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Plasma and Thermal Neutron Diffusions in Two-Dimensional Periodic Structures Analogical to Photonic Crystals

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We theoretically discuss plasma and thermal neutron diffusions in two-dimensional periodic structures with respect to the analogy of photonic crystals. In such structures, there exist the relaxation frequency regions and damping constant regions in which diffusion cannot take place, which may also provide novel contribution in applications utilizing plasma and thermal neutron.

KEYWORDS：plasma, thermal neutron, diffusion, two-dimensional periodic structure

フォトニック結晶との類推による二次元周期構造でのプラズマ、熱中性子拡散

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我々はフォトニック結晶との類似の観点から二次元周期構造でのプラズマ、熱中性子拡散を議論する。そのような構造では拡散が生じない緩和周波数領域や減衰定数領域が存在し、そのことはまたプラズマや熱中性子を用いた応用において斬新な貢献をもたらすかもしれない。

1. Introduction

Recently, photonic crystals with periodic dielectric structures have attracted much attention from both fundamental and practical viewpoints, because novel concepts such as photonic band gaps have been predicted, and various new applications of the photonic crystals have been proposed.[1, 2] In earlier works, two fundamentally new optical principles, that is, the localization of light and the controllable inhibition of spontaneous emission of light were considered to be the most important.

In nuclear fusion and fission, on the other hand, plasma and thermal neutron diffusion are important problems. Especially, the inhibition of diffusion would provide novel applications, because nuclear fusion and fission significantly depend on plasma and thermal neutron diffusion, respectively.[3, 4]

Therefore, we propose the use of periodic structures in diffusion with respect to the analogy of photonic crystals. Indeed, plasma diffusion is described by the diffusion equation, while electromagnetic waves are described by the wave equation, that is, plasma diffusion and light propagation are different physical phenomena. However, diffusion equations with constant relaxation frequencies and damping constants are analogical to wave equations with constant frequencies, and therefore, periodic structures have relaxation frequency regions and damping constant regions in which plasma diffusion cannot take place like inhibition of propagation of electromagnetic waves in photonic band gaps. We define such relaxation frequency regions and damping constant regions as diffusion band gaps. Moreover, there exist complex dispersion relations between relaxation frequencies, damping constants and directions of diffusions. We define such dispersion relations as diffusion band structures.

In this paper, we theoretically demonstrate plasma and thermal neutron diffusion in two-dimensional periodic structures with triangular and square lattices, respectively. We do not consider diffusion perpendicular to two-dimensional planes. In plasma diffusion, two-dimensional periodic structures are assumed to be composed of periodic cylindrical magnetic field, although it is difficult to sustain such structures. In thermal neutron diffusion, on the other hand, two-dimensional periodic structures are assumed to be composed of periodic cylindrical moderators. Such structures could easily be realized.

2. Plasma diffusion

2.1 Theory

In order to obtain diffusion band structures, we start with the diffusion equations of plasma for two-dimensional structures

$$\nabla \cdot [D(\mathbf{r})\nabla N(\mathbf{r},t)] = \frac{\partial N(\mathbf{r},t)}{\partial t}, \qquad (2.1)$$

where D(**r**) is the diffusion coefficient, and N(**r**, t) is the particle number of plasma. For simplification, D(**r**) is assumed to depend only on space. The diffusion coefficient D(**r**)=D(**r**+**R**) is periodic with respect to the lattice vector **R** generated by the primitive translation and it may be expanded in a Fourier series on **G**, the reciprocal vector

$$D(\mathbf{r}) = \sum_{\mathbf{G}} D(\mathbf{G})\exp(i\mathbf{G}\cdot\mathbf{r}), \qquad (2.2)$$

where D(**G**) is a Fourier coefficient of D(**r**).

N(**r**, t) is described by the following equation.

$$N(\mathbf{r},t) = N(\mathbf{r})\exp(-\gamma t), \qquad (2.3)$$

where γ is a relaxation frequency. Using Bloch's theorem, we may expand N(**r**) as

$$N(\mathbf{r}) = \sum_{\mathbf{G}} N(\mathbf{G})\exp\{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}\}, \qquad (2.4)$$

where N(**G**) is a Fourier coefficient of N(**r**) and **k** is the wave vector which indicates the direction of diffusions.

