent on the loading condition, and the mechanism of influence upon the stiffness and strength is clarified. One of the most important results from this study is that the stiffness and strength of V-joint is reduced to 70% of those of T-joint in a severe loading condition. So the reinforcing method by using a stiffener is discussed.

KEY WORDS : (Tubular Offshore Structure) (Three-dimensional Joint) (V-joint) (Stiffness) (Strength) (Deformation) (Theoretical Analysis) (Plastic Node Method) (Reinforcement)

1. Introduction

The offshore structures for oil exploitation are composed of a deck and legs. Most of the legs are tubular (pipe-framed) structure and have little redundancy. The local failure of the leg may lead to overall collapse of the structure. Accordingly, it is very important to clarify the ultimate strength of the leg.

For the analysis of the strength of the leg, detailed information on the strength of a joint is necessary. However, the studies carried out so far are almost limited to the joints of which chord and braces exist in the same plane, such as Y(T)-joint and K(TY)-joint (see **Fig. 1**). While the loading condition of the K(TY)-joint is limited to brace-axial force. The conditions in the rules of API [1] are the same as described above.

In the actual structures, most of joints are of three-dimension. The loads acting from the brace to the chord are not only brace-axial force but also in-plane and out-of-plane bending moments. The mechanical behavior of such three-dimensional joint may differ from a two-dimensional joint.

In this study, a simple(unstiffened) V-joint is investigated as one of basic three-dimensional joints, of which two braces exist in the plane perpendicular to the axis of the chord(see Fig. 1). The mechanical behaviors under various loads(normal force and bending moments) are analyzed by PNM(Plastic Node Method) which is one of the very efficient and accurate FEM for elastic-plastic analysis. The characteristics of the stiffness and strength of the V-joint are clarified, being compared with those of the T-joint which has the same dimensions.

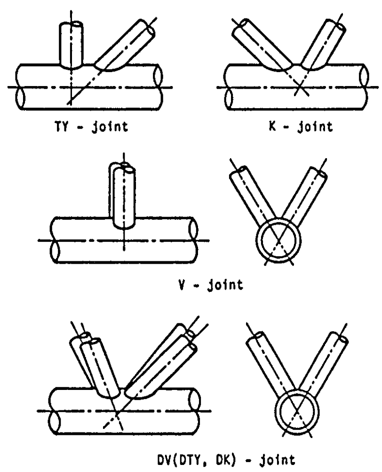

Fig. 1 K(TY), V and DV(DK, DTY)-joints

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2. V-Joint Model in Study

The V-joint model analyzed in this study is shown in **Fig. 2**. The scale is about 1/3 to the actual structure because of possibility of future experiment. The angle between two braces is 60°. The chord is not stiffened and the length is enough not to take the effect of the boundary condition(fix) of both ends. The wall thickness of the chord influences the behavior of the joint largely , and so three different thicknesses are chosen in the model.

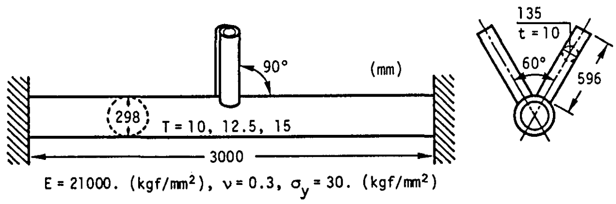


Fig. 2 V-joint model in study

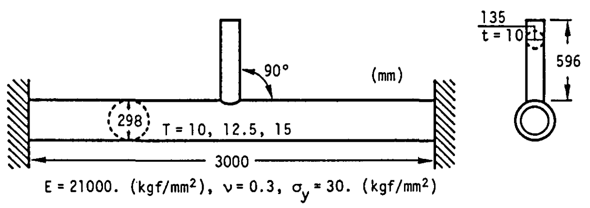


Fig. 3 T-joint model in study

A T-joint exhibits standard characteristics of the joint behavior. The T-joint model is also analyzed, which has the same sizes as the V-joint as shown in **Fig. 3**. The results are used to clarify the characteristics of the stiffness and strength of the V-joint, by comparing them.

3. Loading Conditions

The authors have been analyzing the ultimate strength of offshore structures [2)~4)]. From the results the following facts have been clarified about the stresses induced at the join between the chord and the brace. In the jack-up type, the normal stress produced by brace-axial force is important. In the jacket type, the major stresses are produced by axial force and out-of-plane bending moment from the brace. The stress by in-plane bending moment is not small and not negligible in both types. The shearing stress is small and negligible.

