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A Predicting Method of Welding Residual Stress Using Source of Residual Stress (Report I)†

— Characteristics of Inherent Strain (Source of Residual Stress) —

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Abstract

In this paper, a predicting method of welding residual stress by using the source of residual stress (inherent strain) is proposed. And the validity of the method is demonstrated by numerical experiments with the aid of the finite element method.

Welding residual stress is produced as a result of thermal elasto-plastic behavior. Its source is composed of the thermal strain and/or plastic strain, etc., which is called here inherent strain.

Taking a butt joint as an example, the sources of residual stress are estimated from several joints of different sizes manufactured under the same welding condition. It is found that the distributions of the source of residual stress are almost the same if the sizes of the joint are larger than that of a specific one, which is named a standard size.

It is demonstrated that the residual stress produced in a joint of any size can be predicted accurately by elastic analysis using the inherent strain in the joint of standard size, unless the sizes are too small.

KEY WORDS: (Welding Residual Stress) (Inherent Strain) (Source of Residual Stress) (Butt Welding) (Thermal Stress) (FEM)

1. Introduction

The welding residual stresses are inevitably produced in welded structures. It is known that the performance and safety of welded structures are influenced significantly by the welding residual stresses. Therefore, it is important to estimate the magnitude and distribution of welding residual stresses accurately to investigate their influence.

In contrast with complex calculation by thermal elasto-plastic analysis[1], many researches[2,3,4] have been carried out to develop predicting methods of welding residual stress by simple ways, some of which are concerned with the source of welding residual stress. However, there are no papers which describe the method of determination of the source, except a series of papers[5-10] published by the authors. In these papers, the measuring methods of three dimensional residual stress in a body are developed using the source of residual stress (inherent strain) as a parameter. In other words, the source of residual stress can be determined and the residual stress can be computed by the elastic analysis.

In this report, the characteristics of the source of welding residual stress is studied. It will be noted that the source is almost independent of the size of the joint for the specified material and welding condition if it is quite large. Then, a predicting method of welding residual stresses is proposed, in which the elastic analysis is only necessary on a stress free plate loaded by the above mentioned source independent of the size of plate.

To demonstrate the validity and application of the method, a butt-welded joint is used as the analysis model here.

2. Proposal of a Predicting Method of Welding Residual Stress

2.1 A predicting method of welding residual stress

The welding residual stress is produced in welded joint as a result of plastic deformation caused by non-uniform thermal expansion and contraction due to non-uniform temperature distribution in the welding process. The main influencing factors on the magnitude and distribution of welding residual stress are as follows.

(1) The kind of steel (temperature dependent physical and mechanical properties and the temperature range of phase transformation produced at the cooling process.)

(2) The degree of restraint against thermal expansion and contraction (the type, and sizes of welded joint etc.)

(3) Welding condition (heat input, application of preheating and post heating etc.)

The magnitude and distribution of welding residual stress vary according to the size of the joint, although the kind of steel and welding condition are unchanged. Here,

attention should be paid to the source of welding residual stress. It is known that the source of welding residual stress is the incompatible strain including plastic strain. This incompatible strain is generally called inherent strain.

In case of welding, the source of welding residual stress (inherent strain) is considered to be produced only in the limited portion near the welding line. Therefore, using the same kind of steel and same welding condition, if the size of plate is larger than a certain value, the magnitude and distribution of inherent strain are presumed to be almost the same as those in the joint whose size is infinite. From this, it may be assumed that the inherent strain is the same regardless of the size of plate. The size is one important influencing factor on the welding residual stress, but it can be eliminated from the prediction.

If this inherent strain is used, the welding residual stress can be obtained by performing the elastic analysis instead of complex calculation of the thermal elasto-plastic analysis.

According to the numerical experiment to be mentioned later, the magnitude and distribution of inherent strain approach rapidly to the limiting ones, when the size of plate becomes larger from a certain value to the infinite. For estimation of the limiting ones of inherent strain, it is impractical to use the infinite plate for experiment. Then, it is appropriate to determine the size of a finite plate in which the inherent strain is produced almost the same ones as in the infinite plate. Here, this size of plate is called the standard model and the inherent strain generated in the model is the standard inherent strain.

