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An Efficient Method for Estimation of Reduction of Welding Residual Stresses from Stress-Relief Annealing (Report III) †

- Development of Estimation Equations for Multi-Axial Stress State in Thick Welded Joint -

Keiji NAKACHO* and Yukio UEDA**

Abstract

Stress-relief annealing (SR treatment) is included in the fabrication process of welded structures such as pressure vessels. This study aims to develop a simple analytical method which facilitates the calculation of transient and residual stresses during SR treatment, in order to easily determine the reasonable conditions of SR treatment for the recent large-size structures of high quality thick plates.

In the first report, estimation equations for uni-axial stress states were formulated for relaxation tests at changing and constant temperatures. In the second report, the stresses relaxed by SR treatment in the thick welded joint were analyzed accurately by the finite element method based on thermal elastic-plastic creep theory. The characteristics of the changes of the welding residual stresses in multi-axial stress states were studied in detail. In this report, estimation equations in multi-axial stress states are developed for thick welded joints, based on the above characteristics.

KEY WORDS : (Simple Estimation Method) (Stress-Relief Annealing) (Welding) (Thick Welded Joint) (Transient Stress) (Residual Stress) (Thermal Elastic-Plastic Creep Analysis)

1. Introduction

Stress-relief annealing (hereinafter called SR treatment), whose main purpose is to relieve welding residual stresses, is included in the fabrication process of welded structures such as pressure vessels. Standard conditions for SR treatment are specified in JIS, ASME code, etc. For recent large-size structures constructed of high quality thick plates, the specified conditions require keeping the structures at a high temperature, near 700 °C, for a long time in proportion to the thickness. Repeating this treatment in the fabrication and repair processes may degrade the qualities of the steel and the joint. The main reason for the difficulty in specifying rational conditions for SR treatment is that the real effect of SR treatment on stress relief (that is, the real change of welding residual stresses during SR treatment) is not fully known, especially for thick joints. So severer conditions for SR treatment are specified for safety from the viewpoint of residual stress.

On the other hand, the available theory [1), 2)] of thermal elastic-plastic creep analysis based on the finite element method had been developed, and with the theory, the changes of stresses in the welded joint of a very thick plate due to SR treatment have been computed [2)]. However, the theory and computer program for analysis are so complicated that it is not easy to perform the analysis. Furthermore, for the three-dimensional problem, very long CPU time is necessary for computation even with a super computer. Each analyzed result does not always have the essentials of the behavior or the mechanism.

This study aims to develop an analytical method (as simple as hand calculation) which facilitates the calculation of transient and residual stresses during SR treatment in order to determine easily the reasonable conditions for SR treatment by parametric study. For this purpose, some relations between stresses and strains during SR treatment are idealized, and the estimation equations are developed. In the first report [3)], estimation equations for the uni-axial stress state were formulated for relaxation tests at changing and constant temperatures, which present phenomena similar to the stress relaxation during SR treatment. It was shown that the analytical results of simple calculations using these equations are highly

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accurate. In the second report [4)], the stresses relaxed by SR treatment in thick welded joints were analyzed accurately by the finite element method based on thermal elastic-plastic creep theory. The characteristics of the changes of the welding residual stresses in the multi-axial stress state were studied in detail.

In this report, estimation equations are developed, which can calculate the changes of welding residual stresses of thick welded joints in SR treatment. The simple analysis method for uni-axial stress state shown in the first report is expanded and applied for the multi-axial stress state. The characteristics of stress relaxation phenomena in SR treatment of thick welded joints, obtained in the second study, are then utilized.

2. Characteristics of Stress and Strain States in Thick Welded Joint during SR Treatment

The butt welded joint of thick plane plates, which is in the three-axial stress state, is the object of this study. Estimation equations to obtain the residual stresses after SR treatment by a simple method are derived. In the first report, relaxation tests at changing and constant temperatures were examined. The stress state of this case was uni-axial. During the test, the end of the specimen was moved in the axial direction so that the quantity of change of total strain in the axial direction would agree with the quantity of thermal strain. On the other hand, in the welded joint under SR treatment, the stresses of each point are in multi-axial stress state. The quantity of change of total strain of each point doesn't usually agree with the quantity of thermal strain of the point.

In this report, the changes of stress of the welded joint in such stress and strain states are examined. Estimation equations to obtain transient and residual stresses during SR treatment are developed. The procedure is fundamentally the same as the first report. The equation development will be described in the following chapters, comparing newly derived equations with the equations for the relaxation tests in the first report, and showing the differences between them.

3. Assumptions and Basic Equations

The stress in one direction at one position in the welded joint under SR treatment is considered first (the stresses in the other directions will be described later). This is usually the maximum tensile stress component at the position where the most attention for cracking is needed. The direction is set up as the x direction, and it is treated as $\sigma_x > 0$ (the stress in x direction is positive, that is, tensile), $\varepsilon^e_x > 0$ (the elastic strain in x direction is positive), $\varepsilon^c_x > 0$ (the creep strain in x direction is positive) for convenience. It is assumed that the plastic deformation is not newly produced as in usual SR treatments. As the quantity of change of total strain during SR treatment doesn't always agree with the quantity of thermal strain, the relations between various strains in the x direction and the relations between various strain rates are expressed with the next equations. For the holding stage of temperature, the thermal strain in the equations is set as $\varepsilon^T_x = {}^r\varepsilon^T_x = 0$.