Inserting Eqs. (2. 2)-(2. 4) into Eq. (2. 1) results in the following infinite matrix eigenvalue problem:

$$\sum_{\mathbf{G}'} M_{\mathbf{G},\mathbf{G}'} N(\mathbf{G}') = \gamma N(\mathbf{G}), \qquad (2.5a)$$

where

$$M_{\mathbf{G},\mathbf{G}'} = D(\mathbf{G}-\mathbf{G}')(\mathbf{k}+\mathbf{G})\cdot(\mathbf{k}+\mathbf{G}'). \qquad (2.5b)$$

For numerical purposes, Eq. (2. 5a) is truncated by retaining only a finite number of reciprocal lattice vectors. However, high convergence cannot be obtained from Eq. (2. 2). Thus, we use another method, that is, to calculate the matrix of Fourier coefficients of 1/D(**r**) and then take their inverse in order to obtain required coefficients D(**G-G'**). In the case of triangular lattices, D^{-1}(**G-G'**) is represented as

$$D^{-1}(\mathbf{G}-\mathbf{G}') = \begin{cases} 1/D_{out} + (1/D_{in} - 1/D_{out}) f(\mathbf{G}=\mathbf{G}') \\ 2(1/D_{in} - 1/D_{out}) \dfrac{J_1(|\mathbf{G}-\mathbf{G}'|R)}{|\mathbf{G}-\mathbf{G}'|R} f(\mathbf{G}\neq\mathbf{G}') \end{cases},$$

where $f = 2\pi R^2/\sqrt{3}a^2$, R is the radius of cylindrical magnetic field, a is the lattice constant, and D_{in} and D_{out} are diffusion coefficients inside and outside cylindrical magnetic field, respectively. J_1 is the first-order Bessel function. D(**G-G'**) can be obtained from the inverse matrix of D^{-1}(**G-G'**). Then, errors of eigenfrequencies computed with 441 and 1369 reciprocal vectors are within 1%. Therefore, we calculate diffusion band structure with 441 reciprocal vectors.

This problem corresponds to the transversal electric (TE) mode in two-dimensional photonic

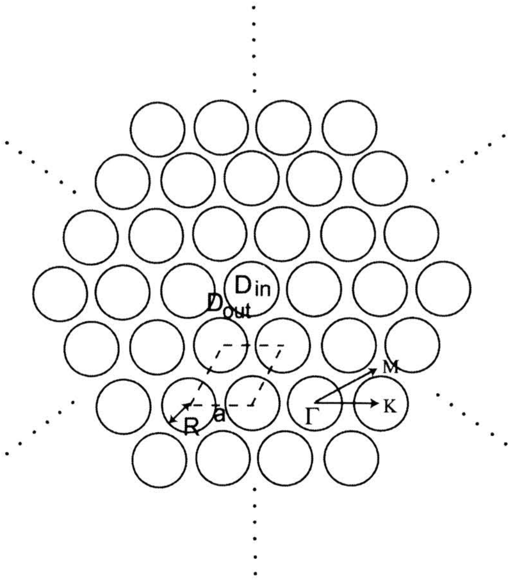

Fig.1. Schematic diagram of a two-dimensional structure with periodic diffusion coefficients. D_{in} and D_{out} indicate diffusion coefficients inside and outside cylindrical magnetic field. R indicates the radius of cylindrical magnetic field. a is the lattice constant of triangular lattices.

crystals. In the case of the TE mode, photonic crystals composed of air rods with triangular lattices in a dielectric substrate possess large photonic band gaps. The velocity of light decreases with increasing dielectric indices, which corresponds to the small diffusion coefficient in plasma diffusion. Therefore, we suppose that diffusion coefficients outside cylindrical magnetic field are smaller than those inside cylindrical magnetic field.

2.2 Results and discussion

In Fig. 1, we show the schematic diagram of structures with periodic diffusion coefficients composed of cylindrical magnetic field with triangular lattices. Magnetic field is perpendicular to two-dimensional planes. The region embedded by dotted lines indicates the unit cell. In Fig. 2, we calculate the diffusion band structure of plasma at D_{in}=0.1 [m^2/s], D_{out}=0.01 [m^2/s] and R/a=0.45. Shaded region indicates the diffusion band gap in which plasma diffusion cannot take place. The Γ, M and K indicate directions of diffusions that are drawn by arrows in Fig. 1.