In this study, a new predicting method of welding residual stress is proposed, taking a full advantage of the standard inherent strain. The following procedure will be taken:

(1) to determine the size of the standard model of the specified steel under the specified welding condition,

(2) to determine the standard inherent strain by experiments (or thermal elasto-plastic analysis),

(3) to calculate the welding residual stress by performing the elastic analysis on an arbitrary size of stress-free plate under the loading of the standard inherent strain.

The validity of the method is to be examined in chapter 3.

2.2 Calculation of inherent strain and welding residual stress

The proposed predicting method is applied taking the above three steps.

As for the step (1), the size of standard model can be determined theoretically. This will be presented in the next report[11]. In this report, by the numerical experiment, the size is discussed later in Chapter 3.

For the steps (2) and (3), the basic theory is described based on the finite element method, taking a butt welded joint in the plane stress state as an example.

Concerning the general theory of predicting welding residual stress using inherent strain as a parameter, the details is described in the original reference[5,10].

In the case of welding, the inherent strain: $|\epsilon^*| = |\epsilon^*_x, \epsilon^*_y, \epsilon^*_{xy}|^T$ is considered to be only produced in the limited region near the weld. However, welding residual stress σ_x (or elastic strain $|\epsilon|$) due to the inherent strain is produced over the whole plate.

This relation is expressed in the following equations.

$$|\epsilon| = [H^*] \ |\epsilon^*| \tag{1}$$

$$|\sigma| = [D] \ |\epsilon| \tag{2}$$

Here,

[D] : Stress-strain matrix,

[H*] : Elastic response matrix.

In the above case, finite elements in which the inherent strain is produced are limited near the welded portion and the number of these elements is assumed to be m. Then, the total number of components of the inherent strains becomes $3m$, since there are three components of each inherent strain.

On the other hand, the number of finite elements in the whole joint is n ($>$ m), and the components of elastic strain (or welding residual stress) become $3n$.

$$3n \ 3n \times 3m \ 3m \tag{1'}$$

$$|\epsilon| = [H^*] \ |\epsilon^*|$$

In order to determine $3m$ components of the inherent strains, Eq.(1) is necessary to be solved conversely giving more than $3m$ known strains for $|\epsilon|$.

In experiments, various methods for measuring elastic strain can be used. For example, by the stress relaxation method, the welding joint with strain gages is cut into small pieces and residual elastic strain is obtained by each strain gage. Residual elastic strain is of the same magnitude and opposite in sign as the observed strain. In the numerical experiment as described later, the elastic strain obtained as a result of the thermal elasto-plastic analysis is used as observed elastic strain $|\epsilon|$.

When the elastic strain $|\epsilon|$ is measured, it is possible that various errors are contained in it. Therefore, according to the condition which minimize the sum of the square of residual, the most probable value of inherent strain $\hat{\epsilon}^*|$ is calculated from the following equation.

$$|\hat{e}*| = ([H*]^T[H*])^{-1}[H*]^T|\epsilon| \qquad (3)$$

Substituting the most probable value of inherent strain $|\hat{e}*|$ into Eq.(2), the most probable value of welding residual stress $|\hat{\sigma}|$ produced at arbitrary positions of the joint can be obtained.

$$|\hat{\sigma}| = [D][H*]|\hat{e}*| \qquad (2')$$

3. Validity and Applicability of the Predicting Method of Welding Residual Stress

In this section, a butt welded joint of thin plate by an instantaneous plane heat source is adopted as the analysis model, and the numerical experiment is performed using the finite element method.

The validity of the proposed predicting method of welding residual stress will be verified.

3.1 Object for analysis

A butt welded joint prepared from two rectangular thin plates is considered as a model for analysis (**Fig. 1**).

The sizes of the plate are as follows. The length, width and thickness of the plate are denoted by 2L, 2B and h (= 6mm: constant), respectively. The material is mild steel, and its temperature dependent physical and mechanical properties are the same ones as in reference (11).

The CO_2 gas shielded arc welding is used. The magnitude of heat input Q is indicated by electric current I = 220(A), voltage V = 24(V), welding speed v = 24(m/h) and the efficiency of heat-input η is assumed to be 75%. The heat input of the instantaneous plane heat source becomes Q = 594 (J/mm).

