GAME THEORETICAL MODELS AND SOLUTION APPROACHES FOR DYNAMIC MODULAR COVERING LOCATION PROBLEMS

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Abstract

Facility location decisions are one of the important elements in the strategic planning of supply chains and they have applications in both private and public sectors. The importance of having optimal facility locations has two main folds. The first reason is that facility location decisions are strategic and they cannot be easily changed. The second reason is that the location of facilities can have a direct impact on other supply chain decisions as well. This thesis addresses the importance of locating emergency facilities by developing the novel game theoretical model for covering facility location problem. First, by defining different structural units of the facilities as the modules, a modular maximal covering location problem having back-up service coverage for demand nodes in a multi-period framework is proposed. Considering back-up services for the demand of points from the modules can increase the service quality that is a vital optimization objective for applications such as emergency services. A heuristic method and a genetic algorithm are developed to solve the computational test problems.

Second, a hybrid covering location problem that integrates the set covering and maximal covering location problem is developed for the first time to benefit from the advantages of these two models in one model. The hybrid model distinguishes between facilities and the structural units of facilities as the modules and locates the facilities according to the coverage concept of set covering location problem to provide service for all demand points. The hybrid model assigns the limited number

of modules using the coverage concept of maximal covering location problem to maximize the total covered demands.

Third, using the developed models in the previous parts, this thesis studies the covering facility location problems from non-cooperative game theoretical approach to investigate the related non-cooperative theoretical and mathematical model of covering model and the solving complexity. The non-cooperative approach is studied using the Stackelberg game perspective for maximal hub covering location problem, which is resulted into a bi-level mathematical formulation of the problem. In the bi-level maximal hub location problem, the freight companies who seek to find the optimal location of the hubs to have the maximal amount of covered demand are the leaders, while the customers looking for the minimum price among different available companies are the followers. Furthermore, the difficulty to deal with the bi-level mathematical model is addressed in order to solve the problem. Two reformulation techniques as dual-based and Karush-Kuhn-Tucker-based reformulation are developed to reformulate the bi-level problem to a single level problem. As the obtained single-level problems are difficult to solve, a Benders-decomposition-based method is proposed and used to solve the test problems and investigate the efficiency of the reformulation techniques and the solution procedure.

The obtained results from numerical experiments and analysis indicate that the integrated mathematical model developed for hybrid covering location problem is capable of improving the coverage percentage compared to the conventional models and other possible integrated models. The case study that is conducted to validate the capability of the hybrid model also approves the applicability of this model in modeling emergency humanitarian logistic systems. Furthermore, the developed maximal hub location problem can reflect the non-cooperation framework for freight companies while they compete to attract more customers by rational behaviors who want to use the player with less price. On the other hand, the developed reformulation and decomposition procedures to solve the problem approves the efficiency of the procedures to solve large scale problems.

The future work for this thesis can be suggested as studying the developed models in the presence of uncertainty from both stochastic and robust optimization point of views. The other direction for future studies is to develop an efficient solving procedure for the developed hybrid covering location model.

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Chapter 1

Introduction

1.1 Facility location problems and supply chain management

Supply chain management (SCM) encompasses the broad range of activities to plan, control and execute the products' flow from materials to production to distribution in an economical and optimized way. Supply chain management contains the comprehensive processes that answers the questions such as: where to locate plants and distribution centers, where to produce, how to produce, and how much to produce at each site, what quantity of products to hold in inventory at each stage of the production, and how to share information among parties with the objectives of cost minimization and customer satisfaction. Figure 1.1 shows the elements of supply chain management and the position of facility location problem among the elements developed by Stevenson [1]. According to this figure, Stevenson [1] introduces nine elements of supply chain decisions as location, forecasting, customers, design, processing, inventory, purchasing, suppliers and logistics. The element of the location deals with finding the optimal location for the facilities. In this regard, Sunil and Peter [2] describe the facilities are a key driver of supply chain performance in terms of responsiveness and efficiency. They indicate that if the companies manufacture and store their products in the same place, it provides efficiency and if the companies locate the facilities close to the customers (there is a need to more facilities in this case), they can achieve higher responsiveness.

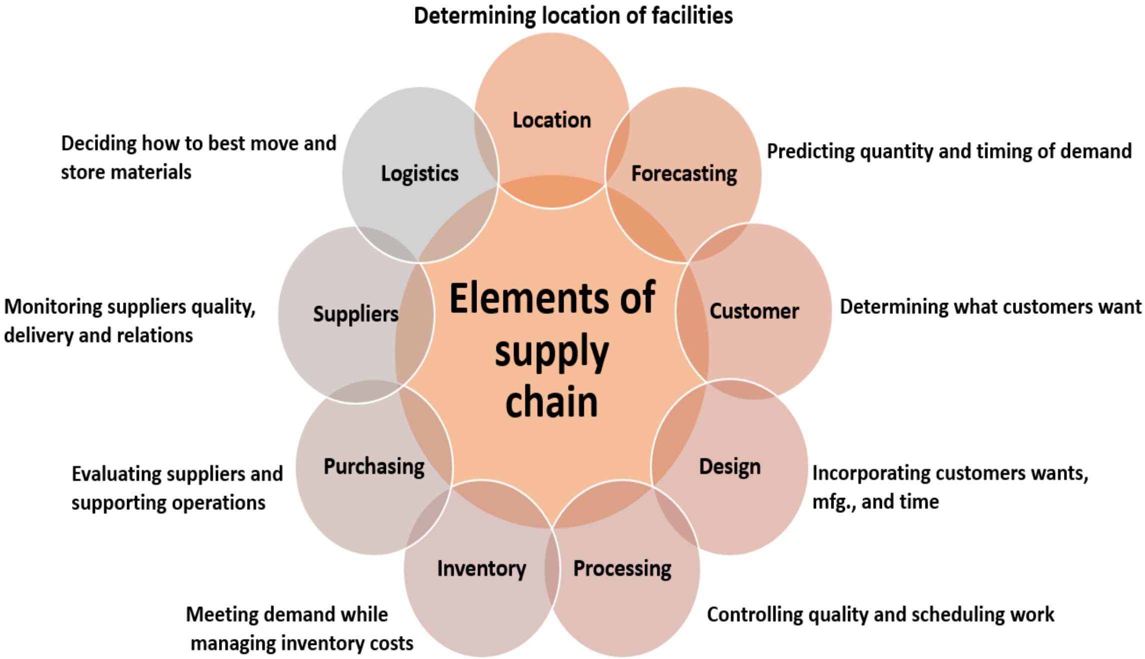

Figure 1.1: The elements of supply chain management and position of facility location problems [1].

There are three planning levels in SCM that are usually distinguished depending on the time horizon as: strategic, tactical and operational. The strategic level entails long-term planning decisions, midterm decisions are related to tactical level and short-term decisions are made at the operational level. One of the most critical strategic level decisions in each supply chain is the decision of the facility locations. The importance of facility location decision becomes apparent by considering the point that the locations of the facilities cannot be changed once they are constructed and the influences, they can have on other decisions of SCM such as transportation, inventory and production quantity.

1. Introduction

In general, in a facility location problem, there is a set of customers with some demands to be served by a set of facilities. In this regard, there are some criteria that have to be taken into account to model and optimize the problem. Demands, distance, time and costs between customers and facilities are of such criterion. The most general solutions of a facility location problem are the locations to open a facility and the allocation of customers or demand points to the opened facilities with the objective of minimizing the total cost (or maximizing the income or profit). In addition to this generic setting, a number of constraints arise from the specific application domain of each facility location category.

The literature of facility location problems can be investigated from different perspectives. Boonmee et al. [3] categorize the deterministic facility location problem in four major groups as: minisum facility location problem, covering problem, minimax facility location problem, and obnoxious facility location problem. The minisum problem seeks to locate a defined number of facilities at most in order to minimize the total transport distance between the demand points and selected facilities. In the same way, the minimax facility location problem locates the predefined number of facilities but minimized the maximum distance between demand points and the located facilities. In contrast to the minisum and minimax problems, the obnoxious facility location problem seeks to have demand points far from facilities and have applications in locating chemical plants, nuclear reactors, garbage dumps, or waste-water treatment plants. The covering location problem seeks to find a location of facilities like fire stations or shelter sites to cover the demand points within distance or time limits. It consists of two main problems as set covering location problem (SCLP) and maximal covering location problem

(MCLP). The SCLP was developed by Toregas et al. [4] and aims to provide coverage for all points by finding the minimum number of facilities to be located. Covering all points is an ideal objective that might not be compatible with all systems, because in most of the management systems the resources or budgetary limitations have to be taken into account. MCLP can be the appropriate model for these conditions when the demand points covered by the predefined number of facilities are maximized that was developed by Church and ReVelle [5]. The basic facility location problems were mostly developed having common characteristics that made them insufficient to cope with many realistic facility location settings such as capacity limitations of the facilities and extending the location decisions through planning time horizons. Therefore, many extensions to the basic problems have been considered and extensively studied. The next sections introduce some of these extensions on covering facility location problems as the main focus of this research belongs to this category of facility location problems.

1.2 Multi-period and capacitated covering location problems

In the basic formulation of covering location problems, i.e., set covering and maximal covering location problem, there is no limitation for the capacity of the located facilities and they are formulated for a single time period. Uncapacitated facility means that service by each facility can be provided limitless as long as demand points are within the coverage standard. However, in most of the real-world applications of covering problems, considering capacity limitations for facilities is a more realistic assumption. Most facilities have limits on service capabilities due to physical, political, structural, regional, and other reasons [6]. Current and Storbeck [7] considered facility capacity restrictions for the covering location problems and introduced the capacitated versions of SCLP and MCLP formulations.

Another restrictive assumption of basic covering location problems is considering the planning through the time horizon called multi-period or dynamic models. So-called dynamic location models consider a multi-period operating context where the demand varies between different time periods [8]. Dynamic models are classified into two categories: explicitly dynamic models and implicitly dynamic models. In explicitly dynamic models, facilities may close and open in time periods in a response to the parameter variation over time. In implicitly dynamic models, a part of the facilities opens in the beginning of the time horizon and the rest of them open throughout the time horizon to take account the increase in demands [9]. Extending the covering location models to a multi-period or a dynamic structure is more common for MCLP [10]- [11] rather than SCLP. The reason is that SCLP provides full allocation of demand points from fixed located facilities and there is no need to alter these locations in different time periods unless the problem is studied in the presence of demand fluctuation [12] and expansion possibilities for capacitated facilities. In multi-period MCLP, decision makers are interested in finding the optimal way of locating a definite number of facilities in different periods. The application of multi-period MCLP can be found in locating emergency service centers in populated regions that on-road accidents may happen and the number of facilities to be located may fluctuate between different periods of time because of daily traffic, weather situation and etc. Moreover, each opened facility at the beginning of a time period can be closed at the end of that time period in a multi-period MCLP [13]. In this regard, Marín et al. [14] addressed a general discrete covering location model in which they considered a finite planning horizon that is partitioned into several time-periods. Because the time periods are not necessarily of the same length and in each period, it is allowed that multiple facilities/equipment can be opened or closed in each location at some costs. Furthermore, Marín et al. [14] assumed that each demand point should be covered by at least a specific number of facilities and the coverage less than the minimum threshold undergoes a time dependent penalty cost. These features of the studied problem result to a different way of demand point allocations to the facilities in each time period.

Bagherinejad and Shoeib [15] studied the multi-period maximal covering location problem in which the total number of facilities that have to be opened, is located gradually over time periods. From this perspective, their model is an implicitly dynamic model. Another characteristic of their developed model is the dynamic capacity for each of the located facilities. As the application of their model is locating ambulances in emergency bases, locating a different number of ambulances in each time period makes it possible to set different levels of capacities in each time period. Vatsa and Jayaswal [16] developed the multi-period capacitated maximal covering problem considering uncertainties in server availability. Their problem addressed allocating doctors to non-operational primary health centers and as the population that needs to be served by the facilities was changing over time, they studied their problem in different time periods.

1.3 Hierarchical and modular location problem

Hierarchical facility location problems use “hierarchy” to describe the problem as the coordination of location decisions for different type facilities in multi-level systems [17]. For example, healthcare systems are one of the most studied systems in the literature. The hierarchy refers to the number of levels in healthcare facility systems such that in a three-level example, there may exist demand points, local clinics, hospitals, and regional hospitals as different levels of facilities. Note that the definition of hierarchy in the hierarchical facility location problem context and hierarchical decision making and optimization context is totally different. In hierarchical facility location problems, the hierarchy refers to different levels of available facilities and have applications in health care systems, solid waste management systems and education systems [17] and each of the applications is composed of different kinds of facility and demand points are allocated to the lower-level facilities and then lower-level facilities are allocated to the higher level facilities. However, in hierarchical decision making, the decisions of strategic, tactical and operational levels are decided in each level of decision making in a hierarchical format and the higher level decisions are used in lower-level decisions [18]. Figure 1.2 depicts an illustration of clinics and hospitals as different levels of hierarchical facility location problem.

Hierarchical facility location problems are classified in two groups based on their objective functions. In the first group, the costs are to be minimized while in the second group the focus is on the configuration of the facilities in order to maximize the demand point coverage. The traditional hierarchical maximal covering location

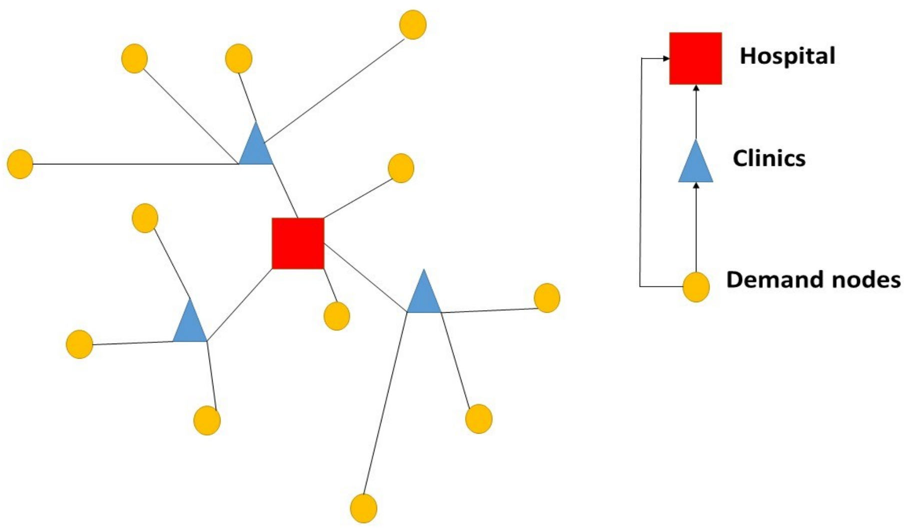

Figure 1.2: Schematic illustration of hierarchical facility location problem.

problem was formulated by Daskin [19] for the first time and Farahani et al. [20] extended the model of hierarchical maximal covering location problem by considering the risk of disruptions. An example of a hierarchical facility in their study is having clinics and hospitals are going to be located within a city at the different level facilities. The objective function is to maximize the expected covered demands while considering the risk of disruptions (disasters such as congestion, earthquakes, floods and adverse weather conditions) for different levels of facilities in the mentioned hierarchy of different facilities.

Modularity is almost a newer concept that is utilized by researchers to attain multi-level systems. In the field of location problems, the modularity concept is used to represent different types of facilities [21] or applied in arcs when there are different kinds of vehicles for transportation [22]. It can generate hierarchical facilities as well. The main difference between modular and hierarchical facility location problems is that modular facility location problems can have more levels of facilities because various types of modules, the number of them and different sizes of modules may create a diversity of multi-level facilities. Modularity is a strategy recognized by academia and industry and plays an important role in the development of sustainable systems. The modular location problem is one of the important research streams of this thesis, which is applied to facility location problems recently. For instance, Addis et al. [23] studied a two-level capacitated facility location problem, where two sets of facilities have to be located in which different devices can be installed at each site that can provide various capacities with different costs. Four kinds of decisions are made in their model, whether or not to open a higher-level facility, whether or not to open an intermediate level facility with a device kind, assigning this intermediate level facility to the higher level one or not and finally allocation of demand points to the intermediate level facilities. Moreover, Correia and Melo [21] presented a multi-period facility location problem with modular capacity that is adjustable according to the flexible demand fulfillment. In their model, customers were divided according to different sensitivity to delivery lead times. They also proposed two mixed integer linear programming formulations and did an extensive numerical study on randomly generated data with different demand patterns. The objective of their model is to find the minimal cost schedule for facility opening and closure, the capacity expansion decisions in each time period, and the allocation of demands to operating facilities in each time period. Yin and Mu [24] proposed two variants of the capacitated MCLP for fixed and unfixed number of facilities to be located assuming the ambulances as the modules and facilities of the emergency service problem. In the first developed model that the number of the facilities to be located in each candidate location is unfixed, the number of facilities (ambulances) to be located can be any integer number in such a way that the sum of all located facilities cannot exceed the available facilities. However, in the second model they developed the number of facilities that can be located in each candidate location is restricted. The difference of two models is that in the first model all the available modules can be located in one location but the second model defines a limitation for the number of located modules in each candidate location. They formulated a static capacitated MCLP and utilized the geographical information system to solve it.

1.4 Hub location problem

In network systems that some physical or non-physical flow needs to be transferred from one node (as origin) to another node (as destination), it is not practical to connect all the nodes with each other. The solution to overcome the problem of connection is to select some of the central nodes as the hubs and conduct the connections through these hubs, as illustrated in Figure 1.3. Figure 1.3 (a) shows the connection between nodes when there is no hubs and Figure 1.3 (b) shows the same nodes connected by using hubs. The problem of finding the optimum location and the number of hubs is called a hub location problem which seeks to locate facilities in potential hub nodes in networks and allocate other non-hub nodes to these hubs. This problem has applications in the design of air transportation

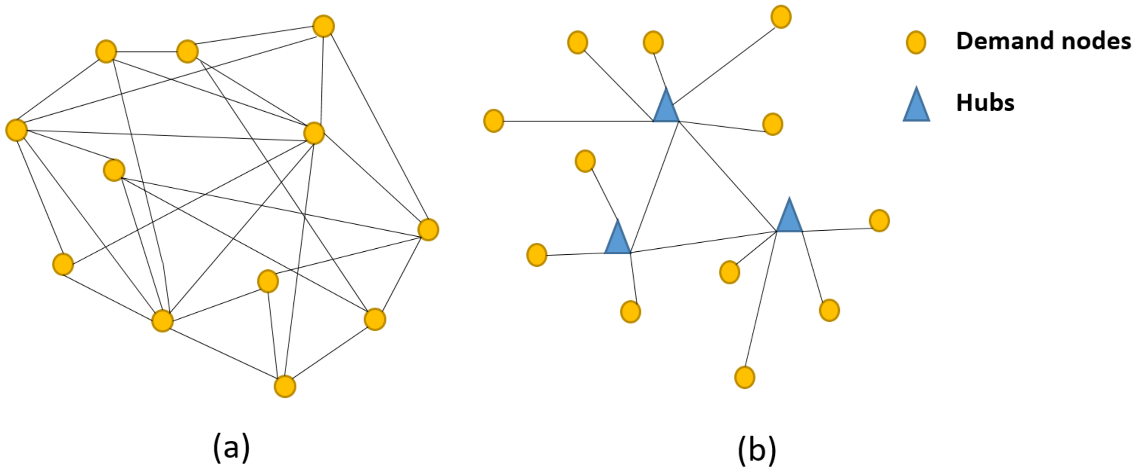

Figure 1.3: (a) non-hub sytem. (b) hub system.

networks, distribution systems for perishable products, postal delivery networks, and tourism routing. Researchers have developed many kinds of hub location problems that can be distinguished based on different criteria such as: solution domain, objective function criterion, allocation type, etc. The literature on hub location problems can be categorized into two main groups concerning the objective function type. The first category belongs to the problems that aim to minimize the costs of locating and operating the hubs, while the second category of problems prioritizes the service levels to the cost issues, mostly known as p-hub center problem and the hub covering problem. The p-hub center problem minimizes the maximum distance, time or cost between hubs and their allocated nodes. The p-hub covering location problem attempts to locate hub facilities in such a way that the origin/ destination pair of two non-hub nodes is covered by a pair of hub nodes under a predefined coverage distance, time or cost. Similar to the covering location models, the p-hub covering location problem has two main variants as hub set covering location problem and the p-hub maximal covering location problem. Hub set covering problems is defined to provide services for all origin/ destination pairs with the objective of minimizing the total number of hubs or hub location costs while p-hub maximal covering location problem maximizes the covered flows in origin/ destination pairs locating a pre-defined number of hubs.