$$\varepsilon_x = (\varepsilon^e_x - \varepsilon^e_{x0}) + \varepsilon^c_x + \varepsilon^T_x \quad (1)$$

Where ε_x : Total strain in x direction produced during SR treatment afresh

$\varepsilon^e_x, \varepsilon^e_{x0}$: Elastic strain in x direction and the initial value

ε^c_x : Creep strain in x direction

ε^T_x : Thermal strain in x direction

Equation (1) may be transformed into

$$\varepsilon^c_x = \varepsilon^e_{x0} - \varepsilon^e_x + \varepsilon_x - \varepsilon^T_x \quad (1)'$$

As the relation between these strain rates

$${}^r\varepsilon^c_x = -{}^r\varepsilon^e_x + {}^r\varepsilon_x - {}^r\varepsilon^T_x \quad (2)$$

Where ${}^r\varepsilon^c_x, {}^r\varepsilon^e_x, {}^r\varepsilon_x, {}^r\varepsilon^T_x$: Rate of each strain (1/min) (Left upper suffix **r** indicates the rate)

Creep strain rate in the upper equation is different depending on the material and the temperature. In this study, the material of the welded joint has been 2 1/4 Cr - 1 Mo steel from the first report. This material obeys strain-hardening theory in the temperature range below 575 °C, and obeys power theory in the temperature range above 575 °C [2)]. The creep constitutive equations for these creep hardening rules are expressed in multi-axial stress state, using the Mises'es form, as follows.

(a) Creep strain rate below 575 °C (Strain-hardening theory)

$$\{{}^r\varepsilon^c\} = (3/2)\, m\, A^{1/m}\, ({}^E\sigma)^{\gamma/m-1} ({}^E\varepsilon^c)^{1-1/m} \{{}^d\sigma\} \quad (3)$$

Where $\{{}^r\varepsilon^c\}$: Creep strain rate vector

${}^E\sigma$: Equivalent stress

${}^E\varepsilon^c$: Equivalent creep strain

$\{{}^d\sigma\}$: Deviatoric stress vector

m, A, γ : Creep constants which have temperature dependency (See Ref. 2)

(b) Creep strain rate above 575 °C (Power theory)

$$\{{}^r\varepsilon^c\} = (3/2)\, \beta\, ({}^E\sigma)^{n-1} \{{}^d\sigma\} \quad (4)$$

Where β, n : Creep constants which have temperature dependency (See Ref. 2)

The component of x direction of Eq. (3) or Eq. (4) becomes the left member of Eq. (2). Those components are

(a) Creep strain rate in x direction below 575 °C
(Strain-hardening theory)

$$^{r}\varepsilon^{c}_{x} = (3/2)\, m\, A^{1/m} (^{E}\sigma)^{\gamma/m-1} (^{E}\varepsilon^{c})^{1-1/m}\ ^{d}\sigma_{x} \quad (5)$$

(b) Creep strain rate in x direction above 575 °C
(Power theory)

$$^{r}\varepsilon^{c}_{x} = (3/2)\, \beta\, (^{E}\sigma)^{n-1}\ ^{d}\sigma_{x} \quad (6)$$

Finally, the relation between the temperature and the time in the heating stage can be expressed as follows when the heating rate is constant.

$$T = T_0 + v\,t \quad (7)$$

Where T : Temperature (°C)
T_0 : Initial temperature (°C)
v : Heating rate (°C/min)
t : Time (min)

Equations (1)', (2), (5), (6) and (7) expressed above are the basic equations.

4. Development of Estimation Equations - 1

Using the basic equations shown in the previous chapter, estimation equations are developed in the same procedure that treated relaxation tests at changing and constant temperatures in the first report. In the development, some coefficients are introduced as follows, and various stresses and strains are converted.

$$^{E}\sigma = S_1 \sigma_x,\quad ^{d}\sigma_x = S_2 \sigma_x,$$
$$^{E}\varepsilon^{c} = S_3 \varepsilon^{c}_{x},\quad \sigma_x = S_4 E \varepsilon^{e}_{x} \quad (8)$$

$$\varepsilon^{c}_{x} = \varepsilon^{e}_{x0} - \varepsilon^{e}_{x} + \varepsilon_x - \varepsilon^{T}_{x} = b_1 (\varepsilon^{e}_{x0} - \varepsilon^{e}_{x}),$$
$$^{r}\varepsilon^{c}_{x} = -^{r}\varepsilon^{e}_{x} + ^{r}\varepsilon_x - ^{r}\varepsilon^{T}_{x} = -b_2\, ^{r}\varepsilon^{e}_{x} \quad (9)$$

The above transformation coefficients have the next roles. The stress state in the relaxation tests was uni-axial, but the state in the welded joint is multi-axial. Coefficients $S_1 \sim S_4$ are transformation coefficients to express the stresses and strains in multi-axial stress state only with a component of x direction. In the relaxation tests, the production of creep strain and the decrease of elastic strain are equal in quantity, but such strain change does not generally occur in the welded joint. Coefficients b_1 and b_2 are introduced as the ratio of the two quantities. Accordingly, each coefficient value becomes as $S_1 = S_3 = S_4 = b_1 = b_2 = 1$, $S_2 = 2/3$ when the stress and strain states agree with the relaxation tests.