The loading conditions are assumed as follows on the basis of the above discussion. That is, brace-axial force and bending moments in two directions are considered in the analysis. The former is the most important load. The latters are the in-plane bending moment which acts in the plane including the chord and the brace, and the out-of-plane bending moment which acts around the axis parallel to the chord axis. These loading conditions are shown with the names in **Figs. 4 and 5**. The V-joint is analyzed for two cases in above each kind of load. One is the case where the directions of the two loads from the two braces are the same. In the other case, the directions are opposite. The magnitudes of the two loads are the same.

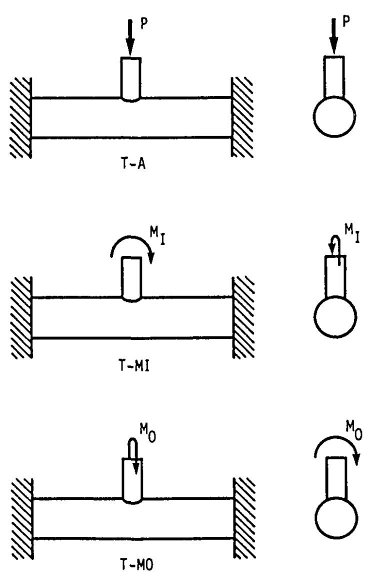


Fig. 4 Loading conditions for T-joint

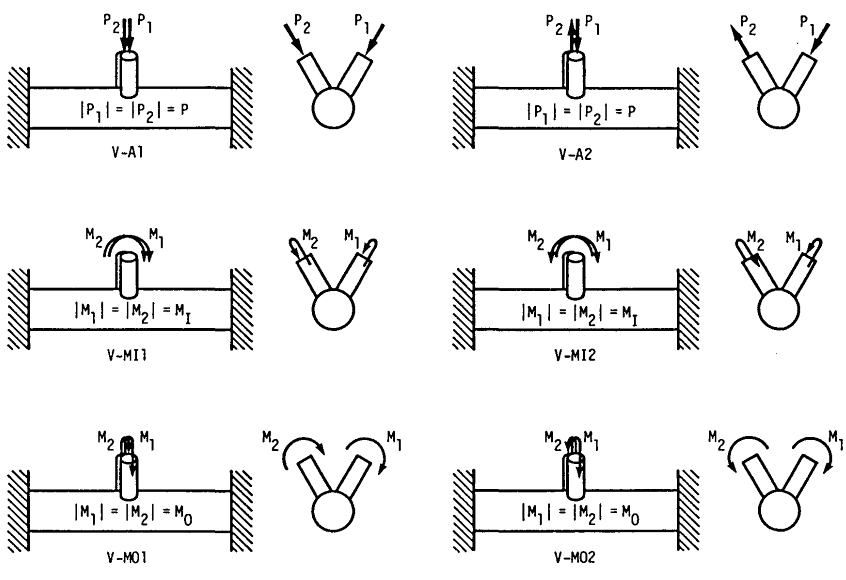


Fig. 5 Loading conditions for V-joint

4. Definitions of Local Stiffness and Yield Strength at Joint

4.1 Definition of stiffness

When a load acts on a chord from a brace, the chord is deformed locally near the joint and also as a beam. Removing the deformation as a beam, local sti^^ness of the joint is determined. The definitions for each kind of load are as follows.

*(a) Stiffness for normal force (see **Fig. 6(a)**)*

$$K_A = P / \delta = P / (\delta_T - \delta_B) \quad (1)$$

where, K_A : Normal stiffness
P : Normal force (equal to the brace-axial force in this joint)
δ : Local normal displacement
δ_T : Whole normal displacement
δ_B : Normal displacement as a beam

*(b) Stiffness for in-plane bending moment (see **Fig. 6(b)**)*

$$K_{MI} = M_I / \theta_I = M_I / (\theta_{IT} - \theta_{IB}) \quad (2)$$

where, K_{MI} : In-plane bending stiffness
M_I : In-plane bending moment
θ_I : Local in-plane rotation
θ_{IT} : Whole in-plane rotation
θ_{IB} : In-plane rotation as a beam

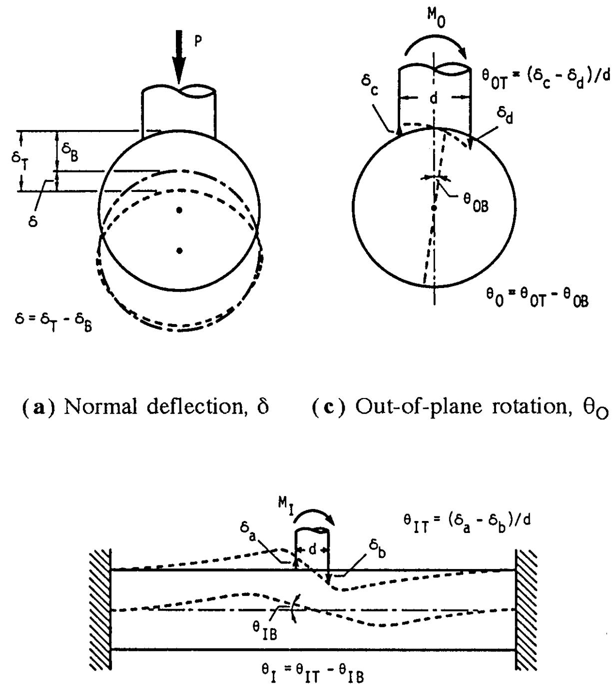
*(c) Stiffness for out-of-plane bending moment (see **Fig. 6(c)**)*