In most of the hub location problems, the demands can just happen from origin/ destination pairs but Karimi and Bashiri [25] studied the case that the demand can raise from all nodes of the network. An example of this kind of problem in real life can be when passengers in an airline network don't want to take a flight more than a specific time. For these kinds of demands, Karimi and Bashiri [25] presented four kinds of covering problems as single allocation and multiple allocations for both hub set covering and p-hub maximal covering location problems. In single allocation problems, each node is restricted to be allocated to only one hub, while in multiple allocation problems the nodes can get service from more than one hub.

Although the coverage distance is a given parameter to develop the covering models, there are some studies that consider the coverage radius as a decision variable. Ebrahimizedeh et al. [26] studied the multi-period hub set covering location problem with such variable coverage radius. The variable coverage is applicable in the telecommunication systems when the area covered by the radio waves depends on the strength of the transmitted waves, and by reinforcing the radiation, larger areas may be covered by the hub node. As a result of such variable coverage having a larger covering radius, it has an impact on the number of located hubs and may reduce the number of needed hubs. Their developed model minimizes the transportation, hub location and coverage costs and each opened hub can have a coverage distance in each time period that is determined as one of the solutions of the problem. In addition, they allow a single allocation of nodes to the hubs, not considering capacity constraints for hubs but allow the opened hubs to be closed and vice versa over time.

1.5 Double coverage and back-up covering location problem

In a system without back-up service, when a call for service occurs, the nearest ambulance or firefighting truck would dispatch. The problem is when all the ambulances or trucks are fully utilized and a call arises asking for service before the others have returned to the base. The solution in this situation can be the back-up service provided from another facility and its resources must be brought to bear. This back-up facility is unlikely to be as close to the incident as the primary ambulance. The need for back-up service seems mandatory designing the service providing systems in regions with high demand. The first attempts to include the back-up coverage for demand nodes was fulfilled by Hogan and ReVelle [27] in their proposed a hierarchical objective set covering model, which was a bi-objective model minimizing the number of located facilities to cover all demand points in the first objective. However, the second objective counted number of the times each demand point was covered more than the first-time coverage and maximizing these numbers. Their developed model had a set of constraints similar to the original set covering location problem but it set the demand points to be covered more than one time. The maximal covering location problem with back-up services was addressed by Pirkul and Schilling [28] for the first time. In their model, they had two kinds of demands and coverage distances for primary and back-up cases. On the other hand, the decision variables of demand node allocations to the facilities were two kinds corresponding to the primary and back-up allocation of demand nodes to the facilities. For the same concept from a different modeling perspective, Başar et al. [29] developed a new model that can provide back-up coverage for demand points by having fewer decision variables. The difference between their model and Pirkul and Schilling [28] model is that in Başar et al.'s model the demand points are not supposed to be covered unless they are allocated to two opened facilities. Another innovative problem modeling the back-up coverage for maximal covering location problem for locating police patrols is presented by Curtin et al. [30] who let the binary conventional variable for facility location take any integer number less than the pre-determined one. Accordingly, the number of police patrols that can be located in areas can be 0, 1, 2 or more but the sum of all located patrols cannot exceed the available resources. The model was tested using a case study in Dallas, TX that the results approved using the optimization problem can improve the number of the incidents covered and the total distance traveled by the located police patrols.

Another study is conducted by Erdemir et al. [31] who considered both air and ground ambulances to provide services for emergency request modeling with both SCLP and MCLP. Three coverage options were possible: only air ambulance, only ground ambulance or teamwork in such a way that the ground ambulance carries the patient to a place that can be reached by air ambulance. Their model took into account the uncertainty in demands and used a greedy heuristic to solve the problem of a case study in New Mexico State.

1.6 Game theory and covering location problem

In the literature of game theory, games are classified into cooperative and non-cooperative games. In non-cooperative games, decision makers act solely and do not admit any promises trying to take advantage of competing with other rivals. Cardinal & Hoefer [32] studied the SCLP from a non-cooperative game theory point of view, in which the players want to satisfy a subset of constraints. They also investigated the existence of exact and approximate Nash equilibria and its computational cost. Konak et al. [33] developed a competitive facility location problem in which multiple competitors aim to maximize their market shares. Their problem is called the Competitive Maximal Covering Location Problem (CMCLP) that is modeled as a Stackelberg game in which the first player and then the other one locate a fixed number of facilities. Furthermore, their work considers multiple competitors, and the objective is on discovering a set of the competitors' decision tuples that are not dominated by any other decision tuples in the solution space.

Different from the non-cooperative games that try to model the competition among the players, the cooperative games try to optimize the objectives by encouraging the players to collaborate. In most of the facility location games, players are assumed to be customers or demand points acting deterministically. In this group of games, given the facilities chosen by the players who are attracted to utilize their nearest facility, customers cooperate to allocate the costs of the facilities to share the cost among themselves. From this perspective, Bergantiños et al. [34] and Bagherinejad et al. [35] studied the cooperation among the customers for the set covering and maximal covering location problems, respectively. Bagherinejad et al. [35] modified the cooperative coverage of demand points in the maximal covering location problem. In this type of cooperative location game that is also called joint coverage games, it is assumed that each facility dissipates a signal whose strength decreases according to a distance decay function. They showed that the case, which the facilities cooperate to cover the points could prepare more coverage for demand points in the applications like non-physical signals.

1.7 Solution methods to solve covering location problems

Since both SCLP and MCLP are NP-hard [36] and commercial software packages are unable to solve such problems in a rational time. This fact makes it essential to develop efficient solving procedures. There are three methods namely exhaustive enumeration, mathematical programming, and heuristic approaches to solve the location problems. Heuristic approaches can solve large size problems, but do not guarantee to obtain an optimal solution [37]. There is only one exact algorithm for the MCLP that is developed by Downs and Camm [38]. They dualized the covering constraints and the problem was convertible to a binary knapsack problem and used subgradient optimization to solve the Lagrangian dual. Finally, as the Lagrangian subproblem had the integrality property, the best bound obtained was equal to the LP relaxation lower bound. Galinier and Hertz [39] proposed three exact algorithms for solving SCLP with potential locations from very large (possibly infinite) sets. Two of the algorithms used a removal and insertion scheme that could determine minimal covers and the third one was utilizing the hitting set to produce the minimum covers. They also developed heuristic versions of these algorithms and analyzed them.

The Lagrangian relaxation-based heuristics have also been applied to many combinatorial optimization problems. The quality of the solution is controlled by the upper and lower bounds provided by the Lagrangian relaxation procedure. Galvão and ReVelle [40] describe a Lagrangian heuristic by relaxing the covering constraints and used subgradient optimization to solve the Lagrangian dual. The greedy heuristic is the most conventional heuristic to solve the MCLP and was developed by Church and ReVelle [5]. The greedy heuristic method adds at each iteration a facility that has the most increase in the objective function value. This method is embedded in a branch-and-bound tree to obtain an optimal integer solution.

Metaheuristics have also been applied to solve covering location problems. A genetic algorithm was utilized by Zarandi et al. [41] to solve large-scale instances of the MCLP with up to 2,500 nodes. Máximo et al. [42] developed a guided adaptive search algorithm and solved instances up to 7,730 nodes. Furthermore, Bilal et al. [43] describe an iterated tabu search heuristic to solve a variant of the set covering location problem.

Decomposition techniques are also applied to solve covering location problems. To solve very large-scale problems of partial SCLP and MCLP up to millions of demand points and obtaining an optimal solution for this huge size of problems, Cordeau et al. [44] presented a Benders decomposition method. The good performance of their method is due to the utilization of a Branch-and-Benders-cut algorithm.

In addition to the studies mentioned above that used different methods to solve MCLP and SCLP, Table 1.1 provides the solution procedures of the other investigated papers in previous sections.

Table 1.1: Solution methods for MCLP and SCLP variants.

Paper	Solution method
Toregas et al. [4]	linear relaxation technique using cuts to generate integer solutions
Church and ReVelle [5]	Greedy hueristic
Marín et al. [14]	Lagrangian heuristic based method
Bagherinejad and Shoeib [15]	Genetic algorithm and bee algorithm
Vatsa and Jayaswal [16]	Benders decomposition-based solution method along with several refinements.
Farahani et al. [20]	Hybrid artificial bee colony
Addis et al. [23]	Dantzig–Wolfe reformulation techniques to develop exact optimization algorithm
Karimi and Bashiri [25]	Two heuristics
Ebrahimizedeh et al. [26]	Genetic algorithm with dynamic stopping criteria and immigration operator, original Genetic algorithm and imperialist competitive algorithm
Pirkul and Schilling [28]	Lagrangian relaxation
Başar et al. [29]	Tabu search
Erdemir et al. [31]	Greedy heuristic, CPLEX and premature CPLEX

1.8 Motivation and research goals

From the investigated topics in previous sections, it can be concluded that covering facility location problems have been studied in the extensive literature considering different concepts. However, it seems that these studies are not sufficient to introduce a model for real-life applications and there is still so much that can be done to improve this category of facility location problems. This importance has been addressed in this thesis and the main research goals are as follows:

Research goal 1: develop covering location models to model the real-life conditions and take the advantages of two separate covering modes in an integrated model.

Sub-goal 1: develop a practical covering facility location problem for applications in real-life conditions such as emergency humanitarian logistics problem.

Sub-goal 2: As the two categories of covering facility location problem (set covering and maximal covering location problem) have different coverage concepts, integrate these two models in one model to improve the service quality for demand points.

Research goal 2: Apply the game theory to the covering facility location problems.

Sub-goal 1: study the covering facility location problems from a non-cooperative perspective and address the specifications of such appropriate model.

In order to fulfill the research goals, firstly an improved formulation for the covering location problem with application in emergency humanitarian logistics is developed. In the same way, the covering facility location problems are combined in a compact mathematical model to benefit from the coverage concept of covering problems. Second, the covering facility location problems are investigated from non-cooperative perspectives. The non-cooperative covering facility location problem resulted in bi-level formulation structures that require computational effort to deal with.

1.9 Contribution of this thesis

The supply chain decisions can be categorized into three kinds of decisions, i.e., strategic, tactical and operational decisions. In this regard, the facility location decisions belong to the strategic and long-term decisions. On the other hand, the decisions can be static and dynamic. The static decisions determine the solutions for only one period of time while the decisions in dynamic models are decided in different time-periods of the planning horizon. The problems in this thesis are mostly addressing the strategic and dynamic decisions as they are determining the location of facilities, module assignment and demand point allocations in different time periods. However, the decisions of facility locations, module assignment and demand point allocations have extended to be studied in both strategic and tactical levels as one of the contributions of this thesis in hybrid covering location model in Chapter 4.

The overall contribution of this thesis can be described from two main viewpoints: 1) problem modeling and 2) theoretical and solutions.

1.9.1 Contribution from problem modeling viewpoint

Figure 1.4 shows the main streams about problem modeling utilized in this thesis. The dynamic and capacitated models, modular facilities, back-up services, set covering location problem, bi-level or Stackelberg models and hub location problem are discussed and introduced in previous chapters. These modeling streams are used to model the problems developed in this thesis. The contribution of this thesis from the problem modeling perspective is as follows:

1. The facilities in maximal covering location problem are developed as modular and capacitated facilities and can provide back-up services for demand points in applications like emergency services. These modules can transfer from one facility to another in different time periods in response to demand requests variation and therefore the allocations of demand points would also change. The objective of this problem is to maximize the total demands covered by primary and back-up modules in all time periods (Chapter 3).
2. Capacitated and dynamic set covering location problem are integrated with dynamic capacitated and modular maximal covering location problem in the hybrid covering location model. The decisions of facilities are determined using set covering location problem as a strategic decision and in tactical

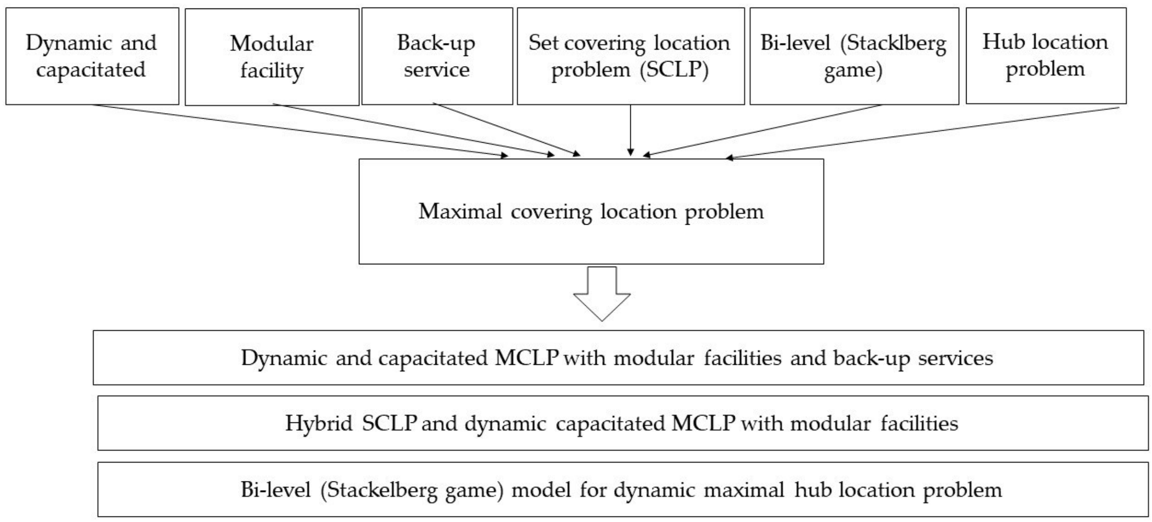

Figure 1.4: Problem modeling contribution of this thesis.

periods the decisions of modules assignment and demand allocations are determined. The problem of integrating set covering and maximal covering location problems is studied for the first time in this thesis (Chapter 4).

3. Maximal hub covering location problem is studied from a non-cooperative perspective that resulted in a bi-level model. In this bi-level model, the freight company who determines the locations of the hubs is the leader of the game and the customers who desire to have the minimum cost for their requested service are the followers. This problem is also a dynamic and capacitated problem and the number of hubs can be expanded in upcoming periods to cover more demands (Chapter 5).

1.9.2 Contribution from theoretical and solution procedure viewpoint

As shown in Figure 1.5, the theories and solution procedures that are utilized in this thesis and to solve the developed models are location theory, non-cooperative game theory, genetic algorithm, Benders decomposition algorithm and heuristic method. The contribution of this thesis from theoretical and solution methods points of view are as follows:

1. The proposed solution procedure composed of a genetic algorithm and a novel heuristic method is used to solve dynamic capacitated maximal covering location problem with modular facilities and back-up services. The novelty of the proposed genetic algorithm is that the constraints of the problem are used to improve the infeasible solutions and convert them into

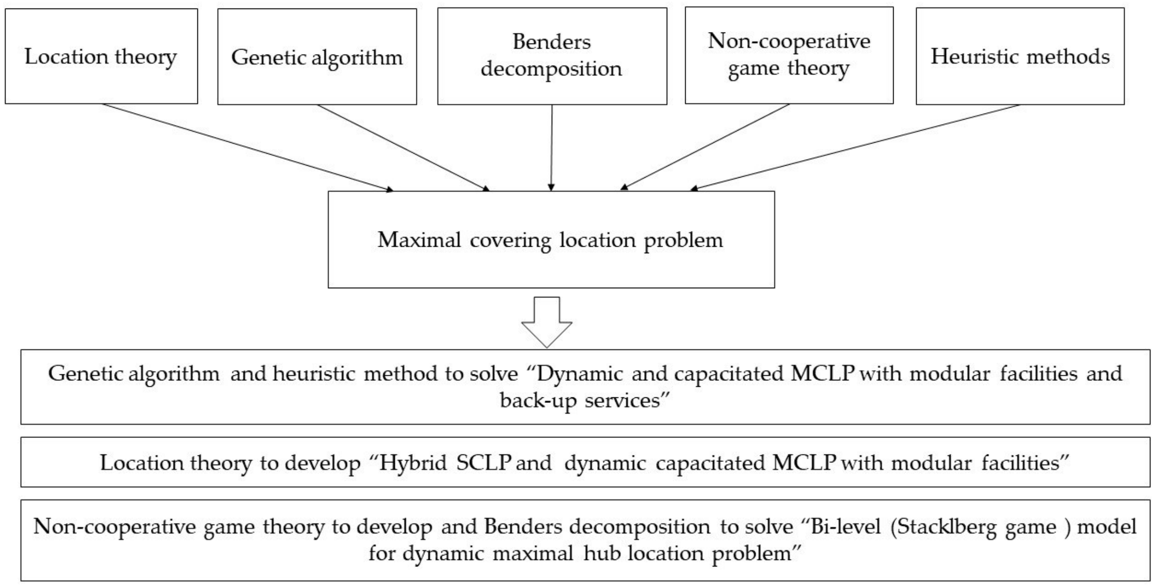

Figure 1.5: Theoretical and solution contribution of this thesis.

feasible solutions. Moreover, the uniform crossover and mutation operators are developed that enables to have more exploitation and exploration of the search space (Chapter 3).

2. The location theory is used to introduce the coverage definitions in hybrid covering location problem that provides full coverage for demand nodes from facilities and a maximal coverage from a limited number of modules (Chapter 4).
3. Using the non-cooperative game theory, the bi-level model for maximal hub covering location problem is developed. To solve this problem, it is firstly reformulated to a single level model using duality theory and Karush-Kuhn-Tucker conditions. Then the single level problems are solved using a Benders decomposition-based method (Chapter 5).

1.10 Structure of this thesis

Before applying the game theoretical approaches to the covering location problem, the problem has been presented using some variants covering location problem. Figure 1.6 shows the relationship between the chapters of this thesis.

In Chapter 1, the background of the thesis is stated. The importance of facility location problems in supply chain management has been addressed. The covering location problems that are the focus of this thesis is introduced as one of the main categories of facility location problems for locating facilities in both public and private sectors. In addition, several extensions developed for covering location problems that are related to the topic of this thesis have been addressed.

In Chapter 2, the basic theories that are utilized in this thesis are presented. These theories are mainly originated from two directions. At first, the theories of the covering location problems are introduced that have the main roles in the developed models in all chapters. After that the related theories to the non-cooperative game theory is presented that are used in Chapter 5 of this thesis.

