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Osaka University
Market size in globalization

Hayato Kato†  Toshihiro Okubo‡

December, 2017

Abstract

A salient feature of the current globalization is a loss of manufacturing in developed countries and rapid industrialization in middle-sized developing countries. This paper aims to construct a simple three-country trade and geography model with different market sizes and non-constant wage rates. The large country fosters industrial agglomeration (geographical concentration) in the early stage of globalization, but loses manufacturing in the later stage of globalization. When losing manufacturing, the large country might be worse off. Thus, the large country might have an incentive to implement welfare-maintaining policies to prevent a loss of manufacturing. All of these results can be explained by market sizes.

JEL classification: F12; F15; F20

Keywords: Agglomeration; Market size; Middle-sized country; Non-constant wages; Industry/welfare maintaining policy

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1 Introduction

In the early nineteenth century, the development of manufacturing caused a sudden shift in hegemony towards the today’s wealthy countries such as those in Western Europe, and away from the empires of China, India and the Middle East, which had to that point dominated the world economy for thousands of years. Pomeranz (2000) called this the “Great Divergence” and discussed the growth acceleration in Europe and the U.S., where manufacturing had been developed. These countries created industrial clusters/cities that lead to high economic growth, and dominated economics, politics, military power and culture all over the world. The economies of developed countries such as the U.S., Japan and those in Europe have grown substantially during the twentieth century, particularly in terms of per-capita income and GDP. However, according to Baldwin (2016), the Great Divergence ended in the 1990s, at which point the global shares of income and manufacturing of the developed countries began to decline. By contrast, some middle-income countries have developed industries rapidly, resulting in strong economic growth. He called this the “Great Convergence” as his book title suggests.

The current trends in globalization are characterized by large international trade flows and high capital mobility, facilitated by a substantial decline in transport costs and tariff barriers and the revolution in information and communication technology. Firms are mobile between countries, and the cites where production takes place are geographically concentrated. Some middle-income countries such as Newly Industrialized Economies (NIEs) and the Association of Southeast Asian Nations (ASEAN) attract productive industries and create a high degree of industrial agglomeration. Amid the Great Convergence in the current globalization, growth paths across middle-income countries have diverged; some middle-income countries have experienced the convergence process and joined the group of developed countries, while other countries have become caught in the middle income trap with low economic growth (Jones, 1997; Bairoch and Kozul-Wright, 1998; Baldwin and Martin, 1999).

Behind the drastic shifts as mentioned above, a serious concern of globalization in developed countries is the loss of manufacturing to developing countries, known as offshoring in North America, delocation in Europe and hollowing-out in Japan. Many
firms have ceased operations in developed countries and moved their manufacturing to developing countries in search of large workforces with lower wages. In the U.S., manufacturing industries facing severe competition by increased imports from low-wage countries saw higher exit rates of plants from the late 1970s to the 1990s (Bernard et al., 2006). Furthermore, the rise of China in the last few decades has negatively impacted the U.S. manufacturing employment and wages (Autor et al., 2013; Autor et al., 2014; Ebenstein et al., 2014; Acemoglu et al., 2016).¹ Autor et al. (2013) find that, of the decline in manufacturing employment between 1990 and 2007, one-quarter could be due to a surge in imports from China. Autor et al. (2014) report that workers in manufacturing industries facing import competition from China earn lower income over the period of 1992 to 2007 than those in other sectors. Political debate on anti-globalism addresses the issue of how to stop firm relocation and keep jobs in developed countries.

To illustrate the rise and fall of manufacturing across countries, we construct a simple three-country trade and geography model with different market sizes. We show that in the early stages of globalization (i.e., high or intermediate levels of trade costs), manufacturing firms are concentrated in the large country, but further progression of globalization (i.e., low trade costs) causes offshoring from the large to the smaller countries. Offshoring might worsen welfare in the large country, which might justify policy intervention. On the other hand, the outcome for the middle country is mixed and depends on its market size.

Relation to the literature. The literature on trade and economic geography has addressed the question of how trade liberalization affects firm locations across countries. The common finding using a variety of standard trade and geography models (Fujita et al., 1999; Baldwin et al., 2003; Fujita and Thisse, 2013) is that lowering trade costs results in geographical concentration of all firms in one region, which is the so-called core-periphery structure. Once all firms are concentrated at the core by agglomeration

¹Section 7.4 links our theoretical results with these empirical findings. The negative impact of increased Chinese import competition on firm performance and labor-market outcome is found in other Western countries: Belgium (Mion and Zhu, 2013), Norway (Balsvik et al., 2015), and 12 European countries (Bloom et al., 2016). See Haskel et al. (2012) and Autor et al. (2016) for comprehensive surveys.
forces, which always dominates dispersion forces, all firms remain at the core even in the case of extremely low trade costs. This standard outcome cannot perfectly explain the above-mentioned consequences of recent globalization; globalization triggers collapse of industrial clusters in developed countries and facilitates industrial development in middle-sized countries. One reason why the standard trade and geography model fails to explain these phenomena comes from its basic theoretical structure: the two-country setting and constant wage rates. To characterize the recent globalization, we relax these assumptions and extend our analysis to a three-country model with wage rates varying in market size and firm share.

The three-country setting in our model can highlight the role of intermediate-sized countries in the agglomeration process. The trade and economic geography literature to date ignores asymmetric country size in a three-country framework, apart from a few studies. A limited number of three country/region models (e.g., Krugman and Livas Elizondo, 1996; Takahashi, 2003; Ago et al., 2006; Saito et al., 2011; Forslid, 2011; Brühlhart et al., 2012; Gaspar et al., 2017) have provided numerous interesting results not found in two-country models. Krugman and Livas Elizondo (1996) develop a model with two domestic regions and one foreign country, and find that lower trade costs against the foreign country leads industries to spread across the two domestic regions. The closest paper to ours is Forslid (2011). He extends the footloose capital (FC) model of Martin and Rogers (1995) to a three-country setting in which the three countries have different market sizes, and firms are mobile across countries. He studies the impact of market size difference on the agglomeration process. As trade costs fall, firms in the small country first relocate to the large country. After all firms in the small country have

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2Extensions to three-country models often provide richer insights than two-country models. Takahashi (2003) finds the possibility of inefficient locations driven by factor mobility in the three-country model. Using a model with a linear demand function, Ago et al. (2006) find that the hub country with good transport access from the other countries could lose manufacturing because of severe competition. The three-country model by Saito et al. (2011) incorporates firm heterogeneity into the model of Krugman and Livas Elizondo (1996). They discuss how a fall in trade costs affects firm locations as well as regional average productivities in the two domestic regions. Brühlhart et al. (2012) empirically examine how regional employment and wages in Austria were affected by the opening of Central and Eastern European markets after the end of the cold war. Then they show that their empirical findings can be supported by a three-region economic geography model. Gaspar et al. (2017) investigate the bifurcation of equilibrium in a symmetric multi-region footloose entrepreneur model.

3See also Matsuyama (2017)’s multi-region model with constant wages and a more general spatial structure.
relocated to the large country, firms in the middle country start relocating. Finally, all firms end up relocating to the large country. In his model, wage rates are normalized and thus agglomeration is simply caused by the interaction of market size difference and trade costs. An implication of Forslid (2011) is how substantial reductions in trade costs and development of infrastructure affect firm location patterns within Europe.

Another important aspect of our paper is non-constant wage rates, i.e., wage rates varying in market size and firm share. The standard economic geography models use the model of Helpman and Krugman (1985), i.e., two-country and two-sector model with the Dixit-Stiglitz monopolistic competition. The model has one monopolistic competitive sector with trade costs (manufacturing sector) and one perfectly competitive sector without trade costs (agricultural or numéraire sector). A crucial mechanism is that the presence of an agricultural good can normalize wage rates between the two countries. The wage equalization can simplify the analysis, but it ignores wage disparities in the globalization process. Thus, to characterize wage rates varying in market size and firm share, we relax the standard assumption by assuming away the tradable numéraire good with no trade costs. Instead, our model introduces a non-tradable numéraire good (infinite trade costs for the agricultural good). In other words, this is an extreme case of Davis (1998), who imposes trade costs on the agricultural sector, thus allowing for non-constant wage rates. The labor market clearing process determines wage rates. As firms geographically concentrate in one country, a rise of labor demand boosts wage rates, which moderates the agglomeration process. In short, non-constant wages operate as a dispersion force.

Non-constant wages in the Dixit-Stiglitz monopolistic competition model have been studied mainly in the literature on the home market effect (Davis, 1998; Head and Ries, 2001; Brülhart et al., 2004; Davis and Weinstein, 1999, 2002; Crozet and Trionfetti, 2008; Takatsuka and Zeng, 2012). The definition of the home market effect is

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4To our knowledge, there are three standard approaches to non-constant wage rates in the literature. One is using a one-sector model: monopolistic competition sector à la Krugman (1980). Recent applications include Takahashi et al. (2013), Zeng and Uchikawa (2014) and Mossay and Tabuchi (2015). Under this approach, the trade balance endogenously determines wage rates between two countries. The second method is to allow for trade costs in the numéraire sector (agriculture) à la Davis (1998) and Takatsuka and Zeng (2012). This drops the assumption of costless trade in the numéraire sector. The third method is to introduce differentiated products in a constant-returns-to-scale perfect competition sector (Head and Ries, 2001). Our model adopts the second approach.
twofold: (1) firm shares in large countries are greater than their market-size (i.e., population or GDP) shares, which is in line with Helpman and Krugman (1985); and (2) wage rates are increasing in market size shares, which is line with Krugman (1980). A main objective of these studies is to determine whether the home market effect exists, whether it is dampened or strengthened by trade cost reduction and how the model assumptions influence the home market effect. In this literature, a model is first constructed, the home market effect is tested using data and then the results are compared across model specifications. By contrast, we are interested in the agglomeration process in the three-country model, the impact of market size difference on firm location patterns (geographical concentration and dispersion), welfare analysis and policies.

Our model is a three-country FC model à la Forslid (2011) with non-constant wage rates and different market sizes. The model can help us understand the consequences of recent globalization, particularly industrial development in middle-sized countries and offshoring in developed countries. We obtain the following results. First, the middle-sized country might develop manufacturing as part of globalization, but this depends on its market size. Second, the large country (and the middle-sized country in some cases) can attract manufacturing despite increased wage rates. This moderates agglomeration process, resulting in loss of manufacturing when trade costs are small. Third, a fall in trade costs causes a collapse of agglomeration and might worsen welfare in the large country. Fourth, to prevent the collapse of agglomeration and worsening welfare, the large country has an incentive to use bilateral trade agreements with the small country rather than the middle country. All of the results are characterized by the market sizes of countries. It is worth emphasizing that these results are not obtained in the multi-region geography model with constant wages by Forslid (2011), where the collapse of full agglomeration is never observed. Likewise, two-region geography models with non-constant wages cannot characterize middle-sized countries (e.g., Crozet and Trionfetti, 2008; Takatsuka and Zeng, 2012). Although there is a limited number of studies dealing with multiple regions and non-constant wages (e.g., Krugman and Livas Elizondo, 1996; Fujita et al., 1999, Ch 15; Brülhart et al., 2012), all of them assume away or do not investigate the impact of market size difference on location patterns and welfare in analytical forms.
The rest of the paper proceeds as follows. The next section constructs the three-country model with non-constant wages. Section 3 derives the long-run equilibrium. Section 4 explores industrial concentration in one or two countries. Section 5 conducts welfare analysis and then Section 6 investigates welfare-maintaining policies. Section 7 discusses our results in details. The final section concludes.

2 Basic model

We construct a simple three-country economic geography model based on the footloose capital (FC) model of Martin and Rogers (1995). The FC model is marked by internationally mobile capital and immobile capital owners and workers. The economy has three countries, indexed by 1, 2 and 3, with two sectors, a tradable manufacturing sector and a non-tradable agricultural sector. As in the standard FC model, there are two factors of production, capital used in the manufacturing sector, and labor used in both the manufacturing and agricultural sectors. The agricultural sector produces a homogeneous good using constant-returns-to-scale technology so that it is subject to perfect competition. Importantly, in contrast to the standard FC model, the homogeneous good in our model is not internationally traded because of infinite trade costs. The manufacturing sector is monopolistically competitive and produces differentiated goods, which are internationally traded with trade costs.

The total amount of the two factors in the world is expressed as $L$ (labor) and $K$ (capital) and Country $i \in \{1, 2, 3\}$ is endowed with $L_i = s_i L$ and $K_i = s_i K$, where the labor and capital shares of Country $i$, $s_i \in (0, 1)$ are identical. Importantly, the share of endowments, $s_i$, is exogenously given and different across the three countries. We assume that Country 1 has the largest market size and that Country 3 has the smallest market size, i.e., $s_1 > s_2 > s_3$. Each household holds labor and capital. The household provides one unit of labor to either sector. Labor is freely mobile between the two sectors so that sectoral wages in a country are equalized. The household in

\footnote{This implies that all countries face identical capital-labor ratios. To highlight the impact of country size, different capital-labor ratios are not allowed in our model.}
Country $i$ owns $K_i/L_i$ units of capital and invests it to create firms and then receives capital returns. Simply, one unit of capital makes one manufacturing firm and thus the total number of firms in the world, denoted as $N$, is equal to that of world capital, i.e., $N = K$. In the long-run equilibrium, capital (i.e., firm) moves to the country in which the highest (operating) profits are made, although the household (i.e., capital owner) cannot move between countries. Capital rewards are repatriated to the country of origin. Consequently, the share of capital employed in Country $i$, denoted as $n_i \in [0, 1]$ (the number is $N_i = n_iK$), is generally different from the initial endowment share, namely $n_i \neq s_i$.

2.1 Demand side

Aggregate utility in Country $i$ takes the following form:

$$cQ_i^{\mu} q_0^{1-\mu},$$

where 

$$Q_i \equiv \left[ \sum_{j=1}^{3} \int_{\theta \in \Omega_j} q_{ji}(\theta)^{\frac{\sigma-1}{\sigma}} d\theta \right]^\frac{\sigma}{\sigma-1}, \quad c \equiv \mu^{-\mu}(1-\mu)^{-(1-\mu)},$$

where $\mu \in (0, 1)$ is the expenditure share on manufacturing goods, $\sigma > 1$ is the elasticity of substitution between varieties of manufacturing goods. $\theta$ indicates a brand of differentiated products and $\Omega$ represents a set of the varieties. $q_0$ is consumption of the non-tradable good, $q_{ji}$ is the quantity of the variety produced in Country $j$ and consumed in Country $i$ for $i, j \in \{1, 2, 3\}$, and $Q$ is a real consumption index of manufacturing goods.

