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Effect of Various Factors on Transverse Shrinkage under Butt Welding[†]

Masakazu SHIBAHARA* and Hidekazu MURAKAWA **

Abstract

In this report, the effect of various factors on welding deformation is investigated in order to predict the transverse deformation in high speed FCB one side butt welding of large steel plates. To classify the problem with respect to the size and the welding condition, the welding on small laboratory specimen and FCB welding of large plate are compared based on governing parameters derived from similarity rule. Further to clarify the effect of the governing parameters on the welding deformation, serial transient thermal-elastic-plastic FEM analyses are performed. In case of small specimen, the validity of the empirical formula proposed by Satoh and Terasaki is examined. Based on the computed results, the effects of the welding conditions, the size of plate and the tab are summarized.

KEY WORDS:(Welding Deformation) (Transverse Shrinkage) (Butt Welding) (Similarity Rule) (Theoretical Prediction)

1. Introduction

Due to the shortage of skilled workers, automated machines and robots are introduced in shipyards. To obtain expected productivity through mechanization, high precision of parts to be assembled must be maintained. However, most technologies employed in shipbuilding are thermal processes such as cutting, forming, welding and straightening. These thermal processes provoke distortions as an inevitable result of localized thermal cycle. Unfortunately, the level of the current knowledge of the geometrical error arising from thermal processes is not high enough to predict or control the distortion. For accurate prediction and effective control of distortion, comprehensive understanding of the mechanical phenomena is crucial. In particular, computational welding mechanics is a key factor for the further innovation of ship building technology.

In this report, the effect of various factors on welding deformation are investigated in order to predict the transverse deformation in high speed FCB one side butt welding of large steel plates. To clarify the problem, the welding on small laboratory specimen and FCB welding of large plate are compared based on governing parameters derived from similarity rule.

Further to clarify the effect of the governing parameters on the welding deformation, serial transient thermal-elastic-plastic FEM analyses are performed. In the case of small specimens, the validity of the empirical formula proposed by Satoh and Terasaki is examined. Based on the computed results, the effects of the welding conditions, the size of plate and the tab are summarized.

2. Empirical Formula for Transverse Shrinkage

Welding distortion can be categorized into longitudinal shrinkage, transverse shrinkage, angular distortion, buckling and other forms. In this study, transverse shrinkage in the butt welding of skin plate by one side welding is investigated based on thermal-elastic-plastic FEM analysis. **Figure 1** shows the relation between the transverse shrinkage S normalized by plate thickness h and heat input parameter Q/h^2 reported by Satoh and Terasaki¹⁾. Where, Q is heat input per unit length of welding line. This figure contains two types of phenomena according to the magnitude of heat input parameter Q/h^2 . When the heat input is small relative to the plate thickness, the melting is limited on the welding side and full thickness is melted when the heat input is large as in the case of one side butt welding. As it is indicated in the figure, the

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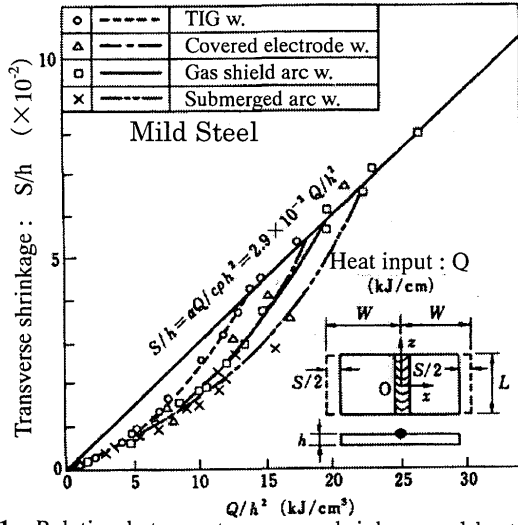


Fig.1 Relation between transverse shrinkage and heat input parameter.

transverse shrinkage is roughly proportional to the heat input when Q/h^2 is large. Based on the laboratory test, Satoh and Terasaki proposed the following equation.

$$S = \alpha Q / c \rho h = S_0 \quad (1)$$

Where,

- α : thermal expansion coefficient
- c : specific heat
- ρ : density

Physically, Eq.(1) implies that the magnitude of the transverse shrinkage S is the exactly same as the total thermal expansion due to heat input Q and it is independent of mechanical properties of the material, such as the yield stress and the Young's modulus. If Eq.(1), proposed for small-scale laboratory test, holds for the real scale butt welding in shipyard, transverse shrinkage of the plate due to butt-welding can be easily estimated. To examine the applicability of Eq.(1), serial computations are conducted in this research.

3. Comparison between Small Scale Test and Real Welding Based on Similarity

3.1 Similarity holds for butt-welding

As it is shown in reference ²⁾, if the magnitude and the distribution of the temperature are the same, similarity holds for the mechanical phenomena in welding of geometrically similar models under the condition that the material is same. Under such situations, it is enough to consider the similarity of the temperature field. According to the similarity rule³⁾, the following dimension-less parameters must be the same.

- (1) parameter for heat input

(average temperature rise)

$$\xi_1 = T_{av}^* = \frac{Q}{c \rho B h T_{melt}} \quad (2)$$

- (2) parameter for travelling speed
(traveling speed relative to the heat conduction in the distance of plate width)

$$\xi_2 = v^* = \frac{c \rho B v}{\lambda} \quad (3)$$

- (3) parameter for heat transfer

$$\xi_3 = \frac{\gamma}{c \rho v} \quad (4)$$

- (4) geometrical parameters

$$\xi_4 = \frac{h}{B}, \quad \xi_5 = \frac{L}{B} \quad (5)$$

Where,

- λ : thermal conductivity
- γ : heat transfer coefficient
- T_{melt} : mechanical melting point
- v : traveling speed
- L : length of plate
- B : breadth of plate

The effects of the above parameters on transverse shrinkage in butt welding are clarified in the following chapters.