The analysis is performed on one quarter of the model due to the symmetries.

3.2 Distribution of inherent strain obtained by thermal elasto-plastic analysis

Even the material and welding condition are specified, the resulting residual stress distribution is not the same when the sizes of butt welded joint change. In contrast with this, it will be studied by numerical analysis how the inherent strain distribution changes with the sizes of the joint and how the distribution converges to a certain limiting one as the sizes become larger. In this study, the following three models will be analysed. The plate thickness of these models is the same, that is h = 6mm.

1. Model M_0 L = 600 mm,   B = 500 mm
2. Model M_1 L = 600 mm,   B = 800 mm
3. Model M_2 L = 1500 mm, B = 500 mm

Under the welding condition mentioned before, the thermal elasto-plastic analysis is performed, in cooperation with thermal conduction analysis. From the resulting residual stress, the inherent strain in each model is computed, assuming that the components of inherent strain $|\epsilon*|$ are only $\epsilon*_x$ and $\epsilon*_y$, and $\epsilon*_{xy}$ is neglected, that is, $|\epsilon*| = |\epsilon*_x, \epsilon*_y, 0|^T$. It will be proven in the next item that this assumption make the analysis simplified,
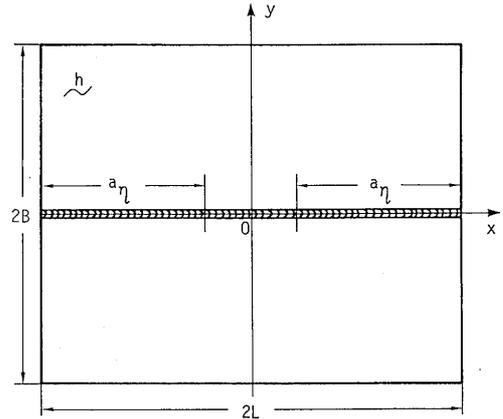

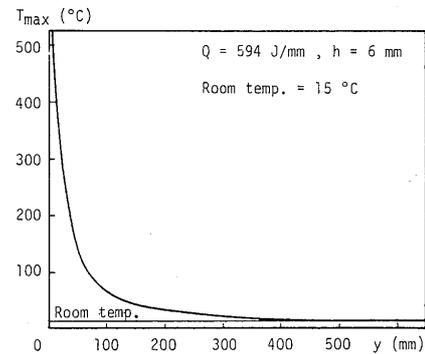**Fig. 1**   Model of butt-welded joint for analysis.

Fig. 2   Maximum temperature in the perpendicular direction to the welded line in infinite plate.
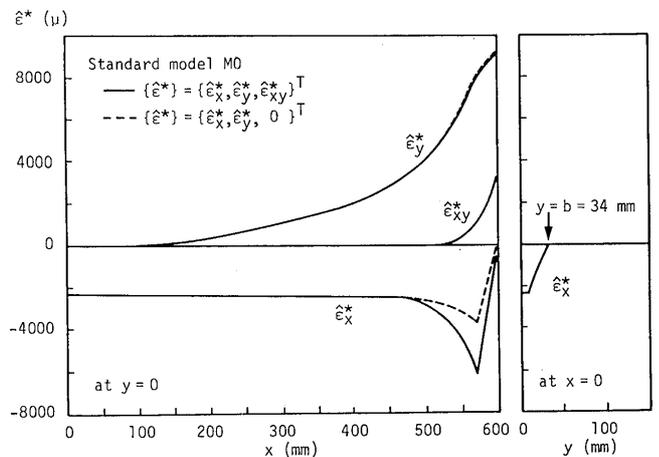

Fig. 3   Distributions of most probable values of inherent strains.

keeping a good accuracy of the predicted welding residual stress.

From the results, the most probable values of inherent strains in the welding direction, $\hat{e}^*_x$ are the same regardless of the sizes of models. The most probable values of inherent strains perpendicular to the weld line, $\hat{e}_y$ somewhat differ depending on the sizes of models. The reason for this is considered to be as follows. e^*_x is produced according to the degree of restraint against thermal expansion in the welding direction near the welding line, while e^*_y largely depends on in-plane stiffness such as L/B.