From the first order conditions, aggregate demand in each variety can be given as

$$q_{ji}(\theta) = \frac{p_{ji}(\theta)^{-\sigma}}{P_i^{1-\sigma}} \mu Y_i,$$

where 

$$P_i \equiv \left[ \sum_{j=1}^{3} \int_{\theta \in \Omega_j} p_{ji}(\theta)^{1-\sigma} d\theta \right]^{\frac{1}{1-\sigma}}.$$

$Y_i$ is national income, $p_{ji}$ is the price of a variety produced in Country $j$ and consumed
in Country $i$ and $P$ is a price index of manufacturing goods. Hereafter, the index of each brand, $\theta$, is suppressed.

In the non-tradable agricultural sector, domestic demand must equal domestic supply:

$$q_{0i} = \frac{(1 - \mu)Y_i}{p_0i},$$

where $p_0$ is the price of the agricultural good.

The national income consists of labor income and capital rewards. Letting $w_i$ be the wage rate in Country $i$ and $r$ be the capital reward, which is identical across countries, national income in Country $i$ becomes $Y_i \equiv w_iL_i + rK_i = s_i(w_iL_i + rK)$. Without loss of generality, the wage rate in Country 3 can be normalized to unity, i.e., $w_3 = 1$.

### 2.2 Supply side

**Non-tradable agricultural sector.** The non-tradable sector uses one unit of labor to produce one unit of the good. The price is determined to eliminate excess profits, implying $p_{0i} = w_i$.

** Tradable manufacturing sector.** Manufacturing firms are subject to monopolistic competition. An individual firm requires one unit of capital as a fixed cost and uses $a = (\sigma - 1)/\sigma$ units of labor to produce one unit of a brand. Profit maximization by a firm yields a constant mark-up of price over marginal cost:

$$p_{ji} = \tau_{ji}p_j = \frac{\tau_{ji}\sigma w_ja}{\sigma - 1} = \tau_{ji}w_j.$$ 

Although firms can supply their local market without incurring trade costs, i.e., $\tau_{jj} = 1$, firms in Country $j$ have to export $\tau_{ji} > 1$ units of a brand to sell one unit in Country $i \neq j$. For the moment, we assume $\tau_{ji}$ to be symmetric in all country pairs, i.e., $\tau_{ji} = \tau$ for $j \neq i$. 

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By substituting these equilibrium prices, operating profits are given by

\[ \pi_j = \sum_{i=1}^{3} (p_{ji} - \tau_{ji} w_{ja}) q_{ji} \]

\[ = \sum_{i=1}^{3} \phi_{ji} \left( \frac{p_j}{P_i} \right)^{1-\sigma} \mu Y_i^\sigma \]

\[ = \frac{\mu w_j^{1-\sigma}}{\sigma} \sum_{i=1}^{3} \phi_{ji} Y_i \Delta_i, \]

where \( \Delta_i = \sum_{k=1}^{3} \phi_{ki} w_k^{1-\sigma} N_k. \)

\( w_k^{1-\sigma} \) is the inverse measure of marginal costs and \( \phi_{ji} \equiv \tau_{ji}^{1-\sigma} \in [0, 1] \) is called the freeness of trade, where higher values mean low trade costs. \( \phi = 1 \) indicates free trade and \( \phi = 0 \) indicates autarky. We note that there are no intra-national trade costs, \( \phi_{ji} = 1 \) if \( j = i \). Operating profits go to capital owners so that capital rewards are given by \( r = \max_{j=1,2,3} \{ \pi_j \} \).

3 Long-run equilibrium

3.1 Location patterns and market size

As capital (i.e., firm) is mobile between countries, the long-run equilibrium is defined as location patterns where international capital movements stop. In the interior equilibrium where firms are active in all countries, operating profits are equalized between the three countries, i.e., \( \pi_1 = \pi_2 = \pi_3 \), which endogenously determines location patterns. In the corner equilibrium, profit equalization partially holds if firms concentrate in two countries, i.e., \( \pi_i = \pi_j > \pi_k \) where \( n_i, n_j \in (0,1) \) and \( n_k = 0 \), or it never holds if firms locate only in one country, i.e., \( \pi_i > \pi_j \) and \( \pi_i > \pi_k \) where \( n_i = 1 \) and \( n_j = n_k = 0 \). Capital rewards can be derived from the market clearing condition in the manufacturing sector:
\[ \sigma \sum_{j=1}^{3} \pi_{j} N_{j} = \mu \sum_{i=1}^{3} (w_{i} L_{i} + r K_{i}), \]

\[ \rightarrow K \left( \sigma \sum_{j=1}^{3} \pi_{j} n_{j} - \mu r \right) = \mu \sum_{i=1}^{3} w_{i} L_{i}, \]

\[ \rightarrow r = \frac{\mu \sum_{i=1}^{3} w_{i} L_{i}}{K(\sigma - \mu)}. \]

where we make use of \( \sum_{i=1}^{3} p_{ji} q_{ji} = \sigma \pi_{j} \) and \( \pi_{j} n_{j} = r n_{j} \) because of \( r = \max_{j \in \{1,2,3\}} \pi_{j} \) for \( n_{j} > 0 \).

Let us now consider the labor market. The non-tradable sector needs \( q_{0j} \) workers. While labor remaining available for the manufacturing sector is \( L_{j} - q_{0j} = L_{j} - (1 - \mu)Y_{j}/w_{j} \), labor demand is given by \( N_{j} \sum_{i=1}^{3} \tau_{ji} a q_{ji} \). Using constant mark-up pricing, labor demand can be re-written as \( N_{j} \sum_{i=1}^{3} \tau_{ji} a q_{ji} = N_{j} \pi_{j} (\sigma - 1)/w_{j} \). The labor market clearing condition in Country \( j \) is given by

\[ L_{j} - (1 - \mu)(w_{j} L_{j} + r K_{j})/w_{j} = (\sigma - 1)\pi_{j} N_{j}/w_{j}. \]

Given firm shares, the labor market clearing conditions determine wage rates, \( w_{1} \) and \( w_{2} \). Plugging capital rewards into these clearing conditions yields the following wage rates:

\[ w_{1} = \frac{s_{3}[(1 - \mu)s_{1} + (\sigma - 1)n_{1}]}{s_{1}[(1 - \mu)s_{3} + (\sigma - 1)n_{3}]}, \]  

(1)

\[ w_{2} = \frac{s_{3}[(1 - \mu)s_{2} + (\sigma - 1)n_{2}]}{s_{2}[(1 - \mu)s_{3} + (\sigma - 1)n_{3}]}. \]  

(2)

The wage rates in Countries 1 and 2 are proportional to their firm shares. More firms increase labor demand, raising wage rates. It is also worth noting that \( w_{3} \) is normalised to one, and hence \( w_{1} \) and \( w_{2} \) can be interpreted as the relative wage rates of Countries 1 and 2 to Country 3. This explains the result that \( w_{1} \) and \( w_{2} \) are decreasing in \( n_{3} \).

Now we consider the interior equilibrium where firms locate in all countries. Let \( v_{ij} = \pi_{i} - \pi_{j} \) be the profit gap between Countries \( i \) and \( j \). Firm shares in the long-run equilibrium are determined by profit equalization:
\[ v_{12} = \frac{\mu}{\sigma K} \left[ \frac{(w_1^{1-\sigma} - \phi w_2^{1-\sigma})Y_1}{n_1w_1^{1-\sigma} + \phi n_2^{1-\sigma} + \phi n_3} + \frac{(\phi w_1^{1-\sigma} - w_2^{1-\sigma})Y_2}{\phi n_1^{1-\sigma} + n_2w_2^{1-\sigma} + \phi n_3} \right. \\
\left. + \frac{\phi(w_1^{1-\sigma} - w_2^{1-\sigma})Y_3}{\phi n_1^{1-\sigma} + \phi n_2^{1-\sigma} + \phi n_3} \right] = 0, \tag{3} \]

\[ v_{13} = \frac{\mu}{\sigma K} \left[ \frac{(w_1^{1-\sigma} - \phi)Y_1}{n_1w_1^{1-\sigma} + \phi n_2^{1-\sigma} + \phi n_3} + \frac{\phi(w_1^{1-\sigma} - 1)Y_2}{\phi n_1^{1-\sigma} + n_2w_2^{1-\sigma} + \phi n_3} \right. \\
\left. + \frac{(\phi w_1^{1-\sigma} - 1)Y_3}{\phi n_1^{1-\sigma} + \phi n_2^{1-\sigma} + \phi n_3} \right] = 0, \tag{4} \]

\[ v_{23} = \frac{\mu}{\sigma K} \left[ \frac{\phi(w_2^{1-\sigma} - 1)Y_1}{n_1w_1^{1-\sigma} + \phi n_2^{1-\sigma} + \phi n_3} + \frac{(w_2^{1-\sigma} - \phi)Y_2}{\phi n_1^{1-\sigma} + n_2w_2^{1-\sigma} + \phi n_3} \right. \\
\left. + \frac{(\phi w_2^{1-\sigma} - 1)Y_3}{\phi n_1^{1-\sigma} + \phi n_2^{1-\sigma} + \phi n_3} \right] = 0. \tag{5} \]

To obtain the equilibrium firm shares, we substitute Eqs. (1) and (2) into the above equilibrium conditions and solve any two equations among Eqs. (3), (4) and (5) for \( n_i \). Although the firm shares in general cannot be derived in an explicit form, those at \( \phi \in \{0,1\} \) exceptionally take a simple explicit form as \( n_i = s_i \). That is, the firm shares of a country in autarky and free trade are equal to its market-size share, i.e., no home market effect. We further investigate the marginal impact of trade cost reduction on the firm shares. At \( \phi = 0 \), we have \( dn_1/d\phi|_{\phi=0} = \sigma(3s_i - 1) \). As the order of country size implies \( s_1 > 1/3; s_3 < 1/3; s_2 \gtrless 1/3 \), Country 1 attracts firms from the other countries \( (dn_1/d\phi|_{\phi=0} > 0) \), whereas firms in Country 3 relocate to the other countries \( (dn_3/d\phi|_{\phi=0} < 0) \). Country 2 may gain or lose firms \( (dn_2/d\phi|_{\phi=0} \gtrless 0) \), depending on whether its market-size share is greater or smaller than one-third.

In the same manner, the marginal impact at \( \phi = 1 \) is negative in Country 1, \( dn_1/d\phi|_{\phi=1} < 0 \), positive in Country 3, \( dn_3/d\phi|_{\phi=1} > 0 \), and either positive or negative in Country 2, \( dn_2/d\phi|_{\phi=1} \gtrless 0 \).\(^\text{7}\) We note that unlike our model, the firm shares are indeterminate at \( \phi = 1 \) in the standard FC model with normalized wages.

\(^6\sum_{i=1}^{3} n_i = 1 \) always holds, and thus we only need to solve two out of the three equations.

\(^7\)The marginal effects at \( \phi = 1 \) take a complex form. See Appendix B for the exact formulas.
From these signs of the marginal impact at $\phi \in \{0, 1\}$, we expect that as $\phi$ becomes higher, $n_1$ first increases and then decreases, while $n_3$ moves in an opposite way. Moreover, $n_2$ is expected to show more complex patterns. As explicit form solutions cannot be derived at $\phi \in (0, 1)$, we rely on numerical simulations. Fig. 1 shows the relationship between $\phi$ and $n_j$ in the interior equilibrium for different combinations of market sizes, confirming our expectations.\(^8\) A horizontal line for each $n_i$ represents the market-size share of Country $i$. At any $\phi \in (0, 1)$, Country 1 (or Country 3) always has a greater (or smaller) share manufacturing share than its market-size share, i.e., $n_1 > s_1 (n_3 < s_3)$. Country 2 may gain manufacturing share like Country 1 (case (I)), lose it like Country 3 (case (III)), or the pattern may be more complex than these two (case (II)). In any case, $n_1$ looks hump-shaped in terms of $\phi$: trade liberalization first attracts firms to Country 1 and then promotes relocation from Country 1 to the other countries. $n_3$ behaves in an opposite way to $n_1$. Trade liberalization first accelerates relocation from Country 3 and then attracts some firms from the other countries. These results are in sharp contrast with the standard FC model (e.g., Forslid, 2011): large countries always attract firms and small countries generally lose firms as $\phi$ rises.

These location patterns are characterized by hump-shaped agglomeration rents and non-constant wages. As thoroughly investigated by Baldwin et al. (2003) and Baldwin and Krugman (2004), hump-shaped agglomeration rents are a key element for a better understanding of agglomeration.\(^9\) Markets are substantially segmented at low $\phi$. Firms do not easily export and thus have an incentive to diversify their production to avoid severe competition in domestic markets. With a high $\phi$, firms can easily export anywhere and prefer to locate in the small country with cheaper costs (i.e., lower wages). With an intermediate $\phi$, firms prefer to locate in the large country to save trade costs.

---

\(^8\)The parameter values that produce these figures are given in Appendix H.

\(^9\)Agglomeration rents are formally defined as a firm’s loss associated with deviating from core to periphery, when full agglomeration occurs. See Section 4 for more details on this point.
The other key element in understanding location patterns is non-constant wage rates. The inflow (or outflow) of firms in a country raises (or reduces) wage rates by increasing (or decreasing) labor demand. Fig. 2 plots the relationship between $\phi$ and wage rates relative to the world average ($\bar{w} = \sum_{i=1}^{3} s_i w_i$), corresponding to Fig. 1. The wage rate in a country is largely proportional to its firm share.$^{10}$ $w_1/\bar{w}$ is hump-shaped, while $1/\bar{w}$ is U-shaped in terms of $\phi$. This indicates that the wage gap between countries first expands and then shrinks in terms of $\phi$. The wage rates

$^{10}$If the world average wage rate is defined as the simple average ($\bar{w} = \sum_{i=1}^{3} w_i/3$), the relative wage rate in each country is not proportional to its firm share.
are internationally equalized when trade costs are prohibitively high \((\phi = 0)\) or zero \((\phi = 1)\).

*Fig. 2.* The impact of trade liberalization on wages.

*Note:* Market-size shares: \((s_1, s_2, s_3) = (I) (0.4, 0.37, 0.23); (II) (0.51, 0.4, 0.09); (III) (0.5, 0.3, 0.2)\) as in Fig. 1.

These findings are summarized as

**Proposition 1.** The firm share in Country 1 is always greater than its market-size share, i.e., \(n_1 > s_1\) for \(\phi \in (0, 1)\). By contrast, the firm share in Country 3 is always smaller than its market-size share, i.e., \(n_3 < s_3\) for \(\phi \in (0, 1)\). Country 2 has a greater or smaller firm share than its market-size share. Firm shares are equal to market-size shares, i.e., \(n_i = s_i\) at \(\phi = 0\) and \(\phi = 1\).

**Proposition 2.** Country 1 always has the highest wage, while Country 3 always has the lowest one, i.e., \(w_1 > w_2 > w_3 = 1\) for \(\phi \in (0, 1)\). The wage rates between countries are equalized at \(\phi = 0\) and \(\phi = 1\).

