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# Three Dimensional Numerical Simulation of Various Thermo-mechanical Processes by FEM (Report II)<sup>†</sup>

— Deformation Analysis of Compressor under Assembly by Shrinkage Fit —

Yukio UEDA\*, Hidekazu MURAKAWA\*\*, Jianhua WANG\*\*\* and Min Gang YUAN\*\*\*\*

## Abstract

*Residual deformations and stresses after shrinkage fit of a hollow cylinder with an internal stator are calculated. The numerical simulations are performed using the thermal-elastic-plastic finite element method. Two idealized models for the shrinkage fitted structure are investigated. In these models the 3-D solid elements with local coordinate system are used. In order to prevent the numerical locking phenomena, the reduced integration method is introduced and its effectiveness is discussed. The calculated radial shrinkages are compared with the measured values obtained by experiment. The results show that good agreement can be obtained if the reduced integration method is applied to the cylinders.*

**KEY WORDS :** (Shrinkage Fit)(Compressor)(Numerical Simulation)(Locking Phenomena)  
(Reduced Integration)(Deformation Analysis)(Finite Element Method)

## 1. Introduction

The shrinkage fit is commonly used to assemble an external hollow cylinder and an internal stator of a rotating machine such as a compressor. The outer diameter of the stator is designed to be a little larger than the internal diameter of the cylinder at room temperature. When the cylinder is heated at a certain temperature, its internal diameter will expand to exceed the outer diameter of the stator which is still kept at room temperature. The stator is inserted into the heated cylinder and these two components are connected by shrinkage of the cylinder during cooling.

Residual deformations and stresses are caused by shrinkage fit, and it may influence the accuracy of the following assembly processes such as welding. Therefore the numerical simulations are performed to predict the deformations due to shrinkage fit. Two models are investigated to simulate the shrinkage fit process, and in these models the 3-D solid elements in which local coordinates are considered are used. The computer program is modified from the 3-D thermal-elastic-plastic FEM which was reported in the previous paper<sup>1)</sup>.

In general, 3-D solid elements give the most accurate results in a mechanical simulation. However, it sometimes cause what is so-called "locking phenomenon"<sup>2)</sup>, especially when the solid elements are used in the bending of a thin plate. In these cases, it would give the smaller deformations if the solid elements were not modified. In order to improve the modelling of shrinkage fit, reduced

integration is introduced in the solid element. Simple curved beam models are investigated using reduced integration. The results show that solid elements with reduced integration give more reasonable deformations than that using solid elements without reduced integration. The degree of the effect of locking also depend on the constraint conditions and the mesh division. Therefore, their effects should be clarified and controlled carefully.

The results obtained by simulations are compared with these by experimental measurements. The computed deformation is found to agree well with the measured values in tendency.

## 2. Coordinate Transformation

To analyze models with axisymmetric geometry such as cylinders or pipes, it is necessary to employ the cylindrical coordinates instead of rectangular Cartesian coordinates. In this case components of stresses and deformations can be decomposed into radial and circumferential directions. The use of local coordinate defined in the cylindrical coordinate also makes it easy to introduced the reduced integration for particular components of strain.

In Fig.1,  $x, y$  are the global coordinates, and  $x', y'$  are the local coordinates defined for each element.  $\alpha$  is the angle between  $x$  and  $x'$  (or  $y$  and  $y'$ ). Then the following relationship between the displacements defined in these

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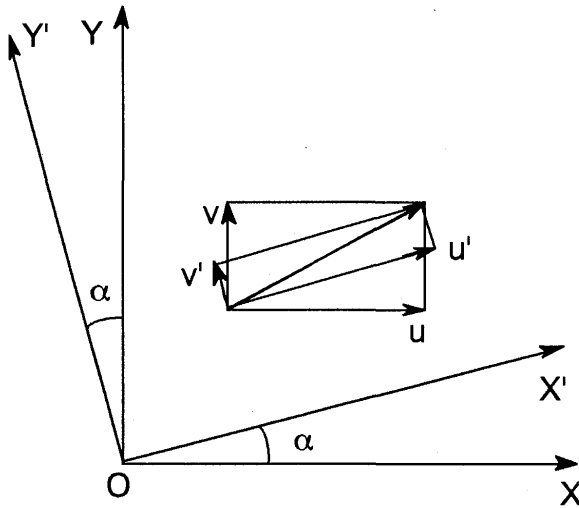