3.2 Comparison between small scale test and real scale welding

Following the definition of the dimension-less parameters, small-scale laboratory tests by Satoh and Terasaki and butt welding of skin plate in shipyard are compared. Figure 2 shows their comparison with respect to T_{av}^* and v^* . As it is seen from Fig.2 the difference between the dimensionless parameter for the travelling speed, v^* , for the small-scale tests and the butt welding in shipyards reaches almost two order. Similarly

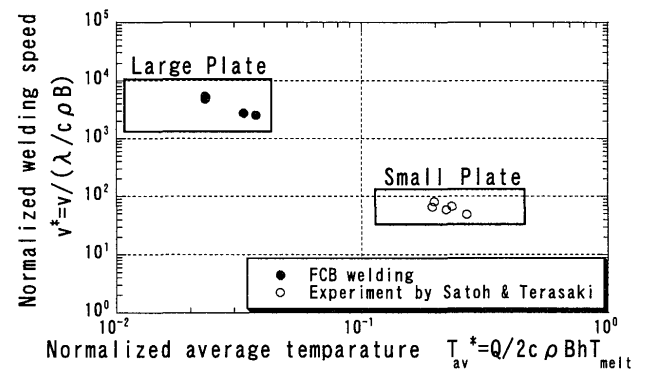
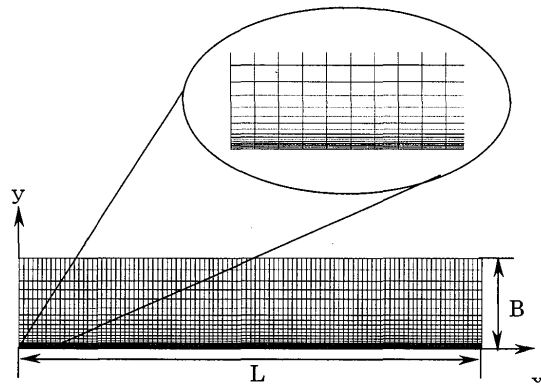
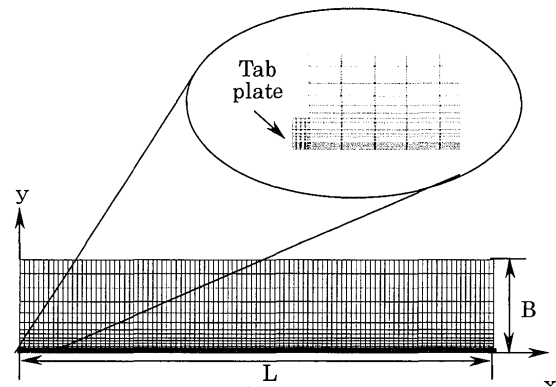


Fig.2 Parameters governing transverse shrinkage in butt welding.



(a) without tab plate.



(b) with tab plates.

Fig.3 FEM mesh divisions.

the difference in the heat input parameter is almost one order. Therefore it is quite natural to expect some differences in mechanical phenomena in the two situations. To clarify the differences, there are two ways. One is the experiment and the other is the computer simulation. In the case of experiment, the cost may be too high if full scale specimens are used. Small scale tests simulating the real welding may not be possible. Because, according to the similarity rule, the travelling speed must be ten times faster if the size of the model is one tenth of the real model. Thus, it is readily understood that the experimental method is not a realistic choice and FEM simulation is an ideal method for the present problem.

Table 1 Welding condition for small plate.

Analysis Case	Heat input (kJ/mm)	Welding speed v(mm/min)	Length×Width×Thick L(mm)×B(mm)×t (mm)
Case- 1	0. 36	250	200×100×5
Case- 2	0. 72	250	200×100×5
Case- 3	1. 44	250	200×100×5
Case- 4	0. 36	500	200×100×5
Case- 5	0. 72	500	200×100×5
Case- 6	1. 44	500	200×100×5
Case- 7	0. 36	1000	200×100×5
Case- 8	0. 72	1000	200×100×5
Case- 9	1. 44	1000	200×100×5
Case- 10	0. 36	500	100×100×5
Case- 11	0. 72	500	100×100×5
Case- 12	1. 44	500	100×100×5
Case- 13	0. 36	500	500×100×5
Case- 14	0. 72	500	500×100×5
Case- 15	1. 44	500	500×100×5
Case- 16	0. 72	1000	100×100×5
Case- 17	0. 72	1000	500×100×5
Case- 18	0. 72	250	100×100×5
Case- 19	0. 72	250	500×100×5

Table 2 Welding condition for large plate.

Analysis Case	Heat input (kJ/mm)	Welding speed v(mm/min)	Length×Width×Thick L(mm)×B(mm)×t (mm)
Case-20	5. 4	250	20000×4000×20
Case-21	10. 8	250	20000×4000×20
Case-22	21. 6	250	20000×4000×20
Case-23	5. 4	500	20000×4000×20
Case-24	10. 8	500	20000×4000×20
Case-25	21. 6	500	20000×4000×20
Case-26	5. 4	1000	20000×4000×20
Case-27	10. 8	1000	20000×4000×20
Case-28	21. 6	1000	20000×4000×20
Case-29	5. 4	2000	20000×4000×20
Case-30	10. 8	2000	20000×4000×20
Case-31	21. 6	2000	20000×4000×20
Case-32	10. 8	250	8000×4000×20
Case-33	10. 8	500	8000×4000×20
Case-34	10. 8	1000	8000×4000×20
Case-35	10. 8	2000	8000×4000×20
Case-36	10. 8	250	4000×4000×20
Case-37	10. 8	500	4000×4000×20
Case-38	10. 8	1000	4000×4000×20
Case-39	10. 8	2000	4000×4000×20

Table 3 Welding condition for large plate with tab.