As is to be described in Section 3.5, in the case of butt weld, e^*_x is the most influencing component of inherent strain which decides the magnitude and distribution of welding residual stress. As $\hat{e}^*_x$ in the three models are same, the residual stresses in models M1 and M2 are calculated by using the same inherent strain as in model MO. The estimated invariant $\hat{s}_\sigma$ for the welding residual stress by the thermal elastic-plastic analysis is 9.7 and 8.7MPa for models M1 and M2.

Accordingly, model MO is considered tentatively as the standard model, for obtaining the inherent strain. Here, this inherent strain may be called the standard inherent strain, since the sizes of standard model will be determined theoretically in the next report[11].

3. Standard inherent strain and the accuracy

The thermal elasto-plastic analysis is performed on model MO. Using the obtained elastic strain for $|\epsilon|$ of Eq. (3), the distribution of inherent strain $|e^*| = |e^*_x, e^*_y, e^*_{xy}|^T$ is computed and shown by solid lines in **Fig. 4(a)**.

In the case of butt weld, the shearing component of inherent strain, e^*_{xy}, is only produced near the end of the plate (Fig. 4(a)). Therefore, even if the inherent strain is calculated assuming $e^*_{xy} = 0$, high accurate reproduction of welding residual stress should be presumed. Then, the number of unknown components of inherent strain becomes 2/3, and time for computing the inherent strain is saved.

Neglecting the shearing component e^*_{xy}, as an approximation, the inherent strain is calculated as $|e^*| = |e^*_x, e^*_y, 0|^T$. This result is also represented by a dotted line in Fig. 4(a). Comparing these results, it may be said that the role of e^*_{xy} is supplemented by e^*_x.

Figure 4(a) shows that the inherent strain exists in a narrow width from the weld line up to y = 34 mm. Furthermore, the distribution of $\hat{e}^*_x$ along the direction of welding is same at each cross-section except near the ends of the plate and the magnitude of e^*_y distributes in a parabola form along the weld line. These distributions are illustrated schematically in **Fig. 4(b)**.

In order to confirm the accuracy of reproduction of welding residual stress by the standard inherent strain $|\hat{e}^*| = |\hat{e}^*_x, \hat{e}^*_y, 0|^T$, welding residual stress is computed by the elastic analysis imposing to model MO in stress-free state. This welding residual stress is compared with that of the thermal elastic-plastic analysis in **Figs. 5(a)** and **(6)**.

The distributions of welding residual stresses at the central section (x = 0, Fig. 5(a)) and at the cross section (Fig. 5(b)) of x = 400mm (200mm inside from the end of the plate) coincide with each other. The estimated invariant $\hat{s}_\sigma$ to the welding residual stress of thermal elastic-plastic analysis in the whole plate is $\hat{s}_\sigma = 6.5$MPa.

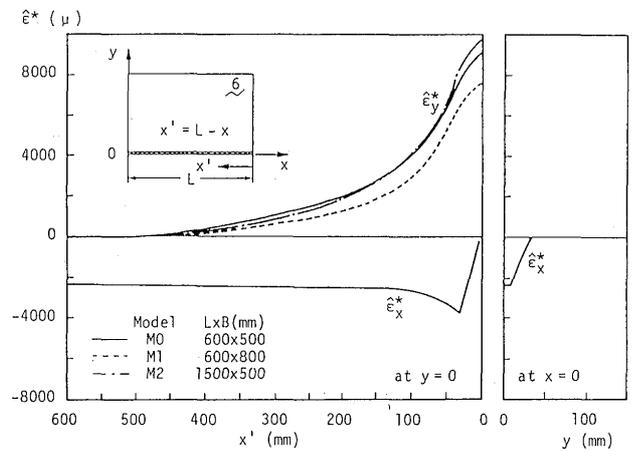As mentioned above, in the case of butt weld, the accuracy of reproduction of welding residual stress is very