Fig.1 Coordinate transformation

coordinates can be introduced.

$$\begin{aligned} u' &= u \cos \alpha + v \sin \alpha \\ v' &= -u \sin \alpha + v \cos \alpha \end{aligned} \quad (1)$$

where,  $u$  and  $v$  are displacements in  $x$ - $y$  coordinate system,

$u'$  and  $v'$  are these in  $x'$ - $y'$  coordinate system.

Equation (1) can be rewritten in matrix form such as,

$$\{u'\} = [L] \{u\} \quad (2)$$

where, the matrix  $[L]$  represents the coordinate transformation matrix.

If no external force is acting, the virtual work theorem can be written as

$$\int \{ \sigma' + \Delta \sigma' \}^T \{ \delta \epsilon' \} dV = 0 \quad (3)$$

Based on Eq.(3) the following discretized equation for FEM is derived,

$$\{\Delta u\}^T [L]^T [K_e'] [L] = - \{f'\} [L] \quad (4)$$

$$\text{where, } [K_e'] = \int [B]^T [D] [B] dV \quad (5)$$

$$\{f'\} = \int \{ \sigma' \}^T [B] dV \quad (6)$$

Introducing the stiffness matrix  $K_e$  and the load vector  $\{f\}$  defined in the global coordinate, ie.,

$$[K_e] = [L] [K_e'] [L] \quad (7)$$

$$\{f\} = \{f'\} [L] \quad (8)$$

Eq.(4) will be rewritten as,

$$[K_e] \{\Delta u\} = - \{f\} \quad (9)$$

Superposing the element stiffness matrix  $K_e$  and nodal load vector  $\{f\}$ , the equilibrium equation for the whole system will be given as follows,

$$[K] \{\Delta U\} = \{F\} \quad (10)$$

From equilibrium Eq.(10), the increments of nodal displacement  $\Delta U$  can be solved, such that

$$\{\Delta u'\} = [L] \{\Delta u\} \quad (11)$$

$$\{\Delta \epsilon'\} = [B] \{\Delta u'\} \quad (12)$$

$$\{\Delta \sigma'\} = [D] \{\Delta \epsilon'\} - \{C\} \Delta T \quad (13)$$

where,  $\{\Delta \epsilon'\}$  : the strain increment in local coordinate,

$\{\Delta \sigma'\}$  : the stress increment in local coordinate.

### 3. Shrinkage Fit and Simulation Models

#### 3.1 A cylinder with internal stator

A cylinder and stator which are connected by shrinkage fit is shown in Fig.2. The outer diameter of the stator is about 0.10 - 0.33 mm larger than the internal diameter of the cylinder depending on the manufacturing tolerance. During the shrinkage fitting process, the cylinder is heated to 200°C and its internal diameter will expand enough so that the stator can be inserted into the cylinder and connects each other after cooling down of the cylinder.

#### 3.2 Material properties

The properties of material (SM41) used in the simulations are as follows,

|                               |   |
|-------------------------------|---|
| Young's modulus (R.T.)        | $E = 215 \text{ GPa}$                         |
| Poisson's ratio               | $\mu = 0.3$                                   |
| Yield stress (R.T.)           | $\sigma_y = 530 \text{ MPa}$                  |
| Thermal expansion coefficient | $\alpha = 13 \times 10^{-6} / ^\circ\text{C}$ |

#### 3.3 Simulation models

In order to predict the residual deformations due to shrinkage fit, two models are used. The overlap between the internal diameter of the cylinder and the outer diameter of the stator is assumed as  $\Delta d = 0.33 \text{ mm}$ . Model-1 consists of the cylinder only and the internal wall of the cylinder which is in contact with the stator is assumed to be fixed. Model-2 includes both cylinder and stator. Thus, it is a more realistic 3-D thermal elastic-plastic model. A thin layer is assumed in between the cylinder and the stator in contact. A quarter of the structure is analyzed because of the symmetry. Figure 3 shows the mesh division of the two models.