Analysis Case	Heat input (kJ/mm)	Welding speed v(mm/min)	Length×Width×Thick L(mm)×B(mm)×t (mm)	Tab size ℓ (mm)×b(mm)
Case-40	10. 8	250	20000×4000×20	200×50
Case-41	10. 8	500	20000×4000×20	200×50
Case-42	10. 8	1000	20000×4000×20	200×50
Case-43	10. 8	2000	20000×4000×20	200×50
Case-44	10. 8	250	20000×4000×20	400×100
Case-45	10. 8	500	20000×4000×20	400×100
Case-46	10. 8	1000	20000×4000×20	400×100
Case-47	10. 8	2000	20000×4000×20	400×100
Case-48	10. 8	250	8000×4000×20	400×100
Case-49	10. 8	500	8000×4000×20	400×100
Case-50	10. 8	1000	8000×4000×20	400×100
Case-51	10. 8	2000	8000×4000×20	400×100
Case-52	10. 8	250	4000×4000×20	400×100
Case-53	10. 8	500	4000×4000×20	400×100
Case-54	10. 8	1000	4000×4000×20	400×100
Case-55	10. 8	2000	4000×4000×20	400×100

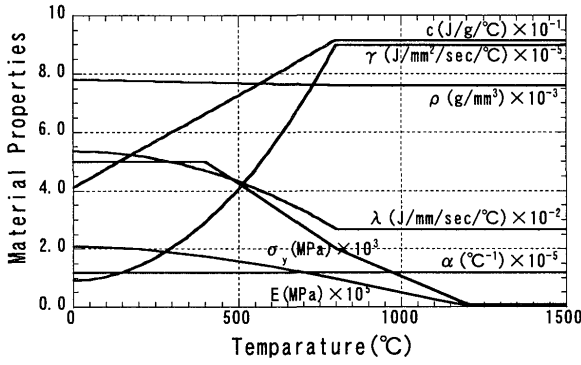


Fig.4 Temperature dependent material properties.

4. FEM Models

For the analysis of butt-welding, the temperature distribution and the stress distribution across the thickness is assumed to be uniform. Thus, two dimensional heat conduction FEM and plane stress thermal-elastic-plastic FEM developed by authors⁴⁾ can be applied. The 4-node isoparametric element is used to model the problem. As shown in Fig.3, two types of mesh divisions are used for cases with and without tab plates. The material properties are assumed to temperature dependent as shown in Fig.4. The serial computation is performed on the cases listed in Tables 1, 2 and 3 for small plate model and large plate model with and without tab plates, respectively.

5. Effect of Yield Stress and Young's Modulus on Transverse Shrinkage

According to Eq.(1) proposed by Satoh and Terasaki, the transverse shrinkage is not influenced by the yield stress. To make sure that this phenomenon is true, the cases with changing yield stress and Young's modulus are computed using case-5 of Table 1 as an example. The computed deformation in the y direction along the edge as illustrated by Fig.5 is shown in Fig.6. As seen from this figure, the transverse shrinkage is nearly independent on the yield stress and slightly influenced by Young's modulus.

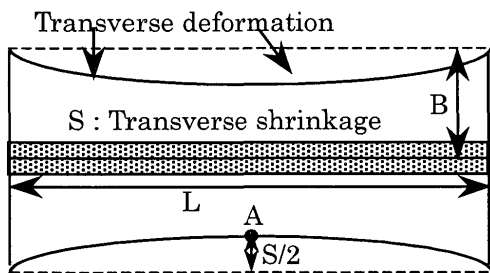


Fig.5 Schematic of deformed configuration of welded plate.

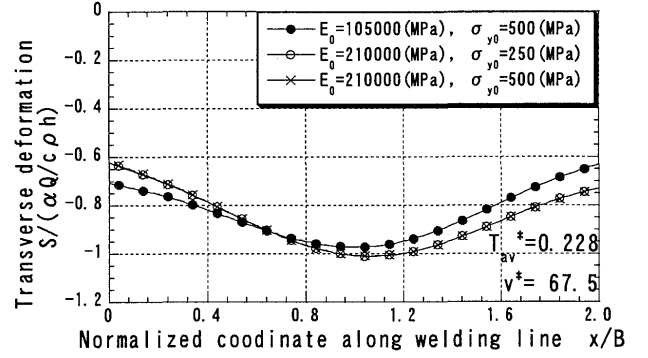


Fig.6 Effects of Young's modulus and yield stress on transverse shrinkage.

6. Transverse Shrinkage in Small Specimen and Large Steel Plate

As shown by Fig.2, experiments using small specimens and FCB welding in shipyard differ by two orders in parameter v^* and one order in parameter T_{av}^* . These differences are large enough to expect fundamental differences between the phenomena in the welding of small and large plates. Thus, serial computations are performed for the cases within two areas indicated in Fig.2 for both small and large plates.

6.1 Transverse shrinkage of small plate

The half breadth of the specimens used by Satoh and Terasaki is 100 mm. The plates are welded by GMAW, GTAW and SMAW. The welding conditions assumed for the simulations are listed in Table.1. Case-5 is the standard or the reference case and the heat input and/or the welding speed in other cases are assumed to be half or double of the standard case. Also, the effect of the aspect ratio is examined for cases with three different lengths of plates.

6.1.1 Effect of welding speed and heat input

Figures 7, 8 and 9 shows the transverse shrinkage of the plates along the edge for cases with small ($v^*=33.8$), medium ($v^*=67.5$) and large ($v^*=135$) welding speeds. The aspect ratio of the plates is 2.0. In each figure, cases with three different welding speeds are compared. The ordinate of the figure is the shrinkage of the plate normalized by the shrinkage S_0 given by Eq.(1). The abscissa represents the longitudinal coordinate of the plate x normalized by the plate width B . It is seen that the transverse shrinkage is nearly equal to S_0 and its distribution roughly symmetric with respect to the center of the plate when

the heat input parameter T_{av}^* is small. The magnitude of the shrinkage increases monotonically toward the end of welding when the welding speed is small. As summarized in Fig.10, transverse shrinkage becomes small compared to S_0 when the heat input parameter T_{av}^* is large.

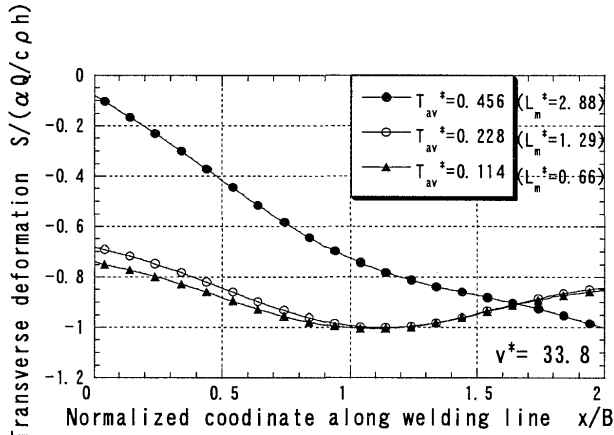


Fig.7 Effect of T_{av}^* on deformation along outer edge of plate ($v^*=33.8$).