Chapter 3 presents a dynamic model and a solution procedure for maximal covering location problem with back-up services for locating emergency facilities location. In this chapter, two main concepts are used to model a real-life condition and to improve the service level for demand nodes. These concepts are the modular arrangement of different fire trucks and ambulances, as an example for modules, and considering back-up services for demand nodes. The mathematical model is a pure 0-1 programming problem. To solve the problem, an efficient genetic algorithm is used because this algorithm has appeared as a strong method to solve integer problems. In addition, a heuristic algorithm is developed that is able to solve problems in efficient computational time. Some test problems are generated and solved by CPLEX, GA and a developed heuristic to be able to compare the results and obtain managerial insights.

In Chapter 4, an extension of covering location problem as a hybrid covering model is presented that utilizes the set covering and maximal covering location problems. The developed model is a multi-period model that considers strategic and tactical planning decisions. The proposed hybrid covering location model

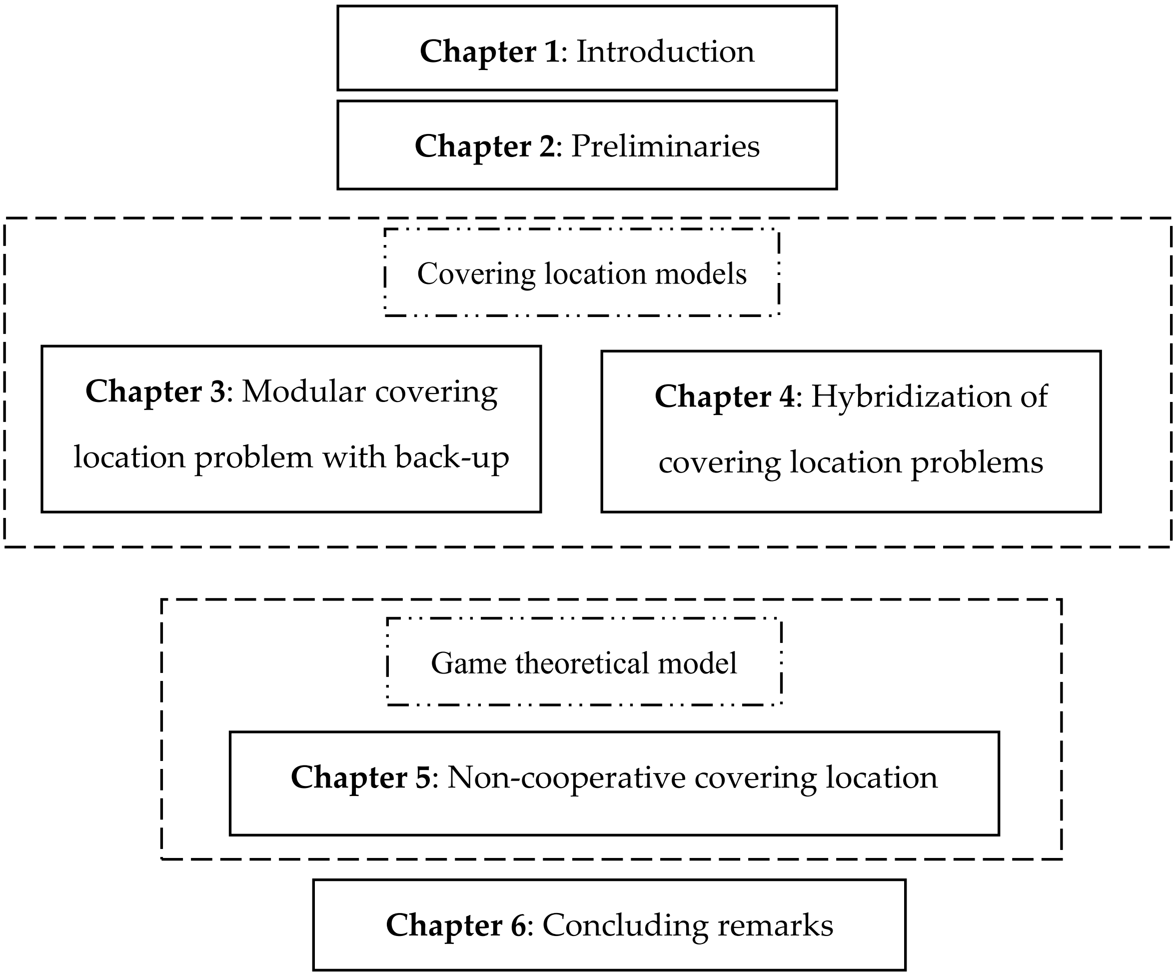

Figure 1.6: The structure of the thesis.

determines the location of the capacitated facilities by using a dynamic set covering location problem as strategic decisions and assigns the constructive units of facilities and allocates the demand points by using dynamic modular capacitated maximal covering location problem as tactical decisions. One of the applications of the proposed model is locating first aid centers in humanitarian logistic services that have been addressed by studying a threat case study in Japan. In addition to validating the developed model, it is compared to other possible combined problems and several randomly generated examples are solved.

Chapter 5 addresses a non-cooperative game as a Stackelberg game for a dynamic maximal hub location covering problem applicable in the freight transportation system. The model has the possibility of having expansion scenarios for the future according to the forecasts of increasing demands. Two expansion scenarios are to add up the number of hubs in the network and to add up more carriers. As the markets are involved in the pricing procedure, the model is a bi-level problem which needs more effort to deal with, for which in this work two reformulations based on Karush-Kuhn-Tucker conditions and duality theory are utilized to reformulate the bi-level problem to a single-level one. To solve the model efficiently, a Benders decomposition-based method is applied and numerical examples are solved to verify the accuracy of the proposed model.

Finally, the concluding remarks are described in Chapter 6.

Chapter 2

Preliminaries

In this thesis, game theoretical approaches for covering location problems along with their solution procedures are addressed. This chapter introduces some fundamentals about the mathematical models for covering location problems, game theoretical approaches as non-cooperative games and solution procedures to solve these problems.

2.1 Covering facility location problem

According to the classifications of facility location problems by Arabani and Farahani [45], facility location problems can be classified as continues, discrete and network location problems. Covering location problems belong to the network location problems and contains two main covering location problems as set covering and maximal covering location problems. These two problems are introduced in details in the next section.

2.1.1 Set covering location problem and maximal covering location problem

Set covering location problem (SCLP) and maximal covering location problem (MCLP) were introduced in 1971 and 1974 by Toregas et al. [4] and Church and ReVelle [5], respectively. In the original set covering location problem, a facility can serve all demand points that are within a given coverage distance from the facility. The problem is finding the minimum number of facilities to ensure that all demand points can be served. In this model, there are no capacity constraints for the facilities. The demand points are represented by the set of $j \in J$, the candidate locations of the facilities are given by the set of $i \in I$. dis_{ij} is the distance between potential facility i and demand point j. Having the coverage distance ρ, one can generate the possible facilities that can cover demand points in a binary parameter σ_{ij} that gets the value 1 if $dis_{ij} \leq \rho$ and 0, otherwise. The decision variable χ_i is a binary variable that is 1 if a facility is located in i and 0, otherwise. The mathematical model for the original SCLP is:

$$\min \sum_{i \in I} \chi_i \tag{2.1}$$

$$\sum_{i \in I} \sigma_{ij} \chi_i \geq 1 \qquad \forall j \in J \tag{2.2}$$

$$\chi_i \in 0,1 \qquad \forall i \in I \tag{2.3}$$

The objective function (2.1) minimizes the number of located facilities and the constraints (2.2) implies that every demand point j needs to be served by at least one facility.

If there is a budget or recourse limitation and one desires to locate φ predefined facilities with the objective of maximizing the covered demand points, the problem is called the maximal covering location problem for which in addition to the introduced variable for SCLP, another decision variable is needed. γ_j is a binary variable that is 1 if demand point j is covered and 0, otherwise. The mathematical model of MCLP is:

$$\max \sum_{j \in J} \gamma_j \tag{2.4}$$

$$\gamma_j \leq \sum_{i \in I} \sigma_{ij} \chi_i \qquad \forall\, j \in J \tag{2.5}$$

$$\sum_{i \in I} \chi_i = \varphi \tag{2.6}$$

$$\chi_i, \gamma_j \in 0,1 \qquad \forall\, i \in I, j \in J \tag{2.7}$$

The objective function (2.4) maximizes the number of covered demands. Constraints (2.5) allow each demand point to be covered only if there is a facility or more in the coverage distance from it. Constraint (2.6) fixes the number of located facilities to be equal to φ. Figure 2.1 shows the difference of the coverage SCLP and MCLP provide. Figure 2.1 (a) is the illustration of SCLP in which all demand points (orange nodes) are covered and for this full coverage, there is a need to have three located facilities. Figure 2.1 (b) is the illustration of MCLP with having only two facilities to be located. As seen in Figure 2.1 (b) all points might not be covered by MCLP.

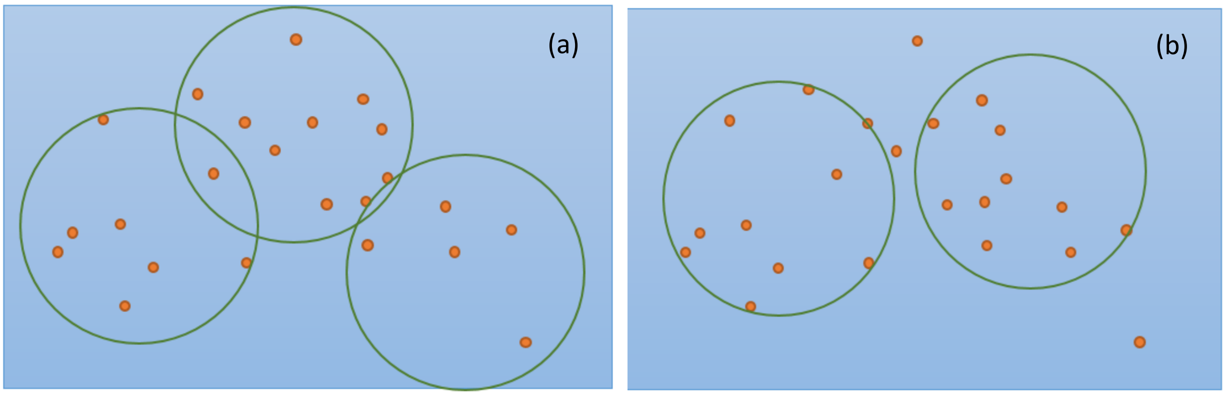

Figure 2.1: Illustration of coverage and located facilities by SCLP and MCLP.

2.2 Non-cooperative game theory

Game theory is a tool to analyze situations like conflicts or cooperation among different players. Non-cooperative games depict the conflict while the cooperative stream contains the cooperation of the players. Stackelberg game is one of the main strategies to analyze the non-cooperative situation among players. The decision-making process is sequential in a way that at first one of the players chooses his strategy who is called the leader of the game. The other player who is called the follower chooses his optimal response according to the leaders announced strategy [46]. If the Stackelberg game is modeled using mathematical models, the problem would have a bi-level structure in which the follower's mathematical model would be included in the constraints of the leader's problem as:

$$\min_{x_u \in X_U, x_l \in X_L} F\,(x_u, x_l) \tag{2.8}$$

Subject to

$$x_l \in \underset{x_l \in X_L}{\operatorname{argmin}} \{f(x_u, x_l): g_j(x_u, x_l) \leq 0, j = 1,2, \dots, J\} \tag{2.9}$$

$$G_k(x_u, x_l) \leq 0, k = 1,2, \dots, K \tag{2.10}$$

where G_k: $X_U \times X_L \rightarrow R$, $k = 1,2, \dots, K$ denote the upper-level constraints, and g_j: $X_U \times X_L \rightarrow R$, $j = 1,2, \dots, J$ represent the lower-level constraints. $x_u \in X_U$ $(, x_l \in X_L)$ is the leader's (follower's) decision variable and X_U (X_L) is the leader's (follower's) decision space. F and f are the objective functions of the leaders and follower, respectively.

2.3 Genetic algorithm

Metaheuristics like tabu search (TS), simulated annealing (SA) and genetic algorithm (GA) are methods applicable to solve a wide range of optimization problems. The metaheuristics general rule to approach the new solutions is to stick to what is known as good up to now. In this case, GA has a big difference with other methods in this way that in GA "good" does not come from the whole solution (as in TS and SA) but comes from the parts of the solution. In other word, it pays attention to the parts of the solution that have made it to be a good solution. GA then uses these parts in its recombination mechanism to produce new solutions [47].

In a typical genetic algorithm, the main mechanisms are: N population size, pc crossover ratio and pm mutation ratio. The main structure of the genetic algorithm is:

0. Encoding scheme: representation is one of the important steps in developing a GA, as it has direct influence on runtime and also crossover and mutation. To have a bigger search space continuous interval between 0 and 1 can be used for each bit and then convert to a binary one whenever it is needed.
1. Creating initial population N.
2. Repair the solutions and create feasible solutions from possibly infeasible solutions. Calculating fitness function for each chromosome in the population.

3. Produce new population. Repeat the following steps for N times to produce new population.

   3.1. Selecting parents from the current population using Roulette wheel selection approach.

   3.2. Applying crossover operator on the selected parents with a ratio of pc and create offspring.

   3.3. Applying mutation operator on offspring with a ratio of pm.

   3.4. Adding new offspring to the population.

4. Select new population based on selection operator.

5. Check the termination criteria, if it is not satisfied go to step 2.

2.3.1 Solution representation

One of the important parts of metaheuristic algorithms is defining a solution representation. This solution representation in GA is called Chromosome. In a 0-1 integer problem the genes of each chromosome consists of 0 or 1 bits as shown in Figure 2.2.

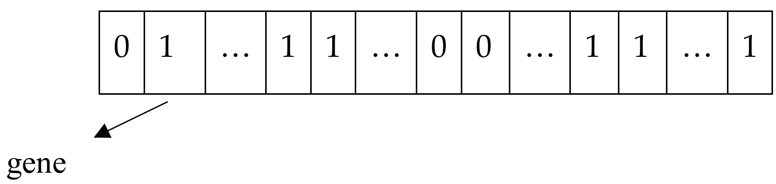

Figure 2.2: Chromosome representation.

2.3.2 Crossover

In genetic algorithm and evolutionary algorithms, crossover operator is for combining the genetic information of two parents to generate new offspring from the population.

First two new chromosomes (same as the initial population was produced) should be produced. We call them R1 and R2. We also call the two chosen parents x1 and x2. The new offsprings are called y1 and y2 which are produced in this way: y1= (R1.*x1) + (R2.*x2) and y2= (R2.*x1) + (R1.*x2), respectively. Figure 2.3 illustrates a small example of a uniform crossover operator. By doing this procedure we have obtained two offspring that contain the continuous values between 0 and 1 and they will be converted to binary values when they will be used in the fitness function to calculate the objective function.

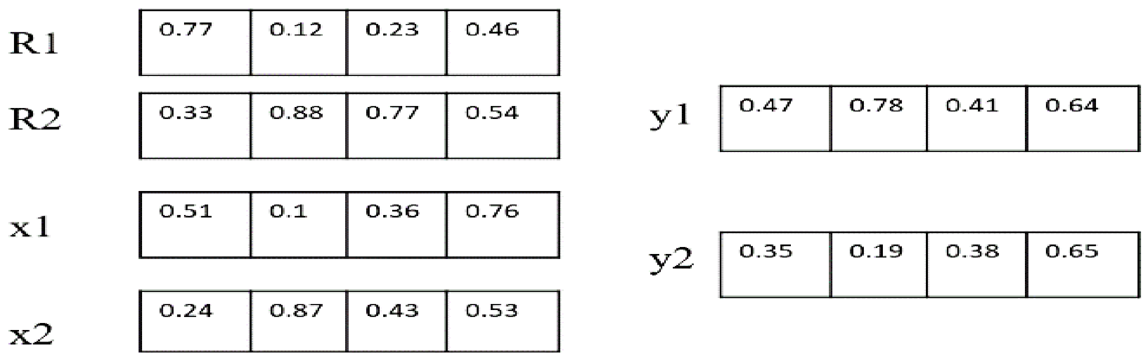

Figure 2.3: A simple illustration of the uniform crossover operator.

2.3.3 Mutation

While crossover tries to converge to a specific point in the landscape, the mutation does its best to explore more areas. One of the main benefits of this operator is to avoid trapping in a local optimum.

To establish the uniform mutation mechanism, we take the following steps:

1. Calculate the number of elements in the chromosome (n).
2. Choose $I = R.n$ random sample genes of n genes in the chromosome. (R is the percent of genes that would be selected to perform mutation operator out of n genes, so I defines the number of genes that we select for mutation. For example, if the number of bits or genes (n) in the chromosome is 300 and 0.01 of them have to be selected, it gives that I=3 genes have to be selected to conduct the mutation).
3. Produce I random values between (-0.1, 0.1).
4. Add or subtract the random values obtained in step 3 to/from the samples chosen in step 2.
5. Stop.

2.4 Benders decomposition

Benders decomposition is a solution method for solving certain large-scale optimization problems. Instead of considering all decision variables and constraints of a large-scale problem simultaneously, Benders decomposition partitions the problem into multiple smaller problems [48]. Since the computational difficulty of optimization problems increases significantly with the number of variables and constraints, solving these smaller problems iteratively can be more efficient than solving a single large problem. Consider the following problem:

$$\min c^T x + f^T y \tag{2.11}$$

$$\text{subject to: } Ax + By \geq b \tag{2.12}$$

$$x \geq 0, y \in Y \subseteq \mathbb{R}^q \tag{2.13}$$

where x and y are vectors of continuous variables having dimensions p and q, respectively, Y is a polyhedron, A, B are matrices, and b, c, f are vectors having appropriate dimensions. Suppose that y variables are “complicating variables” in the sense that the problem becomes significantly easier to solve if y variables are fixed. The complete minimization problem can therefore be written as:

$$\min_{y \in Y} \{f^T y + \min_{x \geq 0}\{c^T x \mid Ax \geq b - By\}\} \tag{2.14}$$

The dual form of the inner problem would be:

$$\max_{u} u^T(b - By) \tag{2.15}$$

$$\text{subject to: } A^T u \leq c \tag{2.16}$$

$$u \geq 0$$

In the Benders' decomposition framework two different problems are solved. A restricted master problem which has the form:

$$\min_{y} z \tag{2.17}$$

$$z \geq f^T y + (b - By)^T \bar{u}_k \qquad k = 1, \dots, K \tag{2.18}$$

$$(b - By)^T \bar{u}_l \leq 0 \qquad l = 1, \dots, L \tag{2.19}$$

$$y \in Y \tag{2.20}$$

k and l refer to different iterations in which different cuts are added to the master ptoblem and subproblems of the form:

$$\min_{u} \; f^T \bar{y} + (b - B\bar{y})^T u \tag{2.21}$$

$$A^T u \leq c \tag{2.22}$$

$$u \geq 0 \tag{2.23}$$

Then the Benders decomposition procedure is the iterative procedure of solving dual subproblem and master problem using the cuts generated from dual subproblem. Figure 2.4 shows an illustration of the Benders decomposition method.