In addition to the elastic strain ε^{e}_{x} in the x direction, the elastic strains ε^{e}_{y} and ε^{e}_{z} are necessary to obtain the stress σ_x in the x direction from elastic strains in the multi-axial stress state. The treatment for that case is complex. So, the equivalent elastic strain ε^{e}_{x1} in the x direction is defined as follows (hereinafter this strain is merely called the equivalent elastic strain). Multiplying this strain ε^{e}_{x1} merely by an elastic coefficient E, σ_x can be calculated like in the case of uni-axial stress state.

$$\varepsilon^{e}_{x1} \equiv \sigma_x / E \quad (10)$$

The relation between this equivalent elastic strain ε^{e}_{x1} and the true elastic strain ε^{e}_{x} in the x direction can be expressed with the next equation using coefficient S_4 in the fourth equation in Eqs. (8).

$$\varepsilon^{e}_{x1} = S_4 \varepsilon^{e}_{x} \quad (11)$$

As a result, the stress σ_x in the x direction can be given from Eq. (10) promptly when ε^{e}_{x1} is obtained. With the introduction of ε^{e}_{x1}, the next two transformations are added.

$$(\varepsilon^{e}_{x10}/S_{40} - \varepsilon^{e}_{x1}/S_4) = S_5 \{(\varepsilon^{e}_{x10} - \varepsilon^{e}_{x1})/S_4\},$$
$$(1/S_4)\, ^{r}\varepsilon^{e}_{x1} + ^{r}(1/S_4)\, \varepsilon^{e}_{x1} = S_6 (1/S_4)\, ^{r}\varepsilon^{e}_{x1} \quad (12)$$

Where ε^{e}_{x10} : Initial value of ε^{e}_{x1} before heating
S_{40} : Initial value of S_4 determined by the stress state before heating

The estimation equations are developed with the following procedure using various equations shown here. At first, on the basis of Eq. (2), creep strain rate of the left side is expressed with Eq. (5) or Eq. (6) depending on the temperature range. Using coefficients in Eqs. (8) and (9), all strain and stress components in the above transformed equations are replaced in the elastic strain ε^{e}_{x} in the x direction. Furthermore, by means of the Eqs. (11) and (12), ε^{e}_{x} is expressed with equivalent elastic strain ε^{e}_{x1}. Transforming these equations, we can separate them into the left members only of material constants and the right members of the functions of equivalent elastic strain ε^{e}_{x1}. And, those equations are integrated by time. In the left members of material constants, the integration by time is replaced with the integration by temperature, based on the relation of Eq. (7). In the right members, the integration by time is replaced with the integration by equivalent elastic strain, based on the relation of $({}^{r}\varepsilon^{e}{}_{x1})dt = (d\varepsilon^{e}{}_{x1}/dt)dt = d\varepsilon^{e}{}_{x1}$. As a result, next two equations are obtained. These equations correspond to Eqs. (7) and (8) in the first report.

(a) In the temperature range below 575 °C

(In obedience to strain-hardening theory)

$$\int_{T_{ss}}^{T_{es}} m A^{1/m} E^{\gamma/m} dT = -\frac{2}{3} v \int_{\varepsilon^{e}{}_{x10s}}^{\varepsilon^{e}{}_{x1}} S_1^{1-(\gamma/m)} S_2^{-1} \times S_3^{(1/m)-1} S_4^{-1/m} S_5^{(1/m)-1} S_6 b_1^{(1/m)-1} b_2 \times (\varepsilon^{e}{}_{x1})^{-\gamma/m} (\varepsilon^{e}{}_{x10} - \varepsilon^{e}{}_{x1})^{(1/m)-1} d\varepsilon^{e}{}_{x1} \quad (13)$$

Where

T_{ss} : The temperature at which creep strain begins to be markedly produced according to strain-hardening theory (In the case of this material, the creep strain is produced according to this strain-hardening theory at first, and it is regarded as $T_{ss} = 400$ °C)

T_{es} : The temperature at which we want to estimate equivalent elastic strain $\varepsilon^{e}{}_{x1}$

$\varepsilon^{e}{}_{x10s}$: Initial value of equivalent elastic strain when creep strain begins to be produced according to strain-hardening theory ($\varepsilon^{e}{}_{x10s} = \varepsilon^{e}{}_{x10}$)

(b) In the temperature range above 575 °C

(In obedience to power theory)

$$\int_{T_{sp}}^{T_{ep}} \beta E^{n} dT = -\frac{2}{3} v \int_{\varepsilon^{e}{}_{x10p}}^{\varepsilon^{e}{}_{x1}} S_1^{1-n} S_2^{-1} S_4^{-1} S_6 b_2 \times (\varepsilon^{e}{}_{x1})^{-n} d\varepsilon^{e}{}_{x1} \quad (14)$$

Where

T_{sp} : The temperature at which creep strain begins to be produced according to power theory (T_{sp} =575 °C)

T_{ep} : The temperature at which we want to estimate equivalent elastic strain $\varepsilon^{e}{}_{x1}$

$\varepsilon^{e}{}_{x10p}$: Initial value of $\varepsilon^{e}{}_{x1}$ when creep strain begins to be produced according to power theory

5. Development of Estimation Equations - 2

The development of estimation equations has advanced to Eqs. (13) and (14), but creep constants m, γ, n and coefficients $S_1 \sim S_6$, b_1, b_2 are contained in the integral term of equivalent elastic strain of right hand members of those equations. Creep constants m, γ and n are assumed to be constant in the domain of integration because their temperature-dependencies are comparatively small, as described in the first report. It was confirmed in the first report that the accuracy of solution hardly drops by this approximation. Coefficients $S_1 \sim S_6$ and b_1, b_2 generally change in the domain of integration too, but those coefficients do not have the functional relation with equivalent elastic strain $\varepsilon^{e}{}_{x1}$. Accordingly, it is impossible to integrate the term of equivalent elastic strain, considering those changes of the coefficients.