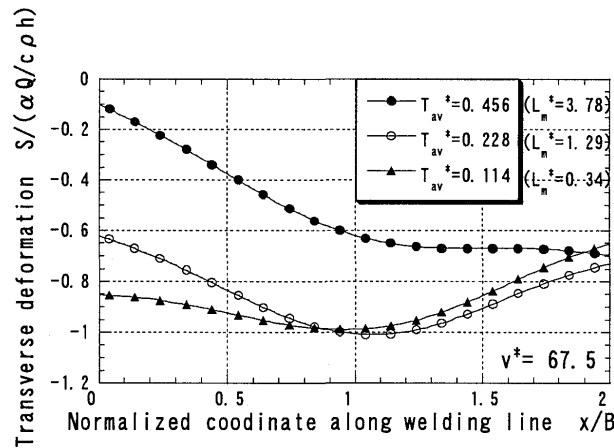


Fig.8 Effect of T_{av}^* on deformation along outer edge of plate ($v^*=67.5$).

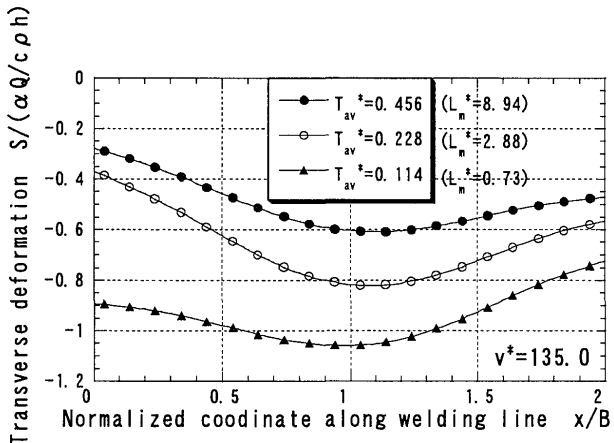


Fig.9 Effect of T_{av}^* on deformation along outer edge of plate ($v^*=135.0$).

6.1.2 Effect of plate length

The effect of the plate length in the cases with different welding speed v^* is summarized in Figs.11, 12, 13 and 14. When the plate is long, such as the case when $L/B=5$, a steady part with almost constant shrinkage is observed in the middle part of the welding. Before and after this steady part, transient zones are observed. The length of the transient zone increases with v^* . When the length of the plate becomes short, the steady part disappears. When the length is very short, such as in the case of $L/B=1$, the transient zones for the end part also disappears. Similar comparisons are made among cases with different heat inputs T_{av}^* in Figs. 15, 16, 17 and 18. Basically similar tendencies are observed in these cases. The length of the transient zone increases with the heat input parameter T_{av}^* . The parameter L_m^* shown in the figures is the molten length L_m normalized by the width of the plate B . As summarized in Fig. 19, the molten length L_m^* increases with welding speed v^* and heat input T_{av}^* . In the figure, the molten length for the FCB welding which will be discussed in the following sections are also shown for comparison. Since the length of the transient zone is expected to be closely related to the molten length L_m^* , the magnitude of the transverse shrinkage at the center of the plate is summarized with respect to the molten length L_m^* in Fig.20. For cases with three different aspect ratios, two curves for which the heat input T_{av}^* and welding speed v^* are changed respectively almost coincide each other. This means that the transverse shrinkage in small specimens is mainly governed by the molten length L_m^* . In case of specimens with small aspect ratio, the magnitude of the shrinkage is generally smaller than S_0 when L_m^* is small and vice versa when L_m^* is large. Also, it is seen that the magnitude of the shrinkage converges to S_0 as L_m^* decreases.

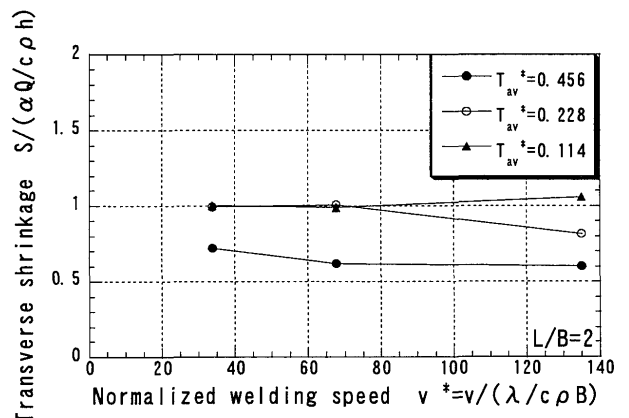


Fig.10 Effect of T_{av}^* and v^* on transverse shrinkage at center section of plate.

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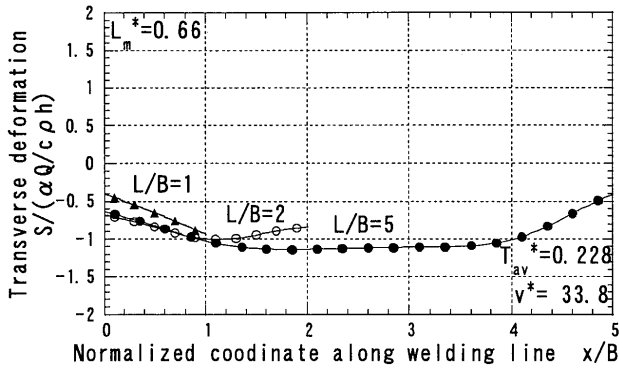


Fig.11 Effect of plate length on deformation along outer edge of plate ($v^*=33.8$).

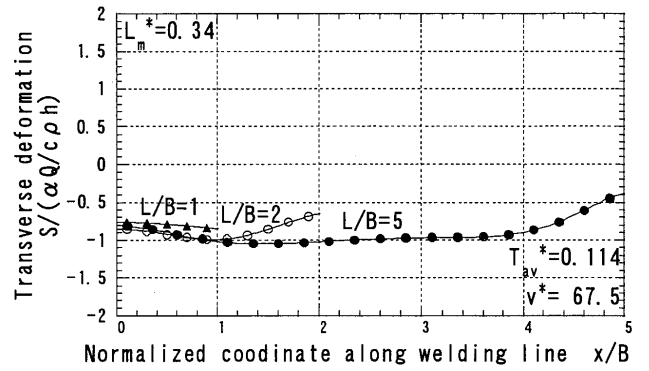


Fig.15 Effect of plate length on deformation along outer edge of plate ($T_{av}^*=0.114$).