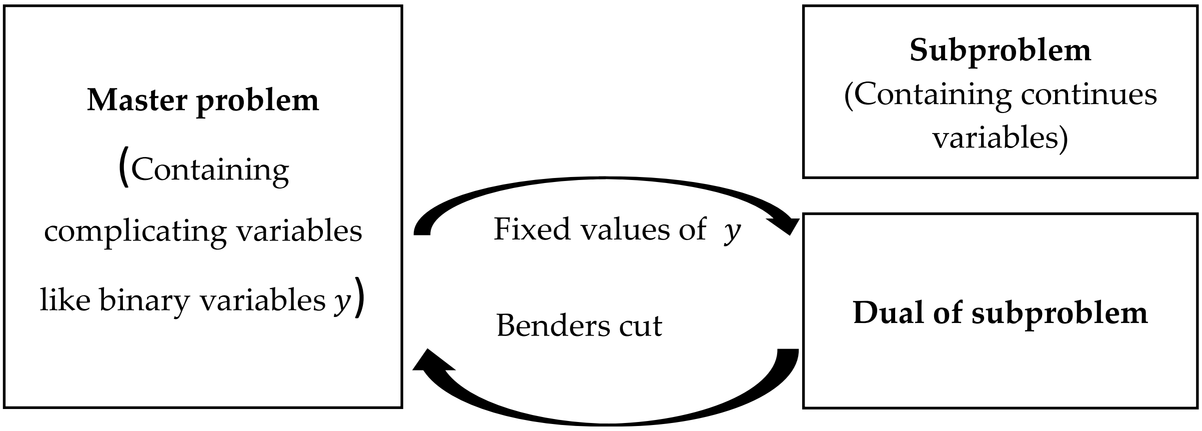

Figure 2.4: Illustration of Benders decomposition method.

The Benders' Decomposition algorithm can be stated as:

{initialization}

y :=initial feasible integer solution

$LB := -\infty$

$UB := +\infty$

while $UB - LB < \varepsilon$ **do**

{solve subproblem}

$\min_{u} \ \{f^T\bar{y} + (b - B\bar{y})^T u \mid \ A^T u \ \le c, u \ge 0\}$

if Unbounded **then**

Get unbounded ray $\bar{u}$

Add cut $(b - By)^T\bar{u} \le 0$ to master problem

else

Get extreme point $\bar{u}$

Add cut $z \geq f^T y + (b - By)^T \bar{u}$ to master problem

$UB := \min \{UB, f^T y + (b - By)^T \bar{u}\}$

end if

{solve master problem}

$\min_{y} \{z | cuts, y \in Y\}$

$LB := z$

end while

Chapter 3

Multi-period modular emergency facilities with back-up services

3.1 Introduction

Emergency facility location problem seeks to find the optimal locations for facilities like fire stations, police stations, emergency departments and roadside emergency services. For a review on location problems for emergency facilities, interested readers may refer to [49]. Emergency facilities mostly consist of constructional units, like different kinds of firefighting trucks in fire stations, ambulances in emergency stations or police cars in police stations. These separable units that specify the size and capacity of facilities are called modules of the facility, which seems inevitable not to consider trying to locate them [21]. Correia and Captivo [50] called the location problems with such capacity constraints, modular capacitated location problems. In their study, they sought to find the location of capacitated facilities with the objective function to give service to the customers at a minimum cost.

Having back-up service, when an emergency call for service occurs, the nearest module serves the demand and would be no more available. If back-up service had been prepared and other call arises for that module before the previous one returns the base, the back-up module can go for the duty. This back-up service might have been provided from the other existing modules in the base or from the other facilities that cover the demand node for back-up service.

The main components of the model in this chapter are multi-period, capacitated facilities and back-up service for demand nodes. In this model, the facilities (for example fire stations' building and location) and the equipment (modules like fire trucks and ambulances) are supposed to have different decision variables, in this way that the demand nodes would be assigned to the modules, and the modules would be assigned to the facilities.

The model development will be outlined in section 3.2 followed by the solution method in section 3.3. The computational experiments are organized in section 3.4. Conclusions are stated in section 3.5.

3.2 Model formulation

In this mathematical model, once the facilities are located, their location cannot be changed, because changing the location would impose a cost. But they can have different arrangements for the module assignments in each time period and in other word, the modules may transfer between facilities according to the demand changes. The size of modules, in this formulation, refers to the number of modules that would be assigned to each facility. In addition, the capacity of each module is related to the amount of service that it can provide, which is an inherent specification for the modules. The notation for our model is as follows:

Indices and sets:

$i \in 1,2,\dots,I$	Index of potential facility sites;
$j \in 1,2,\dots,J$	Index of customers;

$l \in 1,2,\ldots,L$ Index of modules;

$k \in 1,2,\ldots,K$ Index of size;

$t \in 1,2,\ldots,T$ Time periods;

Parameters:

d^{m}_{jlt} Expected demand of node j for primary service of module l at period t.

d^{b}_{jlt} Expected demand of node j for back-up service of module l at period t.

p Number of facilities to be sited.

q_l Total number of available module l.

dis_{ij} Distance between facility i and demand node j.

S^m Maximum service distance for primary services.

S^b Maximum service distance for back-up services.

c^{m}_{ij} Binary parameter which is 1 if $dis_{ij} \leq S^m$, 0 otherwise.

c^{b}_{ij} Binary parameter which is 1 if $dis_{ij} \leq S^b$, 0 otherwise.

c_l Capacity of each module l.

e_{lk} Size of size index k of module l.

Decision variables (binary variables):

z_i 1 if a facility is located at node i, 0 otherwise.

y_{ilkt} 1 if module l with size k is assigned to facility i at period t, 0 otherwise.

x^{m}_{ijlt} 1 if demand node j gets primary service from module l assigned to facility i at period t, 0 otherwise.

x^b_{ijlt} 1 if demand node j gets back-up service from module l assigned to facility i at period t, 0 otherwise.

Thus, the mathematical model can be formally stated as:

$$\max \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}(c^m_{ij}d^m_{jlt}x^m_{ijlt} + c^b_{ij}d^b_{jlt}x^b_{ijlt}) \tag{3.1}$$

$$\sum_{l\in L}\sum_{k\in K}\sum_{t\in T} y_{ilkt} \leq z_i \qquad \forall i \in I \tag{3.2}$$

$$x^m_{ijlt} + x^b_{ijlt} \leq \sum_{k\in K} y_{ilkt} \qquad \forall i \in I, j \in J, l \in L, t \in T \tag{3.3}$$

$$\sum_{j\in J}\left(d^m_{jlt}x^m_{ijlt} + d^b_{jlt}x^b_{ijlt}\right) \leq \sum_{k\in K} e_{lk}c_l y_{ilkt} \qquad \forall i \in I, l \in L, t \in T \tag{3.4}$$

$$\sum_{k\in K} y_{ilkt} \leq 1 \qquad \forall i \in I, l \in L, t \in T \tag{3.5}$$

$$\sum_{i\in I} z_i \leq p \tag{3.6}$$

$$\sum_{i\in I}\sum_{k\in K} e_{lk}y_{ilkt} \leq q_l \qquad \forall l \in L, t \in T \tag{3.7}$$

$$z_i, y_{ilkt}, x^m_{ijlt}, \ x^b_{ijlt} \in \{0,1\} \qquad \forall i \in I, j \in J, l \in L, k \in K, t \in T \tag{3.8}$$

The objective function (3.1) maximizes the covered demand of each service level. The first term calculates the covered demand nodes for primary service while the second term, calculates the demand nodes that are served by back-up service. Constraints (3.2) state that no module is allowed to be assigned to any facility node unless there is a facility located in that node. Constraints (3.3) ensures that each module can just provide one level of service for each demand node. In other words, in each time period, each module can just be dispatched for primary service or back-up service only. Constraints (3.4) are for capacity limits of modules. The right-hand side calculates the total number of allocated demands of primary and back-up services and the left-hand side is the multiplication of the number of assigned modules and the capacity of each module, which is the total capacity that each module can provide. For example, if two ambulances are assigned to a facility and each of them has the capacity of two beds then the total service capacity of the ambulance module for that facility would be four for which the total number of the assigned demands cannot exceed the capacity of each module in each time period. Constraints (3.5) imply that in each facility, at most one size of each module can be located. Constraint (3.6) is for the total number of located facilities. Constraints (3.7) also restrict the number of available modules. Suppose there are just q_l numbers of module l available, the left-hand side of the constraints calculates the total number of this module l that are sited in facilities and this number should not exceed the value q_l. Constraints (3.8) are for the nature of decision variables.

3.3 Solution methodology

3.3.1 Solution methodology based on genetic algorithm

To solve the developed problem, a genetic algorithm is applied in this research, as the evolutionary algorithms have been proven to be one of the best methods to solve facility location models and it is a better approach to solve MCLP, compared to local search procedures such as SA and the TS [11].

GA as an evolutionary algorithm has got many extensions and also being used to solve various kinds of problems like unconstrained and constrained problems. Solving constrained problems needs more effort to design and code the algorithm. There are many techniques to handle constraints and the most well-known one is the penalty function method, in which the violation amount of constraints from the solution is accommodated in the objective function, so that the problem is converted to an unconstrainted one [51]. Some disadvantages have been pointed to this technique and the major one is the difficulty to select the penalty factors. The other method to face the constrained optimization problem which is used in the algorithm design of this thesis is to *repair* the infeasible solutions by modifying them according to the model constraints and create feasible solutions. In this way that in the fitness function calculation procedure, we start from constraint (3.6) to assure about the number of facilities. The constraints checking order would be (3.6), (3.2), (3.5), (3.7), (3.3), and (3.4). For each constraint, the right-hand side and left-hand side is obtained and if the constraint is found feasible, the next constraint will be checked, otherwise, the bits of the solution are modified to satisfy the constraint. After obtaining feasible solutions, the fitness function would be calculated as (3.1).

It is also noteworthy that the GA used is in the category of continuous genetic algorithm, and the reason to use continuous space to generate initial solutions and operate uniform crossover and mutation is because this kind of GA makes the search space bigger and one can have more exploration options. Also, uniform crossover and mutation seem to have better functionality in the continuous GA.

The genetic algorithm presented in section 2.3 is used to solve the test problems. The parameters for pc and pm have been chosen as 0.9 and 0.1 after running the algorithm for different values and these values where selected because they were producing better results for test problems used in this thesis. The termination criteria for this problem set to 70 iterations.

3.3.2 A heuristic method

This section introduces a heuristic algorithm as an auxiliary tool to enable us to evaluate the quality of produced results for computational experiments especially for the cases that CPLEX is unable to reach the optimal objective values. The heuristic starts with using the possibly covered demands to locate the p facilities that can cover the maximum number of demands for primary and back-up service. In the next step the demand nodes, for both kinds of services and each module in each period that can be covered by most facilities are allocated to the p located facilities to obtain the number of necessary modules. The total number of allocated demands for each module and in each period is compared with the existing resources to refrain infeasibility in Step 4. Step 1 of the method, satisfies constraint (3.6), constraints (3.2) and (3.3) are satisfied in Step 2 and constraints (3.4), (3.5) and (3.7) are satisfied in Steps 3 and 4. At the final Step 5, the objective function for the heuristic method (OFV_H) is obtained by obtaining the total number of modules.

The heuristic algorithm is as follows:

Heuristic Algorithm

Step 1. $\mu_{ijlt} = \sum_{jlt}(c_{ij}^{m}\, d_{jlt}^{m} + c_{ij}^{b}\, d_{jlt}^{b})$. Sort μ_i and select p maximum values. i_{pmax} is the set of i that provides p maximum values for μ_i and put the related $z_{i \in i_{Pmax}} = 1$, otherwise $z_i = 0$.

Step 2. Calculate $\phi_{jlt}^{m} = \sum_{i \in i_{Pmax}} c_{ij}^{m}\, d_{jlt}^{m}$ for each j, l, t. Sort ϕ_{jlt}^{m} with respect to j for each l and t, and allocate the p first largest values into $i = 1, \dots, p$ opened facilities and set $x_{ijlt}^{m} = 1$ such that $\sum_{ij} x_{ijlt}^{m} = p$. The selected j is added to the set of j_{pmax}^{m}. Calculate $\phi_{jlt}^{b} = \sum_{i \in i_{Pmax}} c_{ij}^{b} d_{jlt}^{b}$ without the allocated j where $x_{ijlt}^{m} = 1$ for each i, l, t. Sort ϕ_{jlt}^{b} with respect to j for each l and t, and allocate the p first largest values into $i = 1, \dots, p$ opened facilities and set $x_{ijlt}^{b} = 1$ such that $\sum_{ij} x_{ijlt}^{b} = p$. The selected j is added to the set of j_{pmax}^{b}.

Step 3. Set $\sigma_{lt} = 0, \theta_{ilt} = 0, y_{ilkt} = 0$. $\delta_{ijlt} = c_{ij}^{m}\, d_{jlt}^{m}\, x_{ijlt}^{m} + c_{ij}^{b}\, d_{jlt}^{b}\, x_{ijlt}^{b}, j_{max} = j_{pmax}^{m} \cup j_{pmax}^{b}$.

Step 4. For each l, t, iterate $\theta_{ilt} \leftarrow \theta_{ilt} + \delta_{ijlt}$ until $\theta_{ilt} \leq (\max_k e_{lk}) c_l$ for $i \in i_{pmax}$ and $j \in j_{pmax}$.

For each l, t, iterate $e\left\lceil \frac{\theta_{ilt}}{c_l} \right\rceil$, if $k = e, y_{ilkt} = 1$ for $i \in i_{pmax}$ and for $k = 1, \dots, K$. For each l, t, iterate $\sigma_{lt} \leftarrow \sigma_{lt} + \sum_k e_{lk} y_{ilkt}$ until $\sigma_{lt} \leq q_l$ for $i \in i_{pmax}$.

Step 5. Obtain $OFV_H = \sum_{l,t} \sigma_{lt}$

3.4 Computational experiments

To have a better insight of the model and its application in real life, there is a simple illustration of a problem studied in two time periods in Figure 3.1. In this figure black filled nodes are demand nodes for primary service and white filled nodes have been used to refer to back-up service request, also black hexagonals are the facilities located according to variable z_i. The circles around the facilities show the covered area (black is for covered area for primary service and gray one depicts the covered area with back-up service). The numbers around the hexagonals are showing the number of each module assigned to each facility.

In this example, it is supposed that there are three kind of modules, which the total number of each module should be the same in each time period. There may be some nodes colored in black, which are in the primary coverage area of one facility but in the secondary coverage of another facility. It shows that the primary service has been provided from main facility but back-up service has been provided from other one due to insufficient resources.

To generate test problems, the parameters of the problem are generated randomly. The distance between nodes is generated using the Euclidean distance of two-dimensional coordinates having uniform distribution between 1 and 100. The

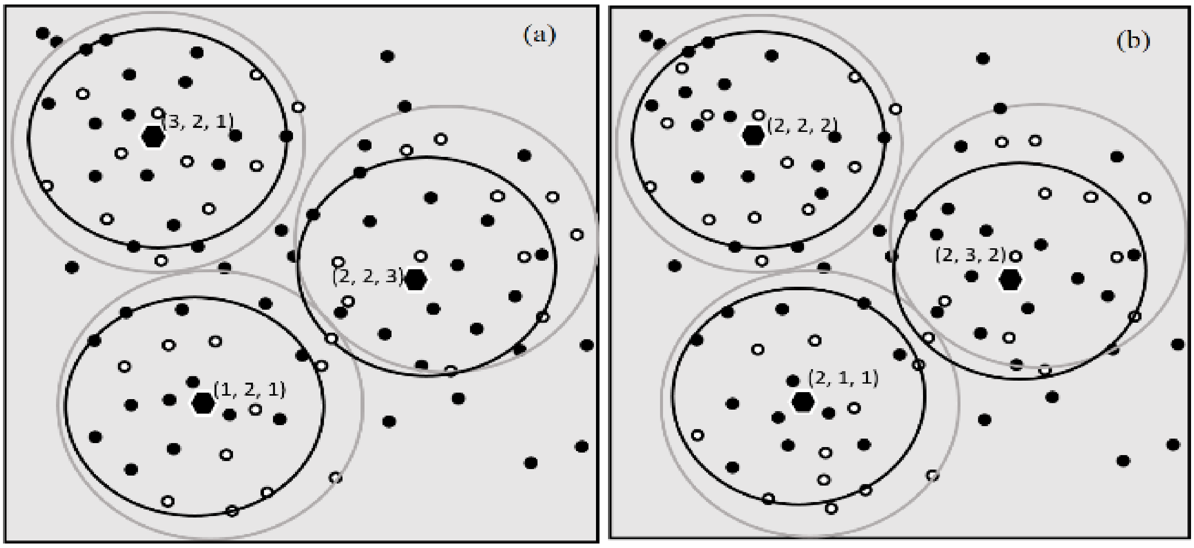

Figure 3.1: Simple illustration of the model. (a) results for period 1, (b) results for period 2.

expected demand for primary service d_{jlt}^{m} and back-up service d_{jlt}^{b} for each demand node j and module l in each time period t are generated using uniform integer distribution between 0 and 2. The covering radius for primary service S^{m} and back-up service S^{b} are set to be 30 and 35, respectively. The number of available modules for each kind of module is generated using a uniform integer distribution for each interval indicated in Table 3.1 and Table 3.2.

It is worth mentioning that all instances are solved using GAMS (CPLEX solver)/MATLAB software on a PC with a 3.4-GHz Core i7-6700 CPU and 8 GB of RAM running Windows 10 (64 bit). Computational results of the problem are summarized in Table 3.1 and Table 3.2.

In Table 3.1 and Table 3.2 the first column contains the information for number of demand nodes and potential locations for facilities. Column p is for the number of facilities to be located, T contains the number of time periods and l refers to the number of available modules. Furthermore, the largest possible size of each module was set to be at most 3 ($e_{lk} = K = 3$). Two columns under "CPLEX", "GA" and "Heuristic" contain the objective function value as "OFV" and computational time as "time" in seconds.

As GA algorithm is a stochastic method that may produce different solution in each time the problem is solved, each test problem is solved 10 times and the results in the Table 3.1 and 3.2 contain the averaged values of these 10 times running for both “OFV” and “time”. The third column under "GA" calculates the gap between the results of CPLEX and GA in percent. Comparing the computational time for the CPLEX and GA, GA does not show any specific superiority. The heuristic also has an acceptable performance as its main application is to investigate the results trend. Figure 3.2 illustrates the OFV values of three solving procedures. The illustrated diagrams give a better insight than the results in Table 3.1. These diagrams show that the CPLEX has the maximum values among the three methods and the heuristic has the lowest values. It also should be added that although the heuristic has the lower values, but its trend is the same as CPLEX, which it was developed for. The GA also is placed between two other diagrams but closer to the CPLEX that approves the ability of this proposed method to obtain satisfactory values. The test problems generated in Table 3.1 may just have some limited practical application and are mostly used to be able to evaluate the solving procedures, which necessitates to develop larger test problems as in Table 3.2 that can have real life applications like a town or city.

These large-scale problems are solved by all methods and the results are summarized in Table 3.2. It should be noted that because of the inherent feature of variables which are all binary variables, the complexity of problems increases exponentially as the problems size increase. For which the CPLEX was able to solve the problems up to 600 nodes (only for some of the test problems of this size), but GA is still able to solve larger problems having satisfying values, where the maximum gap is 0.035. Also, GA is able to solve the problems in a shorter time than CPLEX. The results in Table 3.2 show that for problems up to 600 noes, GA has satisfying values, where the maximum gap is 0.035. Also, GA is able to solve the problems in a shorter time that CPLEX.

The most important observation of the results is related to problems with 800 and 1000 nodes. For these problems, CPLEX is unable to solve them and runs out of

Table 3.1: Computational results for CPLEX and GA for small size problems.