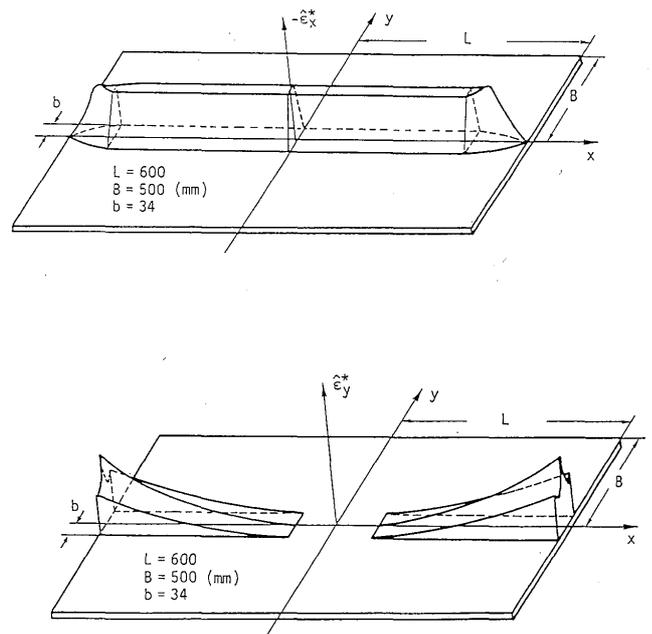**Fig. 4(a)** Distributions of most probable value of inherent strain components.

Fig. 4(b) Schematic representations of distributions of inherent strain components in butt-welded joint.
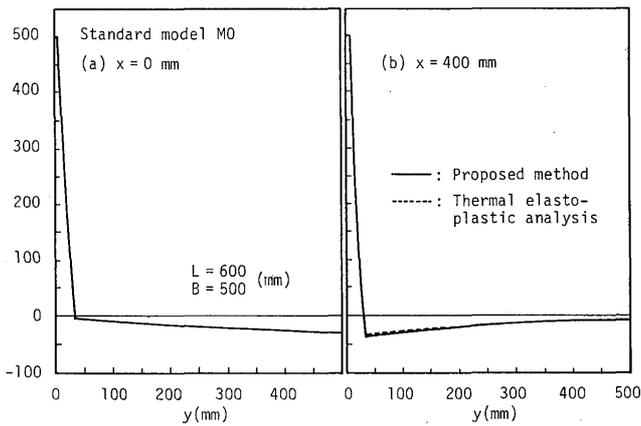
Fig. 5 Welding residual stresses obtain by using standard inherent strains and by thermal elastic-plastic analysis.

good, even if the standard inherent strain without the shearing component is used.

3.4 Adjustment of distribution of the standard inherent strain to different size of plates

When the standard inherent strain is imposed to stress-free models of various sizes for calculation of residual stress, the distribution should be adjusted to fit the size of a model. As for the width direction, the distribution need not be changed since the width of the inherent strain, 2b is much shorter than the plate width 2B.

Along the welding direction, the standard inherent strain distributes uniformly in the middle part of the plate except near the ends, where it has of particular form.

Accordingly, for a plate of different length, the standard inherent strain is adjusted to distribute as follows.

In the case where the length of model, 2L, is larger than $2L_0$, that is $L > L_0$, the distribution of standard inherent strain is cut into two in the middle and extended these adding the same uniform standard inherent strain as in the middle, so that the total length becomes L. In the case

where the length of model 2L is shorter than $2L_0$, that is $L < L_0$, the distribution is made shorter by taking out the middle part of the uniform standard inherent strain to be fit in the total length.

With this adjustment, the elastic analysis is performed on models of the various sizes, models M3 to M6 by imposing the standard inherent strain. The sizes of these models are indicated in the same figure and the resulting welding residual stress at the central section is illustrated in **Fig. 6**. These residual stresses are compared with those obtained by the thermal elasto-plastic analysis. It is observed that the pattern of distribution and the magnitude of welding residual stress can be well reproduced by the proposed method. However, when the width of the plate is 2B = 200mm, the accuracy of estimation of welding residual stress is lowered.

3.5 Components of inherent strain and the accuracy of reproduction of welding residual stress

At first, it is considered how the components of inherent strain influence the distribution pattern of welding residual stress.

Among the components of inherent strain, two components ϵ^*_x and ϵ^*_y are given to model MO separately, and the elastic analysis is performed. The resulting stress distributions are shown in **Fig. 7**.

According to Fig. 7, the most governing component upon the magnitude and distribution of welding residual stress is ϵ^*_x. ϵ^*_y influences directly the inclination of linear distribution of compressive welding residual stress.