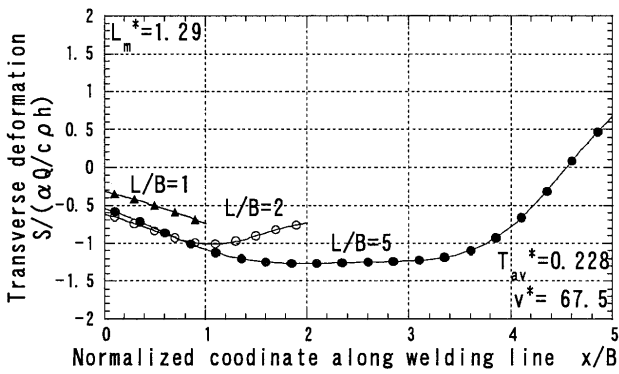


Fig.12 Effect of plate length on deformation along outer edge of plate ($v^*=67.5$).

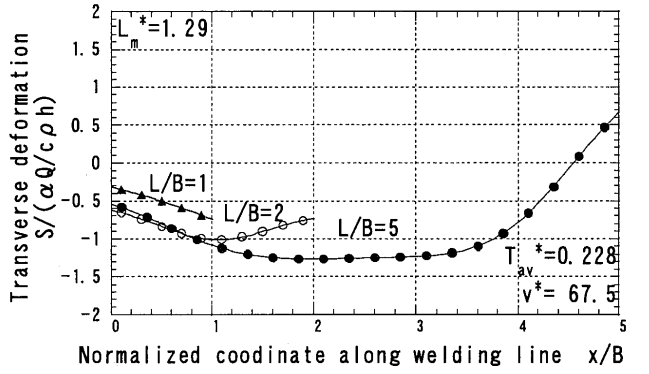


Fig.16 Effect of plate length on deformation along outer edge of plate ($T_{av}^*=0.228$).

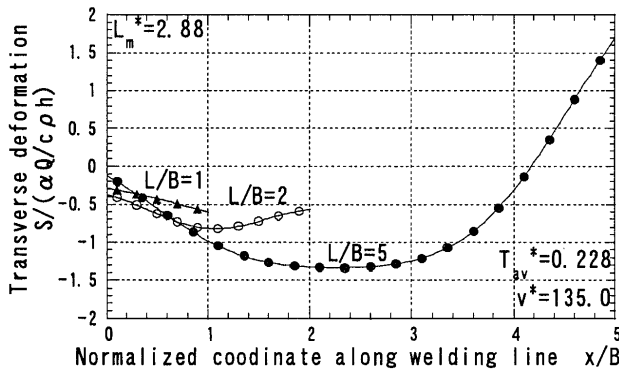


Fig.13 Effect of plate length on deformation along outer edge of plate ($v^*=135.0$).

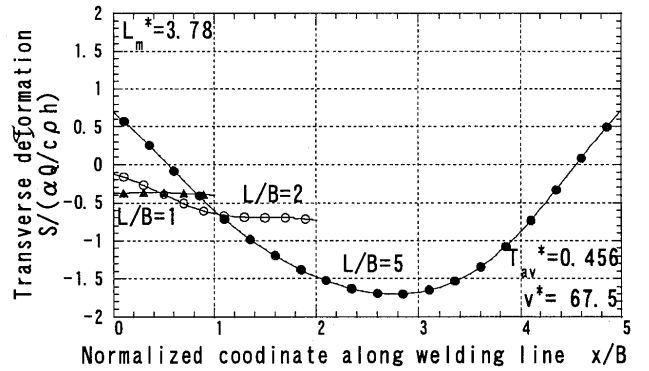


Fig.17 Effect of plate length on deformation along outer edge of plate ($T_{av}^*=0.456$).

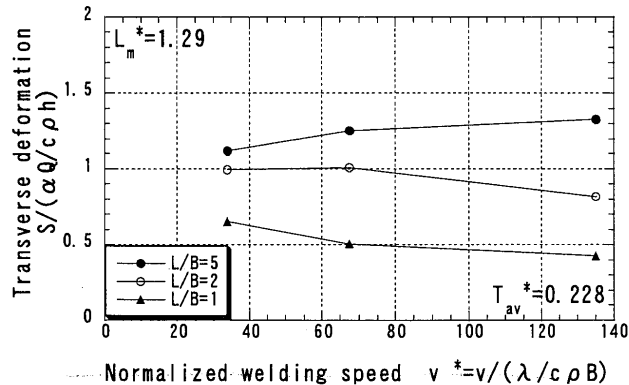


Fig.14 Effect of plate length and v^* on transverse shrinkage at center section of plate.

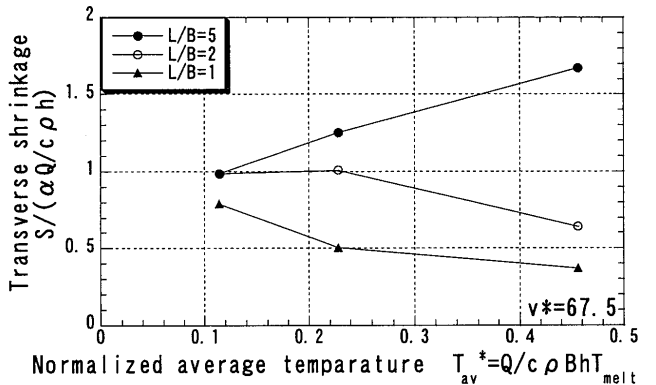


Fig.18 Effect of plate length and T_{av}^* on transverse shrinkage at center section of plate.

