Generation of Extracellular Current in vivo

~ Mechanism of Cardiac Electrophysiology ~

S. Uto, C.T. Maguire* and H. Wakimoto*
5-16-1 Omiya, Asahi-ku, Osaka 535-8585, Japan
Tel: +81-6-6954-4445, Fax: +81-6-6957-2133
E-mail: suto@ee.oit.ac.jp
*Children's Hospital, Harvard Medical School,
Enders Research Building, 320 Longwood Avenue, Boston, MA 02115, USA

An application of quantitative methods in electrophysiology is extensive [1], especially in a field of cardiology [2]. Electrical signal strength of the heart is considerably larger than that from other bioelectric sources; body surface signals of around 5 mV are typical. So, the model simplicity, the relative ease of measurement, and the recognized importance of heart disease attracted the attention of physiologists, physicists, mathematicians and engineers. Simulation of the cardiac body surface potentials may begin with measured isochrones obtained from body surface mapping method and so on [3]. The simulation accuracy depends on an ability to include structural details, ionic behavior, etc.; it has an advantage in that it describes the link between sources and the cellular structure and electrophysiology. As an electrical generator the heart can be approximated by a single dipole with varying magnitude and orientation through the cardiac cycle. The dipole is vector sum of local electrical potentials in a myocardium. We aim to get quantitative relationship between the local electrical potentials and the body surface potentials.

An electrocardiography (ECG) is useful method to measure the body surface potentials. The standard or limb leads are placed at the extremities as shown in Fig.1. Placement is not critical since the extremities are, roughly, isopotential. The

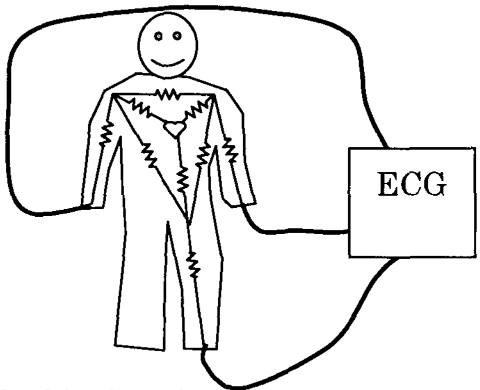
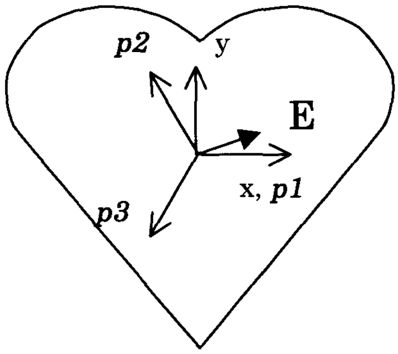


Fig.1 The standard leads and schematic circuits in the body.

electromotive force (emf) with varying magnitude and orientation through the cardiac cycle can be calculated by this measurement. A distributed constant circuit model is more proper in this case, but a lumped constant circuit model is adopted to be simple.



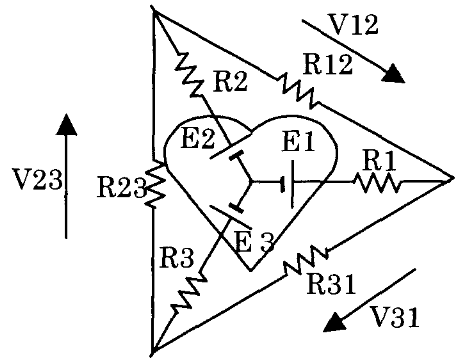


Fig.2　A rectangular coordinate system.　Fig.3 The emf and the equivalent circuit.

A rectangular coordinate system is determined as shown in Fig.2. A vector **E** is the emf due to the heart activation. Vectors **p1**, **p2** and **p3** are unit vectors pointing each ECG electrodes. The magnitude of the projection of **E** onto each unit vectors is given by the dot product as

$$\begin{aligned} E1 &= \mathbf{E}\cdot\mathbf{p1} \\ E2 &= \mathbf{E}\cdot\mathbf{p2} \qquad (1) \\ E3 &= \mathbf{E}\cdot\mathbf{p3} \end{aligned}$$

, where the relation between unit vectors is

$$\mathbf{p1}+\mathbf{p2}+\mathbf{p3}=0 \qquad (2)$$

. The electrical circuit can be writen as shown in Fig.3 with those projections. Using the Kirchhoff's law, we can get a relation as

$$\begin{pmatrix} E1-E2 \\ E2-E3 \\ E3-E1 \end{pmatrix} = \begin{pmatrix} R1+R12+R2 & -R2 & -R1 \\ -R2 & R2+R23+R3 & -R3 \\ -R1 & -R3 & R3+R31+R1 \end{pmatrix} \cdot \begin{pmatrix} \frac{V12}{R12} \\ \frac{V23}{R23} \\ \frac{V31}{R31} \end{pmatrix} \qquad (3).$$

From (2) and (3),

$$\begin{aligned} &\mathbf{p3} = -(\mathbf{p1}+\mathbf{p2}) \\ &\mathbf{E}\cdot\mathbf{p3} = -\mathbf{E}\cdot(\mathbf{p1}+\mathbf{p2}) \quad (4) \\ &E3 = -(E1+E2) \end{aligned}$$

can be derived. Substituting (4) into (1) yields

$$\begin{pmatrix} E1-E2 \\ E1+2E2 \end{pmatrix} = \begin{pmatrix} \frac{R1+R12+R2}{R12}+\frac{R1}{R31} & \frac{R1}{R31}-\frac{R2}{R23} \\ -\frac{R2}{R12}-\frac{R3}{R31} & \frac{R2+R23+R3}{R23}-\frac{R3}{R31} \end{pmatrix} \cdot \begin{pmatrix} V12 \\ V23 \end{pmatrix} \quad (5)$$

, where a relation

$$V12+V23+V31=0 \quad (6)$$

is used.

Equation (5) means that the emf can be determined by ECG signals. The accuracy of the measurement is depends on how to determine the resistances precisely. In the real body, the resistanses must be distributed through the many kind of tissues, so precise measurement is not so easy. The determination of the resistances is one of most important problem in precise ECG.

References

[1] E.R.Kandel, J.H.Schwarts and T.M.Jessell, Principles of Neural Science, McGraw-Hill.

[2] R.Plonsey and R.C.Barr, Bioelectricity, Kluwer Academic / Plenum Publishers.

[3] H.Wakimoto, N.Izumida, Y.Asano, M.Hiraoka, T.Kawara, K.Hiejima, TK.Hirao and F.Suzuki, J.Cardiovasc.Electrophysiol., **11** (2000) 52.