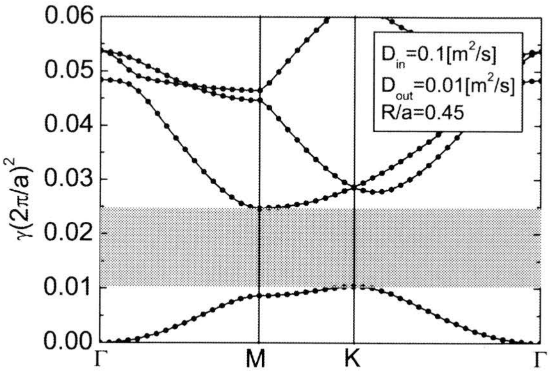

Fig.2. Diffusion band structure of plasma at D_{in}=0.1 [m^2/s], D_{out}=0.01 [m^2/s] and R/a=0.45. A shaded region indicates a diffusion band gap.

In photonic crystals, spontaneous emission with a certain frequency is inhibited by photonic band gaps. Relaxation frequencies significantly depend on electromagnetic energy densities and plasma temperature, that is, it is possible to obtain plasma with a certain relaxation frequency artificially. Therefore, plasma diffusion with a certain relaxation frequency may be inhibited by diffusion band gaps. However, we must control diffusion band gaps in considering that diffusion coefficients depend on external factors such as plasma temperature practically.

Although we focused our attention on two-dimensional structures with periodic diffusion coefficients, three-dimensional structures with periodic diffusion coefficients that possess diffusion band gaps could be realized with respect to the analogy to three-dimensional photonic crystals, which may make it possible to inhibit plasma diffusion in three dimensions.

3. Thermal neutron diffusion

3. 1 Theory

In order to obtain diffusion band structures, we start with the diffusion equation of thermal neutron for two-dimensional structures

$$\frac{1}{t_d(\mathbf{r})\Sigma_a(\mathbf{r})}[\nabla\cdot\{D(\mathbf{r})\nabla\phi(\mathbf{r},t)\} - \Sigma_a(\mathbf{r})\phi(\mathbf{r},t)] = \frac{\partial\phi(\mathbf{r},t)}{\partial t}, \qquad (3.1)$$

where D(**r**), t_d(**r**) and Σ_a(**r**) are the diffusion coefficient, the diffusion time and the absorption cross section, respectively, and ϕ(**r**, t) is the thermal neutron flux. We do not consider thermal neutron sources to investigate properties of two-dimensional structures with periodic moderators. The diffusion coefficient D(**r**)=D(**r**+**R**), the diffusion time t_d(**r**)= t_d(**r**+**R**) and the absorption cross section Σ_a(**r**)= Σ_a(**r**+**R**) are periodic with respect to the lattice vector **R** generated by the primitive translation, and they may be expanded in Fourier series on **G**, the reciprocal vector

$$D(\mathbf{r}) = \sum_{\mathbf{G}} D(\mathbf{G})\exp(i\mathbf{G}\cdot\mathbf{r}), \qquad (3.\ 2a)$$

$$\Sigma_a(\mathbf{r}) = \sum_{\mathbf{G}} \Sigma_a(\mathbf{G})\exp(i\mathbf{G}\cdot\mathbf{r}), \qquad (3.\ 2b)$$

$$\frac{1}{t_d(\mathbf{r})\Sigma_a(\mathbf{r})} = \sum_{\mathbf{G}} (t_d\Sigma_a)^{-1}(\mathbf{G})\exp(i\mathbf{G}\cdot\mathbf{r}), \qquad (3.\ 2c)$$

where D(**G**), Σ_a(**G**) and $(t_d\Sigma_a)^{-1}$(**G**) are the Fourier coefficients of D(**r**), Σ_a(**r**) and 1/t_d(**r**)Σ_a(**r**), respectively.

ϕ(**r**, t) is described by the following equation.

$$\phi(\mathbf{r},t) = \phi(\mathbf{r})\exp(-\lambda t), \qquad (3.\ 3)$$

where λ is a damping constant. Using Bloch's theorem, we may expand ϕ(**r**) as

$$\phi(\mathbf{r}) = \sum_{\mathbf{G}} \phi(\mathbf{G})\exp\{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}\}, \qquad (3.\ 4)$$

where ϕ(**G**) is a Fourier coefficient of ϕ(**r**) and **k** is the wave vector which indicates the direction of diffusion.