The proofs of both propositions are in Appendix A.
At any positive but finite trade costs, the large country always has a larger firm share than its market-size share, whereas the smallest one always has a smaller share than its market-size share. This result is consistent with the standard two-country FC model.\footnote{In a two-country model with non-constant wages, Takatsuka and Zeng (2012) also confirm this point.} However, the location patterns in the middle-sized country are not straightforward. On the demand side, the middle-sized country is more profitable than the small country but less than the large country. On the supply side, the middle country can employ workers at lower wages than the large country, but at higher wages than small country. In sum, market-size and cost (dis-)advantages in the middle country are not decisive enough to generate clear-cut location patterns.

Based on Propositions 1 and 2, our model finds two types of home market effect in terms of firm shares and wage rates.\footnote{The home market effect in our three country setting can be formally stated as follows (Behrens et al., 2009; Zeng and Uchikawa, 2014). (1) A country with a larger domestic market hosts a greater share of firms than its market-size share, i.e., \( n_1/s_1 > n_2/s_2 > n_3/s_3 \); (2) a country with a larger domestic market has a higher wage rate: \( w_1 > w_2 > w_3 = 1 \); and (3) a country with a larger domestic market has a greater trade surplus of manufacturing goods (or the net outflow of capital rewards): \( n_1 - s_1 > n_2 - s_2 > n_3 - s_3 \). Our model supports the first two definitions, (1) and (2).} A larger country has (1) a greater share of firms than its market-size share in a two-factor model with exogenous wages à la Helpman and Krugman (1985) and (2) higher wages in a one-factor model with non-constant wages à la Krugman (1980). We can interpret our results using these two types of home market effects as follows. When trade costs are high, the home market effects both in terms of firm shares and wages are complement. That is, both home market effects accelerate concentration of firms in larger countries. The influx of firms increases labor demand and pushes wages upward in larger countries, which increases manufacturing expenditures. This promotes more relocation to larger countries. By contrast, when trade costs are low, the two home market effects work in opposite ways. The home market effect in terms of firm shares promotes agglomeration, whereas the home market effect in terms of wages dampens agglomeration. This is why we observe an inverted U-shaped (or U-shaped) relationship between firm share and trade freeness in the large (or small) country. Location patterns in the middle country are determined by the counter-balance of the two home market effects. The mechanism of the two types of home market effect is similar to the one argued by Crozet and Trionfetti (2008), who
allow for non-constant wages in a two-country model.\textsuperscript{13}

A few comments are in order. Krugman and Livas Elizondo (1996) model three regions (two domestic regions and one foreign country) and find that high trade costs against the foreign country result in full agglomeration in one of the two domestic regions, while low trade costs result in dispersion between the two regions. Although their model and ours yield a similar result that dispersion force is dominant at low trade costs, mechanisms generating dispersion force are totally different. Krugman and Livas Elizondo (1996) involve urban congestion costs arising from commuting to the city center, while our focus is on wages determined by local labor market.\textsuperscript{14} Moreover, since their model hinges on a quite specific structure (e.g., symmetric regional size and no international factor mobility), it is impossible to highlight the role of asymmetric market size on location patterns and welfare. In a related vein, Brülhart et al. (2012) develop a three-region model where wages depend on firm location patterns. They introduce a non-tradable good, housing, as well as heterogeneous locational preferences as dispersion forces. Although they do not give a full analytical characterization, we expect that their model would generate more dispersed location patterns than ours.

\subsection*{3.2 Industrial development in the middle country}

As shown in Fig. 1, there are three types of location patterns with different development paths in Country 2. This section discusses the country of intermediate size more in more detail. First, to display market size differences in the three-country model, we propose a “market size triangle” (Fig. 3). The vertical axis in Fig. 3 is Country 2’s market size (share), and the horizontal axis is Country 1’s market size (share). Fig. 3 displays all possible combinations of market sizes in the three countries. The shaded triangle

\textsuperscript{13}Crozet and Trionfetti (2008) theoretically predict and empirically confirm that when two countries become more dissimilar in market size, the magnitude of the home market effect in terms of firm share gets stronger because of the mixture of Helpman and Krugman (1985) and Krugman (1980).

\textsuperscript{14}This difference gives rise to different welfare implications. If dispersion force comes from urban congestion costs, it is evident that dispersion evenly mitigates congestion costs and thus raises welfare (Fujita et al., 1999, Ch 18). On the other hand, if wages work as dispersion force as in our model, a shift from the agglomerated to the dispersed configuration could reduce the welfare of some countries, which will be shown in Section 5.
represents our setting of $s_1 > s_2 > s_3$.\textsuperscript{15} As a comparison, a set of market sizes in the standard two-country FC model, i.e., $s_1 > s_2$ and $s_3 = 0$ (Martin and Rogers, 1995; Baldwin et al., 2003, Ch. 5), can be plotted as the dotted line in Fig. 3. Clearly, the line implies a limited set of market sizes. The advantage of our model is that country-size combinations across countries can be plotted in area rather than on-the-line. In this sense, our model allows for many more possible combinations of market sizes, and thus market size differences can be discussed in more depth than in the standard two-country FC model.

Using the market size triangle, Fig. 4 illustrates three types of location patterns in Country 2. Now the shaded triangle area can be divided into (I), (II) and (III), each corresponding to the three location patterns in Figs. 1 and 2. Area (I) in Fig. 4 satisfies $dn_2/d\phi|_{\phi=0} > 0$ and $dn_2/d\phi|_{\phi=1} < 0$. When Countries 1 and 2 are similar in market size, their industrial evolutions take similar paths. Country 2 has a larger firm share than its market-size share and $n_2$ is hump-shaped in terms of $\phi$, as shown in Fig. 1 (I). Area (II) satisfies $dn_2/d\phi|_{\phi=0} > 0$ and $dn_2/d\phi|_{\phi=1} > 0$. When two large countries have similar market sizes but Country 3 is very small, $n_2$ looks inverted S-shaped in terms of $\phi$: Country 2 first gains, then loses and finally regains firms, as shown in Fig. 1 (II). Area (III) satisfies $dn_2/d\phi|_{\phi=0} < 0$ and $dn_2/d\phi|_{\phi=1} > 0$. When Country 1 is much larger than the other countries, Country 2 behaves like Country 3. Country 2 has a smaller share of firms than its market-size share and $n_2$ is U-shaped in terms of $\phi$, as shown in Fig. 1 (III).

Importantly, Fig. 4 plots all possible combinations of market sizes of the three countries. The market size largely affects Country 2’s industrial development path under trade liberalization. One thought experiment is that when Country 2’s market size is $s_2 = 0.4$, we gradually increase Country 1’s size, $s_1$, from 0.4 (equivalently, a gradual decrease in Country 3’s size, $s_3$). For $s_1$ ranging from 0.4 to 0.45, Country 2 has a greater share of firms than its market-size share under any trade costs (Area (I)). For $s_1$ between 0.5 to 0.6, however, Country 2 has a greater share of firms than its marker-size share under high trade costs, but loses many firms under low trade

\textsuperscript{15}By using $s_1 + s_2 + s_3 = 1$, the order of $s_1 > s_2 > s_3 > 0$ implies $s_1 > 1/3$; $s_2 < 1/2$; $s_2 < 1 - s_1$; $s_2 < s_1$; and $s_2 > (1 - s_1)/2$. All of these conditions are satisfied in the shaded triangle excluding its borders.
costs (Area (II)). What this thought experiment tells us is that even if Country 2’s market size is unchanged, a change in Country 1’s size may result in either a gain or loss of manufacturing in Country 2. The industrial development path in Country 2 is determined by its market size relative to those of other two countries. Detailed conditions can be found in Appendix B.

Fig. 3. The market size triangle.

Fig. 4. Three patterns of industrial evolutions in Country 2.
4 Core-periphery structure

Following on from the interior equilibrium, we now explore two types of corner solutions (core-periphery structure): (1) two-country agglomeration, where all firms are in two countries, and (2) full agglomeration, where all firms are in one country. As Proposition 1 states that \( n_1 \geq s_1 \) and \( n_3 \leq s_3 \) hold for \( \phi \in [0, 1] \), Country 3 never hosts all firms while Country 1 may attract all firms (full agglomeration) or Countries 1 and 2 may achieve an agglomeration of firms (two-country agglomeration).

Trade in our model only involves manufacturing goods. Once full agglomeration or two-country agglomeration occurs, how is trade balanced? We can use the analogy of Takahashi et al. (2013), who use a two-country one-sector FC model. Their model relies on the assumption of the standard FC model that one unit of capital creates one manufacturing firm associated with international mobility. As discussed in Takahashi et al. (2013, p.226), when full agglomeration occurs, the country hosting all manufacturing firms is the exporter of manufacturing goods and the importer of capital from the countries without manufacturing firms. Thus, the trade deficit in the countries without firms is compensated by the surplus on the capital account, and vice versa for the country with full agglomeration. More details on our three-country model can be found in Appendix G.

4.1 Two-country agglomeration and full agglomeration

First, two-country agglomeration is investigated. When firms concentrate in both Countries 1 and 2, operating profits are required to be equalize between the two countries, \( \pi_1 = \pi_2 > \pi_3 \). The left panel of Fig. 5 plots \( n_i \). Trade freeness in which two-country agglomeration is sustainable (or unsustainable) is denoted by \( \phi_3 \) (or \( \bar{\phi}_3 \)). When \( \phi \) exceeds \( \phi_3 \), the agglomeration process in Country 1 is accelerated, whereas Country 2 loses firms. No firms locate in Country 3 from \( \phi_3 \) to \( \bar{\phi}_3 \). Then above \( \bar{\phi}_3 \), the higher
wage rate in Country 1 leads to firm relocation from Country 1 to Countries 2 and 3.

Next, we explore full agglomeration, shown in the right panel of Fig. 5. When all firms concentrate in Country 1 at \( \phi_2 < \phi < \phi_2 \), this requires the profit gaps to be \( v_{12}|_{n_1=1} > 0 \) and \( v_{13}|_{n_1=1} > 0 \). Different market sizes result in \( v_{12}|_{n_1=1} > v_{13}|_{n_1=1} \) at \( \phi \in [0, 1) \). This implies that the gains of firms by moving from Country 2 to Country 1 are always larger than those by moving from Country 3 to Country 1.\(^{16}\) Therefore, using the standard method (Baldwin et al., 2003, Ch. 2), solving \( v_{12}|_{n_1=1} = 0 \) for \( \phi \) gives two critical points where firms are indifferent between two countries:

\[
0 = v_{12}|_{n_1=1} = \pi_1 - \pi_2|_{n_1=1} = \frac{\mu L}{\sigma(1 - \mu)K}[(1 - s_1 - \sigma)w_1^{\sigma-1}\phi^2 + (\sigma - s_3w_1^{\sigma-1})\phi - s_2w_1^{\sigma-1}],
\]

where \( w_1 = 1 + (\sigma - 1)/[s_1(1 - \mu)] \). \( v_{12}|_{n_1=1} \) indicates hump-shaped agglomeration rents, i.e., a quadratic function in terms of \( \phi \). We call \( \phi_2 \) the sustain point and \( \phi_2 \) the break point.

Furthermore, one condition on \( \sigma \) is required for full agglomeration. Larger values of \( \sigma \) lead to diversification. This simply means that agglomeration is not possible if increasing returns are small, which is an analogy of the so-called no-black-hole condition (Fujita et al., 1999, Ch. 4). Thus, the degree of differentiation should be high such that \( \sigma \in (1, \bar{\sigma}) \), where \( \bar{\sigma} \) is defined as the critical value of \( \sigma \) for full agglomeration.\(^{17}\) Once \( \sigma \in (1, \bar{\sigma}) \) is satisfied, full agglomeration occurs in Country 1 over a certain range of trade costs, \( \phi_2 < \phi < \phi_2 \).

The right panel of Fig. 5 shows firm shares in terms of \( \phi \). Full agglomeration occurs at an intermediate level of trade costs (\( \phi_2 < \phi < \phi_2 \)). Furthermore, the figure illustrates the order of firm relocation. As trade liberalization proceeds, Country 3 loses all firms first before Country 2 and it re-attracts firms after Country 2. This order reflects the fact that the agglomeration rents transferred from Country 1 to Country 3 are always larger than those to Country 2, i.e., \( v_{13}|_{n_1=1} > v_{12}|_{n_1=1} \).

\(^{16}\)These relocation incentives evaluated at full agglomeration are related to agglomeration rents and relocation costs. See e.g., Baldwin et al. (2003) and Baldwin and Okubo (2006).

\(^{17}\)Eq. (6) indicates that \( v_{12}|_{n_1=1} \) is a quadratic function in terms of \( \phi \). For \( v_{12}|_{n_1=1} = 0 \) to have two solutions for \( \phi \in [0, 1] \), it must hold that (1) the axis of symmetry is in \([0, 1]\) and (2) the discriminant of the equation is positive. These two conditions reduce to \( \sigma \in (1, \bar{\sigma}) \). See Appendix C for details.
Fig. 5. Two-country agglomeration (left) and full agglomeration (right).

Again, market size is a key element in our model. Fig. 6 shows the equilibrium patterns in the market size triangle. The line dividing (a) and (b) is $v_{13|n_1=1} = 0$ and that dividing (b) and (c) is $v_{12|n_1=1} = 0$.\(^{18}\) **Full agglomeration** is more likely to occur when Country 1 dominates with a large market-size share. **Two-country agglomeration** is more likely to arise when Country 2 is relatively large. When Countries 1 and 2 are not large enough, equilibrium is an interior solution.

\(^{18}\)To draw Fig. 6, full agglomeration as well as two-country agglomeration should arise. From our previous discussion, we need to choose small $\sigma$ in order for $v_{12|n_1=1} = 0$ and $v_{13|n_1=1} = 0$ to have two solutions for $\phi \in [0, 1]$. In area (a), it holds that $v_{12|n_1=1} < 0$ and $v_{13|n_1=1} < 0$; in area (b), $v_{12|n_1=1} < 0$ and $v_{13|n_1=1} > 0$; in area (c), $v_{12|n_1=1} > 0$ and $v_{13|n_1=1} > 0$. Note that $v_{12|n_1=1} < 0$ and $v_{13|n_1=1} > 0$ are a sufficient condition for two-country agglomeration.
Here we make a few comments. First, areas (b) and (c) in the market size triangle indicate that there exists a range of trade costs such that two-country agglomeration and full agglomeration occur. The areas do not indicate that two-country or full agglomeration occurs for the whole range of trade costs. Second, our model supports the result of Forslid (2011). In the case of $w_i = 1$ for $i \in \{1, 2, 3\}$, the sustain and break points are given by $\phi_2 = s_2/(\sigma + s_1 - 1) < 1$ and $\overline{\phi}_2 = 1$, implying that full agglomeration always occurs, even if trade costs are very low (close to zero). On the other hand, a critical point in our model involves the wage term $w_1^{\sigma-1}$. As non-constant wages work as a dispersion force, the range of $\phi$ for full agglomeration is smaller than in the standard FC model.\footnote{To be precise, it is easily checked that as long as $v_{12|n_1=1} = 0$ has two solutions in $\phi \in [0,1]$, we have $d\phi_2/dw_1 = \sigma(\sigma-1)\phi_2/(w_1^{\sigma-1}\sqrt{D}) > 0$ and $d\overline{\phi}_2/dw_1 = -\sigma(\sigma-1)\overline{\phi}_2/(w_1^{\sigma-1}\sqrt{D}) < 0$ where $D$ stands for the discriminant of $v_{12|n_1=1} = 0$.} Third, there always exists a range of $\phi$ for full agglomeration if $\sigma$ is in $(1, \tilde{\sigma})$. Lower values of $\sigma$ mean that varieties of manufacturing goods are more differentiated and the agglomeration force is stronger. Even if Country 1 has a slightly larger market-size share than Country 2, Country 1 can attract all firms at intermediate level of trade costs.
Proposition 3. *Our model involves* two-country agglomeration *where all firms con-
concentrate in Countries 1 and 2 and full agglomeration where all firms locate in Country 1. The main drivers for these configurations are market sizes and the degree of differ-
entiation.