$$K_{MO} = M_O / \theta_O = M_O / (\theta_{OT} - \theta_{OB}) \quad (3)$$

where, K_{MO} : Out-of-plane bending stiffness
M_O : Out-of-plane bending moment
θ_O : Local out-of-plane rotation
θ_{OT} : Whole out-of-plane rotation
θ_{OB} : Out-of-plane rotation as a beam

4.2 Definition of yield strength

The definition of the strength of joint is important and various definitions have been made. In this study the yield strength which is considered appropriate for a measure of the strength is defined simply and clearly. That is, as shown in **Fig. 7**, the load at the time when the local stiffness of the joint decreases to 1/10 of the initial value is defined as the yield strength, P_y for normal force or M_y for bending moment. This yield strength can be determined numerically from the load-displacement curve.



(a) Normal deflection, δ (c) Out-of-plane rotation, θ_O

(b) In-plane-rotation, θ_I

Fig. 6 Definitions of local displacements (deflection and rotations) near the joint

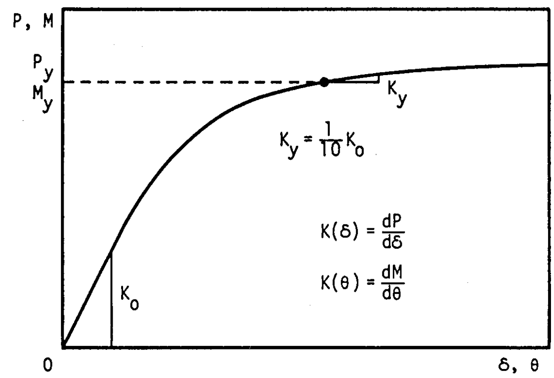


Fig. 7 Definition of yield strength in this study

5. Theoretical Analysis

5.1 Method of theoretical analysis

The theoretical analysis was performed by applying the Plastic Node Method (PNM) [5),6)]. The authors had developed the elastic-plastic element for PNM, which has the same elastic characteristic as the element of Zienkiewicz-Parekh-King [7)]. This element has been applied to the strength analysis of the joint of the offshore structure, and the accuracy of application has been confirmed from the comparison with the experimental result, beam theory, etc [8),9)]. The same element is used here.

The outline of the mesh adopted in this analysis is the following. The differences between the results by the

PNM using the mesh and the solutions by a beam theory are within a few percent in the elastic range, for the joint models shown in Figs. 2 and 3.

	The chord	Each brace
The number of elements	2200 ,	310
The number of nodal points	1100 ,	170
The number of unknowns	6600 ,	1020

The symmetricalness of the model can decrease the above numbers, depending on the loading condition.

The analysis was performed by using the incremental method. The load acting from the brace to the chord was increased little by little. The magnitude of the load increment was small enough to be able to precisely deal with the plastic behavior in each element. In consequence the number of the load increment became 70 to 200 times, depending on the model and the loading condition.

A series of calculations were carried out for 27 cases which are different in the type of the joint, the wall thickness of the chord and/or the loading condition. One of the fastest computer, NEC SX-1, of the computation center of Osaka University was used for the analyses, and the computation time (CPU time) was 2 to 3 hours per a case.

5.2 Analytical results

The load - local displacement curves are represented in **Figs. 8, 9, and 10**. In the cases of the loading conditions [V-A2] and [V-MO1] the displacements of the two joins of two braces are not always equal, but they have the almost same values in the range analyzed in this study, and have the almost same initial stiffness and yield strength.

The deformed states of the nine models with 15 mm chord wall thickness are illustrated in **Fig. 11**, which are different in the type of joint and/or the loading condition. The figures show the deformations at the top and bottom lines, and the cross sections near the center of the chord.

The initial stiffness and the yield strength of each model for each loading condition were determined from the load - local displacement relationship according to the definitions described in Chapter 4. The results are shown in the following chapters.

6. Initial Stiffness of V-Joint

The initial stiffnesses of V-joints are shown for various loads acting from the brace to the chord face, being compared with those of T-joints. Then, the mechanisms how the stiffness of the V-joint increases or decreases depending on the loading condition are clarified, based on the deformed states, etc.