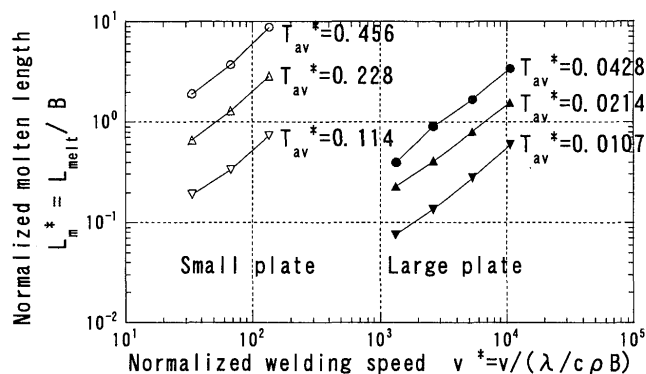


Fig.19 Effect of v^* and T_{av}^* on molten length for small and large plate.

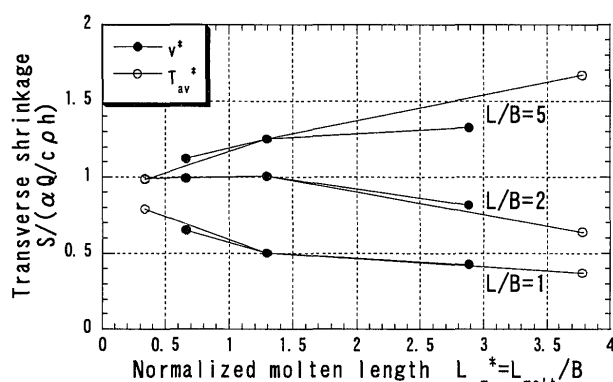


Fig.20 Effect of L_m^* on transverse shrinkage at center section of plate.

6.2 Transverse shrinkage of large plate under FCB welding

In this section, the transverse shrinkage of large plates under FCB welding is investigated. In case of FCB welding, three or four electrodes are used. To simulate this situation, it is assumed that a uniformly distributed rectangular heat source of length 260 mm is moving with constant speed v . Serial computations are performed for the 20 cases with the welding conditions listed in Table 2.

6.2.1 Effect of welding conditions

Figures 21, 22 and 23 show the distribution of shrinkage along the edge of the plate under weldings with different welding speed v^* and heat input T_{av}^* . As observed for small plates, a steady part appears in the middle portion of the welding and the magnitude of shrinkage approaches to S_0 when welding speed becomes small. Also, it is seen that, the shrinkage decreases monotonically toward the end of welding when T_{av}^* is small. When heat input T_{av}^* is large, the distribution of the shrinkage becomes symmetric along the welding line which is similar to the case of small plates. The effect of

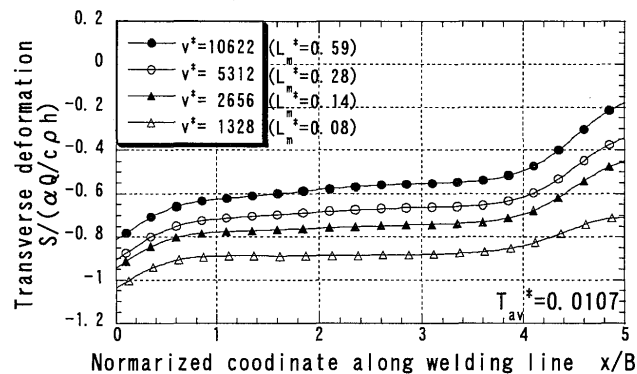


Fig.21 Effect of v^* on deformation along outer edge of plate ($T_{av}^*=0.0107$).

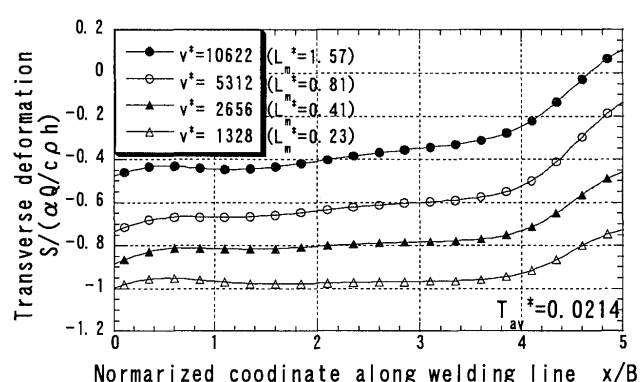


Fig.22 Effect of v^* on deformation along outer edge of plate ($T_{av}^*=0.0214$).

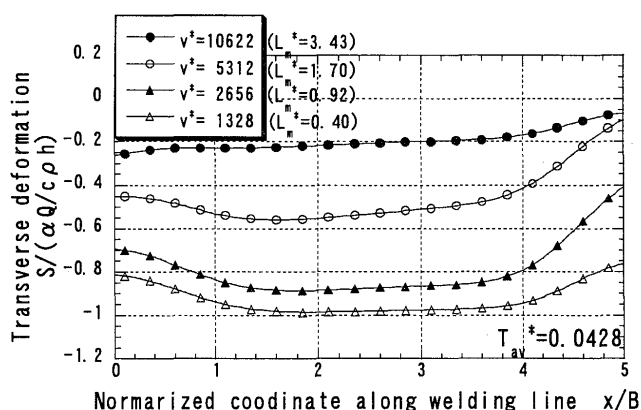


Fig.23 Effect of v^* on deformation along outer edge of plate ($T_{av}^*=0.0428$).

T_{av}^* and v^* on the transverse shrinkage are summarized in Fig.24. The shrinkage increases when v^* becomes small. When $T_{av}^*=0.0428$, the shrinkage seems to converge to S_0 with the decrease of v^* .

To examine the effect of molten length L_m^* , the relation between the shrinkage and L_m^* is summarized in Fig. 25. Since the three lines with different heat input T_{av}^* do not coincide each other, the main factor controlling the shrinkage is not the molten length L_m^* .

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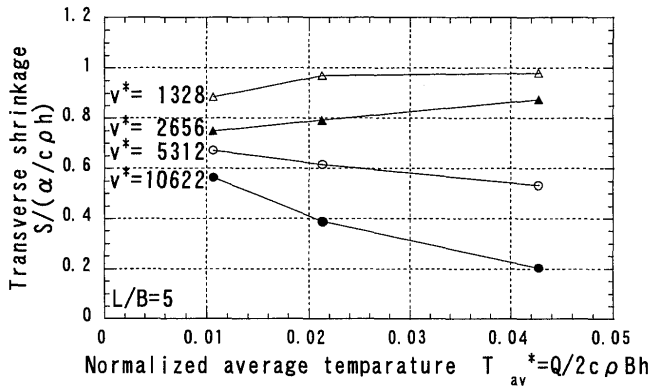


Fig.24 Effect of v^* on transverse shrinkage at center section of plate.