Inserting Eqs. (3. 2)-(3. 4) into Eq. (3. 1) results in the following infinite matrix eigenvalue problem:

$$\sum_{\mathbf{G}'} \Phi_{\mathbf{G},\mathbf{G}'}\phi(\mathbf{G}') = \lambda\phi(\mathbf{G}), \qquad (3.\ 5a)$$

where

$$\Phi_{\mathbf{G},\mathbf{G}'} = \sum_{\mathbf{G}''} (t_d\Sigma_a)^{-1}(\mathbf{G}-\mathbf{G}'') \{D(\mathbf{G}''-\mathbf{G}')(\mathbf{k}+\mathbf{G}'')\cdot(\mathbf{k}+\mathbf{G}') + \Sigma_a(\mathbf{G}''-\mathbf{G}')\}. \qquad (3.\ 5b)$$

For numerical purposes, Eq. (3. 5a) is truncated by retaining only a finite number of reciprocal lattice vectors. However, high convergence cannot be obtained from Eqs. (3. 2a)-(3. 2c). Thus, we use another method, that is, to calculate the matrix of Fourier coefficients of 1/D(**r**), 1/Σ_a(**r**) and t_d(**r**)Σ_a(**r**), and then take their inverse in order to obtain required coefficients D(**G-G'**), Σ_a(**G-G'**) and $(t_d\Sigma_a)^{-1}$(**G-G'**), respectively. In the case of square lattices, for example, $(t_d\Sigma_a)$ (**G-G'**) is represented as

$$(t_d\Sigma_a)(\mathbf{G}-\mathbf{G}') = \begin{cases} t_d^{out}\Sigma_a^{out} + (t_d^{in}\Sigma_a^{in} - t_d^{out}\Sigma_a^{out})f(\mathbf{G}=\mathbf{G}') \\ 2(t_d^{in}\Sigma_a^{in} - t_d^{out}\Sigma_a^{out})\dfrac{J_1(|\mathbf{G}-\mathbf{G}'|R)}{|\mathbf{G}-\mathbf{G}'|R} f(\mathbf{G}\neq\mathbf{G}') \end{cases}$$

where $f = \pi R^2/a^2$, R is a radius of a rod and a is a lattice constant. t_d^{in} and t_d^{out} are diffusion time inside and outside rods, respectively, while Σ_a^{in} and Σ_a^{out} are absorption cross sections inside and outside rods, respectively. J_1 is the first-order Bessel function. $(t_d\Sigma_a)^{-1}$(**G-G'**) can be obtained from the inverse matrix of $(t_d\Sigma_a)$ (**G-G'**). Then, errors of eigenfrequencies computed with 441 and 1369 reciprocal vectors are within 1%. Therefore, we calculate diffusion band structure with 441 reciprocal vectors.

In the case of C, the diffusion coefficient, the diffusion time, the absorption cross section are D^c=0.84[cm], t_d^c=0.017[s] and Σ_a^c=2.4x10^{-4}[cm^{-1}], respectively. In the case of H_2O, on the other hand, the diffusion coefficient, the diffusion time, the absorption cross section are D^{H_2O}=0.16[cm], $t_d^{H_2O}$=2.1x10^{-4}[s] and $\Sigma_a^{H_2O}$=0.0197[cm^{-1}], respectively.

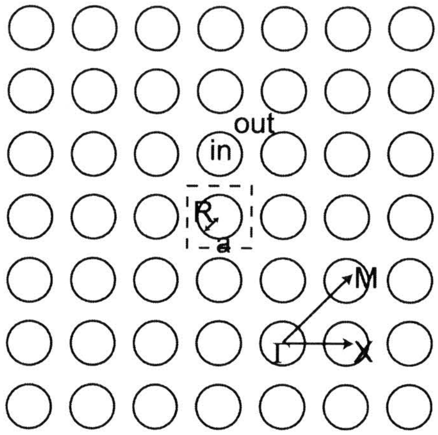

Fig.3. Schematic diagram of a two-dimensional structure with periodic moderators. R is the radius of a rod. a is the lattice constant of square lattices. Arrows indicate the direction of thermal neutron diffusion.