As mentioned in the preceeding section, the accuracy of welding residual stress reproduced by using the standard inherent strain is high even for plates being smaller than the standard model.

The following reason may be considered that the magnitude and distribution of ϵ^*_x are almost the same even the sizes of plate are changed in a certain range, since ϵ^*_x is produced mainly depending upon the degree of
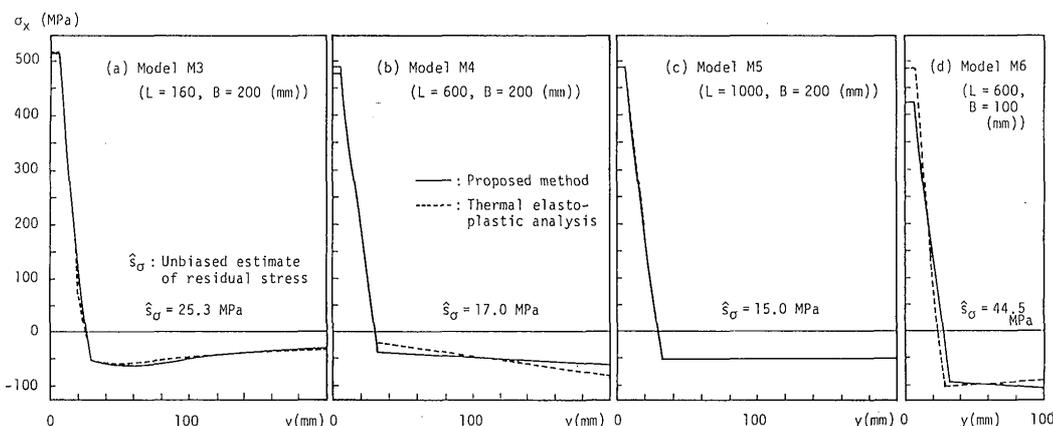

Fig. 6 Reproducibility of welding residual stresses using standard inherent strain.

σ_x (MPa)

Standard model MO
(at x = 0 mm)

——— : $\hat{\epsilon}^*_x$ only
—·— : $\hat{\epsilon}^*_y$ only
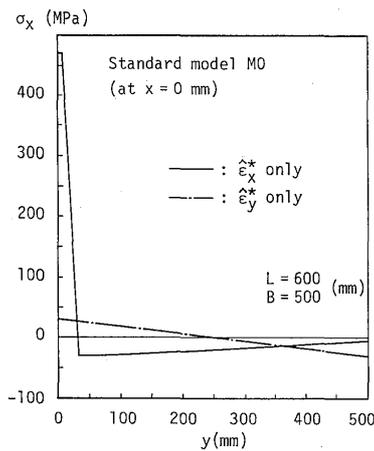
L = 600
B = 500 (mm)

Fig. 7 Stresses produced by $\hat{\epsilon}^*_x$ and $\hat{\epsilon}^*_y$ respectively.

constraint against thermal expansion and contraction in the welding direction, which is almost same if the size of plate is larger.

Next, although the same standard inherent strain is given to the various sizes of plates, the distribution pattern of welding residual stress σ_x is different at each cross section.

The pattern of stress distribution is classified by the ratio between L and B: L/B and illustrated schematically in Fig. 8.

Range I   L/B < 1

Range II   1 < L/B < 4

Range III   4 < L/B

For ranges I to III, the pattern of stress distribution at the central section is named type A, type B and type C for each, respectively.

In range I, the distribution of Type A extends all over the plate length. In range II, type B appears at the central section and type A is observed near the ends. In range III, type C appears at the central section and changes from type B to type A to the ends.

In the case where the length of the joint is shorter than the width, the distribution of type A is produced due to the effect of stress-free end (x = L) or shear lag. Although the influence of shear lag vanishes in the distribution of type B, the influence of in-plane bending due to ϵ^*_y produced near the ends is remarkable, since the plate is not so long.

In type C, the plate is long enough, so that the effect of shear lag, and the influence of bending by ϵ^*_y do not extend to the middle of the plate. This is what is called the plane deformation state and only contributes in producing stresses in uni-axial stress state.

When welding residual stress is predicted by the proposed method, the accuracy of estimation is related

with the pattern of stress distribution rather than the size of the plate.