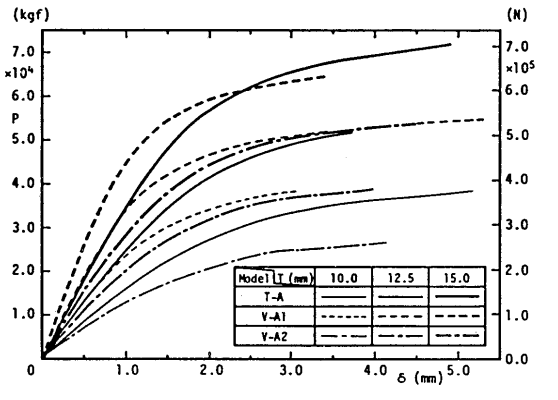


Fig. 8 Normal force (P) - deflection (δ) relationships of V and T-joints

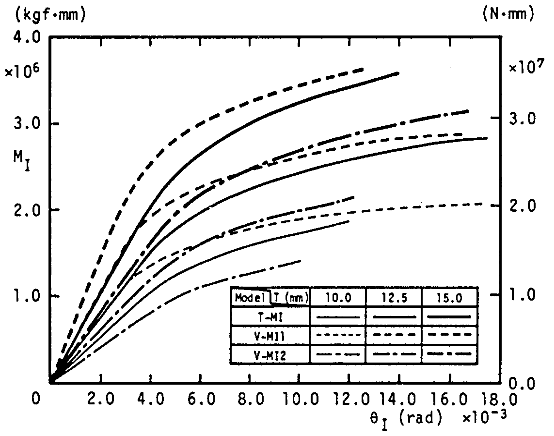


Fig. 9 In-plane bending moment (M_I) - rotation (θ_I) relationships of V and T-joints

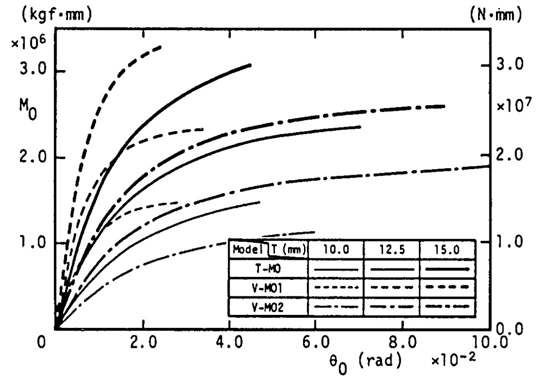
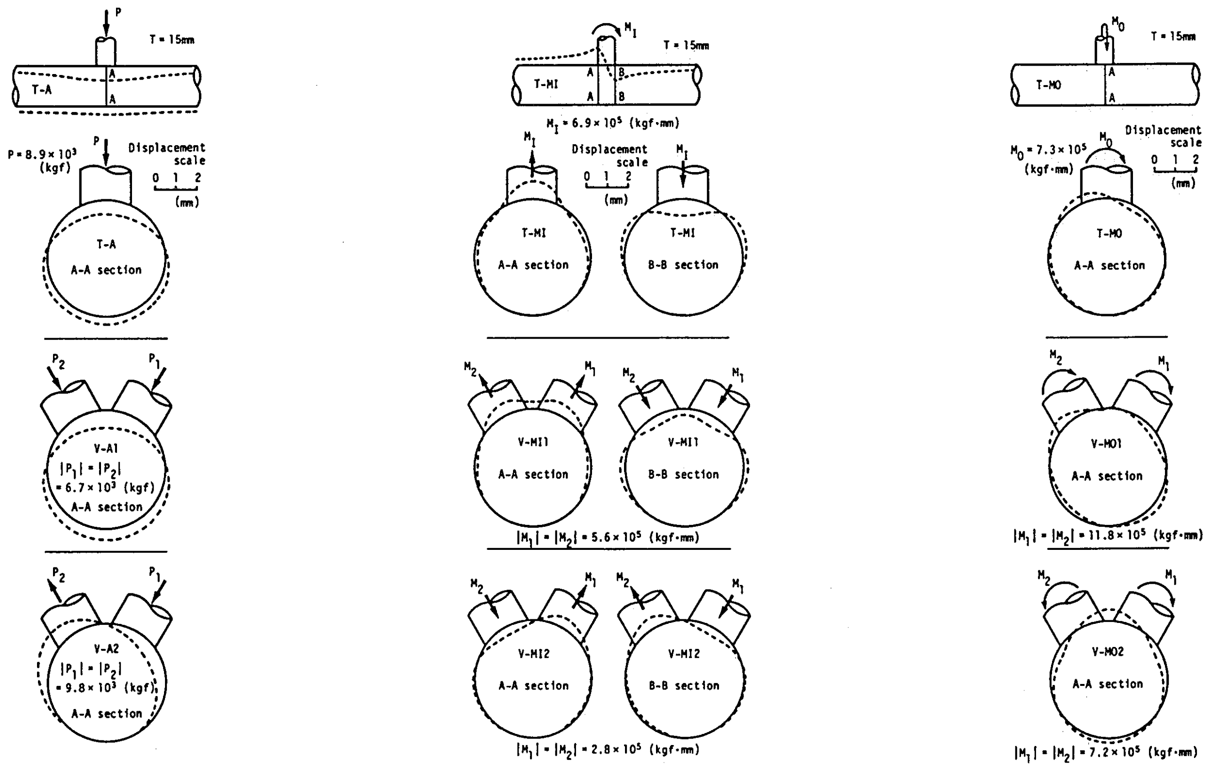