Detailed conditions can be found in Appendix C.

5 Welfare analysis

Each household’s welfare in Country $i$, $U_i$, is measured by real income as follows:

$$U_i = w_i + \frac{\pi(K/L)}{P_iP_{0i}^{1-\mu}},$$

which consists of labor and capital incomes, the price of the non-tradable good and the
price index of the manufacturing goods. It is easy to analytically solve welfare at two extreme cases, i.e., $\phi = 0$ and $\phi = 1$. As shown in Propositions 1 and 2, it holds that
$n_i = s_i$ and $w_1 = w_2 = w_3 = 1$ at these two points. When $\phi = 0$, the only difference
between countries is the price index of the manufacturing goods, which is increasing
in the domestic firm share. The country with the largest (or smallest) market size
enjoys the lowest (or highest) price index. On the other hand, when $\phi = 1$, costless
trade equalizes the price indices internationally, and thus all countries have the same
standard of living. In other words, the welfare gap among countries will eventually
disappear in free trade.

Proposition 4. *If trade costs are prohibitively high, i.e., $\phi = 0$, welfare is the highest
in Country 1 and the lowest in Country 3. If trade costs are zero, i.e., $\phi = 1$, the
welfare levels in all countries converge.*

The proof of the proposition can be found in Appendix D.
As welfare cannot be derived as explicit form solutions at $\phi \in (0, 1)$, we rely on numerical simulations. The right panel of Fig. 7 plots welfare in the interior equilibrium. Two results can be observed: (1) welfare levels in all countries monotonically increase in $\phi$, and (2) Country 1 (or Country 3) always has the highest (or the lowest) welfare.

Next, Fig. 8 plots firm shares and welfare levels in the case of full agglomeration. At $\phi \in [\phi_2, \overline{\phi}_2]$ full agglomeration arises in Country 1. Its welfare is flat because all manufacturing firms locate in Country 1 and thus its welfare is not affected by trade costs. On the other hand, the welfare levels in Countries 2 and 3 are identical because Countries 2 and 3 have no firms and import all their manufacturing goods from Country 1 under the same trade costs. Furthermore, the welfare levels in Countries 2 and 3 increase in $\phi$ by lowering trade costs on imports.

![Graphs showing firm shares and welfare levels in the interior equilibrium and full agglomeration cases.](image-url)
The most notable feature is that welfare in Country 1 could decline once full agglomeration collapses above the break point, i.e., $\phi > \phi_2$, and offshoring occurs: firms leave Country 1. To investigate the deterioration in welfare, it is worthwhile to decompose the impact of trade liberalization on welfare:

$$
\left. \frac{d \log U_1}{d \phi} \right|_{\phi = \phi_2} = \frac{1}{w_1 + \pi (K/L)} \left( \frac{d w_1}{d \phi} + \frac{K}{L} \frac{d \pi}{d \phi} \right) - \frac{\mu}{P_1} \frac{d P_1}{d \phi} \left|_{\phi = \phi_2} \right. - \frac{1 - \mu}{p_{01}} \frac{d p_{01}}{d \phi} \left|_{\phi = \phi_2} \right.,
$$

where $U_1$ is differentiated at the break point, $\phi_2$. Firms start to leave Country 1 at $\phi_2$ (i.e., offshoring occurs), reducing labor demand and thus lowering the wage rate. This reduces household income, which has a negative impact on welfare (the first term). However, this lowers the price of the non-tradable good, which is beneficial in consumption (the fourth term). The capital rewards remain unchanged because the decrease in Country 1’s expenditure on manufacturing goods owing to the decreased wage is offset by the increased expenditure by Country 2 owing to the increased wage (the second term). Finally, the sign of the third term, the effect on the price index of manufacturing goods, is vague. Thus, a further decomposition is given by
\[
\frac{dP_1}{d\phi} \bigg|_{\phi = \bar{\phi}_2} = \frac{\partial P_1}{\partial n_1} \frac{dn_1}{d\phi} + \partial P_1 \sum_{i=1}^{3} \frac{\partial w_1}{\partial n_i} \frac{dn_i}{d\phi} + \frac{\partial P_1}{\partial \phi} \bigg|_{\phi = \bar{\phi}_2}.
\]

(8)

The first term represents the impact of changes in firm shares: a decrease in the number of domestic varieties raises the price index. The second term indicates that offshoring reduces domestic wage rates and thus the price index. The third term is the direct impact of trade cost reductions. This is negligible because all firms are in Country 1 and thus no trade costs are involved. Offshoring raises the price index if the loss from reducing domestic varieties outweighs the gain from lowering domestic production costs and importing cheaper foreign varieties. In sum, the collapse of agglomeration and offshoring of firms might be beneficial in Country 1 by reducing the price of the non-tradable good and decreasing the price index by lower production costs and cheaper import varieties. On the other hand, offshoring could be harmful by reducing the wage rate and increasing the price index by reducing the number of domestic varieties.

Eq. (7) can be re-written as

\[
\frac{d\log U_1}{d\phi} \bigg|_{\phi = \bar{\phi}_2} = \frac{\mu}{\sigma - 1} \left[ 1 - \bar{\phi}_2 w_1^{\sigma-1} - \frac{s_1(\sigma - 1)^2}{(\sigma + s_1 - 1) \{\sigma + s_1(1 - \mu) - 1\}} \right] \frac{dn_1}{d\phi} \bigg|_{\phi = \bar{\phi}_2}.
\]

(9)

A close inspection of the expression in the large square brackets reveals that Country 1 is worse off when \( \sigma \) is close to one and Country 1’s market size is not extremely large. The market size condition for worse off is given by \( s_1 < s_2 + 1/2 \).

Using the market size triangle, Fig. 9 can illustrate patterns of welfare change by offshoring in Country 1. The shaded area in Fig. 9 indicates that full agglomeration occurs at \( \phi \in [\phi_2, \bar{\phi}_2] \), corresponding to area (c) in Fig. 6. Now \( s_1 = s_2 + 1/2 \) splits the full-agglomeration shaded area into two areas, (A) and (B). Area (A) satisfies

---

20We note that once \( \phi \) exceeds \( \bar{\phi}_2 \), firms in Country 1 start moving to Country 2: it holds that \( \frac{dn_1}{d\phi} \bigg|_{\phi = \bar{\phi}_2} < 0 \), \( \frac{dn_2}{d\phi} \bigg|_{\phi = \bar{\phi}_2} > 0 \) and \( \frac{dn_3}{d\phi} \bigg|_{\phi = \bar{\phi}_2} = 0 \).

21Strictly speaking, the effect on the price index also involves \( \sum_{i=2}^{3} \frac{\partial P_1}{\partial n_i} \frac{dn_i}{d\phi} \) and \( \sum_{i=2}^{3} \frac{\partial P_1}{\partial w_i} \sum_{j=1}^{3} \frac{\partial w_i}{\partial n_j} \frac{dn_j}{d\phi} \), but these terms disappear. Our differentiation is evaluated at the point where there are no firms in the smaller countries. See Appendix E for details.
s₁ > s₂ + 1/2 and welfare in Country 1 decreases. Area (B) satisfies s₁ < s₂ + 1/2 and welfare always (weakly) increases under trade liberalization.

The reason for the worsening welfare from offshoring in Country 1 can be explained as follows. As the left panel of Fig. 10 shows, as Country 1 is smaller, full agglomeration is less likely to occur and φ₂ falls. In other words, a collapse of agglomeration and offshoring happens under smaller values of φ.\textsuperscript{22} Once full agglomeration in Country 1 collapses and firm relocation occurs, Country 1 starts importing goods from abroad with trade costs. Lower values of φ₂ indicate greater payments of trade costs associated with imports. In addition, as Country 1 is smaller and Country 2 is larger, the wage (production costs) differential between the two countries is smaller. This reduces Country 1’s benefit from importing from offshoring firms; the products produced in Country 2 do not have low prices. The smaller size of Country 1 and the smaller wage gap with the other countries increase Country 1’s import payments. Thus, greater import payments result in a reduction in Country 1.

To summarize:

**Proposition 5.** Above the break point, φ > φ₂, full agglomeration collapses and offshoring happens in Country 1. If the size of Country 1 is not extremely large, i.e., s₁ < s₂ + 1/2, and the degree of product differentiation is sufficiently high, i.e., σ is close to one, Country 1 experiences a reduction in welfare.

See Appendix E for details and the proof.

\textsuperscript{22}This can be confirmed by numerical simulations.
Fig. 9. Welfare change by offshoring in Country 1.

Fig. 10. Change in the market size of Country 1.
6 Welfare-maintaining policies and trade liberalization

When Country 1 is not substantially large, Country 1 might experience a reduction in welfare following the collapse of full agglomeration at $\phi \simeq \phi_2$. One important question is how Country 1 manages to hamper offshoring when trade costs decrease. One policy solution might be to levy a prohibitive tax on firm relocation or prohibit capital mobility. This could prevent welfare losses from offshoring in Country 1.

However, following a common assumption in the trade and geography literature, we retain the assumption of free mobility of firms under trade liberalization. What (second-best) policies are feasible in Country 1? One feasible policy is a rise of the break point, $\phi_2$, without increasing trade costs or regulating capital mobility. This can sustain full agglomeration and postpone the collapse of agglomeration and offshoring. More specifically, one possibility is to reduce trade costs with a specific country. Let us imagine a situation where trade costs can be reduced more between a particular pair of countries by a bilateral trade agreement. If Country 1 ratifies a trade agreement, for example, with Country 2 in further reducing trade costs, the bilateral trade freeness between Countries 1 and 2 is $\alpha(\geq 1)$ times higher than the freeness of trade between Countries 1 and 3. Namely, we denote $\phi_{12} = \phi_{21} = \alpha \phi$; $\phi_{13} = \phi_{31} = \phi_{23} = \phi_{32} = \phi$, where $\phi_{ij}$ stands for freeness of trade from $i$ to $j$.

Suppose that Country 1 ratifies bilateral trade agreements with Country $j \in \{2, 3\}$. The bilateral agreement alters all sustain and break points, which we re-define as $\bar{\phi}_{ij}^2$ and $\bar{\phi}_{ij}^3$. A marginal improvement in trade freeness between a pair of countries affects the critical points as follows:

$$\left. \frac{d\phi_{12}^2}{d\alpha} \right|_{\alpha=1} < 0, \quad \left. \frac{d\phi_{12}^3}{d\alpha} \right|_{\alpha=1} < 0,$$

$$\left. \frac{d\phi_{13}^2}{d\alpha} \right|_{\alpha=1} < 0, \quad \left. \frac{d\phi_{13}^3}{d\alpha} \right|_{\alpha=1} > 0,$$

where $\bar{\phi}_{ij}^1|_{\alpha=1} = \bar{\phi}_2$ and $\bar{\phi}_{ij}^2|_{\alpha=1} = \bar{\phi}_2$ hold. Fig. 11 compares the industrial evolutions associated with welfare levels in different trade agreements between different country pairs. Fig. 11 illustrates that bilateral trade agreements with Countries 2 and 3 at the
sustain point promote full agglomeration, \( \phi_{13}^{1} < \phi_{12}^{12} < \phi_{2}^{2} \). This implies that lowering bilateral trade costs the large and the smaller countries fosters agglomeration in the large country. Bilateral trade agreements between Countries 2 and 3 result in a similar effect, i.e., \( \phi_{23}^{23} < \phi_{2}^{2} \).

On the other hand, at the break point, \( \phi_{2} \), we see different outcomes across country pairs, \( \phi_{12}^{12} < \phi_{2} < \phi_{13}^{13} \). Country 1’s trade agreement with Country 2 promotes the collapse of full agglomeration by offshoring and then reduces welfare in Country 1. As firms in Country 1 are concerned about higher wages, once trade costs between Countries 1 and 2 fall further, they have greater incentive to relocate to Country 2 because of lower wages and the middle-country market size. By contrast, bilateral trade-costs reductions between Countries 1 and 3 postpone the collapse because Country 1 has better access to Country 3 but has a much larger market. Considering the higher trade costs with Country 2 and the small market size in Country 3, firms are more likely to stay at Country 1 In sum, if Country 1 attempts to maintain full agglomeration, Country 1 should agree on freer trade with the smallest country rather than the middle country.

Finally we have one note. Country 1’s bilateral trade agreement always improves welfare in Countries 2 and 3. Intuitively, both countries benefit from cheaper imports from Country 1 because of lower trade costs. In addition, Country 2 attracts more firms from Country 1, raising wage rates. This means that if Country 1 offers a bilateral trade agreement, Countries 2 and 3 always accept it.

The discussion is summarized as follows:

**Proposition 6.** At the sustain point, \( \phi_{2} \), Country 1’s bilateral trade agreements with Country 2 or 3 both accelerate full agglomeration. By contrast, at the break point, \( \phi_{2} \), Country 1’s bilateral trade agreement with Country 2 causes the collapse of agglomeration, while an agreement with Country 3 sustains full agglomeration. When trade costs decline, Country 1’s trade agreement with Country 3 can prevent the deterioration of its welfare.

\(^{23}\)The first inequality, \( \phi_{13}^{1} < \phi_{12}^{12} \), does not generally hold. At \( \alpha \) close to one, \( \phi_{2}^{12} \) may be greater or smaller than \( \phi_{13}^{1} \).
The proof of the proposition is in Appendix F.

Fig. 11. Bilateral trade agreements in Country 1.

7 Discussions and Extensions

This section discusses the comparison of our model with a constant-wage model and the generality of our results. Our theoretical results are linked to recent experiences of the U.S. economy.

7.1 Comparison with a three-country constant-wage model

To make our contribution perfectly clear, we compare our non-constant-wage model with Forslid (2011)'s constant-wage model. Our model can reduce to Forslid (2011)'s model by setting $w_1 = w_2 = 1$. Figs. 12 and 13 show the evolution of equilibrium firm
share (left panel) and individual welfare (right panel) under non-constant wages (thick line) and constant wages (thin line).