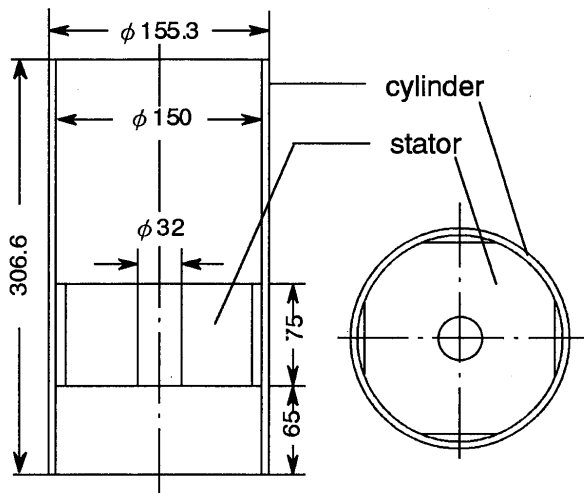


Fig.2 Components of shrinkage fit

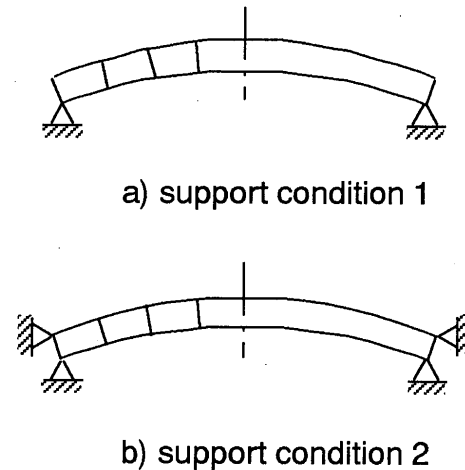


Fig.4 simple curved beam models

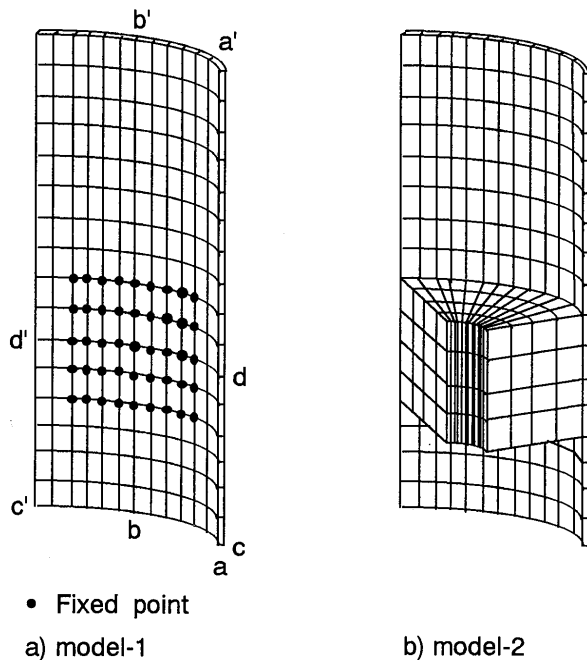


Fig.3 Simulation models for shrinkage fit

proposed to prevent it<sup>3)</sup>.

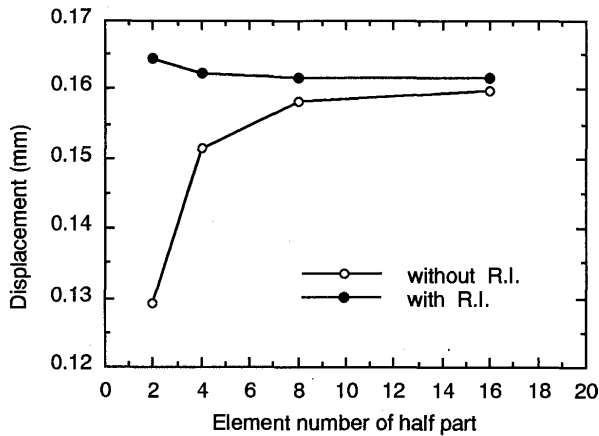
In this study, because the thickness of the cylinder is thin, the locking phenomenon may occur in some part of the model where bending deformation is dominant. In order to investigate the possibility of this phenomenon and its effects, a simplified model is used as shown in Fig.4. A narrow circular cylindrical shell is fixed at two ends. Two kinds of supporting conditions are assumed. The displacements at center when the temperature cools down from 188 °C to 20°C are calculated by the following methods,

- (1) using solid elements without reduced integration,
- (2) using solid elements with reduced integration applied to the terms involving transverse shear strains of  $\gamma_{xy}$ ,  $\gamma_{xz}$  ( $x$  is direction of the thickness) in the local coordinates.