Coefficients $S_1 \sim S_6$ were introduced because the stress state was not uni-axial, but multi-axial. And, coefficients b_1 and b_2 were introduced because the quantity of production of creep strain and the quantity of decrease of elastic strain aren't equal. The relation between these coefficients and those quantities (stresses, strains) are investigated here, and the changes of stresses and strains are idealized to continue to develop the equations.

5.1 Idealizations of changes of stresses and strains

5.1.1 Relation between coefficients $S_1 \sim S_6$ and stress change

Coefficients $S_1 \sim S_6$ change with stresses in a complicated fashion, and it is impossible to predict the changes and to express them. The result of theoretical analysis by the finite element method was shown in the second report. Transient stresses in the welded joints of thick plates during SR treatment were shown in Fig. 6 ~ Fig. 9. When there is a comparatively large tensile stress component in addition to the maximum tensile stress, the magnitudes of stress decrease, keeping the ratio of the magnitudes almost constant (See Figs. 8 (a) and 9 (a) in the second report). So it is assumed that the stress ratio doesn't change. In this case, coefficients $S_1 \sim S_4$ continue to take constant values respectively. When the initial stress (welding residual stress) component ratio is given, the values of coefficients S_1, S_2 and S_4 can be obtained from Eq. (8) promptly. Coefficient S_3 is expressed as follows, in the case that Eqs. (3) and (4) of Mises'es form are used as creep constitutive equation in the multi-axial stress state.

$$S_3 = \left[\frac{2}{9}\left\{\left(1 - \frac{{}^{d}\sigma_y}{{}^{d}\sigma_x}\right)^2 + \left(\frac{{}^{d}\sigma_y}{{}^{d}\sigma_x} - \frac{{}^{d}\sigma_z}{{}^{d}\sigma_x}\right)^2 + \left(\frac{{}^{d}\sigma_z}{{}^{d}\sigma_x} - 1\right)^2\right\} + \frac{4}{3}\left\{\left(\frac{\tau_{xy}}{{}^{d}\sigma_x}\right)^2 + \left(\frac{\tau_{yz}}{{}^{d}\sigma_x}\right)^2 + \left(\frac{\tau_{zx}}{{}^{d}\sigma_x}\right)^2\right\}\right]^{1/2} \quad (15)$$

Where ${}^{d}\sigma_x$, ${}^{d}\sigma_y$, ${}^{d}\sigma_z$: Deviatoric stresses in respective directions

The other coefficients S_5 and S_6 become $S_5 = S_6 = 1$ because the value of coefficient S_4 doesn't change.

5.1.2 Relation between coefficients b_1, b_2 and strain change

The relation between coefficients b_1, b_2 and strain change is examined next. From Eq. (9), this relation doesn't have the direct connection with the stress state, that is, uni-axial stress state or multi-axial stress state. So, it can be investigated in the uni-axial stress state. Here it is examined using the analytical model in uni-axial stress state shown in Fig. 1. Figure 1(a) shows the butt welding of thin plates. As a result, in the section of the model, residual stress σ_1 (tensile stress) and σ_2 (compressive stress) in the welding direction are produced and there is no stress in the other directions. Figure 1(b) shows the case where two thick plates are welded with multi-layer butt welding, symmetrical top and bottom. As a result, only residual stress σ_1 (tensile stress) and σ_2 (compressive stress) in the welding direction are produced. The welded joints like the ones above, where the residual stresses are in a uni-axial stress state and the stresses are in self-balance, may be represented by the mechanical model shown in Fig. 1(c).

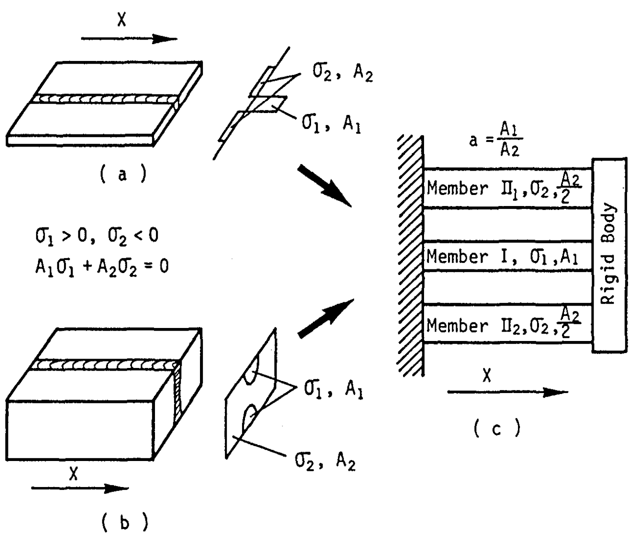


Fig. 1 Idealized mechanical model of butt welded joint

The sectional area of the central member I in the model is A_1, and the total sectional area of members II of the both sides is A_2. Both ends of members I and II are connected by a rigid wall and a rigid body respectively. Tensile stress σ_1 is produced in the member I, and compressive stress σ_2 is produced in the members II of both sides. These stresses are in self-balance in the model, that is, $\sigma_1 A_1 + \sigma_2 A_2 = 0$. The material of the model is 2 1/4 Cr - 1 Mo steel.