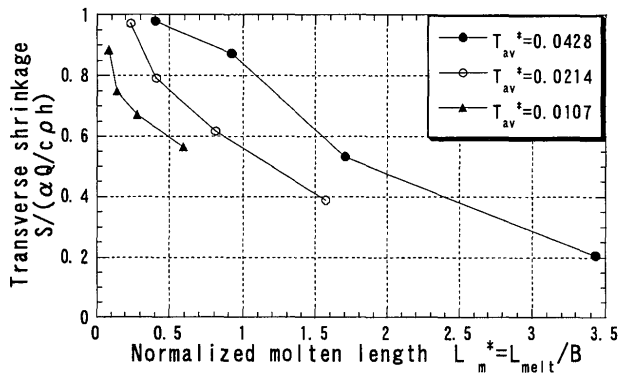


Fig.25 Effect of L_m^* on transverse shrinkage at center section of plate.

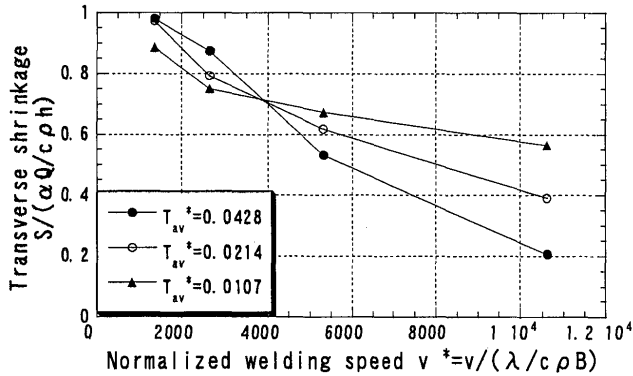


Fig.26 Effect of v^* on transverse shrinkage at center section of plate.

Thus in Fig.26, the shrinkage is plotted against the welding speed v^* . It is observed as a general tendency, that the shrinkage decreases with the increase of v^* . This tendency becomes strong when the heat input T_{av}^* is large. From the comparison between Fig. 25 and Fig. 26, it is seen that the welding speed v^* is more influential to the shrinkage in case of FCB welding.

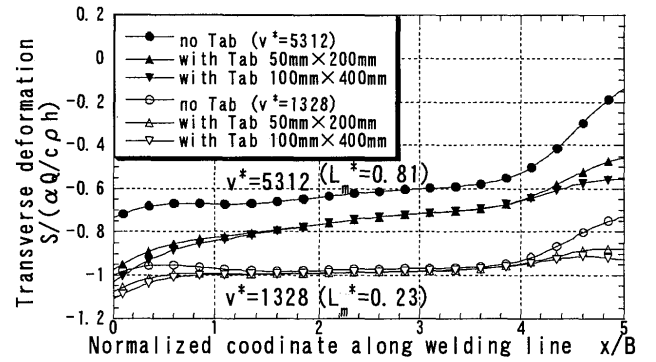


Fig.27 Effect of tab plate on deformation along outer edge of plate.

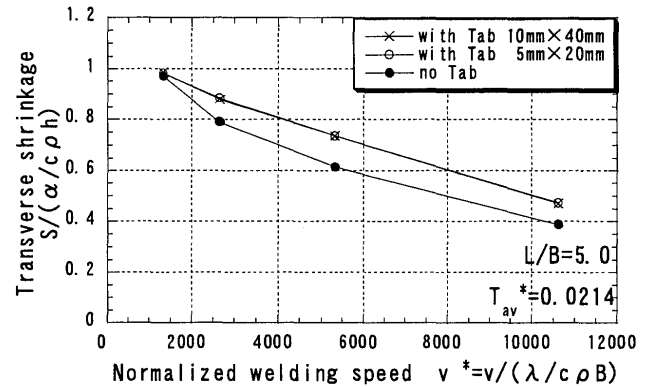


Fig.28 Effect of tab plate and v^* on transverse shrinkage at center section of plate.

6.2.2 Effect of tab plate

As observed from Figs. 21, 22 and 23, the magnitude of shrinkage decreases near the end of welding due to the opening deformation of the plate when the welding reaches the end and the constraint to close the groove is removed. In the real practice, tab plates are attached at both starting and ending of welding to prevent hot cracking due to opening deformation. Therefore, the influence of tab plate on the transverse shrinkage is examined.

Assuming that the welding between tab plates and the main plate is full penetration welding, the FEM mesh division shown in Fig. 3-(b) is used. Computations are performed for plates with no tab and plates with small or large tab plate under the welding conditions listed for case-40 to case-47 in Table 3. The size of the small and the large tab plates are 200 mm in width, 50 mm in length and 400 mm in width, 100 mm in length, respectively. The transverse shrinkages for cases with and without tab plate are compared in Fig. 27. It is seen from this figure that the local variation of the shrinkage at both ends of the welding is significantly

reduced by tab plates. Figure 28 shows the effect of tab plates on the shrinkage at the center of the plate. It can be seen that the size of tab plates has small influence on the shrinkage. When the welding speed is small, the shrinkage is not influenced by tab plates. But, it increases due to the constraint from the tab plates when the welding speed is large.

6.2.3 Effect of plate length

The effect of plate length on the transverse shrinkage of plates with tab is summarized in Figs 29 and 30. Computations are performed for cases 48 to 55 with four different welding speeds v^* . In these serial computations, heat input T_{av}^* and the sizes of the plate and the tab are kept same. The distribution of the transverse shrinkage along the welding is summarized in Fig.29. The shrinkage at the starting end of the welding is almost the same regardless of the plate length. As a general characteristic, the magnitude of shrinkage decreases monotonically towards the finishing end. Figure 30 shows the influence of plate length on the shrinkage at the center of the plate. The magnitudes of the shrinkage tend to converge as the length of the plate increases and the converged value approaches to S_0 when v^* becomes small.