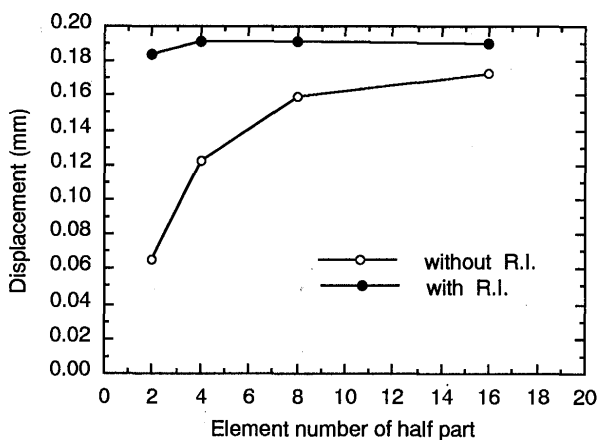
Figure 5 shows the calculated results of displacements with different number of Finite Element divisions. It can be seen that the number of mesh division has the obvious effects on the displacements in the case of the elements without reduced integration. However, the number of mesh division has little effects on the displacements in the case of with reduced integration. The difference between without and with reduced integration is large if the number of elements is less, and decreases with the increase of the element number. Because it is difficult to further increase the elements in case of 3-D problems due to the limitation of computer memory, the reduced integration should be introduced. On the other hand, the support conditions at two ends also give the significant influence to the calculated results, so it should be taken into consideration in modeling the problem.

#### 4. Reduced Integration and Its Effectiveness

Finite Element Method has been recognized as a versatile tool for structural analysis including thermal-elastic-plastic problems. However, it sometimes exhibits abnormal behavior, which is often called locking phenomenon. Such phenomena can be observed as overestimation of stiffness in plate bending analysis with shear deformation and in shell analysis. Since the locking phenomena seriously reduce the accuracy of the analysis, reduced integration method and mixed method have been

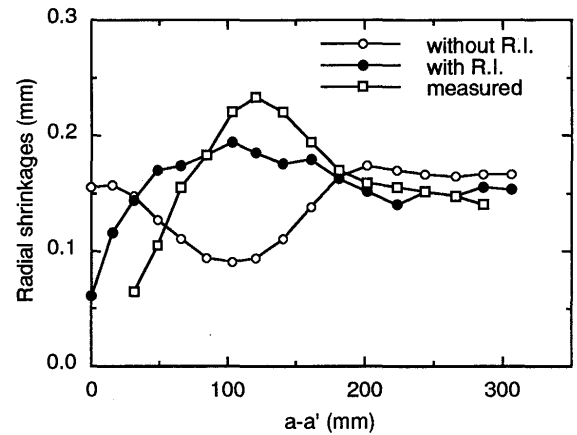


a) support condition 1

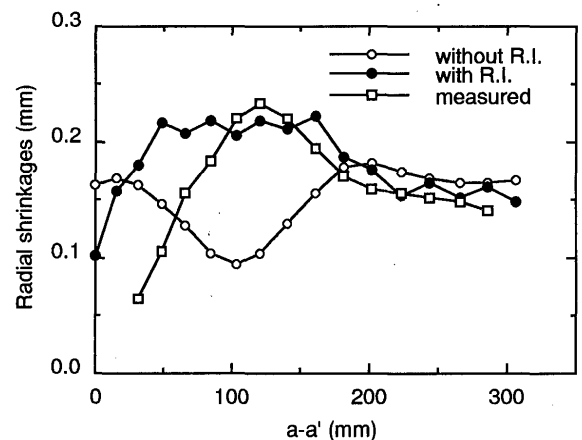


b) support condition 2

Fig.5 Calculated displacements with number of elements



a) model-1



b) model-2

Fig.6 Radial shrinkages on line a-a'