The case where SR treatment is performed on the model which has the above residual stresses will be discussed. In each member, the produced total strain in the axial direction does not generally agree with the quantity of thermal strain in the axial direction. (Two kinds of strains agree only in the case of $A_1 = A_2 \rightarrow \sigma_1 = -\sigma_2$, and the model shows the same behavior as the relaxation test). In other words, the quantity of decrease of elastic strain is different from the quantity of production of creep strain. But, the ratio of quantities of production of creep strain and decrease of elastic strain becomes a constant during SR treatment because creep constants γ and n are assumed to be constant. In this case, coefficient b_1 agrees with b_2, and they continue taking the following constant value in each member.

(a) In the temperature range that the material obeys strain hardening theory

$$b_1 = b_2 = (1+a)/(1+a^{\gamma}) \equiv b_{1s} \quad \text{for member I}$$
$$b_1 = b_2 = (1+a^{-1})/(1+a^{-\gamma}) \quad \text{for member II} \qquad (16)$$

Where $a = A_1 / A_2$

A_1 : Sectional area of the domain where tensile stress is produced

A_2 : Sectional area of the domain where compressive stress is produced

(b) In the temperature range that the material obeys power theory

$$b_1 = b_2 = (1+a)/(1+a^{n}) \equiv b_{1p} \quad \text{for member I}$$
$$b_1 = b_2 = (1+a^{-1})/(1+a^{-n}) \quad \text{for member II} \qquad (17)$$

There is no external force acting on the joint in SR treatment of real structures, and the stresses are in self-balance as in the section of this model. So, coefficients b_1 and b_2 are assumed to take constant values in the temperature range of each creep law.

5.2 Approximate estimation equations

In the above section 5.1, the creep constants and the changes of stresses and strains in SR treatment were idealized as follows.

1) In the creep constants, m, γ and n take constant values in the temperature range for each creep law.

2) Coefficients S_1 ~ S_6 always take constant values, and coefficients b_1 and b_2 take constant values in the temperature range for each creep law.

As a result, the integrations of the right hand members of Eqs. (13) and (14) become possible, and the approximate values can be obtained (In the integration of the left hand members, the creep constants, m, γ and n are assumed to be constant like in the right hand members, to unify the treatment). The right hand member of Eq. (13) doesn't have a simple analytical solution. But, the terms related to coefficients $S_1 \sim S_6$, b_1 and b_2 can be transferred outside the integral as constants. Equivalent elastic strain ε^e_{x1} is normalized by dividing by the initial value ε^e_{x10s}, and the quantity of initial elastic strain is taken outside the integral. The result is shown as Eq. (18). Otherwise, a simple analytical solution can be obtained for the right hand member of Eq. (14). Creep constant n is $n \neq 1$ as described in the first report. The analytical solution is taken as Eq.(20) under this condition.

(a) In the temperature range below 575 °C

(In obedience to strain-hardening theory)

$$m \int_{T_{ss}}^{T_{es}} A^{1/m} E^{\gamma/m} dT = -C_s v (\varepsilon^e_{x10s})^{(1-\gamma)/m} \times \int_1^{{}_n\varepsilon^e_{x1}} ({}_n\varepsilon^e_{x1})^{-\gamma/m} (1 - {}_n\varepsilon^e_{x1})^{(1/m)-1} d\,{}_n\varepsilon^e_{x1} \quad (18)$$

Where

$$C_s = C_{ss} C_{sb}$$
$$C_{ss} = (2/3) S_1^{1-(\gamma/m)} S_2^{-1} S_3^{(1/m)-1} S_4^{-1/m}$$
$$C_{sb} = b_1^{1/m} \quad (19)$$

${}_n\varepsilon^e_{x1}$: Normalized equivalent elastic strain

$${}_n\varepsilon^e_{x1} = \varepsilon^e_{x1} / \varepsilon^e_{x10s}$$

(b) In the temperature range above 575 °C

(In obedience to power theory)

$$\int_{T_{sp}}^{T_{ep}} \beta E^n dT = -C_p \frac{v}{1-n} \{(\varepsilon^e_{x1})^{1-n} - (\varepsilon^e_{x10p})^{1-n}\} \quad (20)$$

Where

$$C_p = C_{ps} C_{pb}$$
$$C_{ps} = (2/3) S_1^{1-n} S_2^{-1} S_4^{-1}$$
$$C_{pb} = b_1 \quad (21)$$

Let us compare the above Eqs. (18) and (20) with Eqs. (11) and (12) for the relaxation test at changing temperature shown in the first report. Those are the same equations except coefficient C_s or C_p being multiplied on the right member.