					CPLEX		GA			Heuristic	
i, j	p	T	l	q_l	OFV	time	OFV	time	%gap	OFV	time
200	10	3	3	[15,25]	179	8	175	28	0.02	168	0.2
				[20,30]	226	8	211	29	0.06	180	0.2
			4	[15,25]	234	13	227	28	0.02	222	0.2
				[20,30]	293	33	274	29	0.06	240	0.2
		5	3	[15,25]	302	21	292	33	0.03	280	0.2
				[20,30]	379	24	357	36	0.05	300	0.2
			4	[15,25]	394	58	383	45	0.02	370	0.3
				[20,30]	479	36	469	47	0.02	400	0.3
300	20	3	3	[30,50]	363	25	349	35	0.03	339	0.5
				[40,60]	450	26	426	36	0.05	360	0.5
			4	[30,50]	463	32	448	43	0.03	447	0.6
				[40,60]	591	43	567	47	0.04	480	0.6
		5	3	[30,50]	604	45	589	56	0.02	565	0.7
				[40,60]	751	70	735	57	0.02	600	0.7
			4	[30,50]	785	111	764	61	0.02	745	0.7
				[40,60]	974	68	935	64	0.04	800	0.7

Table 3.2: Computational results for CPLEX and GA for large size problems.

					CPLEX		GA			Heuristic	
$i = j$	p	T	l	q_l	OFV	time	OFV	time	%gap	OFV	time
500	30	5	3	[50,70]	905	536	882	164	0.025	865	3.7
				[65,85]	1112	407	1103	178	0.008	900	4.2
			4	[50,70]	1185	514	1172	179	0.010	1145	4.7
				[65,85]	1485	399	1480	188	0.003	1200	4.6
		7	3	[50,70]	1267	400	1233	184	0.026	1211	4.2
				[65,85]	1285	422	1259	202	0.020	1260	4.5
			4	[50,70]	1659	1130	1634	199	0.015	1603	6.5
				[65,85]	2050	889	2039	210	0.005	1680	6.4

600	40	5	3	[70,90]	1205	310	1190	182	0.012	1165	6.4
				[80,100]	1355	331	1330	180	0.018	1200	6.1
			4	[70,90]	1585	562	1566	203	0.011	1545	8.1
				[80,100]	1785	708	1722	233	0.035	1600	8.2
		7	3	[70,90]	1687	947	1680	218	0.004	1631	8
				[80,100]	1897	1091	1850	216	0.024	1680	7.8
			4	[70,90]	-	-	2477	286	-	2163	11.1
				[80,100]	-	-	2622	292	-	2240	13.2
800	50	5	3	[95,110]	-	-	1550	360	-	1485	19.1
				[100,120]	-	-	1610	377	-	1500	18.6
			4	[95,110]	-	-	2123	410	-	1980	22.5
				[100,120]	-	-	2223	421	-	2000	22.9
		7	3	[95,110]	-	-	2210	380	-	2079	28.1
				[100,120]	-	-	2266	398	-	2100	25.1
			4	[95,110]	-	-	2800	466	-	2772	32.4
				[100,120]	-	-	2930	485	-	2800	32.6
1000	60	5	3	[110,130]	-	-	1699	620	-	1655	27.2
				[115,140]	-	-	1852	613	-	1795	27.3
			4	[110,130]	-	-	2280	701	-	2185	36.3
				[115,140]	-	-	2400	712	-	2395	38.3
		7	3	[110,130]	-	-	2411	845	-	2317	37.9
				[115,140]	-	-	2633	845	-	2513	37.6
			4	[110,130]	-	-	3170	968	-	3059	49.8
				[115,140]	-	-	3393	980	-	3353	50.4

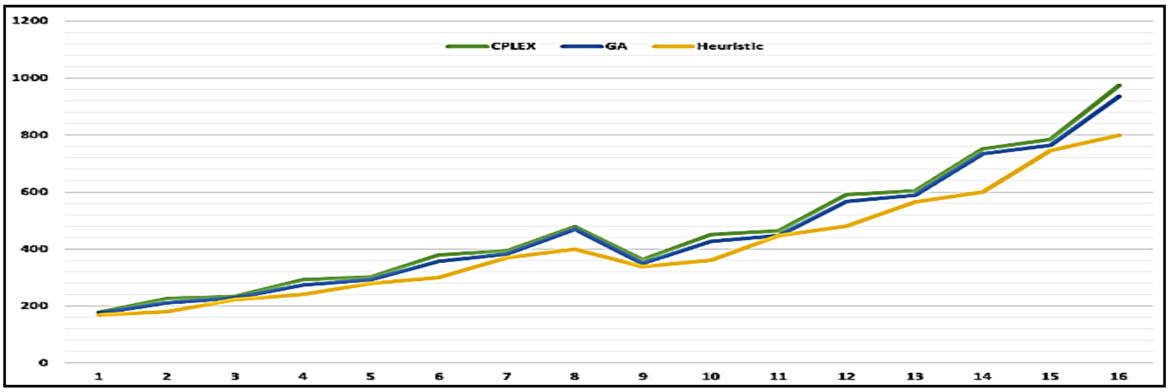

Figure 3.2: OFV for CPLEX, GA and Heuristic for small size problems.

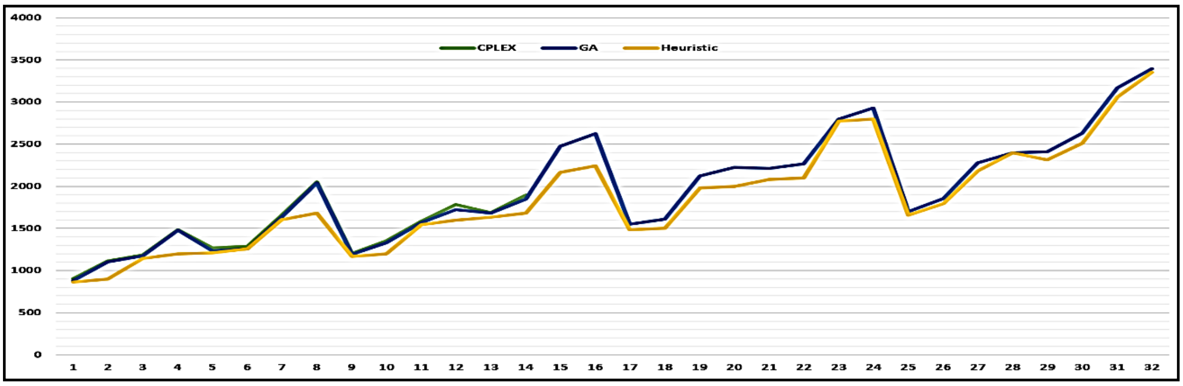

Figure 3.3: OFV for CPLEX, GA and Heuristic for large size problems.

memory, but GA has still satisfactory performance at a reasonable computational time. The main purpose for proposing the heuristic method was to provide a comparing tool to evaluate the performance of GA for the cases that CPLEX in unable to solve. According to the values in Table 3.2 and the Figure 3.3, that depicts these results in a schematic format, it is apparent that the heuristic has kept its correct trend and is producing well enough values as a base to evaluate the GA. This fact approves that one can rely on the proposed GA to solve larger problems and obtain satisfactory results.

3.5 Summary

In this chapter, an extension of maximal covering location problem (MCLP) has been developed for locating emergency facilities, composed of discrete structural components. A genetic algorithm was utilized to solve the problem because of this metaheuristic's strength to solve binary optimization problems and other extension of MCLP. In addition, a heuristic method has been proposed to assess the results obtained from GA. The computational experiments are generated and solved by CPLEX, GA and Heuristic to be able to compare the results and obtain managerial insights. The computational results illustrate a very small gap in the objective function values for the test problems that could be solved with exact method. For larger size problems that CPLEX was unable to solve them, GA produced solutions in short computational time and was able to solve the problems of 800 and 1000 nodes.

Chapter 4

Hybrid set covering and dynamic modular covering location problem: Application to an emergency humanitarian logistics problem

4.1 Introduction

To benefit from the advantages of SCLP and MCLP, in this study, the idea of hybridization of these two models in an integrated model is addressed. The hybrid model is a multi-period model consist of strategic and tactical planning decisions. Strategic planning decisions include the location of the capacitated facilities, and tactical planning decisions include module assignment and demand points allocation. Figure 4.1 shows a schematic illustration of strategic and tactical decisions that are taken in each time period. Strategic decisions reflect the long-term goals that are taken to retain the system more viable. Tactical decisions are made to meet mid time goals that contribute to strategic decisions. Tactical decisions can be taken to respond with a faster action compared to strategic decisions [52] and [53]. It is supposed that the facilities are only the piece of land with basic infrastructures that would be located using SCLP that cover all demand points in each strategic period in response to fluctuating demand having capacitated facilities. After determining the located sites as the facilities, in each tactical period, the limited number of each

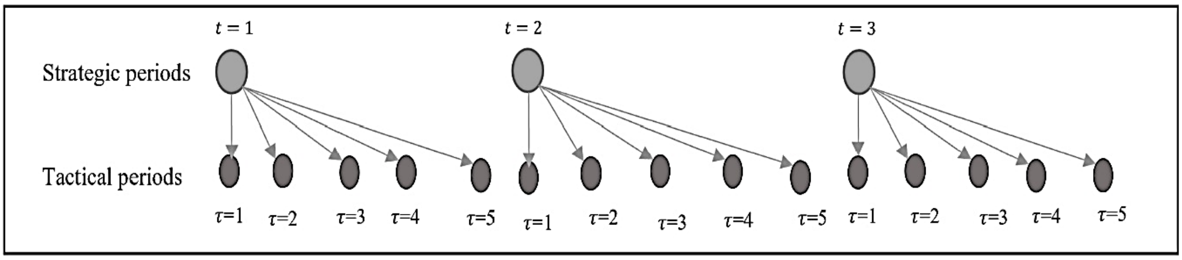

Figure 4.1: Strategic and tactical decision differentiation in the proposed model.

module kind would be assigned to the facilities using modular MCLP to maximize the covered demands. As mentioned in Chapter 1, one of the main features of the hierarchical or modular concept is that the decisions of assignment can be made only for one time period and the decisions can change for the later periods. This feature allows having a different arrangement of modules in each facility and in each tactical period in terms of module type, number and size. Using this integrated framework, trying to cover all demand points in the upcoming strategic periods, more facilities can also be located as extension decisions as a response to rising demand. As an integrated model, the objective of the proposed model is to maximize the profit gained from the income of covering demand points and the cost of minimizing the fixed costs of facilities.

Similar to covering location problems, one of the main applications of the hybrid covering model is locating aid centers or evacuation sites in the humanitarian relief situation. The definition of the facility in this study is similar to the one that is used in real management system, i.e., a public facility mainly with large yards, such as parks, schools or parking lots [54] that can be utilized as an aid center in most of the disasters because of their low vulnerability. These kinds of facilities are usually equipped with essential needs such as water, electricity, etc., and are announced as shelter locations beforehand to the residents. Different kinds of modules can be considered such as ambulances, trucks, helicopters, first aid units, food providing units, sleeping tents, shower rooms and etc. The importance of modularization of resources and services becomes apparent when in most of the disasters the whole area is exposed to be damaged and if the facilities were located having the full equipment, they might have been out of order due to the disaster itself. Modularization can also have a high impact on budget management as the modules can easily be dispatched to the other affected areas in the upcoming future. The other application of hybrid covering location problem is in locating hospitals, distribution centers and integrated production planning and warehouse location problems [55].

The main contribution of this study is summarized as follows.

- In this chapter, it is tried to benefit from the coverage concept of SCLP to locate the facilities (with the aim of providing access to the facilities for all demand points) and MCLP to locate the service providing units (with the aim of maximizing the coverage of demand nodes by the modules respecting the limited number of modules) in an integrated model.
- The integrated model is capable of improving the service quality and exploiting the limited number of modules in a better way compared to the non-integrated approach.
- Studying covering location problems in different decision levels as strategic and tactical decisions is not conducted before in the literature.
- In spite of the modeling advantage of modularity in providing multi-level facilities, it has received very limited attention in covering location problems. In

this model, modular capacitated MCLP is developed to assign the service providing units to the facilities.

- A threat case study as an application of the developed hybrid model is studied and other variants of possible hybridization models are presented and compared through numerical examples.

The remainder of the chapter is categorized as follows. In section 4.2, the literature review is presented. A review of SCLP and MCLP is included in section 4.3. The mathematical formulation of the problem is presented in section 4.4. In section 4.5, the mathematical models of comparable models are discussed. Section 4.6 contains an application of the models and a case study with the results of the numerical examples. Finally, the chapter is concluded in section 4.7.

4.1. Literature review

A significant part of the literature of SCLP focuses on the application of SCLP in real-world problems. Vianna [56] considered the application of the SCLP in the optimization of gas detectors in process plants as a 0–1 integer programming model in order to calculate the best location and the minimum number of gas detectors in a facility site. Another application of SCLP is to locate the traffic counting stations that are used to monitor the traffic flow of transportation vehicles in highways. This application was studied by Vieira et al. [57] with the objective of minimizing the total number of stations to cover all origin and destination nodes of the network for which they proposed a hybrid algorithm based on exact, heuristic and hybrid approaches that could solve an acceptable fraction of instances to find the optimal solution.

Furthermore, Park et al. [58] presented the flight plans for unmanned aerial vehicles that can be very useful for rescue operations in disaster situations using quadratic constraints. As the SCLP in presence of quadratic constraints is a complex problem to solve even small size problems, the authors have developed an approximation model that was solved by a branch and price algorithm. The optimal number and locations of pharmaceutical warehouses is another application of SCLP that was studied by Mokrini et al. [59]. They also conducted a sensitivity analysis to show how different coverage distances can affect the number of warehouses and the network configuration.

There are also many applications of MCLP in modeling real-life situations. Locating bicycle sharing stations in this way that users take the bikes, use them, and then return the bikes at the same or any other located stations were addressed by Muren et al. [60]. They considered lower bound for the workload of each station that could also lead to improved results quality. By the appearance of rechargeable electric vehicles and addressing the need to build charging stations for these kinds of vehicles, Dong et al. [61] formulated and solve the problem of locating vehicles charging stations using MCLP and took into account the spatial information statistic of charging demand as a stochastic process. Similar to the application studied by Park et al. [58], Chauhan et al. [62] studied the problem of assigning the drones used for carrying the delivery packages in commercial services. Chauhan et al. [62] used MCLP to formulate their problem considering package weight, battery and coverage constraints and developed a heuristic method to solve the problem with sensitivity analysis of the problem parameters. Furthermore, Nilsang et al. [63] studied the location of ambulance bases by utilizing MCLP and the real-time information obtained from social media like tweeter. They have studied the reallocation of ambulances in response to demand fluctuation in a dynamic framework and validated the model by applying it to the emergency medical services in Bangkok.

One of the assumptions of the basic MCLP is the binary coverage; that supposes a demand point to be covered completely if it is located within a critical distance or travel time of the facility and the if the demand point is outside of the critical distance or travel time it cannot be covered by the facilities. Most of the researchers have found this assumption to be too restrictive, especially for the applications in emergency systems like the current study. Berman et al. [64] , Drezner et al. [65] and Karasakal and Karasakal [37] are among those who studied the MCLP by modeling coverage as a gradual or partial coverage which means the coverage provided to a demand point decreases gradually with increasing distance or travel time from a facility. Berman et al. [66] have extended the gradual coverage to the case that the coverage can be provided from multiple facilities with applications in cell phone tower service providers. They tested several methods to solve the developed model such as greedy heuristic method, tabu search, ascent heuristic and tangent line approximation method. Locating undesirable facilities like nuclear plants in the presence of gradual coverage was addressed by Khatami and Salehipour [67]. As the objective of undesirable facilities is contrary to the common commercial facilities, the model is called a minimal covering location problem that seeks to locate the facilities in places that covers a minimum number of residents.

Although some modeling ideas have been studied for both SCLP and MCLP, all of these studies have considered SCLP and MCLP as separate models. To the best of our knowledge, there is no study that has modeled SCLP and MCLP in an

Table 4.1: Related papers in the literature (classified based on modelling ideas).

Paper	Model S, M, H	Period S, M	Coverage type B, G, C	Facility type S, M	Data Modelling D, S, F, R	capacity constraint C, N, M	decision levels S, T, O
Toregas et al. [4]	S	S	B	S	D	N	S
Rajagopalan et al. [12]	S	M	B	S	D	N	S
Eiselt and Marianov [68]	S	S	G	S	D	N	S
Berman et al [69]	S	S	G	M	D	N	S
Church and ReVelle [5]	M	S	B	S	D	N	S
Bagherinejad et al. [34]	M	S	G, C	S	D	N	S
Farahani et al. [20]	M	S	B	M	D	N	S
Coco et al [70]	M	S	B	S	R	N	S
Yin and Mu [24]	M	S	B	M	D	C	S
Berman et al [66]	M	S	G, C	S	D	N	S
Vatsa and Jayaswal [16]	M	M	B	S	S	C	S
Zhang et al. [71]	M, S	S	B	S	F	N	S
Erdemir et al. [31]	M, S	S	B	M	D	N	S
HCLP	H	M	G	M	D	C, M	S, T

Model: S (SCLP), M (MCLP), H (Hybrid). Period: S (Single period), M (Multi-period). Coverage Type: B (Binary), G (Gradual), C (Cooperative). Facility type: S (Single), M (Multiple/ Modular). Data Modelling: D (Deterministic), S (Stochastic), F (Fuzzy), R (Robust). Capacity Constraints: C (Capacitated), N (Non-capacitated), M (Module capacity). Decision level: S (Strategic), T (Tactical), O (Operational).

integrated model. Table 4.1 reviews important models in the areas related to our study. Although there is no study combining SCLP and MCLP in one model, they are related to this model at least one perspective. The decision level (last column of the table) of the models might be not specified directly in the papers but as the facility location problems generally belong to strategic decision levels, they have classified as strategic decision models. The last row of Table 4.1 illustrates the contribution of this research compared to the literature.

4.2 Hybrid covering location problem formulation

In this section, a mixed integer linear programming model for the hybrid covering location problem (HCLP) is presented.

Some of the most important decisions in the proposed model are as follows:

- Location, number and establishment time of facilities is strategic periods during the planning horizon.
- Type and number of each module assigned to the located facilities in tactical periods of each strategic period during the planning horizon.
- Percentage of allocating demand of points to the assigned modules in tactical periods of each strategic period during the planning horizon.

The main assumptions to develop the model are as follows:

- The problem is studied in a multi-period framework. The total planning horizon is classified into two types of periods as strategic periods and tactical periods. Each strategic period is composed of several tactical periods with different kinds of decisions to be made.
- The facilities are supposed to be a piece of land or site, equipped with some initial infrastructures. The locations of these facilities are going to be decided only in strategic periods using the coverage concept of SCLP to determine the minimum number of facilities to be located with the aim of covering all demand

points. Once a facility is opened in a strategic period, it cannot be closed and should continue its operation in future periods. In addition, the facilities are supposed to be capacitated and the number of facilities can be expanded in response to the demand variation in the upcoming strategic periods.

- We suppose that there are different kinds of service providing units, namely the modules of facilities that are limited in terms of numbers and capacities. These modules can move to the facilities and should be assigned to the facilities in tactical periods. The optimal decisions of module assignment to the facilities are supposed to obey the coverage concept provided with multi-period modular MCLP. The arrangement of modules in facilities can be varied in different tactical periods according to the points' demand fluctuation in order to maximize the amount of total covered demands.
- Each module comes in different sizes. It can be chosen from different sizes to increase the service quality offered to demand points to overcome the service shortages or having idle units.
- The modules are portable and they can be transferred among the facilities when there is more request in another facility. The transferability is an important specification of modularity design that yields to flexibility in the system and reduces costs. The portability of most modules helps to provide a good level of service to demand points without having to provide more modules.
- It is supposed that covering the demand points by the modules obeys the gradual coverage concept using a partial coverage function. In gradual coverage function, the demand points inside the full coverage radius can be covered completely, but by increasing the coverage radius the amount of coverage

decreases and the points outside the partial coverage radius are supposed not to be covered.