The accuracy of estimation is good enough in the cases of types A and C. However, it is slightly lowered in the case of type B, where the distribution pattern is transient between types A and B. One of the reason is that ϵ^*_y of the standard model is used regardless of the ratio L/B.

Considering the distribution pattern of the standard inherent strain, this predicting method can be applicable accurately for L = B > 100 mm, and the accuracy of estimation is as follows. (Fig. 9)

(1) Range 1

In the rang of L/B $\geqslant$ 1 or B $\geqslant$ 500 (mm), the accuracy of estimation is good enough over the whole length of plate.

(2) Range 2

In the range of 1 < L/B < 4 and B < 500 mm, the stress distribution at the central section becomes type B, and the accuracy of estimation is slightly lowered. However, at the ends of the plate, it becomes type A and the accuracy is improved.

(3) Range 3

In the range of 4 < L/B and B < 500 mm, the stress distribution becomes type C and type A in the sections near the center and the ends, and the accuracy is good enough. However, it becomes type B in the section between the center and the ends and the accuracy is slightly lowered.
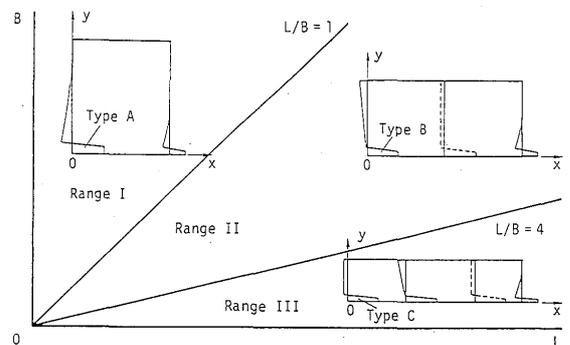
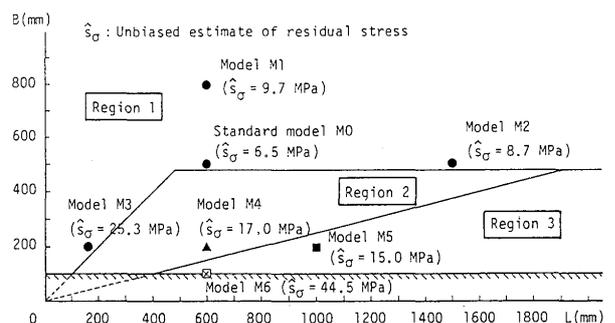**Fig. 8** Ferture of welding residual stress distributions in three ranges of L/B.

Fig. 9 Applicability of proposed method.

From the results, as the inference mentioned in the preceding section, once the inherent strain of the standard model is estimated, the residual stress can be estimated by imposing the standard inherent strain to the stress-free plate of various sizes and performing the elastic analysis. The residual stress obtained by the method is demonstrated to be accurate by the numerical experiment using the finite element method.

Consequently, the validity and applicability of the predicting method of welding residual stress proposed in this study is confirmed.

4. Conclusions

Summing up the obtained results in this paper, the following conclusions can be drawn.

(1) A predicting method of welding residual stress by using the source of residual stress (inherent strain) is developed. The validity and applicability of the method is demonstrated by taking the butt weld as an example.

That is, when the kind of steel and welding conditions are specified and the sizes of the welded joint become larger to some degree, the inherent strain produced at the plate (standard model) is almost the same one in the infinite plate. The inherent strain (standard inherent strain) produced in such finite plate is enough to be calculated only once. Thermal elastic-plastic analysis is not necessary to be performed on in spite of various changes of sizes of members. Welding residual stress is easy to be calculated by elastic analysis giving the predicted standard inherent strain. This is the characteristic and advantage of this new predicting method of welding residual stress and the accuracy is good enough.

(2) In the case of a butt weld, by using standard inherent strain: $| \epsilon^* | = | \epsilon^*_x, \epsilon^*_y, 0 |^T$ of shearing inherent strain $\epsilon^*_{xy} = 0$, the distributions and magnitudes of welding residual stresses are reproduced accurately.

(3) The longitudinal component of inherent strain, ϵ^*_x, which governs the distributions and magnitudes of welding residual stress. The component perpendicular to the weld line ϵ^*_y has a great influence on the slope of linear distribution of compressive welding residual stress.

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