A similar equation will be derived for the holding stage of SR treatment next. The holding temperature is more than 575 °C for this material. So it is treated only for the case of power theory. The procedure of the equation development in this case is fundamentally the same as the derivation of Eq. (20) for the heating stage. It is not necessary that the integration by time is changed into the integration by temperature. All the material constants are constant because the temperature doesn't change. Coefficients $S_1 \sim S_6$ and b_1, b_2 are assumed to be constant as in the heating stage. As a result, the next equation can be derived.

$$\varepsilon^e_{x1} = \{(\varepsilon^e_{x10pc})^{1-n} + C_p^{-1} (n-1) \beta E^n t\}^{1/(1-n)} \quad (22)$$

Where ε^e_{x10pc} : Initial value of ε^e_{x1} in the holding stage

t : Time after start of holding (min)

let us compare Eq. (22) with Eq. (13) for the relaxation test at constant temperature shown in the first report. These are the same equations except coefficient C_p^{-1}.

6. Estimation Method of Transient and Residual Stresses

6.1 Calculation procedure

Equations (18), (20) and (22) are the same equations as Eqs. (11), (12) and (13) for the relaxation tests in the first report fundamentally. They are different only in that coefficient C_s, or C_p, is multiplied in the right hand side of Eqs. (18), (20) and (22). Accordingly, if the values of those coefficients are known, the stress change in SR treatment can be obtained with the same calculation procedure, Fig. 2, as the first report for the relaxation tests. Comparison of behaviors between the relaxation test and the SR treatment will be possible too. Three results of numerical calculation for three integrations included in Eqs. (11) and (12) in the first report can be utilized for Eqs. (18) and (20) just as they are. The results are shown as Tables 1, 2 and 3 again here.

Coefficients C_s and C_p consist of coefficients $S_1 \sim S_4$, b_1 and creep constants m, γ and n. For creep constants m, γ and n, the mean values in each temperature range are used as in the preparation of the three tables mentioned above. Coefficients $S_1 \sim S_4$ are assumed to be calculated from the initial stresses in SR treatment, namely, the welding residual stresses.

The remaining coefficient b_1, the influence for the stress change will be examined, and the introduction to the calculation will be considered last.

6.2 Numerical setting of coefficient b_1

The analytical model of Fig. 1(c) is used again here. The stress state in this model is uni-axial. Coefficients C_s and C_p are expressed as follows.

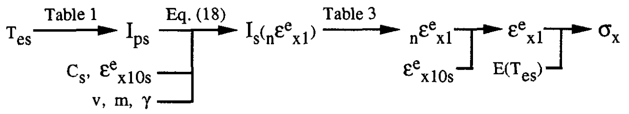
A. For Strain-Hardening Creep Theory

Eq. (18) may be expressed as

$$-C_s^{-1}(m/v)(\varepsilon^e_{x10s})^{(\gamma-1)/m} \cdot I_{ps} = I_s$$

where

$$I_{ps} \equiv \int_{T_{ss}}^{T_{es}} A(T)^{1/m} E(T)^{\gamma/m} dT, \quad I_s \equiv \int_1^{{}_n\varepsilon^e_{x1}} ({}_n\varepsilon^e_{x1})^{-\gamma/m} (1 - {}_n\varepsilon^e_{x1})^{(1/m)-1} d_n\varepsilon^e_{x1}$$



B. For Power Creep Theory

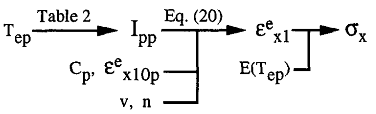
B - 1 At the heating stage

Eq. (20) may be expressed as

$$-C_p^{-1}(1-n)/v \cdot I_{pp} + (\varepsilon^e_{x10p})^{1-n} = (\varepsilon^e_{x1})^{1-n}$$

where

$$I_{pp} \equiv \int_{T_{sp}}^{T_{ep}} \beta(T) E(T)^n dT$$



B - 2 At the holding stage

Eq. (22) is expressed as

$$\varepsilon^e_{x1} = \{(\varepsilon^e_{x10pc})^{1-n} + C_p^{-1}(n-1)\beta E^n t\}^{1/(1-n)}$$

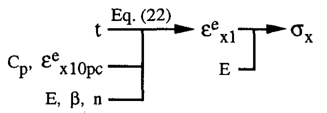


Fig. 2 Flowchart of calculations by simple estimation method

T_{es}	I_{ps}	T_{es}	I_{ps}	T_{es}	I_{ps}
401	1.2884E+15	461	3.1953E+18	521	6.7715E+20
403	4.2382E+15	463	3.8227E+18	523	8.0926E+20
405	7.7656E+15	465	4.5726E+18	525	9.6713E+20
407	1.1983E+16	467	5.4691E+18	527	1.1557E+21
409	1.7027E+16	469	6.5408E+18	529	1.3812E+21
411	2.3058E+16	471	7.8218E+18	531	1.6506E+21
413	3.0269E+16	473	9.3531E+18	533	1.9725E+21
415	3.8893E+16	475	1.1183E+19	535	2.3571E+21
417	4.9204E+16	477	1.3371E+19	537	2.8168E+21
419	6.1532E+16	479	1.5986E+19	539	3.3660E+21
421	7.6274E+16	481	1.9112E+19	541	4.0223E+21
423	9.3900E+16	483	2.2849E+19	543	4.8064E+21
425	1.1497E+17	485	2.7315E+19	545	5.7434E+21
427	1.4017E+17	487	3.2654E+19	547	6.8629E+21
429	1.7030E+17	489	3.9035E+19	549	8.2005E+21
431	2.0632E+17	491	4.6661E+19	551	9.7988E+21
433	2.4939E+17	493	5.5777E+19	553	1.1708E+22
435	3.0089E+17	495	6.6672E+19	555	1.3989E+22
437	3.6245E+17	497	7.9693E+19	557	1.6715E+22
439	4.3606E+17	499	9.5257E+19	559	1.9972E+22
441	5.2407E+17	501	1.1385E+20	561	2.3863E+22
443	6.2928E+17	503	1.3608E+20	563	2.8512E+22
445	7.5507E+17	505	1.6265E+20	565	3.4066E+22
447	9.0545E+17	507	1.9441E+20	567	4.0702E+22
449	1.0852E+18	509	2.3236E+20	569	4.8629E+22
451	1.3001E+18	511	2.7771E+20	571	5.8100E+22
453	1.5571E+18	513	3.3191E+20	573	6.9414E+22
455	1.8643E+18	515	3.9668E+20	575	8.2930E+22
457	2.2315E+18	517	4.7409E+20		
459	2.6705E+18	519	5.6660E+20		