Fig. 10 Out-of-plane bending moment (M_O) - rotation (θ_O) relationships of V and T-joints

6.1 Stiffness under brace-axial force

The upper part of **Fig. 12** represents the initial stiffness value with reference to the chord wall thickness for the case where the brace-axial(normal) force acts to the joint. Regardless of a change of the wall thickness, the strongest stiffness is observed in the loading condition [V-A1], in which the compressive axial forces act from two braces to the V-joint. The second is the loading condition [T-A] for the T-joint. The stiffness becomes



(a) Deformations by normal force P

(b) Deformations by in-plane bending moment M_I

(c) Deformations by out-of-plane bending moment M_O

Fig. 11 Deformations of the chord

weakest in the loading condition [V-A2], in which a compressive axial force acts from one brace , and a tensile axial force acts from another brace.

The reason for a considerably large difference of the stiffnesses is discussed here. The features of the deformation and stress induced in the chord wall are clearly illustrated in **Table 1** for these three loading conditions.

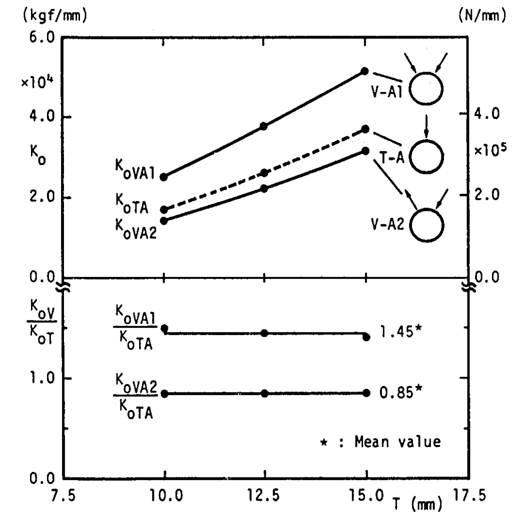


Fig. 12 Local initial stiffnesses of V and T-joints for normal force

In the loading condition [V-A1], the compressive forces from the two braces act as the in-plane compressive forces to the region on the chord wall between the respective welded zones of the two braces. The in-plane displacement under the action of this in-plane force is hard to develop in the chord wall. Therefore the displacement in the loading direction becomes smaller than the case of the T-joint in which the compression force is applied from

Table 1 Comparisons of stiffness, strength and mechanical behavior for normal force

Model & loading condition / Mechanical characteristic	V-A1	T-A	V-A2
Initial stiffness ratio	(K_{oVA1}/K_{oTA}) 1.45 $\gg$	(K_{oTA}/K_{oTA}) 1.0 $>$	(K_{oVA2}/K_{oTA}) 0.85
Yield strength ratio	(P_{yVA1}/P_{yTA}) 0.94 $\doteqdot$	(P_{yTA}/P_{yTA}) 1.0 $>$	(P_{yVA2}/P_{yTA}) 0.71
Mechanism	Compression P_2 P_1	P	P_2 P_1

only one brace, that is, the stiffness becomes higher than the one of the T-joint. In the loading condition [V-A2], the forces from the two braces induce the same deformation mode. Therefore the deformation develops more easily than the T-joint, and the stiffness reduces.