## 5. Results of Finite Element Simulations

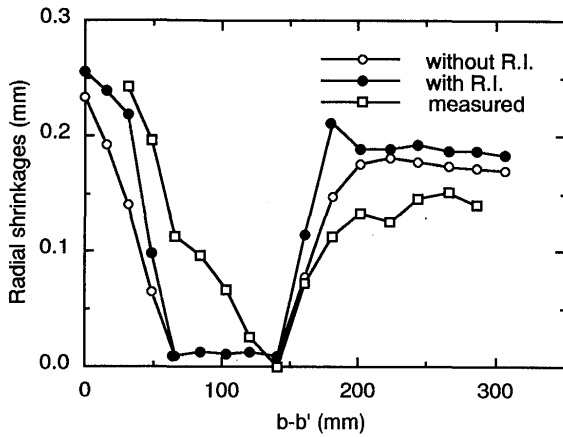
Figures 6-a) and b) show the radial shrinkage of model-1 and model-2 along a-a' line (Fig.3), respectively. The measured geometry is also shown for comparison. It is interesting to notice in Fig.6 that the shape of the deformation due to shrinkage computed without reduced integration is opposite compared to that calculated with reduced integration and measured one. The reason is the locking phenomenon caused in the thin cylinder parts where the bending deformation is dominant. Both of the two models show that reduced integration can prevent the locking phenomena and the computed deformation agree well with the experimental results.

Figures 7-a) and b) show the radial shrinkage of model-1 and model-2 along b-b' line (Fig.3), respectively. The measured geometry is shown for comparison. Because the middle parts of this line is in contact with

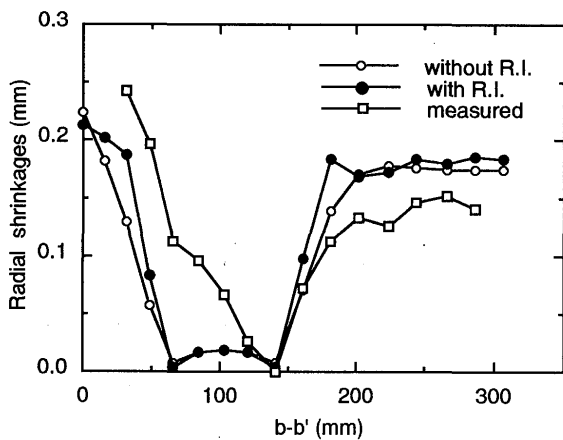
stator and bending deformation is not dominant, both curves show the same shape and agree well with the measured results.

Figures 8-a) and b) show the radial deformation along lines c-c' and d-d' of model-1 and model-2, respectively. The variations in the radius of the cylinder due to the shrinkage fit are about 0.1-0.2 mm. Three dimensional view of the residual deformation of model-1 is shown in Fig.9.

Figure 10 shows the distributions of residual stresses in circumferential direction in model-2. The residual stresses are very large and reach the tensile yield stress in the cylinder parts which are in direct contact with the stator. Because of the sensitivity of reduced integration on the constraint conditions, it is necessary to control the accuracy by careful modeling. Further theoretical investigation and the experimental verification on locking