(1.0000E+05 = 1.0000×10^5)

where

$$I_{ps} \equiv \int_{T_{ss}}^{T_{es}} A(T)^{\frac{1}{m}} E(T)^{\frac{\gamma}{m}} dT, \quad T_{ss} = 400 \ (°C)$$

Table 1 Calculated result of the integration of the left hand side of Eq. (18)

T_{ep}	I_{pp}	T_{ep}	I_{pp}	T_{ep}	I_{pp}
577	2.8825E+09	619	2.5238E+11	661	2.3693E+12
579	6.1080E+09	621	2.8373E+11	663	2.6128E+12
581	9.7162E+09	623	3.1848E+11	665	2.8793E+12
583	1.3751E+10	625	3.5701E+11	667	3.1706E+12
585	1.8261E+10	627	3.9970E+11	669	3.4888E+12
587	2.3302E+10	629	4.4696E+11	671	3.8362E+12
589	2.8932E+10	631	4.9927E+11	673	4.2149E+12
591	3.5219E+10	633	5.5713E+11	675	4.6276E+12
593	4.2237E+10	635	6.2109E+11	677	5.0767E+12
595	5.0068E+10	637	6.9175E+11	679	5.5651E+12
597	5.8803E+10	639	7.6978E+11	681	6.0957E+12
599	6.8542E+10	641	8.5588E+11	683	6.6714E+12
601	7.9396E+10	643	9.5084E+11	685	7.2955E+12
603	9.1488E+10	645	1.0555E+12	687	7.9712E+12
605	1.0495E+11	647	1.1707E+12	689	8.7021E+12
607	1.1994E+11	649	1.2976E+12	691	9.4917E+12
609	1.3662E+11	651	1.4372E+12	693	1.0343E+13
611	1.5516E+11	653	1.5906E+12	695	1.1261E+13
613	1.7578E+11	655	1.7591E+12	697	1.2250E+13
615	1.9869E+11	657	1.9441E+12	699	1.3312E+13
617	2.2414E+11	659	2.1470E+12	700	1.3872E+13

(1.0000E+05 = 1.0000×10^5)

where

$$I_{pp} \equiv \int_{T_{sp}}^{T_{ep}} \beta(T)\, E(T)^n dT, \quad T_{sp} = 575 \ (°C)$$

Table 2 Calculated result of the integration of the left hand side of Eq. (20)

${}_n\varepsilon^e_{x1}$	$-I_s$	${}_n\varepsilon^e_{x1}$	$-I_s$	${}_n\varepsilon^e_{x1}$	$-I_s$
0.99	1.3052E-06	0.66	5.6983E-01	0.33	1.7724E+03
0.98	9.4448E-06	0.65	7.0337E-01	0.32	2.4468E+03
0.97	3.1066E-05	0.64	8.6831E-01	0.31	3.4077E+03
0.96	7.4075E-05	0.63	1.0722E+00	0.30	4.7908E+03
0.95	1.4816E-04	0.62	1.3248E+00	0.29	6.8035E+03
0.94	2.6531E-04	0.61	1.6379E+00	0.28	9.7670E+03
0.93	4.4032E-04	0.60	2.0269E+00	0.27	1.4186E+04
0.92	6.9147E-04	0.59	2.5108E+00	0.26	2.0866E+04
0.91	1.0413E-03	0.58	3.1141E+00	0.25	3.1112E+04
0.90	1.5175E-03	0.57	3.8676E+00	0.24	4.7083E+04
0.89	2.1543E-03	0.56	4.8110E+00	0.23	7.2408E+04
0.88	2.9936E-03	0.55	5.9948E+00	0.22	1.1333E+05
0.87	4.0866E-03	0.54	7.4841E+00	0.21	1.8083E+05
0.86	5.4963E-03	0.53	9.3628E+00	0.20	2.9472E+05
0.85	7.2996E-03	0.52	1.1739E+01	0.19	4.9173E+05
0.84	9.5908E-03	0.51	1.4755E+01	0.18	8.4202E+05
0.83	1.2485E-02	0.50	1.8594E+01	0.17	1.4843E+06
0.82	1.6123E-02	0.49	2.3498E+01	0.16	2.7032E+06
0.81	2.0678E-02	0.48	2.9786E+01	0.15	5.1080E+06
0.80	2.6359E-02	0.47	3.7877E+01	0.14	1.0065E+07
0.79	3.3427E-02	0.46	4.8331E+01	0.13	2.0817E+07
0.78	4.2195E-02	0.45	6.1897E+01	0.12	4.5534E+07
0.77	5.3052E-02	0.44	7.9579E+01	0.11	1.0638E+08
0.76	6.6473E-02	0.43	1.0273E+02	0.10	2.6885E+08
0.75	8.3040E-02	0.42	1.3321E+02	0.09	7.4728E+08
0.74	1.0346E-01	0.41	1.7354E+02	0.08	2.3365E+09
0.73	1.2864E-01	0.40	2.2721E+02	0.07	8.4822E+09
0.72	1.5963E-01	0.39	2.9905E+02	0.06	3.7443E+10
0.71	1.9780E-01	0.38	3.9581E+02	0.05	2.1593E+11
0.70	2.4478E-01	0.37	5.2701E+02	0.04	1.8345E+12
0.69	3.0262E-01	0.36	7.0617E+02	0.03	2.8760E+13
0.68	3.7385E-01	0.35	9.5264E+02	0.02	1.3795E+15
0.67	4.6162E-01	0.34	1.2944E+03	0.01	1.0164E+18