The differences among the magnitudes of stiffness according to the loading conditions enlarge with an increase of the chord wall thickness as indicated in the upper figure of Fig. 12. Here the magnitude of stiffness of the T-joint is considered as the standard. The dimensionless stiffnesses are shown in the lower figure of Fig. 12, which are calculated by dividing each value by the stiffness of the T-joint with the same thickness. It turns out that the dimensionless stiffnesses (stiffness ratios) are almost constant for a change of the chord wall thickness. The same tendency is observed in the other

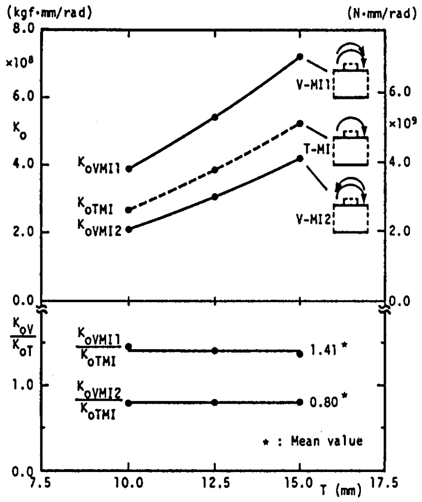


Fig. 13 Local initial stiffnesses of V and T-joints for in-plane bending moment

Table 2 Comparisons of stiffness, strength and mechanical behavior for in-plane bending moment

Model & loading condition / Mechanical characteristic	M_2, M_1 V-MI1	M_I T-MI	M_2, M_1 V-MI2
Initial stiffness ratio	(K_{oVMI1}/K_{oTMI}) 1.41 ≫	(K_{oTMI}/K_{oTMI}) 1.0 >	(K_{oVMI2}/K_{oTMI}) 0.80
Yield strength ratio	(M_{yVMI1}/M_{yTMI}) 0.97 ≒	(M_{yTMI}/M_{yTMI}) 1.0 >	(M_{yVMI2}/M_{yTMI}) 0.83
Mechanism	Tension M_2 M_1 Compression	M_I	A M_2 A B M_1 B

loading conditions and also in the yield strength as shown below.

6.2 Stiffness under in-plane bending moment

The initial stiffness of joint under the in-plane bending moment acting from the brace is illustrated in **Fig. 13.** The large differences are observed in these stiffnesses, and the order of magnitude is dependent upon the following loading conditions [V-MI1], [T-MI] and [V-MI2]. The features of the deformation and stress induced in the chord wall are simply illustrated in **Table 2** for these three loading conditions. The reason for the considerably large differences of the stiffnesses is the same as in the case of the brace-axial force as shown in Table 2.

6.3 Stiffness under out-of-plane bending moment

The initial stiffness under the out-of-plane bending moment is illustrated in **Fig. 14**. The large differences

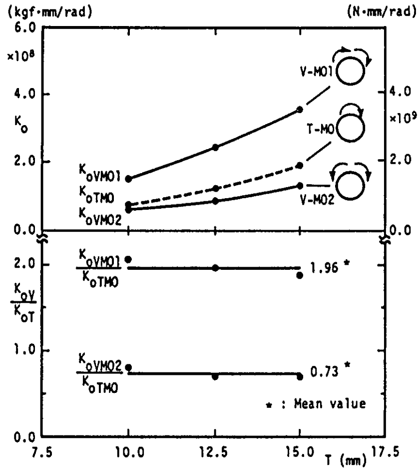


Fig. 14 Local initial stiffnesses of V and T-joints for out-of-plane bending moment

Table 3 Comparisons of stiffness, strength and mechanical behavior for out-of-plane bending moment

Model & loading condition / Mechanical characteristic	M_2, M_1 V-MO1	M_0 T-MO	M_2, M_1 V-MO2
Initial stiffness ratio	(K_{oVMO1}/K_{oTMO}) 1.96 ≫	(K_{oTMO}/K_{oTMO}) 1.0 >	(K_{oVMO2}/K_{oTMO}) 0.73
Yield strength ratio	(M_{yVMO1}/M_{yTMO}) 1.01 ≒	(M_{yTMO}/M_{yTMO}) 1.0 >	(M_{yVMO2}/M_{yTMO}) 0.78
Mechanism	M_2 M_1 Shearing	M_0	M_2 M_1

are observed in these values, and the order of magnitude may be described by the loading conditions as follows : [V-MO1], [T-MO] and [V-MO2]. The feature of the deformation induced in the chord wall are illustrated in **Table 3** for these three loading conditions. The reason for the considerably large differences of the stiffnesses is the same as in the cases treated above.

The bending stiffness under the out-of-plane moment is small, that is, 30 to 45 % of the one for the in-plane moment.

7. Yield Strength of V-Joint

The yield strengths of V-joints are shown for the cases where various loads act from the brace to the chord face, being compared with those of T-joints. The magnitude of the yield strength of V-joint is fundamentally dependent upon the loading condition in the same manner as those for the stiffness described in the Chapter 6.

7.1 Yield strength under brace-axial force

The yield strength of joint under the brace-axial force is illustrated in the upper part of **Fig. 15**, of which horizontal axis indicates the chord wall thickness. The stiffness under the loading condition [V-A1] is considerably larger than the one of the loading condition [T-A]. The yield strength of [V-A1] is almost the same or rather small, being compared with the one of [T-A]. In fact, the axial compressive forces with the same magnitude are transmitted from the two braces in the case of the loading condition [V-A1], and the resultant force acting to the whole joint is $\sqrt{3}$ times of the value shown in this figure. In this point of view the yield strength under the loading condition [V-A1] is larger than under the loading condition [T-A], similarly to the stiffness. The yield strength in the loading condition [V-A2] is considerably smaller than those strengths. These reasons are the same as in the case of the stiffness as shown in Table 1.