The notation for the model is as follows.

Indices:

i index of candidate facility locations;

j index of demand points;

l index of modules;

k index of sizes;

t index of strategic time periods;

τ index of tactical time periods;

Sets:

I Set of candidate facility locations;

J Set of demand points;

L Set of modules;

K Set of sizes;

T Set of strategic time periods;

$\mathcal{T}_t$ Set of tactical time periods in strategic period t;

Parameters:

$d_{jlt\tau}$ Demand of point j for service of module l in strategic period t and tactical period τ.

q_l Number of available modules for module l at each period.

dis_{ij} Distance between facility i and demand point j.

δ_t Maximum service distance at strategic period t.

g_{ij} Coverage level provided by facility i to demand point j.

$$g_{ij} = \begin{cases} 1 & if \ dis_{ij} \leq S \\ \xi(dis_{ij}) & if \ S \leq dis_{ij} \leq S' \\ 0 if dis_{ij} \geq S' \end{cases}$$

$\xi(dis_{ij})$ Partial coverage function, where $0 < \xi(dis_{ij}) < 1$.

S Full coverage distance.

S' Partial coverage distance.

e_{ijt} Binary parameter which is 1 if $dis_{ij} \leq \delta_t$, 0 otherwise.

f_i Cost of locating a facility at facility location i.

c_l Capacity of each module l per each size.

o_{lk} The kth size for module l

h_i Capacity of facility i.

$a_{jlt\tau}$ Earned income from providing service of module l to demand point j in strategic period t and tactical period τ.

Decision variables:

z_{it} 1 if a facility is located at i in strategic period t, 0 otherwise.

$y_{ilkt\tau}$ 1 if the kth size of module l is assigned to facility i in strategic period t and tactical period τ, 0 otherwise.

$x_{ijlt\tau}$ The percentage of demand point j allocated to the module l of facility i in strategic period t and tactical period τ.

To locate the facilities by using the coverage concept of SCLP in different strategic periods, it is supposed that $d_{jl1\tau} \leq d_{jl2\tau} \leq \cdots \leq d_{jl|T|\tau}\ \forall\, j, l, \tau$ and $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_{|T|}$. This assumption is mandatory for modeling the problem, which implies that the demands are assumed to be increasing during the time horizon and while the coverage radius is fixed or decreasing in response to the increasing demand. We formulate the hybrid covering location problem (HCLP) as follows:

$$\max \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}\sum_{\tau\in \mathcal{T}_t} a_{jlt\tau} g_{ij} d_{jlt\tau}\, x_{ijlt\tau} - \sum_{i\in I}\sum_{t\in T} f_i z_{it} \tag{4.1}$$

$$\sum_{i\in I} e_{ijt} z_{it} \geq 1 \qquad \forall j \in J, t \in T \tag{4.2}$$

$$z_{it} \leq z_{it+1} \qquad \forall i \in I, t \in T \tag{4.3}$$

$$\sum_{k\in K} y_{ilkt\tau} \leq z_{it} \qquad \forall i \in I, l \in L, t \in T, \tau \in \mathcal{T}_t \tag{4.4}$$

$$x_{ijlt\tau} \leq \sum_{k\in K} y_{ilkt\tau} \qquad \forall i \in I, j \in J, l \in L, t \in T, \tau \in \mathcal{T}_t \tag{4.5}$$

$$\sum_{i\in I} x_{ijlt\tau} \leq 1 \qquad \forall j \in J, l \in L, t \in T, \tau \in \mathcal{T}_t \tag{4.6}$$

$$\sum_{i\in I}\sum_{k\in K} o_{lk}\, y_{ilkt\tau} \leq q_l \qquad \forall l \in L, t \in T, \tau \in \mathcal{T}_t \tag{4.7}$$

$$\sum_{j\in J} d_{jlt\tau}\, x_{ijlt\tau} \leq \sum_{k\in K} o_{lk}\, c_l\, y_{ilkt\tau} \qquad \forall i \in I, l \in L, t \in T, \tau \in \mathcal{T}_t \tag{4.8}$$

$$\sum_{j\in J}\sum_{l\in L} d_{jlt\tau}\, x_{ijlt\tau} \leq h_i\, z_{it} \qquad \forall i \in I, t \in T, \tau \in \mathcal{T}_t \tag{4.9}$$

$$z_{it}, y_{ilkt\tau} \in \{0,1\},\ 0 \leq x_{ijlt\tau} \leq 1 \qquad \forall i \in I, j \in J, l \in L, k \in K, t \in T, \tau \in \mathcal{T}_t \tag{4.10}$$

The objective function (4.1) maximizes the profit that is the income gained from covering demand points subtracting the cost of locating facilities and assigning modules to the facilities. Constraints (4.2) are the well-known constraint of SCLP that indicates all the points in each strategic period should be allocated to the facilities. Constraints (4.3) state that if a facility is opened in a strategic period, it should continue its operation for the forthcoming periods. Constraints (4.3) imply that modules can just be assigned to the open facilities and in each facility only one size of each module is allowed, while constraints (4.4) imply the same concept for the demand points in this way that the demand points can just be allocated to the assigned modules in each tactical period of strategic periods. Constraints (4.5) indicate that the total percentage of coverage provided for each demand point from all facilities should not exceed 1. Constraints (4.6) set the total number of modules assigned to the facilities to be less than the available number of modules, i.e., q_l. Constraints (4.7) and (4.8) are capacity constraints of the modules and facilities, respectively. Constraints (4.10) set the variables of location and module assignment to be binary variables while the variables of demand allocation are set to be continuous.

4.3 Comparison with other models

There are other potential ways to combine SCLP and MCLP to shape the hybrid model. In the developed model of HCLP, SCLP is used to locate the facilities as strategic decisions and MCLP is used to find the assignment of modules and demand allocation as tactical decisions. In the same way, MCLP-MCLP may refer to the case that both strategic and tactical decisions are determined using MCLP, SCLP-SCLP refers to the case where both strategic and tactical decisions are determined using SCLP and finally MCLP-SCLP refers to the model that strategic decisions are taken using MCLP and tactical decisions by using SCLP. These four models are comparable because the solution variables are the same in all four models (except the number of located facilities that can be obtained from SCLP). Besides the same solution, the goal of covering problems, which is to cover more demand points can be extracted from all four models as the coverage percentage. In this section, the mathematical model for each of these variants is developed and in section 4.7 the numerical examples are conducted to evaluate these models in terms of coverage they provide and efficiency (elapsed time). Figure 4.2 shows these different possible hybridization problem structures.

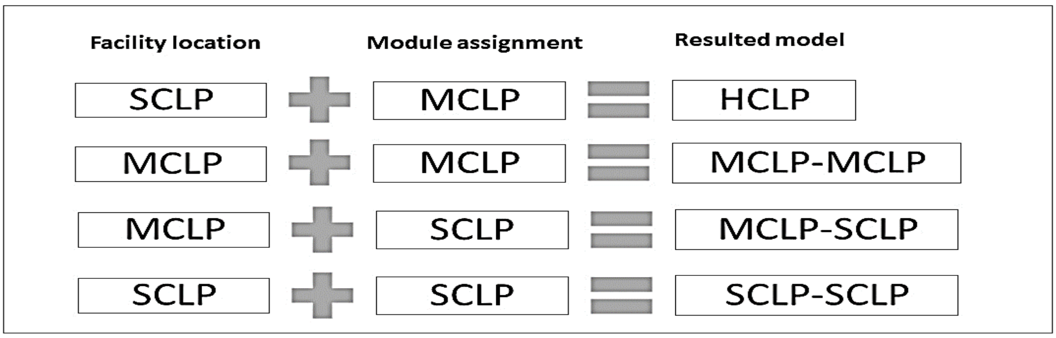

Figure 4.2: The structure of different possible hybridization models.

4.3.1 MCLP-MCLP

MCLP-MCLP locates the facilities utilizing MCLP and the decisions for module assignment and demand points allocation are determined by MCLP as well. In contrary to the HCLP that gives the solution to both numbers and location of the facilities, the number of the facilities is a parameter and the problem only finds the optimal location of these predefined number of facilities in MCLP-MCLP. In addition, while the goal of HCLP is to cover all demand points by the facilities, this goal is not valid anymore in MCLP-MCLP and it only seeks to maximize the total covered demand points. The main difference of HCLP and MCLP-MCLP is the constraint (4.2) that should be substituted by constraint (4.12) that implies the total number of located facilities in each strategic period cannot exceed the number defined beforehand. The mathematical formulation of the MCLP-MCLP can be modified as follows:

$$\max \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}\sum_{\tau\in \mathcal{T}_t} a_{jlt\tau} g_{ij} d_{jlt\tau} x_{ijlt\tau} \tag{4.11}$$

$$\sum_{i\in I} z_{it} \leq p_t \qquad \forall t \in T \tag{4.12}$$

(4.3) – (4.10).

where p_t is the number of predefined facilities for each strategic period that is defined by the decision-makers as a parameter. Note that having the number of located facilities depending on the time periods is to keep the expansion capabilities of the model. Otherwise, it can be a fixed number for all periods.

4.3.2 MCLP-SCLP

This model's structure exploits the coverage concept of covering problems in a contrary format to HCLP model. In MCLP-SCLP, it is supposed that facilities obey the coverage concept of MCLP having specified numbers as strategic decisions, while the minimum number of modules has to be determined by SCLP in order to provide full coverage for each point from each module type as tactical decisions. The mathematical model of MCLP-SCLP has a similar formulation to MCLP-MCLP for facility location part but the difference is in constraint (4.7) that should be replaced with constraint (4.14) that indicated each module type should be assigned to the opened facilities in a way that can provide full coverage for each demand point in each tactical period of strategic period. As a result, the objective of the MCLP-SCLP maximizes the profit obtained from the income of covering demand points and the cost of assigning the modules. The mathematical formulation of MCLP-SCLP is as follows:

$$\max \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{t\in T}\sum_{\tau\in \mathcal{T}_t} a_{jlt\tau} g_{ij} d_{jlt\tau} x_{ijlt\tau} - \sum_{i\in I}\sum_{l\in L}\sum_{k\in K}\sum_{t\in T}\sum_{\tau\in \mathcal{T}_t} b_{ilkt\tau} y_{ilkt\tau} \qquad (4.13)$$

$$\sum_{i\in I}\sum_{k\in K} e'_{ij} y_{ilkt\tau} \geq 1 \qquad \forall j \in J, l \in L, t \in T, \tau \in \mathcal{T}_t \qquad (4.14)$$

(4.12), (4.3) – (4.6), (4.8) – (4.10).

where $b_{ilkt\tau}$ is the cost of assigning the size k of module l to the facility i at tactical period τ of strategic period t and e'_{ij} is the binary parameter which is 1 if $dis_{ij} \leq S$, 0 otherwise.

4.3.3 SCLP-SCLP

In SCLP-SCLP both facility location and modules assignment decisions are determined using the coverage concept of SCLP, in a way that it is desired to provide full coverage of points from facilities in strategic periods and from each module type in tactical periods of strategic periods. In contrary to the objective function of other defined models, the objective of SCLP-SCLP minimizes the cost of facility location and module assignment. To follow SCLP's theoretical perspectives, the objective function of maximizing the coverage of the demands. The mathematical formulation of the SCLP-SCLP model is as follows:

$$\min \sum_{i \in I} \sum_{t \in T} f_i z_{it} + \sum_{i \in I} \sum_{l \in L} \sum_{k \in K} \sum_{t \in T} \sum_{\tau \in \mathcal{T}_t} b_{ilkt\tau} y_{ilkt\tau} \qquad (4.15)$$

(4.2) – (4.6), (4.8) – (4.10), (4.14).

4.4 Experimental tests

4.4.1 Case study; application of HCLP in humanitarian logistic services

According to the latest report of the Centre for Research on the Epidemiology of Disasters (CRED) [72], "in 2019, at least 396 natural disasters were reported, killed 11,755 people, affected 95 million others and costing nearly 130 billion US dollars. Floods, storms and droughts accounted for almost 99% of the total number of affected people." More importantly, the report indicates that "the number of events in 2019 was slightly over the average of the last 10 years." The highest priority in these kinds of situations is to help the survivors. Despite the unknown occurrence time and the place of the natural disasters, emergency preparedness and response activities should be conducted as pre-disaster and post-disaster actions. One of the applications of the proposed HCLP model is locating aid centers and module assignment that can improve the impact of strategic and tactical relief operations in humanitarian situations caused by disasters such as earthquakes, floods, storms, wars, medical epidemic emergencies and droughts. According to the Disaster Operations Management (DOM) framework [73], disaster operations are usually categorized into four main phases as mitigation, preparedness, response, and recovery as shown in Figure 4.3. The application of HCLP in the humanitarian facility location problem, in which the location of the capacitated aid centers is determined as strategic decisions belongs to the preparedness activity phase. In the response phase, the assignment of the service providing units to the located aid centers with the objective of maximizing the covered demand of affected people is

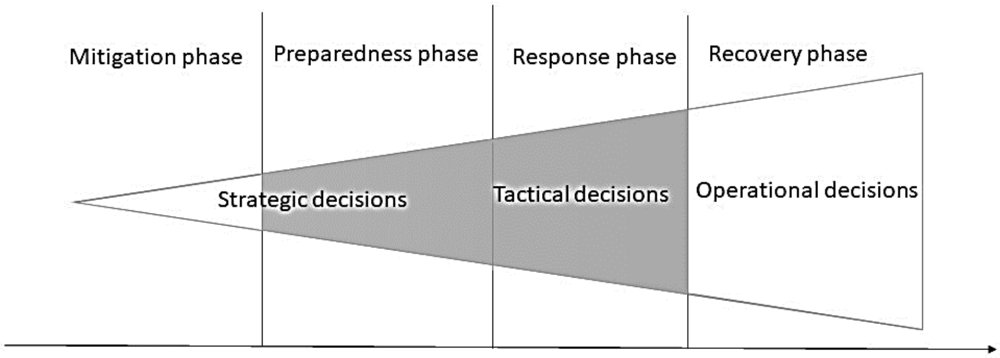

Figure 4.3: Different disaster operational phases and supply chain decision categories.

determined. From Figure 4.3, the decisions during mitigation and preparedness activities can be regarded as strategic decisions, the decisions during the response activity can be regarded as tactical decisions and in the same way, the decisions during recovery phase are assumed as operational decisions. The gray filled part illustrates the domain of HCLP in humanitarian services, which includes the strategic and tactical decisions of preparedness and response phases.

When a disaster happens in any region, the government or any responsible organization can dispatch the limited modules (trucks, helicopters, medical services, mobile kitchens, shelter tents and etc.) to the located facilities to start service operations there. When the modules fulfill their operations, they can be dispatched to be assigned to the other facilities of other affected regions according to the demand requests. In this case study, it is supposed that in each strategic period one of the areas in south-central (R1), north-central (R2), and center (R3) of Japan are affected by a disaster requesting for services provided by the modules in tactical periods. Each tactical period is equivalent to one month and each strategic period is composed of three tactical periods. The north-central part (R2) can correspond to the Japan 2011 earthquake. The number of 160 points matching the cities with more than 150,000 inhabitants according to census results and latest estimates [74] is considered as demand points and also as the potential locations for locating facilities. There are four kinds of modules with four possible sizes, three strategic periods each composed of three tactical periods. Parameters q_l, c_l, f_i, and h_i are generated randomly using uniform distribution between (30, 50), (200000, 300000), (700000, 900000), and (4000000, 6000000), respectively. In this case study, the threat scenarios are designed in this way that firstly one disaster hits the south-central (R1) in the first strategic period, the second disaster occurs in the north-central part (R2) in the second strategic period and in the third strategic period, it is the central part (R3) that is affected by a disaster and needs the modules assignment.

Figure 4.4 shows the demand points in green color and the located facilities in blue in the last strategic period. The arrows show the flow of the modules from previously affected areas to the newly affected one to provide service. The blue circles have been used to demonstrate the affected areas. The solution of the problem for these threats the first area (R1) affected by the disaster could be covered 86.8%, the coverage for the second area (R2) was 99.6% and in the last affected area (R3) the coverage of demand points was 43.3% by the limited number of available modules. Table 4.2 includes the results of this case study in the second row. The coverage percent is calculated as:

$$\frac{\sum_{i,j,l,t,\tau} g_{ij} d_{jlt\tau} x_{ijlt\tau}}{\sum_{j,l,t,\tau} d_{jlt\tau}}$$

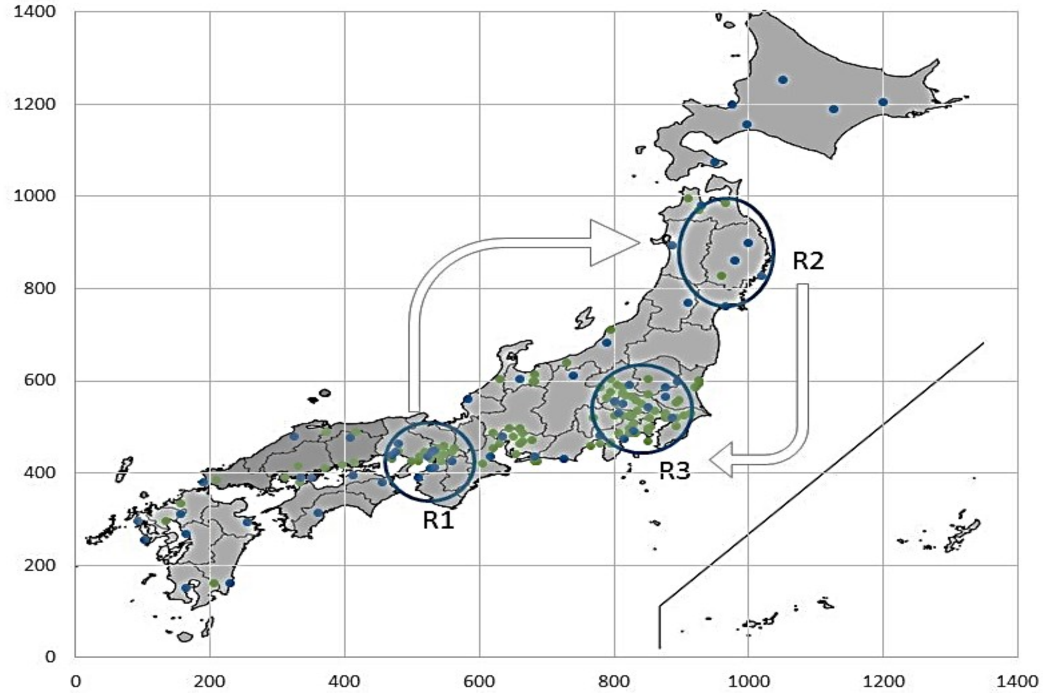

Figure 4.4: The demand points and facilities located at the last strategic period.

The second column of Table 4.2 shows the operating facilities in each strategic period for R1, R2, and R3. The column "MZ" and "MY" contain the average number of located facilities and modules respectively. Columns "Obj" and "Time" include the objective value and elapsed time to solve the problem. Other rows of Table 4.2 contain the results for some sensitivity analysis of capacity and cost parameters. In the second solved problem, the capacities of the modules have been decreased, which has resulted in significant coverage percentage for all regions and objective value. The number of located facilities does not change so much, but the problem tries to provide more coverage by assigning more modules. In the third problem, the effect of increasing the facility location cost has been the main purpose. The results

Table 4.2: Computational results of case study and sensitivity analysis.