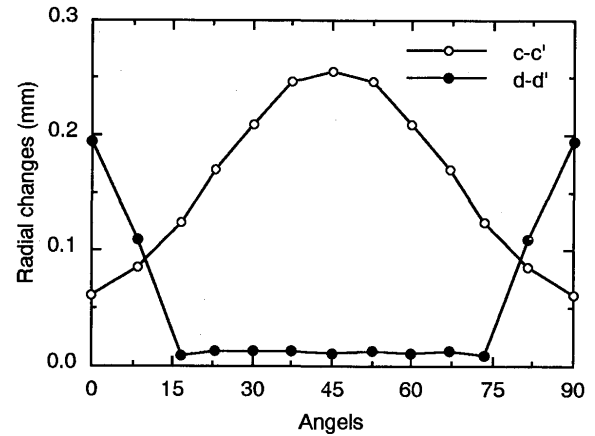


a) model-1

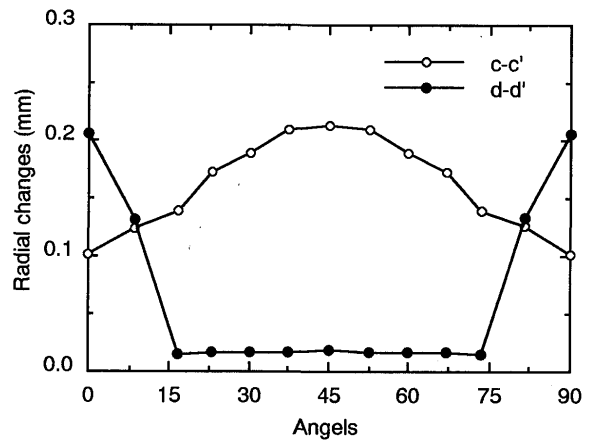


b) model-2

Fig.7 Radial shrinkages on line b-b'



a) model-1



b) model-2

Fig.8 Radial changes on curve c-c' and curve d-d'

phenomena and reduced integration are expected in the next step, especially when the reduced integration is applied to welding problems.

## 6. Conclusions

- (1) The general 3-D thermal-elastic-plastic FEM-program has been modified to consider local coordinate so that it can be applied to axisymmetric thin structures. In this paper, shrinkage fit of a cylinder with stator has been analyzed as an example of practical 3-D problem and satisfactory results are obtained.
- (2) The locking phenomenon and reduced integration method were studied. They should be considered in this problem because the thickness of cylinder is thin. The differences between the deformations calculated with reduced integration and without it depend on the fineness of the Finite Element division and the

constraint conditions.

- (3) Two models have been proposed to simulate the mechanical behaviour of the shrinkage fit. The residual deformations and stresses of the structure can be predicted by 3-D thermal-elastic-plastic Finite Element Method. The calculated results of radial shrinkages using reduced integration agree well with the measured geometry of the cylinder.
- (4) After shrinkage fit, the original circular shape of the cylinder is changed and the variations in its radius are about 0.1-0.2 mm.
- (5) The residual stresses of the part of the cylinder which is in direct contact with stator can reach tensile yield stress.

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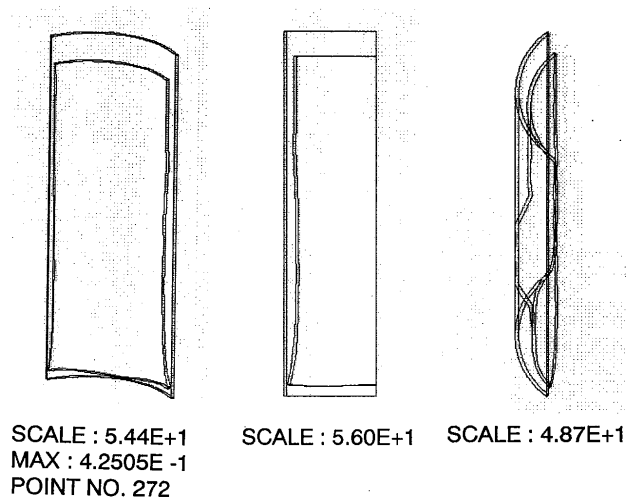


Fig.9 Three dimensional view of the residual deformations of model-1

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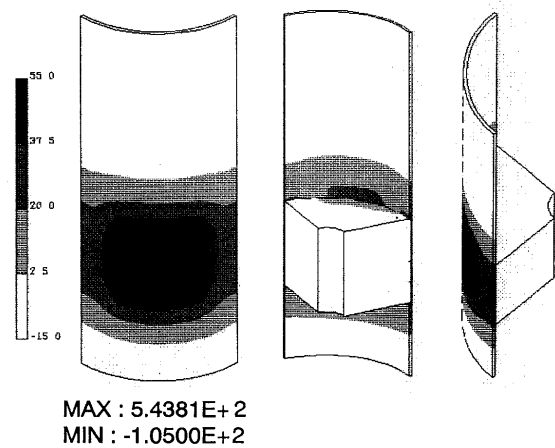


Fig.10 Distributions of residual stresses in model-2

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