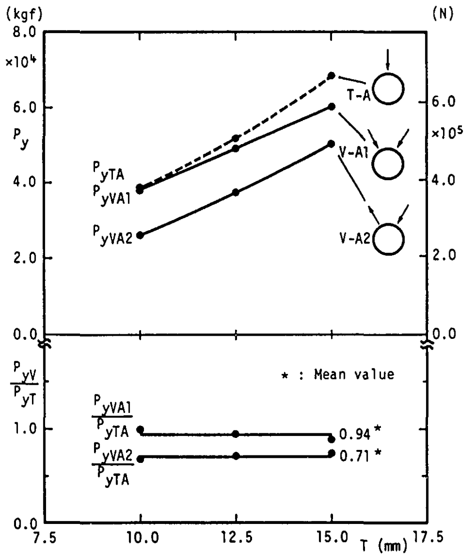


Fig. 15 Local yield strengths of V and T-joints for normal force

Here the magnitude of the yield strength of T-joint is considered as the standard. The dimensionless yield strengths are shown in the lower part of Fig. 15, which are calculated by dividing each value by the strength value of the T-joint with the same chord wall thickness. It turns out that the dimensionless yield strengths (strength ratios) are almost constant for a change of the chord wall thickness, similarly to the stiffness. The same tendency is observed in the other loading conditions.

7.2 Yield strength under in-plane bending moment

The yield strength of joint under the in-plane bending moment acting from the brace is illustrated in **Fig. 16**. The stiffness under the loading condition [V-MI1] is considerably larger than the one under the loading condition [T-MI]. Their yield strengths are almost the same. The actual behavior is the same as explained in the foregoing section, that is, the in-plane moments with the same magnitude are transmitted from the two braces in the case of the loading condition [V-MI1], and the resultant moment acting to the whole joint is $\sqrt{3}$ times of the magnitude shown in this figure. The yield strength under the loading condition [V-MI2] is smaller than those strengths. These reasons are the same as in the case of the stiffness as shown in Table 2.

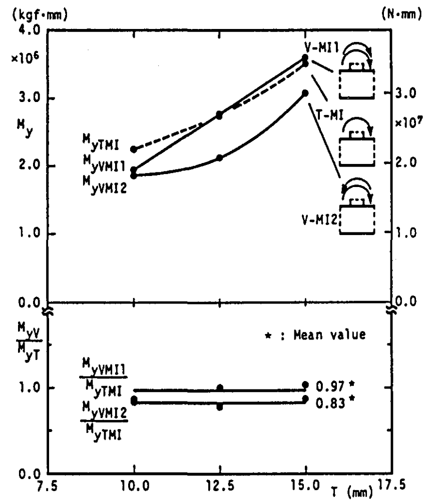


Fig. 16 Local yield strengths of V and T-joints for in-plane bending moment

7.3 Yield strength under out-of-plane bending moment

The yield strength under the out-of-plane bending moment is illustrated in **Fig. 17**. The stiffness under the loading condition [V-MO1] is considerably larger than that under the loading condition [T-MO]. Their yield strengths are almost the same. In the loading condition [V-MO1], the resultant moment acting to the whole joint is twice as much as the magnitude shown in this figure. The yield strength under the loading condition [V-MO2] is considerably smaller than those strengths. These reasons are the same as in the case of the stiffness as shown in Table 3.

The bending yield strength under the out-of-plane moment is a little smaller, that is, 70 to 80 % of the one under the in-plane moment. The same tendency is observed in the T-joint.