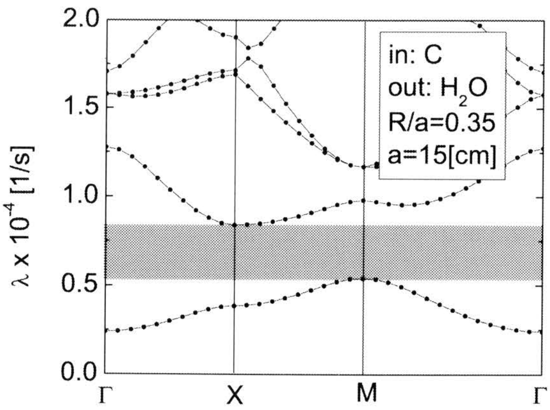

Fig.4. Diffusion band structure of thermal neutron when moderators inside and outside rods are C and H_2O, respectively. The radius of a rod is R/a=0.35, where a=15[cm]. A shaded region indicates a diffusion band gap.

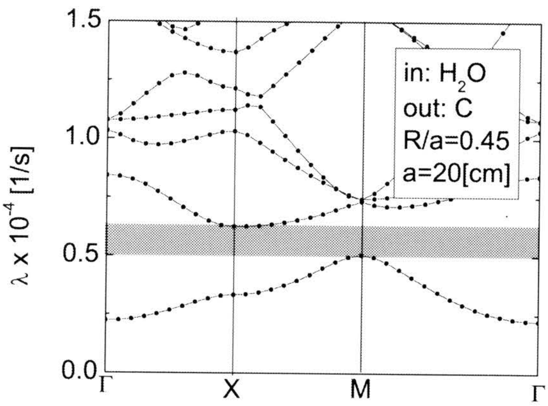

Fig.4. Diffusion band structure of thermal neutron when moderators inside and outside rods are H_2O and C, respectively. The radius of a rod is R/a=0.45, where a=20[cm]. A shaded region indicates a diffusion band gap.

3.2 Results and discussion

In Fig.3, we show the schematic diagram of two-dimensional structures with periodic moderators composed of rods with square lattices. The region embedded by dotted lines indicates the unit cell. Arrows indicate directions of diffusion.

In Fig. 4, we calculate the diffusion band structure of thermal neutron when moderators inside and outside rods are C and H_2O, respectively. The radius of a rod is R/a=0.35, where a=15[cm]. A shaded region indicates the diffusion band gap in which thermal neutron diffusion cannot take place. The Γ, X and M indicate the direction of diffusion that is drawn by arrows in Fig. 3. As shown in Fig. 3, there exists a cutoff damping constant in the diffusion band structure. The cutoff damping constant is 2410 [1/s].

In Fig. 5, moreover, we calculate the diffusion band structure of thermal neutron when moderators inside and outside rods are H_2O and C, respectively. The radius of a rod is R/a=0.45, where a=20[cm]. A shaded region indicates the diffusion band gap in which thermal neutron diffusion cannot take place. As shown in Fig. 5, there exists a cutoff damping constant in the diffusion band structure. The cutoff damping constant is 2230[1/s].

In photonic crystals, the spontaneous emission with a certain frequency can be inhibited by photonic band gaps. Thermal neutron diffusion occurs by thermal neutron flux spreading in the real space. However, thermal neutron flux with a certain damping constant in diffusion band gaps cannot spread because of the lack of solutions in diffusion band structures. Therefore, thermal neutron diffusion with a certain damping constant may be inhibited by diffusion band gaps.

Although we focused our attention on two-dimensional structures with periodic moderators in this paper, three-dimensional structures with periodic moderators that possess diffusion band gaps could be realized from the viewpoint analogical to three-dimensional photonic crystals, which may make it possible to inhibit thermal neutron diffusion in three dimensions.

4. Conclusions

In conclusion, we theoretically demonstrated plasma and thermal neutron diffusions in two-dimensional periodic structures with respect to the analogy of photonic crystals. In such structures, there exist the relaxation frequency regions and damping constant regions in which diffusion cannot take place, which may provide novel contribution in applications utilizing plasma and thermal neutron diffusion.

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