A crucial difference from our result is that under constant wages Country 1 weakly monotonically increases its firm share as trade costs decline. Since firms can always hire workers at a constant wage rate, agglomeration in Country 1 does not discourage further inflow of firms and thus Country 1 can maintain full agglomeration at low trade costs. Consequently, different equilibrium paths between the two models give rise to different welfare implications. Under non-constant wages, Country 1 could reduce welfare when full agglomeration collapses (Proposition 5). By contrast, under constant wages, the collapse of full agglomeration never happens and thus Country 1 never deteriorates welfare (Forslid, 2011). Accordingly, Country 1 does not need to take any welfare-maintaining policies as discussed in Section 6.

Another subtle but important difference is in equilibrium path at high trade costs. Under non-constant wages, Country 1 attracts more firms than under constant wages at low $\phi$ (roughly, $\phi \in [0, 0.15]$ in the left panel of Fig. 12 and $\phi \in [0, 0.55]$ in the left panel of Fig. 13). As emphasized in Section 3.1, our model incorporates two types of home market effect (larger firm shares and higher wages in larger countries). When trade costs are high, both of the two effects work in favor of agglomeration in the large country. As trade costs decline, the home market effect of higher wages in the large country turns to moderate agglomeration force and thus firms are spatially dispersed.
Fig. 12. Non-constant wage (thick lines) vs. constant wage (thin lines) in the interior case.

*Note:* The left (or right) panel shows the equilibrium firm share (or welfare). Horizontal lines in the left panel are the market-size shares.

Fig. 13. Non-constant wage (thick lines) vs. constant wage (thin lines) in the full agglomeration case.

*Note:* The left (or right) panel shows the equilibrium firm share (or welfare). Horizontal lines in the left panel are the market-size shares.
7.2 More-than three countries

Our main results highlight two points: (i) complex location patterns in the middle country and (ii) the collapse of full agglomeration in the large country. Most of the conditions related to these points are analytically derived and characterized by market size. These results can be obtained in a general model with more-than three countries.

Appendix I shows equilibrium location patterns in a four-country model. The middle countries are now defined as the second and third largest countries. In parallel to the three-country model, equilibrium location patterns are determined by market size and summarized in the market-size triangle/rectangle. The largest (or smallest) country always shows an inverted U-shaped (or U-shaped) equilibrium path, while the middle-sized countries involve more patterns determined by their positions in the triangle/rectangle.

The results concerning full agglomeration also hold in a general $J(> 3)$-country model (see Appendix J). We can confirm that the market sizes of the largest and the second largest countries determine the collapse of full agglomeration and its welfare impact. When full agglomeration is possible, the second-largest country is the second-most attractive location for firms following the largest country. As firms compare the profits in these countries, the market sizes of the top-two countries are crucial for the results.

7.3 Multiple industries with different labor intensities

Our model in the main analysis has one manufacturing industry subject to monopolistic competition. Another extension is multiple manufacturing industries with different labor intensities (different unit labor requirements). Our main results remain unchanged (see Appendix K). As long as consumers put an equal expenditure share on each manufacturing industries, equilibrium firm share in each industry exhibits qualitatively the same pattern as in the single-manufacturing model (see Fig. A7). All firms in each
industry have the same labor intensity and thus face the same marginal costs. Hence, their equilibrium location patterns mimic those in the single-manufacturing model.

Furthermore, to highlight inter-industry linkages, one could introduce intermediate goods and model an input-output structure (see Fujita et al., 1999, Ch 15). In this case, we expect that firms in more labor intensive industries would avoid agglomerating in the largest country and move to the smallest country to seek lower wages. On the other hand, firms in less labor intensive industries would be less sensitive to a rise of wages so that they would agglomerate in the largest country to seek greater demand.

7.4 The U.S. economy

Although our analysis is based on a highly stylized model, it helps better understand the current U.S. economy facing globalization. We have shown that as trade liberalization proceeds, the large country loses manufacturing, leading to lower wages. According to the sectoral level analysis by Acemoglu et al. (2016), the import penetration from China contributed around 10% of the decline in manufacturing employment in the 2000s. This number could be around 20% when taking into account indirect impact from inter-industry linkages. Using the individual worker data from 1984 to 2002, Ebenstein et al. (2014) find that occupational switching from manufacturing led to relatively large wage declines. These findings suggest that the proliferation of globalization since the mid-1980s, which Baldwin (2016) calls the “second unbundling,” has put downward pressure on employment and wages in the U.S. manufacturing industry.

Another issue is the U.S. attitude toward trade policies. Proposition 6 states that the large country facing the collapse of agglomeration prefers to reduce trade costs with the small country rather than the middle country. Evaluating the process of free trade agreement negotiations in the early 2000s, Schott (2004) concludes that “the U.S. has put too much effort on low risk, low reward (in both economic and political terms) negotiations with small countries around the globe” (p.373). Our result could rationalize the incentive.
8 Conclusion

We construct a three-country FC model with non-constant wages to illustrate the Great Divergence and the Great Convergence in globalization. Focusing on market size differences across countries, our model can explain full agglomeration in large countries, as in the Great Divergence, as well as offshoring in developed countries and industrial development in middle-sized countries, as in the Great Convergence. We find three types of location patterns in the middle-sized country as well as full and two-country agglomerations. Our three-country model provides much richer location patterns than the standard two-country model. Wage rates depending on market size and firm share are a main driver for offshoring in developed countries, which raises a possibility of declining welfare. All of these results are characterized by market size.

A possible extension is empirical analysis. One might test how relative market size crucially affects the rise and fall of manufacturing in developing and developed countries in interaction with trade costs and wage rates. We leave this topic to future research.
Appendix A. Proofs of Propositions 1 and 2

Following the previous literature (e.g., Fujita et al., 1999), the simple dynamics of firm migration are given as follows:

\[ \dot{n}_i = n_i (1 - n_i) \left( \pi_i - \sum_{j=1}^{3} s_j \pi_j \right), \quad \text{for } i \in \{1, 2, 3\}, \]

where a dot represents the time derivative. Namely, \( n_i \) increases if Country \( i \) offers higher profits than an average profit weighted by market size.\(^{24}\) Using the expression of \( v_{ij} \), we can rearrange the above equation as

\[ \dot{n}_1 = n_1 (1 - n_1) (s_2 v_{12} + s_3 v_{13}), \]
\[ \dot{n}_2 = n_2 (1 - n_2) (s_3 v_{23} - s_1 v_{21}), \]
\[ \dot{n}_3 = -n_3 (1 - n_3) (s_1 v_{13} + s_3 v_{23}), \]

where we make use of \( v_{ij} = -v_{ji} \).

**Firm shares.** Evaluating \( v_{12}, v_{13} \) and \( v_{23} \) at \( (n_1, n_2) = (s_1, s_2) \) gives

\[ v_{12}|_{(n_1, n_2)=(s_1, s_2)} = \frac{\phi \mu L (1 - \phi) (s_2 - s_1)}{K (\sigma - \mu) [s_1 + \phi (s_2 + s_3)] [s_2 + \phi (s_1 + s_3)]} \geq 0, \]
\[ v_{13}|_{(n_1, n_2)=(s_1, s_2)} = \frac{\phi \mu L (1 - \phi) (s_1 - s_3)}{K (\sigma - \mu) [s_1 + \phi (s_2 + s_3)] [s_3 + \phi (s_1 + s_2)]} \geq 0, \]
\[ v_{23}|_{(n_1, n_2)=(s_1, s_2)} = \frac{\phi \mu L (1 - \phi) (s_2 - s_3)}{K (\sigma - \mu) [s_2 + \phi (s_1 + s_2)] [s_2 + \phi (s_1 + s_3)]} \geq 0, \]

where equality holds at \( \phi = 0 \) and \( \phi = 1 \). \( v_{12} \geq 0 \) and \( v_{13} \geq 0 \) imply that firms in Countries 2 and 3 are ready to move to Country 1, i.e., \( \dot{n}_1 \geq 0 \), and thus it holds that \( n_1 \geq s_1 \) in the long-run (or steady-state) equilibrium. Similarly, \( v_{13} \geq 0 \) and \( v_{23} \geq 0 \) imply that firms in Country 3 always find it profitable to relocate to either Country 1 or 2, i.e., \( \dot{n}_3 \leq 0 \), and thus it holds that \( n_3 \leq s_3 \) in the long-run equilibrium.

\(^{24}\) The following discussion goes through if one takes a simple average of profits, rather than a weighted average.
Firm shares at two extreme cases: \( n_i|_{\phi=0} \) and \( n_i|_{\phi=1} \). Evaluating profits at \( \phi = 0 \) gives

\[
\begin{align*}
\pi_1 &= \frac{\mu w_1^{1-\sigma} (w_1 L_1 + rK_1)}{\sigma w_1^{1-\sigma} N_1} = \frac{\mu (w_1 L_1 + rK_1)}{\sigma w_1^{1-\sigma} N_1} = \frac{\mu s_1 (w_1 L + rK)}{\sigma w_1^{1-\sigma} n_1 K}, \\
\pi_2 &=\frac{\mu w_2^{1-\sigma} (w_2 L_2 + rK_2)}{\sigma w_2^{1-\sigma} N_2} = \frac{\mu (w_2 L_2 + rK_2)}{\sigma w_2^{1-\sigma} N_2} = \frac{\mu s_2 (w_2 L + rK)}{\sigma w_2^{1-\sigma} n_2 K}, \\
\pi_3 &= \frac{\mu (L_3 + rK_3)}{\sigma N_3} = \frac{\mu s_3 (L + rK)}{\sigma n_3 K}.
\end{align*}
\]

Solving \( \pi_1 - \pi_2 = 0 \) and \( \pi_1 - \pi_3 = 0 \) for \((n_1, n_2)\) yields \((n_1, n_2) = (s_1, s_2)\).

Similarly, evaluating profits at \( \phi = 1 \) gives

\[
\begin{align*}
\pi_1 &= \frac{\mu w_1^{1-\sigma} (w_1 L_1 + rK_1) + (w_2 L_2 + rK_2) + (L_3 + rK_3)}{\sigma w_1^{1-\sigma} N_1 + w_2^{1-\sigma} N_2 + N_3}, \\
\pi_2 &= \frac{\mu w_2^{1-\sigma} (w_1 L_1 + rK_1) + (w_2 L_2 + rK_2) + (L_3 + rK_3)}{\sigma w_1^{1-\sigma} N_1 + w_2^{1-\sigma} N_2 + N_3}, \\
\pi_3 &= \frac{\mu (w_1 L_1 + rK_1) + (w_2 L_2 + rK_2) + (L_3 + rK_3)}{\sigma w_1^{1-\sigma} N_1 + w_2^{1-\sigma} N_2 + N_3}.
\end{align*}
\]

Equating \( \pi_1 \) and \( \pi_2 \) with \( \pi_3 \) gives \( w_1 = w_2 = 1 \), which implies \((n_1, n_2) = (s_1, s_2)\).

Wage rates. The above discussion shows that \( w_1 = w_2 = 1 \) holds at \( \phi = 0 \) and \( \phi = 1 \). In the following, we consider the case where \( \phi \in (0, 1) \). Suppose \( w_1 > w_2 \) holds, then we have

\[
w_1 > w_2, \\
\Rightarrow [(1 - \mu)s_1 + (\sigma - 1)n_1]/s_1 > [(1 - \mu)s_2 + (\sigma - 1)n_2]/s_2, \\
\Rightarrow n_1 s_2 = n_1(1 - s_1 - s_3) > s_1(1 - n_1 - n_3) = s_1 n_2.
\]

where we make use of \( n_1 > s_1 \) and \( n_3 < s_3 \). Noting that \( n_1(1 - s_1 - s_3) > s_1(1 - s_1 - s_3) \) and \( s_1(1 - n_1 - n_3) > s_1(1 - n_1 - n_3) \), the above inequality is satisfied if

\[
s_1(1 - s_1 - s_3) > s_1(1 - n_1 - s_3), \\
\Rightarrow n_1 > s_1.
\]

This inequality holds from the previous discussion, and thus \( w_1 > w_2 \) holds for \( \phi \in (0, 1) \).
Similarly, $w_2$ is compared with $w_3 = 1$:

$$w_2 > 1,$$

$$\rightarrow s_3[(1 - \mu)s_2 + (\sigma - 1)n_2] > s_2[(1 - \mu)s_3 + (\sigma - 1)n_3],$$

$$\rightarrow s_3n_2 = s_3(1 - n_1 - n_3) > n_3(1 - s_1 - s_3) = n_3s_2.$$ 

Noting that $s_3(1 - n_1 - n_3) > n_3(1 - s_1 - s_3)$, the above inequality is satisfied if

$$n_3(1 - s_1 - n_3) > n_3(1 - s_1 - s_3),$$

$$\rightarrow s_3 > n_3.$$ 

This inequality holds from the previous discussion, and thus $w_2 > 1$ holds for $\phi \in (0, 1)$. We conclude that $w_1 \geq w_2 \geq 1$ holds for $\phi \in [0, 1]$.

**Appendix B. The marginal impact of trade liberalization at $\phi \in \{0, 1\}$**

We derive the slope of equilibrium firm shares at the two endpoints, i.e., $\phi \in \{0, 1\}$. The path of industrial development in Country 2 follows from these signs.

Profit equalizations are given as follows:

$$v_{12}(n_1, n_2, \phi) = \pi_1 - \pi_2 = 0,$$

$$v_{13}(n_1, n_2, \phi) = \pi_1 - \pi_3 = 0,$$

$$v_{23}(n_1, n_2, \phi) = \pi_2 - \pi_3 = 0.$$ 

As the firm share in Country 3 is given by $n_3 = 1 - s_1 - n_2$, we do not need to consider $v_{23} = \pi_2 - \pi_3$. Thus, the two equations are differentiated with respect to trade freeness and can be rearranged in matrix form:

$$\begin{bmatrix} \frac{\partial v_{12}}{\partial n_1} & \frac{\partial v_{12}}{\partial n_2} \\ \frac{\partial v_{13}}{\partial n_1} & \frac{\partial v_{13}}{\partial n_2} \end{bmatrix} \begin{bmatrix} \frac{dn_1}{d\phi} \\ \frac{dn_2}{d\phi} \end{bmatrix} = \begin{bmatrix} -\frac{\partial v_{12}}{\partial \phi} \\ -\frac{\partial v_{13}}{\partial \phi} \end{bmatrix}.$$ 

The system of equations is solved in terms of $dn_1/d\phi$ and $dn_2/d\phi$. At the extreme, when $\phi = 0$, it holds that $n_i = s_i$. This can simplify the expressions to

$$\frac{dn_1}{d\phi} \bigg|_{\phi=0} = \sigma(3s_1 - 1) > 0, \quad \frac{dn_2}{d\phi} \bigg|_{\phi=0} = \sigma(3s_2 - 1) \geq 0.$$ 

By our assumption, the large country has more than one-third of the world endowment
\((s_1 > 1/3)\). Intuitively, with very small \(\phi (\simeq 0)\), Country 2 gains firms if its market-size share exceeds one-third.