Problem data	Operating facilities in R1, R2 and R3	Coverage %	MZ	MY	Obj	Time
q_l: U(30,50) cp_l: U(200k,300k) f_i: U(70k,90k) h_i: U(4M, 6M)	R1: Amagasaki, Izumi, Kakogawa, Kawanishi,Kishiwada, Nara, Sakai, Suita, Takatsuki, Wakayama, Yao. R2: Aomori, Iwaki, Morioka, Sendai. R3: Atsugi, Funabashi, Hino, Hitachinaka, Kawaguchi, Maebashi, Nagareyama, Noda, Odawara, Oyama, Saitama.	R1: 86.8% R2: 99.6% R3: 43.3%	51.6	33.9	256M	165
q_l: U(30,50) cp_l: U(20k,30k) f_i: U(70k,90k) h_i: U(4M, 6M)	R1: Amagasaki, Kakogawa, Kishiwada, Nara, Sakai, Suita, Takatsuki. R2: Aomori, Fukushima, Hachinohe, Iwaki, Koriyama, Morioka, Sendai. R3: Atsugi, Funabashi, Hino, Hitachinaka, Kawaguchi, Maebashi, Odawara, Oyama, Saitama.	R1: 8.6% R2: 37.5% R3: 4.3%	50.6	39.9	22M	71
q_l: U(30,50) cp_l: U(20k,30k) f_i: U(100k,150k) h_i: U(4M, 6M)	R1: Amagasaki, Izumi, Kakogawa, Kishiwada, Nara, Sakai, Suita, Takatsuki, Wakayama, Yao. R2: Aomori, Iwaki, Morioka, Sendai. R3: Atsugi, Funabashi, Hino, Hitachinaka, Kawaguchi, Maebashi, Nagareyama, Odawara, Oyama, Saitama.	R1: 86.7% R2: 99.6% R3: 43.3%	51.3	33.9	250M	330
q_l: U(30,50) cp_l: U(20k,30k) f_i: U(100k,150k) h_i: U(4M, 6M)	R1: Akashi, Amagasaki, Higashiosaka, Himeji, Hirakata, Ibaraki, Itami, Izumi, Kakogawa, Kawanishi, Kishiwada, Kobe, Kyoto, Nara, Neyagawa, Nishinomiya, Okayama, Osaka, Otsu, Sakai, Suita, Takarazuka, Takatsuki, Toyonaka, Uji, Wakayama, Yao. R2: Aomori, Fukushima, Hachinohe, Iwaki, Koriyama, Morioka, Sendai. R3: Ageo, Atsugi, Chiba, Chigasaki, Chofu, Fucho, Fujisawa, Funabashi, Hachioji, Hino, Hiratsuka, Hitachinaka, Ichihara, Ichikawa, Isesaki, Kamakura, Kashiwa, Kasukabe, Kawagoe, Kawaguchi, Kawasaki, Kodaira, Koshigaya, Kuki, Kumagaya, Machida, Maebashi, Matsudo, Mitaka, Mito, Nagareyama, Narashino, Niiza, Nishitokyo, Noda, Odawara, Ota, Oyama, Sagamihara, Saitama, Sakura, Sayama, Soka, Tachikawa, Takasaki,Tochigi, Tokyo,Tsukuba, Urayasu, Utsunomiya,Yachiyo.	R1: 39.5% R2: 47.6% R3: 33.9%	93	35.1	140M	1046

show that the increase in facility location cost does not have that much effect on the coverage percentage and assigned modules, but the number of located facilities and as a result, the objective value decrease. In addition, the last solved problem investigates the effect of the decrease in facilities capacity.

According to the results when the capacity of facilities decreases more facilities have been located, but even this increase in the number of facilities cannot compensate for the reduction in the coverage percentage. On the other hand, locating more facilities imposes costs, which is reflected in the objective value reduction.

4.4.2 Numerical results

In this section, some test problems have been generated randomly with different sizes to examine the performance of the developed model. For this purpose, the test problems have been designed in two main directions. In the first experiment, it is

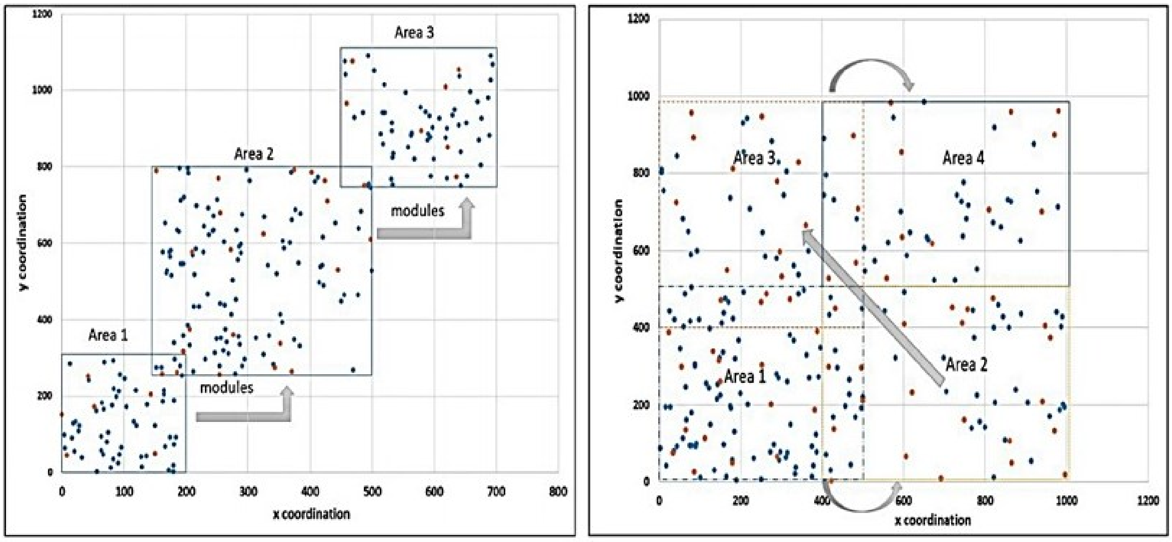

Figure 4.5: A schematic illustration of three region and four region test problem.

Table 4.3: Computational results of test problems for three regions experiment.

$i = j$	Demand scenario	t	τ	l	k	q_l	cp_l	h_i	f_i	Obj	Coverage %	Time
50	Low	3	2	2	2	U(4,7)	U(150,250)	U(500,1000)	U(400,800)	3,042.1	86%	4
	High	3	2	2	2	U(4,7)	U(150,250)	U(500,1000)	U(400,800)	4,359.5	65%	3
100	Low	3	3	2	3	U(7,10)	U(200,250)	U(1000,1500)	U(400,800)	17,251.2	84%	213
	High	3	3	2	3	U(7,10)	U(200,250)	U(1000,1500)	U(400,800)	21,936.2	62%	249
150	Low	3	3	3	3	U(10,15)	U(200,250)	U(1500,2000)	U(400,800)	41,056.5	86%	1013
	High	3	3	3	3	U(10,15)	U(200,250)	U(1500,2000)	U(400,800)	50,697.4	62%	1073
200	Low	3	3	4	3	U(15,20)	U(300,350)	U(1000,1500)	U(400,800)	25,134.5	30%	1029
	High	3	3	4	3	U(15,20)	U(300,350)	U(1000,1500)	U(400,800)	26,740	22%	1072
250	Low	3	4	4	3	U(20,30)	U(300,350)	U(1000,1500)	U(400,800)	24,545.6	21.9%	2169
	High	3	4	4	3	U(20,30)	U(300,350)	U(1000,1500)	U(400,800)	27,159.4	14.1%	1479
	Low	3	4	4	3	U(20,30)	U(300,350)	U(1500,2000)	U(400,800)	28,235	24%	1203
	High	3	4	4	3	U(20,30)	U(300,350)	U(1500,2000)	U(400,800)	31,482.5	16%	1355
	Low	3	4	4	3	U(20,30)	U(300,350)	U(2000,3000)	U(400,800)	28,235	24%	1169
	High	3	4	4	3	U(20,30)	U(300,350)	U(2000,3000)	U(400,800)	31,482.5	16%	1399
	Low	3	4	4	3	U(20,30)	U(350,450)	U(1000,1500)	U(400,800)	25,744.6	22.6%	1343
	High	3	4	4	3	U(20,30)	U(350,450)	U(1000,1500)	U(400,800)	28,396	14.6%	1111

supposed that there are three regions affected by disasters with two kinds of high and low demand scenarios in three strategic periods. In the second experiment, there are four regions affected by the disasters with high and low demand scenarios in four strategic periods. A schematic illustration of three regions and four regions test problems together with the located facilities and module movement flow is depicted in Figure 4.5 for the case with 250 demand points. In each kind of experiments, the dimensions of the test problems are augmented gradually. Tables 4.3 and 4.4 contain the results of test problems. The problems are solved using GAMS (CPLEX solver) software (24.1.2) on a PC with a 3.4-GHz Core i7-6700 CPU and 8 GB of RAM running Windows 10 (64 bit).

Table 4.4: Computational results of test problems for four regions experiment.

$i = j$	Demand scenario	t	τ	l	k	q_l	cp_l	h_i	f_i	Obj	Cover age %	Time
50	Low	4	2	2	2	U(4,7)	U(150,250)	U(400,800)	U(400,800)	715.1	82%	6
	High	4	2	2	2	U(4,7)	U(150,250)	U(400,800)	U(400,800)	2,660.9	76%	5
100	Low	4	3	2	3	U(7,10)	U(200,250)	U(1000,1500)	U(400,800)	13,751.9	68%	82
	High	4	3	2	3	U(7,10)	U(200,250)	U(1000,1500)	U(400,800)	18,627.9	62%	126
150	Low	4	3	3	3	U(10,15)	U(200,250)	U(1500,2000)	U(400,800)	49,496.5	83%	499
	High	4	3	3	3	U(10,15)	U(200,250)	U(1500,2000)	U(400,800)	60,916.8	76%	672
200	Low	4	3	4	3	U(15,20)	U(200,250)	U(1500,2000)	U(400,800)	66,291.6	55%	1083
	High	4	3	4	3	U(15,20)	U(200,250)	U(1500,2000)	U(400,800)	77,956.4	46%	1076
250	Low	4	4	4	3	U(20,30)	U(300,450)	U(1500,2000)	U(400,800)	63,323.5	52%	1115
	High	4	4	4	3	U(20,30)	U(300,450)	U(1500,2000)	U(400,800)	76,925	43%	1757

It must be noted that all parameters of the test problems are designed in a way that there would be no redundant constraints. The results for all test problems approve that problems of low scenarios result in less objective values, but a higher percentage of demands can be covered in the low scenario problems compared to the high demand scenario problems. By increasing the size of the problems, the elapsed time also increases for both kinds of problems in Tables 4.3 and 4.4. For the problem of the size 250 points of three regions, we have conducted some sensitivity analysis. As mentioned, the first two problems of low and high demands are the problems that all the constraints are active. By increasing the capacity of facilities, in second and third problems the constraints of facilities capacity become redundant, so that changing these parameters has no more effect on the solutions. In the last problem, the capacities of the facilities are set to be active and the capacities of the modules have been increased that these changes have yielded to increase in objective value and coverage percentage, though the difference is not much significant. Comparing the results of coverage percentage for two tables, it becomes clear that for large-scale problems the problems of four regions could provide significantly better coverage. The reason for the difference in parameter values in two kinds of problems (Tables 4.3 and 4.4) is to avoid having redundant constraints. GAMS was able to solve the problems of 250 demand points. However, this size of the problem is not regarded as small size because considering the modules and sizes and two kinds of time periods, the real size of problems are in the category of large-size problems.

4.4.3 Model validation and comparison results

To validate the developed model, two approaches are deployed. In the first approach, the solutions of hybrid covering location problem will be compared with the results of the conventional separate models separately for some of the test problems from Table 4.3. In order to do this, we compare the results of HCLP with the problem in which the location of the facilities would be selected by SCLP separately, using the second term of the objective function (4.1) (minimizing the cost of facility location) subject to constraints (4.2), (4.3), (4.6) and (4.9). The solutions of the located facilities are used and fixed in the second problem to assign the modules and allocated the demand points using the first term of the objective function (4.1) (maximizing the total covered demands) subject to constraints (4.4) – (4.10).

The computational results are illustrated in Table 4.5 in which the columns under "Hybrid" and "Con." include the results of the hybrid covering location problem and the described conventional procedure, respectively. Rows of "Obj" show the objective value of income minus fixed cost of facility locations for both methods. Furthermore, the last row of Table 4.5 shows the results for the total number of the assigned modules in all time periods out of the predefined amounts of available modules. For example, the value 58/66 means that the problem has used 58 modules out of 66 available modules. The higher is this proportion; the problem has used the available resources in a better way. The results of this conducted comparison show that using the integrated location decisions of SCLP and MCLP in a united model can improve the objective value and this fact can be interpreted as the quality of provided services. It is important to note that the less value of facility cost in the conventional approach cannot be an advantage, as far as the total objective value is not better than hybrid approach and these values are calculated and included in the table to have the evaluation of cost differences in both approaches. The results of Table 4.5 make it apparent that the hybrid approach has significant superiority to the conventional approach regarding objective value, coverage percentage and exploiting the available modules.

The same problems of the previous section in Tables 4.3 and 4.4 are solved for problems MCLP-MCLP, MCLP-SCLP, and SCLP-SCLP. It is important to note that first HCLP was solved and obtained the total number of facilities that is determined to be opened and then used these numbers and run the experiments for the problems that need to have the number of facilities to be located, i.e., MCLP-MCLP and MCLP-SCLP.

To be able to compare the models, two criteria have been investigated. The first one that is in alignment with the optimization criteria as equity or fairness [76] that

Table 4.5: Computational results for comparing performance of hybrid and conventional approaches.

	50 high		100 high		150 high		200 high	
	Hybrid	Con.	Hybrid	Con.	Hybrid	Con.	Hybrid	Con.
Obj	**4,359**	1,276	**21,936**	10,956	**50,697**	10,126	**26,740**	9,689
Facility cost	5,258	2,470	5,025	2,559	11,230	2,470	11,712	2,409
Coverage %	**65.7%**	25.6%	**61.9%**	31%	**62%**	12.8%	**22%**	0.7%
Total modules	**58/66**	28/66	**120/126**	72/126	**304/306**	108/306	**216/621**	144/621

Table 4.6: Computational results to compare different variants for three regions.

$i = j$	Demand scenario	$p_{\|T\|}$	$HCLP$				$MCLP - MCLP$				$MCLP - SCLP$				$SCLP - SCLP$			
			%	Z	Y	T	%	Z	Y	T	%	Z	Y	T	%	Z	Y	T
50	Low	3	86	3.3	4.3	6	79	3	3.8	2	82	3	5.5	4	0	3	4.6	0.8
		4	-	-	-	-	89	3.3	4.2	7	91	4	6	11	-	-	-	-
	High	3	65	3.6	4.3	2	53	3	3.8	2	59	3	5.6	2	0	3	4.2	0.7
		4	-	-	-	-	61	3.3	4.3	4	74	4	7	11	-	-	-	-
100	Low	3	84	3.6	6.5	213	75	3	5.3	219	Inf				41	100	197	1006
		4	-	-	-	-	82	4	6	904	Inf				-	-	-	-
	High	3	62	3.6	6.6	255	50	3	5.3	268	Inf				42	100	197	1006
		4	-	-	-	-	59	4	6.3	180	Inf				-	-	-	-
150	Low	8	86	8	10.7	10013	RE				NS				39	150	301	1015
	high	7	62	7.6	11.2	10473	RE				NS				40	150	301	1019
200	Low	7	30	7	6	1064	RE				NS				44	200	399	1051
	High	7	22	7	6	1050	RE				NS				44	200	399	1052
250	Low	22	22	22	5.3	1583	RE				NS				40	250	499	1094
	High	22	14	22	5.3	1505	RE				NS				40	250	499	1103

Inf: Infeasible

RE: Resource exceeded

NS: No Solution

we interpreted it here as the percentage of coverage provided for demand points calculate as the total amount of covered demand divided by the total amount of demands as explained earlier. The next investigated optimization criterion is the efficiency of models as the elapsed time to obtain the results. The columns under "Z" and "Y" contain the average number of facilities in all strategic periods and the average number of the assigned modules in all tactical periods, respectively. Tables 4.6 and 4.7 contain the results of these evaluations. To complete the tables and solve the problems of MCLP-MCLP, and MCLP-SCLP, there was needed to have the number of facilities to be located. For this purpose, firstly the problem HCLP was solved and its solutions of the located facilities in the last strategic period ($|T|$) were used in MCLP-MCLP and MCLP-SCLP with two different values; one with the higher number (to provide even more facilities) and the other one with the optimal solution of HCLP. Solving problems for the values of $p_{|T|}$ smaller than the values in Tables 4.6 and 4.7 would not result in better solutions. According to the results of coverage percentage for HCLP, MCLP-MCLP, and MCLP-SCLP, HCLP could provide the highest coverage percentage almost for all test problems with an optimum number of facilities. Only in three cases with the number of facilities more than the optimal number, the coverage percentage was better (one case of MCLP-MCLP and two cases of MCLP-SCLP).

Notice that we have excluded SCLP-SCLP from these comparisons because we believe that the solution, which sets all the demand points to be a facility is not a practical solution. Among the three variants of MCLP-MCLP, MCLP-SCLP and SCLP-SCLP, it was expected that SCLP-SCLP can provide the best coverage as it does not have any budgetary limitations. However, the results show that this problem has a good performance neither for coverage nor the number of located facilities and modules. The problem MCLP-SCLP has good performance for the only one size problem that it could solve and for the rest of the problems it was either infeasible or no solution was found by the solver. Among all three variants, MCLP-MCLP is the only problem that was successful in obtaining solutions. Although the coverage percentage is not better than HCLP, it has overall acceptable performance. However, this problem was not able to solve larger problems and the general-purpose solver required higher computational effort. Concerning the optimization criteria mentioned above, the results obtained from test problems show that in terms of both equity (coverage percentage) and efficiency (elapsed time), the problem HCLP outperforms other variants and can provide better coverage in a reasonable time for different size of test problems.

Table 4.7: Computational results to compare different variants for four regions.