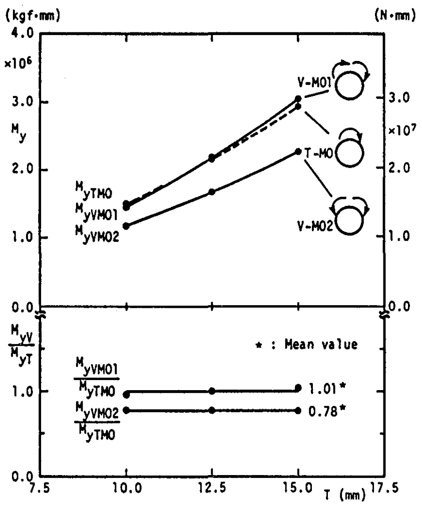


Fig. 17 Local yield strengths of V and T-joints for out-of-plane bending moment

8. Method of Reinforcement of V-Joint

8.1 Influence of loading condition upon stiffness and yield strength

The mechanisms of influence of the loading conditions upon the stiffness and yield strength of the V-joint are similar in all cases, as described in Chapters 6 and 7. If the load transmitted from one brace acts so as to restrict the deformation of the chord around the welded zone of another brace, the stiffness and yield strength of the V-joint are larger than those of the T-joint in which the load is transmitted from only one brace. On the contrary, if these forces amplify the respective deformation each other, the stiffness and yield strength of the V-joint are smaller than those of the T-joint. The loading conditions [V-A1], [V-MI1], [V-MO1] are the former(stronger) cases. The loading conditions [V-A2], [V-MI2], [V-MO2] are the latter(weaker) cases.

An attention should be paid to the fact that the stiffness and yield strength of the V-joint decrease to about 70 to 85 % of those of the T-joint in the severe and serious loading conditions for the V-joint. The ratios of these characteristic values are almost constant, even if the chord wall thickness changes considerably. For the safe design of the offshore structure, the stiffness and strength of the V-joint have to be evaluated for these severe cases.

8.2 Consideration of method of reinforcement of V-joint

A method of reinforcement to increase the stiffness and yield strength of the V-joint is considered on the basis of the results of computation and discussions described above. The main purpose of the reinforcement is to increase the stiffness and strength which are smaller than the T-joint under the loading conditions [V-A2], [V-MI2] and [V-MO2].

The most common method is to provide a ring, a rib and a gusset. Here some basic methods of reinforcement shown in **Table 4** are examined. The stiffener is considered to most effectively restrict the deformation of the chord in the plane of the stiffener. The deformation states under the loading conditions [V-A2], [V-MI2] and [V-MO2] are shown in Fig. 11 and Tables 1, 2, 3. When the stiffener restricts the deformation, the stiffness and yield

Table 4 Effectiveness of various stiffeners on stiffness and strength of V-joint

Type of stiffener / Loading condition	Ring Type-R1	Ring Type-R2	Gusset Type-G	Center rib Type-C1	Center rib Type-C2	Center rib Type-C3
Normal force V-A2	○	○		○	×	
In-plane bend. V-MI2	○	×		○	×	
Out-of-plane V-MO2	○	○		×	○	

○ : Effective, × : A little effective

strength increase greatly, especially those in the in-plane of the stiffener. On the other hand, if the stiffener is installed at the position where the magnitude of deformation is small, the effect is thought to be small.

Each reinforcement is evaluated, estimating the restraint effect of the stiffener on the deformation. The effectiveness of these reinforcements is classified broadly into two grades as indicated in Table 4. Sufficient effective method is shown by the " O " mark, and low effective one shown by the " X " mark. The stiffeners of Type-R1 is estimated to work most efficiently.

9. Conclusion

In this study, a simple(unstiffened) V-joint of tubular offshore structures was investigated as one of three-dimensional joints. The mechanical behaviors under various loads (normal force and bending moments) were analyzed by PNM(Plastic Node Method) which is one of the very efficient and accurate FEM for elastic-plastic analysis.

Based on the results, the characteristics of the stiffness and strength of the V-joint were clarified, being compared with those of the T-joint which has the same dimensions. The main achievements are as follows.

(1) The stiffness and yield strength of the V-joint were evaluated for various loading conditions by the analysis, and compared with those of the T-joint.

(2) The influences of the loading conditions upon the stiffness and yield strength of the V-joint are similar in all cases. If the load transmitted from one brace acts to restrict the deformation of the welded zone of another brace respectively, the stiffness and yield strength of the V-joint become greater than the ones of the T-joint in which the load is applied from only one brace. On the contrary, if they act to amplify the deformation each other, the stiffness and yield strength of the V-joint become smaller than the ones of the T-joint.

(3) It should be paid attention that the stiffness and yield strength of the V-joint decrease to about 70 to 85 % of the T-joint in the severe and serious loading conditions (that is, the latter case in the above (2)) for the V-joint. The reduction ratios of these characteristic values are almost constant, even if the chord wall thickness changes considerably. They may be used to estimate the stiffness and yield strength of the V-joint from those of the T-joint.

(4) For the safe design, the stiffness and strength of the V-joint have to be evaluated accurately for the cases where they are weaker than the ones of the T-joint as above-mentioned description.

(5) Lastly, the method of reinforcing the V-joint is considered for the severe loading conditions. As an effective method, reinforcement by two rings is selected, referring to the deformation states of the chords.

The mechanisms of influence of loading condition upon the stiffness and strength and the discussion about the method of reinforcement for the V-joint can be applied to the other complex joints such as K-joint(TY-joint), DV-joint(DK-joint, DTY-joint).

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