From the fact that the sum of firm shares is equal to one: \(\sum_{i=1}^{3} n_i = 1\), we have \(\sum_{i=1}^{3} \frac{dn_i}{d\phi} = 0\) for any \(\phi\). The slope of the firm share in Country 3 is thus given by
\[
\frac{dn_3}{d\phi} \bigg|_{\phi=0} = - \left( \frac{dn_1}{d\phi} + \frac{dn_2}{d\phi} \right) \bigg|_{\phi=0} = \sigma (3s_3 - 1) < 0.
\]

Next, the slopes at \(\phi = 1\) are derived in the same manner. By using the fact that \(s_1 \in (1/3, 1)\), \(s_2 \in (0, 1/2)\) and \(s_3 \in (0, 1/3)\), the signs of the slopes are determined as follows:
\[
\frac{dn_1}{d\phi} \bigg|_{\phi=1} = \frac{s_1 (\sigma - \mu) f(s_1, s_2)}{(\sigma - 1)^2} < 0, \quad f(s_1, s_2) = 2s_1^2 + (2s_2 - 3)s_1 + 2s_2^2 - 2s_2 + 1 < 0,
\]
\[
\frac{dn_2}{d\phi} \bigg|_{\phi=1} = \frac{s_2 (\sigma - \mu) g(s_1, s_2)}{(\sigma - 1)^2} \geq 0, \quad g(s_1, s_2) = 2s_1^2 + (1 - s_2)[1 - 2(s_1 + s_2)] \leq 0,
\]
\[
\frac{dn_3}{d\phi} \bigg|_{\phi=1} = \frac{s_3 (\sigma - \mu) h(s_1, s_2)}{(\sigma - 1)^2} > 0, \quad h(s_1, s_2) = 2s_1^2 - (s_1 + s_2)(1 - 2s_2) > 0.
\]

We note that the signs of the slopes at \(\phi \in \{0, 1\}\) only depend on market sizes.

**Appendix C. Proof of Proposition 3**

*Conditions for full agglomeration.* Suppose that all firms locate in Country 1 and have no incentive to relocate to the other countries. The operating profits evaluated at \(N_1 = K\) are given by
\[
\pi_1|_{n_1=1} = \frac{\mu}{\sigma K}(Y_1 + Y_2 + Y_3),
\]
\[
\pi_2|_{n_1=1} = \frac{\mu}{\sigma K} \left( \frac{1}{w_1} \right)^{1-\sigma} \left[ \phi Y_1 + \frac{Y_2}{\phi} + Y_3 \right],
\]
\[
\pi_3|_{n_1=1} = \frac{\mu}{\sigma K} \left( \frac{1}{w_1} \right)^{1-\sigma} \left[ \phi Y_1 + Y_2 + \frac{Y_3}{\phi} \right],
\]
where \( w_1 = 1 + (\sigma - 1)/[s_1(1 - \mu)] \). We note that \( w_2 = 1 \) at \( n_1 = 1 \). From these equations, the necessary and sufficient conditions for full agglomeration are given by

\[
\begin{align*}
\nu_{12}|_{n_1=1} &= \pi_1 - \pi_2|_{n_1=1} = \Omega \cdot F_{12}(\phi) > 0, \\
\nu_{13}|_{n_1=1} &= \pi_1 - \pi_3|_{n_1=1} = \Omega \cdot F_{13}(\phi) > 0,
\end{align*}
\]

where \( F_{12}(\phi) \equiv -(\sigma + s_1 - 1)w_1^{\sigma - 1}\phi^2 + (\sigma - w_1^{\sigma - 1}s_3)\phi - w_1^{\sigma - 1}s_2, \)

\( F_{13}(\phi) \equiv -(\sigma + s_1 - 1)w_1^{\sigma - 1}\phi^2 + (\sigma - w_1^{\sigma - 1}s_2)\phi - w_1^{\sigma - 1}s_3, \)

and where \( \Omega \equiv \mu L/[\phi \sigma(1 - \mu)K] \) is a positive constant.

As it holds that \( -(\phi s_2 + s_3) \geq -(\phi s_3 + s_2) \) because of \( s_2 > s_3 \), we have \( F_{13}(\phi) \geq F_{12}(\phi) \) for \( \phi \in [0, 1] \). This allows us to focus only on \( F_{12}(\phi) > 0 \). Because it holds that \( F_{12}(0) = -w_1^{\sigma - 1}s_2 < 0 \) and \( F_{12}(1) = \sigma(1 - w_1^{\sigma - 1}) < 0 \) and \( F_{12} \) is a quadratic function in terms of \( \phi \), the condition for \( F_{12}(\phi) > 0 \) is required to reach a maximum in \( \phi \in (0, 1) \). This is equivalent to two conditions: (i) the axis of symmetry of \( F_{12} \) is in \( \phi \in (0, 1) \), and (ii) the discriminant \( D \) for \( F_{12}(\phi) = 0 \) is positive.

Condition (i) is given as follows:

\[
\frac{\sigma - w_1^{\sigma - 1}s_3}{2w_1^{\sigma - 1}(\sigma + s_1 - 1)} \in (0, 1),
\]

\[
\rightarrow 0 < \sigma - w_1^{\sigma - 1}s_3 < 2w_1^{\sigma - 1}(\sigma + s_1 - 1),
\]

\[
\rightarrow w_1^{\sigma - 1}s_3 < \sigma < G(\sigma) \equiv w_1^{\sigma - 1}[s_3 + 2(\sigma + s_1 - 1)].
\]

We can show that the second inequality, \( \sigma < G(\sigma) \), always holds for \( \sigma > 1 \). Using the Taylor approximation in \( w_1^{\sigma - 1} \approx 1 + (\sigma - 1)^2/[s_1(1 - \mu)] \), we can confirm that \( G(\sigma) \) is greater and steeper than \( \sigma \) at \( \sigma = 1 \), i.e., \( 1 < G(1) = 2s_1 + s_3 \) and \( 1 < G'(1) = 2 \). It suffices to check that the slope of \( G(\sigma) \) is always positive and increasing in \( \sigma > 1 \):

\[
G'(\sigma) = \frac{2[3\sigma^2 - (5 + s_2 - s_1)\sigma + s_2 - \mu s_1 + 2]}{s_1(1 - \mu)} > 0,
\]

\[
G''(\sigma) = \frac{2(6\sigma - 5 + s_1 - s_2)}{s_1(1 - \mu)} > 0.
\]

Hence, condition (i) only requires the first inequality, \( w_1^{\sigma - 1}s_3 < \sigma \).

Condition (ii) is given by

\[
D \equiv (\sigma - w_1^{\sigma - 1}s_3)^2 - 4s_2(\sigma + s_1 - 1)w_1^{\sigma - 1} > 0.
\]

These two conditions reduce to

\[
\sigma > H(\sigma) \equiv w_1^{\sigma - 1}\left[s_3 + 2\sqrt{s_2(\sigma + s_1 - 1)}\right].
\]
Under the above condition, the smaller root of \( F_{12}(\phi) = 0 \) corresponds to \( \phi_2 \) and the larger one to \( \bar{\phi}_2 \). We can confirm that the following inequalities hold for \( \sigma > 1 \):

\[
1 > H(1) = s_3 + 2\sqrt{s_1s_2}, \quad 1 > H'(1) = \sqrt{s_2/s_1},
\]

\[
H'(\sigma) = \frac{s_2[5\sigma^2 - 2(5 - 2s_1)\sigma + 5 - s_1(3 + \mu)] + 2s_3(\sigma - 1)\sqrt{s_2(\sigma + s_1 - 1)}}{s_1(1 - \mu)\sqrt{s_2(\sigma + s_1 - 1)}} > 0,
\]

\[
H''(\sigma) = \frac{s_2[15\sigma^2 - 6(5 - 4s_1)\sigma + 8s_1^2 - (25 - \mu)s_1 + 15] + 4s_3(\sigma + s_1 - 1)\sqrt{s_2(\sigma + s_1 - 1)}}{2s_1(1 - \mu)(\sigma + s_1 - 1)\sqrt{s_2(\sigma + s_1 - 1)}} > 0.
\]

Hence, \( H(\sigma) \) crosses \( \sigma \) from below at some \( \sigma > 1 \). We define such \( \sigma \) as \( \tilde{\sigma} \):

\[
\tilde{\sigma} = \min_{\sigma > 1} \arg \left[ \sigma - H(\sigma) = 0 \right].
\]

In other words, if \( \sigma \) is in \( (1, \tilde{\sigma}) \), full agglomeration occurs at intermediate trade costs such that \( \phi \in [\bar{\phi}_2, \tilde{\phi}_2] \). Note that \( \tilde{\sigma} \) depends on the market sizes.

**Conditions for two-country agglomeration.** A sufficient condition for two-country agglomeration is (i) \( v_{12}|_{n_1 = 1} < 0 \) and (ii) \( v_{13}|_{n_1 = 1} > 0 \). From the previous discussion, condition (i) requires \( \sigma < H(\sigma) \), i.e., \( \sigma > \tilde{\sigma} \). By using the same analogy as before, condition (ii) reduces to the following inequality:

\[
\sigma > H^*(\sigma) \equiv w_1^{\sigma^{-1}} \left[s_2 + 2\sqrt{s_3(\sigma + s_1 - 1)}\right],
\]

where \( w_1 = 1 + (\sigma - 1)/[s_1(1 - \mu)] \). As it can be verified that \( H^*(\sigma) \) crosses \( \sigma \) from below at \( \sigma > 1 \) in the same manner as before, condition (ii) turns out to be \( \sigma \in (1, \sigma^*) \) where \( \sigma^* \) is defined as

\[
\sigma^* = \min_{\sigma > 1} \arg \left[ \sigma - H^*(\sigma) = 0 \right].
\]

As it holds that \( H(\sigma) > H^*(\sigma) \) for \( \sigma > 1 \),\(^{25}\) \( \sigma^* \) is greater than \( \tilde{\sigma} \). Combining the two conditions yields \( \sigma \in [\tilde{\sigma}, \sigma^*] \). If this condition holds, Countries 1 and 2 attract all firms at intermediate trade costs: \( \phi \in [\tilde{\phi}_3, \bar{\phi}_3] \).

**Order of move.** The right panel of Fig. 5 shows that as trade gets freer, Country 3 loses firms before Country 2, i.e., \( \tilde{\phi}_3 < \tilde{\phi}_2 \). Under further trade cost reduction, Country

\(^{25}\)To establish this inequality, it suffices to check that \( s_3 + 2\sqrt{s_2(\sigma + s_1 - 1)} > s_2 + 2\sqrt{s_3(\sigma + s_1 - 1)} \):

\[
s_3 + 2\sqrt{s_2(\sigma + s_1 - 1)} - \left[s_2 + 2\sqrt{s_3(\sigma + s_1 - 1)}\right] = \left[2\sqrt{s_1 - s_2} + (\sqrt{s_2} + \sqrt{s_3})\right](\sqrt{s_2} - \sqrt{s_3}) > 0.
\]
2 first regains manufacturing firms and then Country 3 follows. We can confirm the order of the moves analytically by looking at the relocation tendencies of the firms in Country 1. The previous discussion tells us that \( v_{13|n_1=1} > v_{12|n_1=1} \) holds for \( \phi \in (\phi_2, \bar{\phi}_2) \), which can be illustrated in Fig. A1. At \( \phi \) close to but lower than \( \phi_2 \), \( v_{12|n_1=1} < 0 \) and \( v_{13|n_1=1} > 0 \) hold. This means that a firm in Country 1 is ready to move to Country 2 but has no incentive to move to Country 3. The figure suggests that there is a range of trade costs where Country 2 hosts some firms, and Country 3 does not. Therefore we can conclude that all firms in Country 3 leave before those in Country 2 do so. The analogous argument applies to the case of \( \phi \) close to but higher than \( \bar{\phi}_2 \); we can also conclude that firms move back to Country 2 before Country 3.

![Agglomeration rents](image)

**Fig. A1.** Agglomeration rents.

### Appendix D. Proof of Proposition 4

**Welfare comparison in Country 1.** We prove that Country 1 has a higher welfare level in free trade \( (\phi = 1) \) than in full agglomeration \( (n_1 = 1) \). With free trade, Country
1’s welfare is
\[ U_1|_{φ=1} = \frac{w_1 + π(K/L)}{P_1^{\mu} p_{01}^{1-\mu}}|_{φ=1}, \]
where \( w_1|_{φ=1} = 1, \quad π|_{φ=1} = \frac{μL}{(σ - μ)K}; \)
\[ P_1|_{φ=1} = K^{\frac{1}{1-σ}}, \quad p_{01}|_{φ=1} = 1. \]

Under full agglomeration, Country 1’s welfare is given by
\[ U_1|_{n=1} = \frac{w_1 + π(K/L)}{P_1^{\mu} p_{01}^{1-μ}}|_{n=1}, \]
where \( w_1|_{n=1} = 1 + \frac{σ - 1}{s_1(1 - μ)}, \quad π|_{n=1} = \frac{μL}{(1 - μ)K}; \)
\[ P_1|_{n=1} = K^{\frac{1}{1-σ}} w_1|_{n=1}; \quad p_{01}|_{n=1} = w_1|_{n=1}. \]

Compared with free trade, Country 1 in full agglomeration is better off in terms of wages and capital rewards (\( w_1|_{n=1} > w_1|_{φ=1}; \quad π|_{n=1} > π|_{φ=1} \)), but worse off in terms of the prices of both manufacturing and non-tradable goods (\( P_1|_{n=1} > P_1|_{φ=1}; \quad p_{01}|_{n=1} > p_{01}|_{φ=1} \)). Further inspection reveals that the former positive effects on wages and capital rewards are always dominated by the latter negative effects on prices:
\[ U_1|_{φ=1} - U_1|_{n=1} = \frac{μ(σ - 1)(1 - s_1)}{(σ - μ)[σ + s_1(1 - μ) - 1]} > 0. \]

Thus, welfare in free trade is always higher than under full agglomeration.