(1.0000E+05 = 1.0000×10^5)

where

$$I_s \equiv \int_1^{{}_n\varepsilon^e_{x1}} ({}_n\varepsilon^e_{x1})^{-\frac{\gamma}{m}} (1 - {}_n\varepsilon^e_{x1})^{\frac{1}{m}-1} d_n\varepsilon^e_{x1}$$

Table 3 Calculated result of the integration of the right hand side of Eq. (18)

$$C_s = C_{sb} = b_1^{1/m}$$

$$C_p = C_{pb} = b_1 \qquad (23)$$

Coefficient b_1 of the upper equation is the function of creep constant γ or n, and the sectional area ratio **a** as shown in Eqs. (16) and (17). It is assumed that creep constants γ and n, are constants. So, the values of coefficients C_{sb} and C_{pb} are calculated for the change in the sectional area ratio, **a,** in two temperature ranges. It is usually for the tensile residual stress that the stress change in SR treatment attracts attention. In this model, this is the change of σ_1 produced in member I. The sectional area ratio **a** is smaller than 1 in the usual joint. So, the values of coefficients C_{sb} and C_{pb} of member I were calculated in the range of $0.01 \leqq \mathbf{a} \leqq 1.0$. The result is shown in Table 4. Coefficient b_1 exists in the range from 1 to 1.5. Coefficient C_{sb} exists in the range from 1 to 3. And, coefficient C_{pb} exists in the range from 1 to 1.5. The influences for the stress change are investigated next.

Creep Law	Coeff. \ $a = \frac{A_1}{A_2}$	0.01	0.1	0.2	0.4	0.6	0.8	0.9	1.0
Strain-hardening	b_{1s}	1.01	1.10	1.20	1.36	1.40	1.26	1.14	1.
	C_{sb}	1.03	1.30	1.64	2.32	2.52	1.90	1.43	1.
Power	$b_{1p} = C_{pb}$	1.01	1.10	1.20	1.38	1.47	1.33	1.18	1.

Table 4 Relation between coefficients b_{1s}, C_{sb}, b_{1p}, C_{pb} and sectional area ratio **a**

Coefficient C_{sb} is multiplied on the right hand side of Eq. (18). When the temperature of the model rises to T_{es}, the left hand side of Eq. (18) takes m times the integrated value shown in Table 1. The total value of the right hand side is determined on the condition equal to the left hand side, and the quantity of change of elastic strain can be known using Table 3. When coefficient C_{sb} changes in the range from 1 to 3, the integrated value of elastic strain shown in Table 3 changes over the range from 1 to 1/3. The estimated value of elastic strain changes within only a few % of the initial value even if the integrated value of elastic strain decreases to 1/3, which is obvious from Table 3. The effect of coefficient C_{pb} is similar. Accordingly, the influence of coefficient b_1 on the stress change is small in the welded joint in self-balance, which is subjected to no external force, and which is represented by this analytical model. However, in this study, the next two values are set up for coefficient b_1 to consider the influence. Two values are $b_1 = 1$ and $b_1 = 2$. In the former case, somewhat smaller stress is estimated than the real magnitude because the influence of b_1 is ignored. In the latter case, somewhat larger stress is estimated conversely. In the next report, the stress change in SR treatment will be estimated for these two cases.

7. Conclusion

In the first report, a simple method of stress analysis was developed for relaxation tests at changing and constant temperatures. Estimation equations were formulated in the uni-axial stress state. In the second report, the stresses relaxed by SR treatment in the thick welded joint were analyzed accurately by the finite element method based on thermal elastic-plastic creep theory. The characteristics of the changes of the welding residual stresses in the multi-axial stress state were studied in detail.

In this report, the simple analysis method for uni-axial stress state shown in the first report was expanded to be applied for the multi-axial stress state. The characteristics of stress relaxation phenomena in SR treatment of thick welded joints, which were obtained in the second study, were utilized then. The developed estimation equations can calculate the changes of welding residual stresses in thick welded joint during SR treatment as simply as by hand calculation.

In the next report, the applicability of this method will be examined for the thick welded joint which was analyzed accurately by the finite element method in the the second report.

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