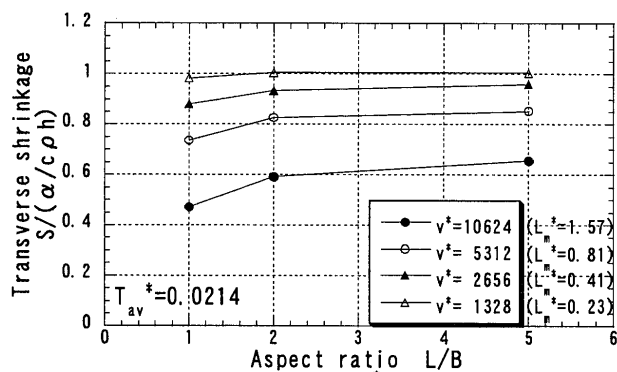


Fig.29 Effect of plate length and v^* on transverse shrinkage at center section of plate.

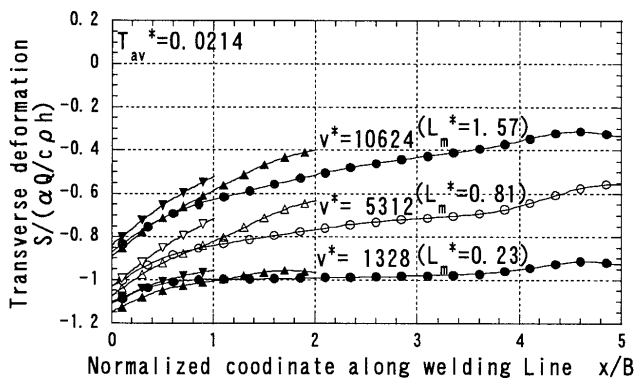


Fig.30 Effect of plate length and v^* on deformation along outer edge of plate.

6.2.4 Transient deformation

To understand the physical phenomena during the welding, the transient deformations in the transverse direction at the center of the plate (point A in Fig. 5) are examined. Figures 31 and 32 show the time histories of the transverse deformation for small specimens and large plates under FCB welding, respectively. In both figures, the effect of welding speed is shown for cases with the same dimensions and heat input. The time measured from the start of the welding is shown as abscissa. In case of small plates shown in Fig. 31, no expansion during the heating process is observed except for the case when the welding speed is very large. Thus, the transverse deformation is mainly produced by the thermal shrinkage during the cooling stage and the total shrinkage becomes the same as the thermal contraction corresponding to the heat input. It is worth noting that the knuckle point in the curve corresponds to the moment when the welding torch reaches the finishing end and the opening of groove happens.

In case of FCB welding shown in Fig. 32, significant expansion during the heating process is observed when the welding speed is large. The magnitude of shrinkage after the complete cooling becomes smaller compared to S_0 due to the expansion in

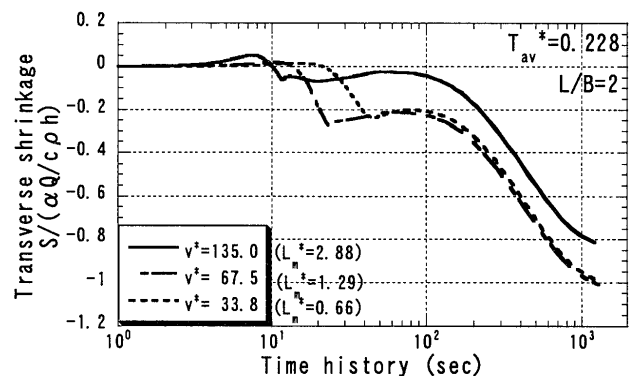


Fig.31 Transient deformation at center section of plate for small plate.

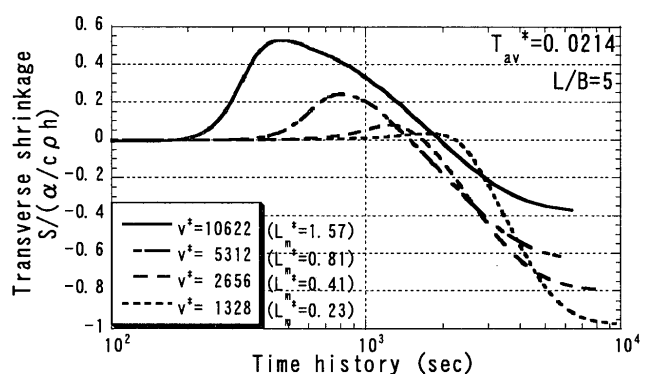


Fig.32 Transient deformation at center section of plate for large plate.

the heating process. The expansion disappears when the welding speed is small and the magnitude of shrinkage becomes almost S_0 . In the transient deformation of FCB a welding knuckle point as in case of small specimen is not observed. This implies that the local deformation near the finishing end has no effect on that at the center of the plate.

For both small specimen and FCB welding, the molten length L_m^* increases with v^* . With the increase of L_m^* , the constraint acting on the area near welding torch becomes small and thermal expansion during heating process is allowed. This may be one possible explanation of the phenomena why the shrinkage decreases when the welding speed is large.

7. Conclusions

Through serial FEM simulations, the following conclusions are drawn for the transverse shrinkage under butt welding.

- (1) Based on the similarity rule, parameters T_{av}^* and v^* are derived as governing factors of welding deformation under butt-welding.
- (2) When welding of small test specimens and FCB welding in shipyard are compared, the differences in v^* and T_{av}^* are two and one order of magnitude, respectively.
- (3) The effect of yield stress on the transverse shrinkage is negligible and that of Young's modulus is small.
- (4) In case of small specimens, the magnitude of shrinkage is greater than S_0 when plate length is large. While, it is generally smaller than S_0 when plate is short. The magnitude of shrinkage converge to S_0 when molten length L_m^* becomes small. L_m^* decreases with v^* and T_{av}^* .
- (5) In case of FCB welding, the distribution of shrinkage is different from that of small specimen and the magnitude is mainly influenced by welding speed v^* .
- (6) Tab plates are effective for controlling the local deformation at starting and finishing of the welding. They also influence the shrinkage at the center part of the plate unless welding speed is too small.

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