Welfare comparison between the three countries at \( φ = 0 \). From the fact that we have \( n_i = s_i \) and \( w_i = 1 \) for \( i \in \{1, 2, 3\} \) at \( φ = 0 \), country \( i \)’s welfare is calculated as
\[ U_i|_{φ=0} = \frac{1 + π(K/L)}{P_i^{\mu} . 1^{1-μ}} = N_i^{\frac{μ}{σ}} [1 + π(K/L)], \]
where \( π = \frac{μL}{(σ - μ)K}. \)
As \( N_1 > N_2 > N_3 \) holds at \( φ = 0 \), we have \( U_1 > U_2 > U_3 \) at \( φ = 0 \).
Appendix E. Proof of Proposition 5

Derivation of Eq. (9). Noting that \( \frac{dn_1}{d\phi} + \frac{dn_2}{d\phi} = 0 \) and \( dn_3/d\phi = 0 \) hold at \( \phi = \bar{\phi}_2 \), we have

\[
\left. \frac{dw_1}{d\phi} \right|_{\phi = \bar{\phi}_2} = \sum_{i=1}^{3} \left. \frac{\partial w_1}{\partial n_i} \frac{dn_i}{d\phi} \right|_{\phi = \bar{\phi}_2} = \frac{\sigma - 1}{s_1(1 - \mu)} \left. \frac{dn_1}{d\phi} \right|_{\phi = \bar{\phi}_2} < 0,
\]

\[
\left. \frac{d\pi}{d\phi} \right|_{\phi = \bar{\phi}_2} = \frac{\mu}{K(\sigma - \mu)} \sum_{i=1}^{3} \sum_{j=1}^{3} L_i \left. \frac{\partial w_i}{\partial n_j} \frac{dn_j}{d\phi} \right|_{\phi = \bar{\phi}_2} = \frac{\mu}{K(\sigma - \mu)} \left( L_1 \left. \frac{\partial w_1}{\partial n_1} - L_2 \frac{\partial w_2}{\partial n_2} \right|_{\phi = \bar{\phi}_2} \right) \left. \frac{dn_1}{d\phi} \right|_{\phi = \bar{\phi}_2} = 0,
\]

where the expressions for wages and capital rewards are given in Section 3.1. Substituting these into the first two terms and the fourth term in Eq. (7) yields

\[
\frac{1}{w_1 + \pi(K/L)} \left( \left. \frac{dw_1}{d\phi} + \frac{K}{L} \frac{d\pi}{d\phi} \right|_{\phi = \bar{\phi}_2} \right) = \frac{\sigma - 1}{\sigma + s_1(1 - \mu)} \left. \frac{dn_1}{d\phi} \right|_{\phi = \bar{\phi}_2} < 0, \quad (E.1)
\]

\[
\left. \frac{1 - \mu}{p_0} \frac{dp_0}{d\phi} \right|_{\phi = \bar{\phi}_2} = \frac{(1 - \mu)(\sigma - 1)}{\sigma + s_1(1 - \mu) - 1} \left. \frac{dn_1}{d\phi} \right|_{\phi = \bar{\phi}_2} < 0. \quad (E.2)
\]

As for the price index term \( P_1 \), we have

\[
\left. \frac{\partial P_1}{\partial n_1} \right|_{\phi = \bar{\phi}_2} = \left( w_1^{1-\sigma} - \bar{\phi}_2 \right) K \left. \frac{P_1^\sigma}{1 - \sigma} \right|_{\phi = \bar{\phi}_2} > 0,
\]

where we can verify that \( w_1^{1-\sigma} - \phi > 0 \) holds at \( \phi = \bar{\phi}_2 \) by showing that (i) \( F_{12}(w_1^{1-\sigma}) < 0 \) and (ii) \( w_1^{1-\sigma} \) is greater than the axis of symmetry of \( F_{12} \).

Further calculations reveal

\[
\left. \frac{\partial P_1}{\partial n_2} \right|_{\phi = \bar{\phi}_2} = \left( \bar{\phi}_2 w_2^{1-\sigma} - \bar{\phi}_2 \right) K \left. \frac{P_1^\sigma}{1 - \sigma} \right|_{\phi = \bar{\phi}_2} = 0,
\]

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\[
\frac{\partial P_1}{\partial w_1} \sum_{i=1}^{3} \frac{\partial w_1}{\partial n_i} \frac{d n_i}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = \frac{\partial P_1}{\partial w_1} \frac{d n_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = \frac{K^{\frac{1}{1-\sigma}}(\sigma - 1)}{s_1(1-\mu)} \frac{d n_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2} < 0,
\]

\[
\frac{\partial P_1}{\partial w_2} \sum_{i=1}^{3} \frac{\partial w_2}{\partial n_i} \frac{d n_i}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = \frac{\partial P_1}{\partial w_1} \frac{d n_2}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = \bar{\phi}_2 N_2 w_2^{-\sigma} P_1^{\sigma} \frac{d n_2}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = 0,
\]

\[
\frac{\partial P_1}{\partial \phi} \bigg|_{\phi = \bar{\phi}_2} = (N_2 w_2^{-\sigma} + N_3) \frac{P_1^{\sigma}}{1-\sigma} \frac{d n_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = 0,
\]

where we make use of the fact that at \( \phi = \bar{\phi}_2 \), it holds that \( N_1 = K \), \( N_2 = N_3 = 0 \) and \( w_2 = 1 \).

Substituting these into Eq. (8) gives

\[
\frac{d P_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = \left[ \frac{K P_1^{\sigma}(w_1^{-\sigma} - \bar{\phi}_2)}{1-\sigma} + \frac{K^{\frac{1}{1-\sigma}}(\sigma - 1)}{s_1(1-\mu)} \right] \frac{d n_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2}.
\]

Accordingly, the third term in Eq. (7) reduces to

\[
\frac{\mu}{P_1} \frac{d P_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2} = \frac{\mu}{1-\sigma} \left[ 1 - \bar{\phi}_2 w_1^{-\sigma-1} - \frac{(\sigma - 1)^2}{\sigma + s_1(1-\mu) - 1} \right] \frac{d n_1}{d \phi} \bigg|_{\phi = \bar{\phi}_2}.
\]

From Eqs. (E.1) to (E.3), Eq. (7) can be re-expressed as Eq. (9).

**Conditions for worse off in Country 1.** From Eq. (9), the condition for worse off is given as follows:

\[
I(\sigma) \equiv 1 - \bar{\phi}_2 w_1^{-\sigma-1} - \frac{s_1(\sigma - 1)^2}{(\sigma + s_1 - 1)[\sigma + s_1(1-\mu) - 1]} > 0.
\]
Thus, we get
\[ I(1) = 0, \quad I'(1) = 0, \]
\[ I''(1) = \frac{2[1 + 2(s_2 - s_1)]}{(1 - \mu)s_1(s_1 - s_2)} > 0. \]

If \( s_1 < s_2 + 1/2 \) holds, then \( I > 0 \) immediately holds. That is, if Country 1 is not extremely large and \( \sigma \) is close to unity, its welfare level starts decreasing once offshoring starts.

**Appendix F. Proof of Proposition 6**

\( \phi_{12}^{12} \) and \( \phi_2^{12} \) are derived by \( v_{12}^{12} = 0 \):
\[
v_{12}^{12} = \Omega' \left[ -\alpha^2 w_1^{\sigma-1}(\sigma + s_1 - 1)\phi^2 + \alpha(\sigma - w_1^{\sigma-1}s_3)\phi - w_1^{\sigma-1}s_2 \right] = 0,
\]
where \( \Omega' \equiv \mu L / \left[ \alpha \phi (1 - \mu) \sigma K \right] \) is a positive constant. The marginal impact of a bilateral trade agreement on the critical points are given by
\[
\left. \frac{d\phi_{12}^{12}}{d\alpha} \right|_{\alpha=1} = -\frac{\sigma - w_1^{\sigma-1}s_3 - \sqrt{D}}{2w_1^{\sigma-1}(\sigma + s_1 - 1)} = -\phi_2 < 0,
\]
\[
\left. \frac{d\phi_2^{12}}{d\alpha} \right|_{\alpha=1} = -\frac{\sigma - w_1^{\sigma-1}s_3 + \sqrt{D}}{2w_1^{\sigma-1}(\sigma + s_1 - 1)} = -\bar{\phi}_2 < 0,
\]
where \( D \equiv (\sigma - w_1^{\sigma-1}s_3)^2 - 4w_1^{2(\sigma-1)}s_2(\sigma + s_1 - 1) \).

Similarly, \( \phi_{13}^{13} \) and \( \bar{\phi}_2^{13} \) are the solutions of \( v_{12}^{13} = 0 \):
\[
v_{12}^{13} = \Omega' \left[ -\alpha w_1^{\sigma-1}(\sigma + s_1 - 1)\phi^2 + (\alpha\sigma - w_1^{\sigma-1}s_3)\phi - \alpha w_1^{\sigma-1}s_2 \right] = 0.
\]
We have
\[
\left. \frac{d\phi_{13}^{13}}{d\alpha} \right|_{\alpha=1} = -\frac{s_3 \left( \sigma - w_1^{\sigma-1}s_3 - \sqrt{D} \right)}{2\sqrt{D}(\sigma + s_1 - 1)} = -\left( \frac{w_1^{\sigma-1}s_3}{\sqrt{D}} \right) \phi_2 < 0,
\]
\[
\left. \frac{d\bar{\phi}_2^{13}}{d\alpha} \right|_{\alpha=1} = \frac{s_3 \left( \sigma - w_1^{\sigma-1}s_3 + \sqrt{D} \right)}{2\sqrt{D}(\sigma + s_1 - 1)} = \left( \frac{w_1^{\sigma-1}s_3}{\sqrt{D}} \right) \bar{\phi}_2 > 0.
\]
Appendix G. Balance of payments

Following the analogy of Takahashi et al. (2013), we first consider the trade balance of the manufacturing good in the world. We then show trade deficits in a country is compensated by a surplus on the capital account. The income spent in Country 1 on the manufacturing goods must equal its expenditures:

\[ \mu Y_1 = \mu (w_1 L_1 + r K_1) = N_1 p_{11} q_{11} + N_2 p_{21} q_{21} + N_3 p_{31} q_{31}, \]

Isomorphic expressions are applied to Countries 2 and 3. Trade deficit terms are extracted as follows:

\[ \mu (w_1 L_1 + r K_1) = N_1 p_{11} q_{11} + N_2 p_{21} q_{21} + N_3 p_{31} q_{31} \]
\[ = N_1 (p_{11} q_{11} + p_{12} q_{12} + p_{13} q_{13}) + (N_2 p_{21} q_{21} - N_1 p_{12} q_{12}) + (N_3 p_{31} q_{31} - N_1 p_{13} q_{13}) \]
\[ = N_1 (\sigma \pi_1) + (N_2 p_{21} q_{21} - N_1 p_{12} q_{12}) + (N_3 p_{31} q_{31} - N_1 p_{13} q_{13}), \]

Note that the result of the constant mark-up pricing is used from the second to the third line. Rearranging the above equation as well as the corresponding equations for Countries 2 and 3 gives

\[ (N_2 p_{21} q_{21} - N_1 p_{12} q_{12}) + (N_3 p_{31} q_{31} - N_1 p_{13} q_{13}) = \mu w_1 L_1 + r (\mu s_1 - \sigma n_1) K, \quad (G.1) \]
\[ (N_1 p_{12} q_{12} - N_2 p_{21} q_{21}) + (N_3 p_{32} q_{32} - N_2 p_{23} q_{23}) = \mu w_2 L_2 + r (\mu s_2 - \sigma n_2) K, \quad (G.2) \]
\[ (N_1 p_{13} q_{13} - N_3 p_{31} q_{31}) + (N_2 p_{23} q_{23} - N_3 p_{32} q_{32}) = \mu w_3 L_3 + r (\mu s_3 - \sigma n_3) K, \quad (G.3) \]

where we make use of \( \pi_i n_i = r n_i \) because of \( r = \max_{i \in \{1,2,3\}} \{\pi_i\} \) for \( n_i > 0 \). The left hand side represents the trade deficit of Country \( i \) (the net imports of \( i \)).

The trade deficit in Country 1, for example, equals the sum of the trade surpluses of Countries 2 and 3. The world trade surplus against Country 1 is the sum of Eq. (G.2) and Eq. (G.3):

\[ (N_1 p_{12} q_{12} - N_2 p_{21} q_{21}) + (N_1 p_{13} q_{13} - N_3 p_{31} q_{31}) \]
\[ = [\mu w_2 L_2 + r (\mu s_2 - \sigma n_2) K] + [\mu w_3 L_3 + r (\mu s_3 - \sigma n_3) K], \quad (G.4) \]

By equating Eq. (G.1) with \((-1) \times \) Eq. (G.4), we have

\[ \mu w_1 L_1 + r (\mu s_1 - \sigma n_1) K = -[\mu w_2 L_2 + r (\mu s_2 - \sigma n_2) K] + \mu w_3 L_3 + r (\mu s_3 - \sigma n_3) K, \]

which reduces to \( r = \mu \sum_{i=1}^{3} w_i L_i / K (\sigma - \mu) \). This equation holds when the equilibrium capital rewards clear the world manufacturing market (see Section 3.1). As the sum of
all trade deficits equals the sum of all trade surpluses across countries, we can confirm that world trade is balanced.

Next, we examine how capital account surplus is offset by trade deficit. The labor market clearing condition in Country 1 is

\[
[(1 - \mu)Y_1/p_{01}] + N_1(q_{11} + \tau q_{12} + \tau q_{13}) = L_1,
\]

\[
\rightarrow [(1 - \mu)Y_1/w_1] + [(\sigma - 1)\pi_1 N_1/w_1] = L_1,
\]

\[
\rightarrow (1 - \mu)(w_1 L_1 + r K_1) + (\sigma - 1)\pi_1 N_1 = w_1 L_1,
\]

\[
\rightarrow \mu w_1 L_1 = [(1 - \mu)rs_1 + (\sigma - 1)\pi_1 n_1]K
\]

Substituting this into Eq. (G.1) yields

\[
(N_2 p_{21} q_{21} - N_1 p_{12} q_{12}) + (N_3 p_{31} q_{31} - N_1 p_{13} q_{13}) = \mu w_1 L_1 + r(\mu s_1 - \sigma n_1)K
\]

\[
= r[(1 - \mu)s_1 + (\sigma - 1)n_1]K + r(\mu s_1 - \sigma n_1)K
\]

\[
= r(s_1 - n_1)K,
\]

where the right hand side in the last line represents the rewards to capital employed in the other countries.

**Appendix H. Parameter values**

The figures in the main text are derived using the following parameter values:

Figs. 1 and 2: \( \sigma = 2; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (I)(0.4, 0.37, 0.23); (II)(0.51, 0.4, 0.09); (III)(0.5, 0.3, 0.2). 
Figs. 3 and 4 do not depend on specific parameter values, except for the ordering of country size: \( s_1 > s_2 > s_3 > 0. \)
Fig. 5: \( \sigma = 1.2; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (0.45, 0.33, 0.22)(two-country agglomeration, left panel); (0.56, 0.3, 0.14)(full agglomeration, right panel). \)
Fig. 6: \( \sigma = 1.2; \mu = 0.3; K = L = 20. \)
Fig. 7: \( \sigma = 2; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (0.5, 0.3, 0.2). \)
Fig. 8: \( \sigma = 1.2; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (0.56, 0.3, 0.14). \)
Fig. 9 does not depend on specific parameter values.
Fig. 10: \( \sigma = 1.4; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (0.63, 0.2, 0.17)(solid); (0.66, 0.2, 0.14)(dashed); (0.7, 0.2, 0.1)(dotted). \)
Fig. 11: \( \sigma = 1.2; \mu = 0.3; K = L = 20; \alpha = 1.2; (s_1, s_2, s_3) = (0.53, 0.28, 0.19). \)
Fig. 12: \( \sigma = 2; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (0.55, 0.3, 0.15). \)
Fig. 13: Parameter values are the same as those in Fig. 12 except for \( \sigma = 1.2. \)
Appendix I. Industrial evolution of middle countries in a four-country model

This appendix shows how equilibrium firm shares evolve as trade costs decline in the four-country model. We can confirm that our main results are robust: middle-sized countries see complex location patterns depending on their relative market sizes.

To illustrate the feasible set of market size, we need to map the three-dimensional plane, i.e., \((s_1, s_2, s_3)\) (note that \(s_4 = 1 - s_1 - s_2 - s_3\)), to a two-dimension plane. We should pin down one of the three market sizes and here fix \(s_3\) as a constant denoted by \(s_3\). Figs. A2, A3 and A4 show all possible combinations of market sizes in the four countries, given \(s_3 = 0.1, 0.2\) and 0.24999..., respectively.\(^{26}\) We can observe a market size triangle in Fig. A4 that was seen in Fig. 3. However, the feasible is not necessarily triangular; we see a ‘market size rectangle’ in Figs. A2 and A3.

---

\(^{26}\)By using \(s_1 + s_2 + s_3 + s_4 = 1\), the order of \(s_1 > s_2 > s_3 > s_4\) implies \(s_1 > 1/4\); \(2s_3 < s_2 + s_3 < 2/3\); \(s_2 + s_3 < s_1 + s_3\); \(s_2 + s_3 < 1 - s_1\); and \(s_2 + s_3 > 1 - s_1 - s_3\). All of these conditions are satisfied in the shaded rectangle/triangle excluding borders.
Following the same manner as Appendix B, the slope of equilibrium firm shares at two endpoints, $\phi = 0$ and $\phi = 1$ are derived. At $\phi = 0$ (infinite trade costs), we have

$$\left. \frac{dn_1}{d\phi} \right|_{\phi=0} = \sigma(4s_1 - 1) > 0,$$

$$\left. \frac{dn_2}{d\phi} \right|_{\phi=0} = \sigma(4s_2 - 1) \geq 0,$$

$$\left. \frac{dn_3}{d\phi} \right|_{\phi=0} = \sigma(4s_3 - 1) \geq 0,$$

$$\left. \frac{dn_4}{d\phi} \right|_{\phi=0} = \sigma(4s_4 - 1) < 0.$$
with \( s_1 \in (1/4, 1); \ s_2 \in (0, 1/2); \ s_3 \in (0, 1/3); \) and \( s_4 \in (0, 1/4). \) At the outset of opening trade, the largest country gains while the smallest country loses firms. Whether the middle countries gain or lose firms depends on their own market size. At \( \phi = 1 \) (zero trade costs), we have

\[
\frac{dn_i}{d\phi} \bigg|_{\phi=1} = s_i(\sigma - \mu)f_i(s_1, s_2, s_3), \quad i \in \{1, 2, 3, 4\},
\]

\[
f_1(s_1, s_2, s_3) = 2s_1^2 - [3 - 2(s_2 + s_3)]s_1 + 2s_2^2 - 2(1 - s_3)s_2 + 2s_3^2 - 2s_3 + 1 < 0,
\]

\[
f_2(s_1, s_2, s_3) = 2s_1^2 - 2(s_1 + s_3)(1 - s_2 - s_3) + (1 - s_2)(1 - 2s_2) \geq 0,
\]

\[
f_3(s_1, s_2, s_3) = 2s_1^2 - 2(s_1 + s_3)(1 - s_2 - s_3) + (1 - s_3)(1 - 2s_3) \geq 0,
\]

\[
f_4(s_1, s_2, s_3) = 2s_1^2 - (s_1 + s_2)[1 - 2(s_2 + s_3)] - s_2(1 - 2s_2) > 0.
\]

When the economy is close to free trade, the largest country loses while the smallest country gains firms. The middle countries may gain or lose firms depending on their market sizes relative to those of other countries. We note that the signs of the slopes at \( \phi \in \{0, 1\} \) only depend on market sizes.

From these conditions, we can characterize how firm shares evolve as trade costs decline. Using the market size triangle/rectangle, the representative equilibrium paths are reported in Figs. A5 and A6. In all combinations of market size in the two figures, we observe an inverted-U shape (or U shape) in terms of \( \phi \) in Country 1 (or Country 4). By contrast, Countries 2 and 3 exhibit more complex patterns. In the four-country model, the industrial evolution of middle countries is determined by their relative market size, as we emphasized in the three-country model.
Fig. A5. Patterns of equilibrium path in the market size rectangle at $\bar{s}_3 = 0.1$.

Fig. A6. Patterns of equilibrium path in the market seize triangle at $\bar{s}_3 = 0.27$. 

Industrial evolution in terms of $\phi$

(I) $n_2$

(II) $n_2$

(III) $n_2$

In all three cases:

$n_1$

$n_3$

$n_4$
Appendix J. The largest country in a $J(>3)$ country model

This appendix re-examines the equilibrium features of the largest country in a $J(>3)$-country model. The market sizes are sorted in descending order: $s_1 > s_2 > \cdots > s_J$. In the main text, we have shown that (i) there exist corner solutions where at least one of the countries loses its industry (Proposition 3) and (ii) the largest country could decrease its welfare (Proposition 5).

Given firm shares, wage rate clearing of labor market in country $i \in \{1, 2, \ldots, J-1\}$ takes:
\[
 w_i = \frac{s_J[(1-\mu)s_i + (\sigma-1)n_i]}{s_i[(1-\mu)s_J + (\sigma-1)n_J]},
\]
while wage rate in country $J$ is normalized to unity, $w_J = 1$.

**Conditions for full agglomeration.** The operating profits at full agglomeration in Country 1 are given as
\[
\begin{align*}
\pi_1|_{n_1=1} &= \frac{\mu}{\sigma K} \sum_{j=1}^{J} Y_j, \\
\pi_i|_{n_1=1} &= \frac{\mu}{\sigma K} \left( \frac{1}{w_1} \right)^{1-\sigma} \left[ \phi Y_1 + (Y_i/\phi) + \sum_{j \neq 1,i}^{J} Y_j \right], \quad i \in \{2, 3, \ldots, J\},
\end{align*}
\]
where $w_1 = 1 + (\sigma - 1)/[s_1(1-\mu)]$. Full agglomeration is the long-run equilibrium if and only if the following conditions hold:
\[
\begin{align*}
v_{1i}|_{n_1=1} &= \pi_1 - \pi_i|_{n_1=1} = \Omega \cdot F_{1i}(\phi) > 0, \quad i \in \{2, 3, \ldots, J\},
\end{align*}
\]
where $F_{1i}(\phi) \equiv -\left(\sigma + s_1 - 1\right)w_1^{\sigma-1}\phi^2 + \left(\sigma - w_1^{\sigma-1}(1-s_1-s_i)\right)\phi - w_1^{\sigma-1}s_i$.

and where $\Omega \equiv \mu L/[\phi \sigma (1-\mu)K]$ is a positive constant. This implies that no firms in Country 1 has an incentive to move. It can be verified that $F_{1k+1}(\phi) \geq F_{1k}(\phi)$ holds for $\phi \in [0, 1]$ and $k \in \{2, 3, \ldots, J-1\}$. Thus we just need to check $F_{12}(\phi) > 0$ as in Appendix G. The small root of $F_{1k}(\phi) = 0$ is denoted as $\phi_{k}$ and the large one as $\phi_{k}$. By applying the same analogy, full agglomeration arises at intermediate trade costs $[\phi_{12}, \phi_{2}]$. 
if $\sigma$ is in $(1, \sigma^*_1)$. $\sigma^*_1$ is defined by
\[
\sigma^*_1 \equiv \min_{\sigma > 1} \arg[\sigma - H_2(\sigma) = 0],
\]
where $H_2(\sigma) \equiv w_1^{\sigma-1} \left[ (1 - s_1 - s_2) + 2\sqrt{s_2(\sigma + s_1 - 1)} \right]$.

We note that $H_2(\sigma)$ and $H(\sigma)$ defined in Appendix C are identical due to $s_3 = 1 - s_1 - s_2$ in $H(\sigma)$. A crucial determinant of $\sigma^*_1$ is the market sizes of the largest and the second largest countries. The same can be true for thresholds, $\overline{\sigma}_2$ and $\overline{\sigma}_3$.

**Conditions for j-country agglomeration.** We refer to the situation where countries from 1 to $j$ ($< J$) attract all firms as $j$-country agglomeration. $j = 1$-country agglomeration corresponds to full agglomeration in our main text. A sufficient condition for $j$-country agglomeration is (i) $v_{1k}|_{n_1=1} < 0$ for $k \in \{2, 3, \ldots, j\}$ and (ii) $v_{1l}|_{n_1=1} > 0$ for $l \in \{j, j + 1, \ldots, J\}$. Following the discussion in Appendix C, we obtain $j$-country agglomeration at intermediate trade costs such that $\phi \in [\overline{\phi}_{j+1}, \overline{\phi}_{j+1}]$ if $\sigma$ is in $[\sigma^*_{j-1}, \sigma^*_j)$. $\sigma^*_j$ is defined by
\[
\tilde{\sigma}^*_j \equiv \min_{\sigma > 1} \arg[\sigma - H^*_j(\sigma) = 0],
\]
where $H^*_j(\sigma) \equiv w_1^{\sigma-1} \left[ (1 - s_1 - s_{j+1}) + 2\sqrt{s_{j+1}(\sigma + s_1 - 1)} \right], \ j \in \{2, 3, \ldots, J - 1\},$
and where $w_1 = 1 + (\sigma - 1)/[s_1(1 - \mu)]$. What determines $j$-country agglomeration is the market sizes of Countries 1 and $j + 1$.

By applying the same analogy of the three-country model, the order of firm movement proceeds as follows; as trade costs decline, countries except for Country 1 lose and then re-attract firms in a descending order of country index.

**Conditions for worse off in Country 1.** In the three-country model, whether Country 1 decreases its welfare in the collapse of agglomeration is determined by the sign of Eq. (9). We note that $dn_1/d\phi + dn_2/d\phi = 0$ and $dn_3/d\phi = \cdots = dn_J/d\phi = 0$ hold at $\phi = \overline{\phi}_2$. It can be verified that the counterpart of Eq. (9) in the $J$-country model takes the same expression as Eq. (9):
\[
\frac{d \log U_1}{d\phi} \bigg|_{\phi = \overline{\phi}_2} = \frac{\mu I(\sigma) \ dn_1}{\sigma - 1} \bigg|_{\phi = \overline{\phi}_2},
\]
where $I(\sigma) \equiv 1 - \overline{\phi}_2 w_1^{\sigma-1} - \frac{s_1(\sigma - 1)^2}{(\sigma + s_1 - 1)\{\sigma + s_1(1 - \mu) - 1\}} \geq 0$. 

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$\phi_2$ is defined as the large root of

$$F_{12}(\phi) \equiv -(\sigma + s_1 - 1)w_1^{\sigma-1}\phi^2 + [\sigma - w_1^{\sigma-1}(1 - s_1 - s_2)]\phi - w_1^{\sigma-1}s_2 = 0,$$

where $1 - s_1 - s_2 = \sum_{j=3}^J s_j$. We can check that $I(1) = I'(1) = 0$ and $I''(1) > 0$ hold, implying that the conditions for Country 1 decreasing its welfare are identical as those in the three-country model. That is, if $s_1 < s_2 + 1/2$ holds and $\sigma$ is close to unity, $I(\sigma)$ is positive and thus the sign of the above derivative becomes negative. The condition involves the market sizes of the largest and the second largest countries.

**Appendix K. Multiple manufacturing industries with different labor intensities**

This appendix considers two manufacturing industries (subject to monopolistic competition), called A and B, with different labor intensities. Suppose that industry-A is more labor intensive than industry-B. An industry-A (or B) firm have a unit labor requirement of $a_A(\sigma - 1)/\sigma$ (or $a_B(\sigma - 1)/\sigma$; $a_A > a_B$). To highlight different labor intensities, the expenditure share on each industrial goods is assumed to be identical, i.e., $\mu_A = \mu_B = \mu/2$. All other settings are the same as in the single-manufacturing model.

Given firm shares, the operating profit of an industry-$k \in \{A, B\}$ firm is given by

$$\pi_{ki} = \frac{\mu w_i^{1-\sigma}}{2\sigma} \sum_{j=1}^3 \phi_{ij} Y_j / \Delta_{kj},$$

where $\Delta_{kj} \equiv \sum_{l=1}^3 \phi_{lj} w_i^{1-\sigma} N_{kl}$,

where the new subscript $k$ stands for the variables of industry-$k$. We note that labor intensity, $a_k$, does not show up in the above expression. Although $a_k$ appears in the price of individual brand ($p_{kij} = \tau_{ij} a_k w_i$) and in the price index ($P_{kj} = (\sum_{l=1}^3 N_{kl} p_{klj}^{1-\sigma})^{1/\sigma}$), these $a_k$s are canceled out.

It can be verified that capital rewards are common to all countries, $r = \mu \sum_{i=1}^3 w_i L_i / (\sigma - \mu)K$. As in the main text, the equilibrium wage rate is derived from labor market clearing conditions:

$$w_i = \frac{s_i [(1 - \mu) s_i + (\sigma - 1)(n_{Ai} + n_{Bi})]}{s_i [(1 - \mu) s_3 + (\sigma - 1)(n_{A3} + n_{B3})]}, \quad \text{for } i \in \{1, 2\},$$

while $w_3$ is normalized to unity. $a_k$ does not enter the above expression because it is reflected on individual price.
Solving $\pi_{k1} = \pi_{k2} = \pi_{k3}$ for $(n_{k1}, n_{k2})$ gives firm shares in the long-run equilibrium. $n_{Ai}$ is expected to behave exactly in the same way as $n_{Bi}$ because $\pi_{Ai}$ and $\pi_{Bi}$ have symmetric expressions. In fact, this can be confirmed in the left panel of Fig. A7. Moreover, the sum of firm shares over industry in each country ($n_{Ai} + n_{Bi}$) coincides with the firm share in the single-manufacturing model ($n_i$) as shown in the right panel of Fig. A7. As consumers equally split their expenditures to two industries, firm shares are also divided equally regardless of different labor intensities.

![Fig. A7.](image)

**Fig. A7.** The impact of trade liberalization on firm shares in the two-manufacturing model.

*Note:* Parameter values are $\sigma = 2; \mu = 0.3; K = L = 20; (s_1, s_2, s_3) = (0.4, 0.37, 0.27)$. 
References