$i = j$	Demand scenario	$p_{\|T\|}$	$HCLP$				$MCLP-MCLP$				$MCLP-SCLP$				$SCLP-SCLP$			
			%	Z	Y	T	%	Z	Y	T	%	Z	Y	T	%	Z	Y	T
50	Low	4	82	4.5	5.2	6	82	3.5	4.7	10			NS		0	5	7.5	1
		5	-	-	-	-	89	4.5	5	8	76	5	8.1	3	-	-	-	-
	High	4	76	4.5	5.5	3	76	3.5	5.4	10			NS		0	5	7.25	2
		5	-	-	-	-	82	4.5	5	11	70	5	8.5	3	-	-	-	-
100	Low	5	68	5.5	6.3	84	63	3.7	5.2	98	25	5	11.4	397	21	6	12.9	292
		6	-	-	-	-	68	4.5	6.3	100	27	6	11	584	-	-	-	-
	High	5	62	5.7	6.5	53	61	4.2	6.6	45			NS		30	7	15.8	485
		6	-	-	-	-	60	4.2	6.1	32			NS		-	-	-	-
150	Low	7	83	7.5	10	499	76	5.7	8.5	1020			NS		28	21.7	26.4	1023
		8	-	-	-	-	72	4.7	8.1	1015			NS		-	-	-	-
	High	7	76	8	10.1	672	62	4.7	8.5	1025			NS		28	150	301	1029
		8	-	-	-	-	48	3.7	6.3	1022	22	8	24	1020	-	-	-	-
200	Low	14	55	14	11.6	1083			RE				NS		30	200	399	1062
	High	14	46	14	11.7	1076			RE				NS		31	200	399	1094
250	Low	21	52	21.7	15.1	1115			RE				NS		27	250	500	1131
	High	21	43	21.7	15.1	1757			RE				NS				RE	

RE: Resource exceeded

NS: No Solution

The number of variables is the same in all variant problems, but the number of constraints is different that has an influence on the efficiency of the results. As the constraints of (4.3) – (4.6), (4.8) – (4.10) and (4.14) are the same in all problems we name this set of constraints Δ and our comparison takes into account the rest of the constraints in each problem. The number of constraints and an example of the problem with 50 demand points studied as the first problem in Table 4.3 is included in Table 4.8. In our test problems, the number of modules, strategic and tactical periods do not take large values, but it is the number of demand points that has a considerable impact by augmenting the size of problems. Therefore, it can be concluded that SCLP-SCLP has the largest number of constraints and the order after SCLP-SCLP is MCLP-SCLP, HCLP, and MCLP-MCLP. However, the computational times of problems and the fact that MCLP-MCLP/ MCLP-SCLP have exceeded resources/no solution results for most of the test problems, the difference in the number of constraints does not have a significant impact on the quality of results or computational time.

Table 4.8: Comparing total number of constraints in HCLP, MCLP-MCLP, MCLP-SCLP and SCLP-SCLP.

Problem	Number of constraints	Example
HCLP	$\Delta + \lvert J\rvert\lvert T\rvert + \lvert L\rvert\lvert T\rvert\lvert \mathcal{T}_t\rvert$	$\Delta + 50 \times 3 + 2 \times 3 \times 2 = \Delta + 162$
MCLP-MCLP	$\Delta + \lvert T\rvert + \lvert L\rvert\lvert T\rvert\lvert \mathcal{T}_t\rvert$	$\Delta + 3 + 2 \times 3 \times 2 = \Delta + 15$
MCLP-SCLP	$\Delta + \lvert T\rvert + \lvert J\rvert\lvert L\rvert\lvert T\rvert\lvert \mathcal{T}_t\rvert$	$\Delta + 3 + 50 \times 2 \times 3 \times 2 = \Delta + 603$
SCLP-SCLP	$\Delta + \lvert J\rvert\lvert T\rvert + \lvert J\rvert\lvert L\rvert\lvert T\rvert\lvert \mathcal{T}_t\rvert$	$\Delta + 50 \times 3 + 50 \times 2 \times 3 \times 2 = \Delta + 750$

4.5 Summary

To address the facility location problem in a disaster relief situation a novel model combining the advantages of two major covering location problems was developed. The coverage concept of two major covering location problems, set covering location problem and maximal covering location problem, was utilized to develop the model of the hybrid covering location problem. In the developed HCLP, the location of facilities is determined by using SCLP and the limited number of modules providing different services can be assigned to the facilities to provide services in tactical periods. To investigate the capability of the developed hybrid covering location problem, an application for it was introduced as locating aid centers in humanitarian relief services. A case study using real data for demand points in Japan was used together with some more randomly generated test problems. The results of the studied problems show that the developed mathematical model can obtain accurate solutions compatible with the real situations and the assumptions of the model.

Furthermore, the other possible combinations of covering location problems were developed as the variants of the main hybrid covering location problem. To evaluate four developed models, some test problems were generated and solved for all variants. The computational results approve that the main developed hybrid model can outperform the other three variants in terms of coverage percentage, solution quality and feasibility of the solutions.

One important fact about the developed hybrid covering location problem is that it can be solved with commercial solver (GAMS) for problems of an acceptable size of real-life situations, which is an important specification for problems arising in disaster situations that need quick responses. However, in most of the problems studied with other researches, the difficulty to solve the problems to obtain the solutions is a barrier to be applicable in disaster situations. The main purpose of this chapter was to introduce the basic and original framework of hybrid covering location problem. While the problem developed in this chapter can be used to be coupled with other decisions of the supply chain such as inventory management and vehicle routing. In addition, it can be studied as a two-stage or a multi-stage stochastic programming model that can be a future direction for research.

Chapter 5

Non-cooperative game for multi-period *p+q* maximal hub location problem for freight transportation planning with rational market

5.1 Introduction

Significant progress has been made in the hub location decisions due to recent globalization of freight transportation. Buying goods from other countries is a usual transaction in recent globalization. Transportation companies are enlarging their business in quantity and size to deliver domestic and international orders to customers. To deal with increasing demands and gain more market share, these companies should exploit some strategies in building hub locations for the future. Transportation companies can choose two main expansion strategies to be able to survive in the future competitive business environment. One of the expansion strategies would be to add up the number of hubs, which they are using to carry the demands through. By adding more hubs to the network, companies can connect more nodes and give better services to their customers. Other expansion plan is to buy or lease more carriers to be able to carry more orders. These two expansion plans are included in the mathematical model for the time periods of the planning horizon.

No one can deny the importance of customers in delivery services. If customers are not pleased with the services, they will get service from the rivals and you will lose the profit, reputation and the market share. As customers' participation in today's competitive environment is inevitable, they should participate in the pricing process. This participation of customers is included in the model's constraints by minimizing the cost paid by customers, which creates a bi-level problem.

The model studied in this chapter focuses on the increasing demand in the future. Expansion plans according to the demand increase are included in the model, in which by participating the customers in the decision-making process the model has become a bi-level model, maximizing the freight company's profit in the upper-level and minimizing the cost paid by customers in the lower-level. A Benders decomposition-based method and two reformulations have been conducted to solve the problem and obtain solutions for numerical examples.

Our main contribution is summarized as follows. A dynamic $p+q$ hub location model for freight transportation planning with rational market is developed. The problem is formulated as a mixed-integer bi-level optimization problem. Two approaches are conducted to reformulate it into a single-level problem. An efficient Benders decomposition-based algorithm is developed and these approaches are compared from computational experiments.

This reminder of the chapter is organized as follows. A literature review is presented in section 5.2. The bi-level problem with rational markets is developed in section 5.3. In section 5.4 the bi-level problem is transformed into a single-level problem with a dual based reformulation and KKT conditions. An efficient method based on Benders decomposition method is proposed to solve the problem in section 5.6. Computational experiments and sensitivity analyses are presented in section 5.7. Finally, in the last section of this chapter is concluding with an overview of the findings discussed.

5.2 Literature review

Since its introduction in 1970s, bi-level optimization has got tremendous research consideration and it has been broadly used in problems with hierarchical decision making. These hierarchical decision-making problems arise so often in transportation planning and network capacity expansion [76], government policy making [77] and revenue management [78]. The study by Garcia-Herreros et al. [79] formulated the capacity expansion planning as a bi-level optimization to model decision-making structure, which exists between producers and consumers in the industry. They have formulated their problem as a mixed-integer bi-level linear program in which the upper-level maximizes the profit of a producer and the lower-level minimizes the cost paid by markets. The lower-level problem is a linear problem assigning demands of the customers to the cheapest offer of the companies while in the upper-level problem, the problem gives solution to expansion planning variables that are mixed integer variables. They also reformulated their bi-level optimization problem as a single-level problem to be able to solve the problem to obtain solutions for their test problems.

Hemmati and Smith [80] considered the competitive set covering problem as a two-player Stackelberg game that both of the players try to maximize their profit. In this problem in both levels of the problem, there are binary variables. This kind of problem occurs in situations like facility location games. To solve their problem, they developed a cutting-plane algorithm and showed that their solving procedure has superiority to other available procedures.

Most recently, Zhang et al. [81] have examined hub location and plane assignment problems for the air-cargo delivery service. They presented two mixed integer programming models. The difference between these two models is related to how they manage the number of visiting hubs for giving service to each origin-destination pair. Since the problem was NP-hard, they developed a two-stage hybrid algorithm to solve large-scale test problems. In this algorithm, the first stage finds the solution for main variables by heuristics and in the second stage the algorithm finds the solution to the remained variables by a commercial solver. They used real-life data to solve numerical examples to test the efficiency of the models and solving approach.

The above-mentioned works are very noteworthy attempts, especially because of their application to deal with an important real-life fact transportation. To the authors' best knowledge, there are rare studies on multi-period hub location problem by Gelareh et al. [82], which is one of the first models developed for the uncapacitated multi-period multiple allocation hub location problem. They have tried to include many features of real-life situations in their work, especially features related to land transportation facts. In a precious attempt they presented a metaheuristic algorithm, which can give high-quality solutions in a reasonable time. They also developed an extension of Benders decomposition technique to solve the test problems and compare the results for both procedures.

5.3 Problem definition

The *p+q* dynamic maximal hub location problem for freight transportation planning is described as follows:

Suppose there is a freight company who determines the hub location planning and the required number of transportation vehicles, even operating in air freight transportation or road or other kind of carriers. The carriers that are used by the company to transfer the demand between nodes are called transportation vehicles or carriers. This company uses the hub and spoke system to traverse the flow among nodes and has predefined decisions on the number of established hubs when demand of transportation is given.

The company has some rivals that are operating in the same business environment and even may use the same hubs that our company is using. All rivals are in the same level from our company's point of view. This means that in total the whole market share is in two parts: our company's market share and the rest that goes to the rivals as one amount.

In addition, the company has information about the increasing demand in the future and is eager to gain benefit through establishing some extension plans. This expectation on the future increasing demand and the necessity to gain advantage of this situation has led to study the problem as a dynamic model.

The company has some dominant customers that have great power and can control the prices by sourcing the demand to the rivals. This ability of the customers is included in the constraints of the model that has made the model to be a bi-level one. The freight company sells the service of flow by sales price depending on origin and destination and customers will buy the service with the sales price. The freight company makes its decisions in the upper-level problem as the leader and the follower in the lower-level, represents the response of markets that select freight companies as providers, with the unique interest of satisfying their demands at the lowest cost. The leader in the first step defines its price and in the next step, the follower reacts according to the prices of the available companies to choose the cheapest one.

The nodes of the network are shown by $N = R \cup U = \{1, \dots, n\}$, U refers to the set of nodes that are controlled by the company while R refers to the set of nodes controlled by the rivals. i, j, k, m are used to refer to the nodes of this graph in which i, j refer to spokes and k, m refer to hubs. The markets place an order to be traversed from origin node i to destination node j. The company has freedom in choosing the hubs to carry this order and also the order can be divided and carried using different hubs and seeks to maximize the flow carried by carriers to maximize the income. If the company does not have enough capacity or possibility to transport the order and the order has been already booked, then the company can outsource it to a sister company. It has been achieved by using the parameter o_{km} that indicates 1 if there are outsourcing services between nodes i and j and 0 otherwise.

Up to here our problem is a $p + q$ dynamic multi-allocation maximal hub location problem (p is the number of hubs to be utilized in the first period of time and the q is the number of hubs that the company is going to add to its current hubs as expansion plan) in which markets are rational and have the possibility to select their providers according to their own interest, i.e., price.

Given the information of travel time (distance) between n nodes, the price paid by customers, demand of transportation and capacity of the carriers, fixed costs of locating a hub, market share for the competitors, rival's hub location, the number of carrier type l in the link $k - m$ for rivals, binary parameter $g_{ikmj} \in \{0,1\}$ whether or not the demand of origin-destination $i - j$ can be serviced according to the time (distance) needed to travers from hubs k and m depending on the discount factor α, the number of hubs at the first period, the number of hubs as expansion hubs, deadline traveling time (or distance), that is used as covering criteria in which if the time (distance) needed to travel from an origin to its destination through assigned hubs is more than this deadline, the related demand will remain unserved, the decisions to be made in this model are: In the upper-level problem; which hubs are going to be opened at the start of planning horizon, and which hubs will be located in the future periods? Which hubs are operating in each period? Other decisions are related to the number and type of carriers in each period. In addition, in the lower-level problem; the amount of flow that is going to be carried through the hubs would be decided.

Note 1. The company's expansion plan is about opening new hubs or buying/leasing new carriers in future periods. Furthermore, when the company opens a hub either at the start of planning horizon or in expansion allowed periods, it cannot close them through the planning horizon.

Note 2. The company and the rivals may share hubs or not. For example, two or more companies may benefit hub k but it can be either the rival company or our company who is using the hub m.

Important note 3. The markets are involved in the pricing decision of the carried flow by having the power to choose the cheapest company. This rational behavior of the markets, i.e., minimizing their costs by selecting the cheapest service, is included in the constraints of the problem, which creates a bi-level optimization problem. In this model, the leader (the freight company) maximizes its profit, while the follower (the market) in the lower-level problem minimizes the price paid by them, with regard to demand, capacity of hubs and carriers and market share constraints.

5.1.1. Illustrative example

Figure 5.1(a) and (b) depict two feasible solutions of an instance where the number of nodes $N = 10$, the number of transportation vehicle types $l = 4$, the number of initial hubs $p = 2$, the number of expansion hub $q = 2$. This example is studied in the total planning horizon $T = 4$, but here just the solutions for $t = 1$ and $t = 3$ are depicted. The circles are market nodes and the green hexagonal are used to refer to the companies established hubs, the yellow one is for rival's hubs and the dark green is for the hubs that are used by both our company and rival. One can track the flow by the colors of the edges, different colors are used for each demand of customers to be carried from an origin node to its destination node. Also, the shapes trapezoid, rectangle, and triangle refer to different kinds of carriers. These carriers differ in capacity and other efficiency factors. In Figure 5.1(a), the rival's hub is located on node 5 and the demand of node 4 is just covered by this rival. Hub 10 is utilized by both companies, and hub 6 is managed by our company. For nodes that carry large

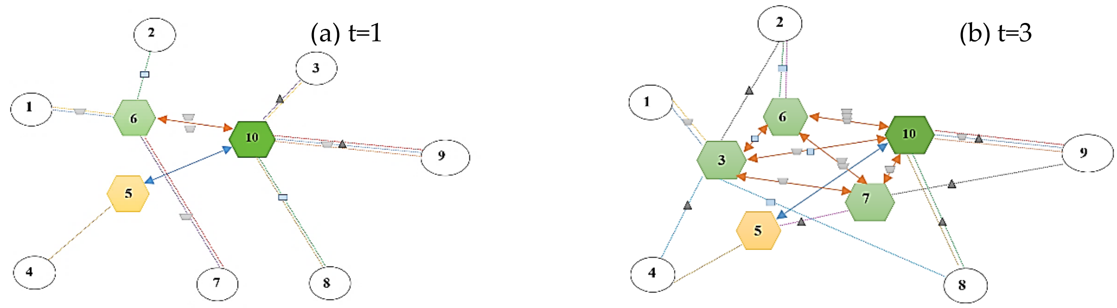

Figure 5.1: Two feasible solutions of an instance where $N = 10, l = 4, p = 2, q = 2$.

flow, more than one vehicle is assigned. In Figure 5.1(b), because of predicted increasing demand, the company decides to have an expansion plan and increase the number of hubs to 4 and also buy/lease more carriers. In this model, because the company does not have any information about the rival's plan, no change has happened in his hubs. In this period, the company has also entered in the market number 4 and the flow of this market is carried by hub 3. The hubs are interconnected and a huge amount of flow is traversing through them. It should be noted that in this illustration the distance between nodes is not considered and this is just a schematic try to have a view of the model application.

5.4 Mathematical formulation

The definition of the model parameters and decision variables are listed in the following:

Indices

i, k, m, j Index of nodes.

l	Index of transportation vehicle type. Index l_0 is exclusively used to represent the outsourcing carrier no matter what specific type it is.
t	Index of planning horizon, t=0 refers to the start of planning horizon that initial hubs are allowed. $t = 1,..,T$ is used to refer to periods that expansion is allowed in them.

Parameters

c_{ij}	Travel time (distance) of each pair of n nodes connected by an arc(i,j).
rev_{ijt}	Price paid by the market for carrying one Kg weight from node i to node j in period t.
d_{ijt}	Markets demand to be carried from origin i to destination j at period t.
cap_l	The capacity of transportation vehicle type l.
$capc_k$	The capacity of node k as chosen to be hub.
E_k	Fixed cost of locating a hub at node k at the first period of the planning horizon.
F_{kt}	Fixed cost of locating a hub at node k at period t .
A_{kmlt}	Hourly transportation cost of using transportation vehicle type l, from node k to node m at period t.
F'_{kt}	Operational cost of hub located at node k at period t.
MSh_r	Market share for the rival company r.
xr_{kml}	The number of carrier type l in the link $k - m$ for rival.
α	Discount factor to pass by hubs on the route of $i - j$, where $0 \leq \alpha \leq 1$.
p	The number of hubs at the start of planning horizon.
q	The number of hubs decided as expansion.
β	The deadline traveling time (distance) from node i to node j, set to assess the coverage.

o_{km} 1 if there are outsourcing services between nodes i and j, 0 otherwise.

$budg_t$ Available budget at period t.

g_{ikmj} Binary parameter, which indicates whether the origin–destination pair $(i-j)$ distance using the hubs $k-m$ can be covered by β as:

$$g_{ikmj} = \begin{cases} 1 & if\ c_{ik} + \alpha c_{km} + c_{jm} \leq \beta \\ 0 & otherwise \end{cases}$$

Decision variables

Upper-level decision variables

H_k Binary variable, 1 if node k is selected as an initial hub at the start of planning horizon.

v_{kt} Binary variable, 1 if node k is selected as an expansion hub at period t.

h_{kt} Binary variable, 1 if node k is operating as a hub at period t.

x_{kmlt} The number of carrier type l in the link $k-m$ at period t. x_{kml_0t} can be interpreted as outsourcing amount on link $k-m$.

Lower-level decision variables

y_{ikmj} The amount of flow carried from node i to node j through hubs $k-m$ at period t.

The problem is formulated as a bi-level optimization problem in (5.1) – (5.22). It should be highlighted that the upper objective takes into account the prices of the main studying company while the lower objective contains the prices offered from all companies performing in the business. The first term in the objective function in (5.1) maximizes the income gained from carrying flow through the hubs that are controlled by the leader, while the second and third terms are for the fixed cost of establishing initial hubs and expansion hubs, the fourth term is related to operational cost of hubs and the fifth term is for the transportation cost of the self-owned carriers and outsourcing. Constraint (5.2) determines the primary number of established initial hubs to be p. Constraints (5.3) determines the expansion number of established hubs to be q. Constraints (5.4) imply that each expansion hub can be opened only once throughout the planning horizon. Constraints (5.5) and (5.6) together enforce that once a hub is selected as an initial or expansion hub, it should be used for whole time periods. Constraints (5.7) state that no carrier can be assigned to link $k-m$ unless one of these nodes are operating as hub (M is a large positive number). Constraints (5.8) mean that unless there is not outsourcing service for the link $k-m$, no outsourcing is allowed for that link. Constraints (5.9) are related to this assumption that usually the number of the carriers operating in backward direction is the same as that in the forward direction. Constraints (5.10) are for the budget limitation of buying or leasing the carriers. Constraints (5.11) and (5.12) define the variables in the upper-level problem as binary and positive integer variables.

The objective function (5.13) as the lower-level objective, minimizes the cost paid by the markets. Constraints (5.14) and (5.15) imply that no flow can be carried through nodes unless they use hub/hubs. M used in these constraints is a large positive number. Constraints (5.16) imply that the total flow from node i to node j at each period t should be equal to the demand of that origin and destination. Constraints (5.17) make sure that the total capacity of employed carriers should not be violated. In this constraint the left-hand side is the total flow occurring on link $k-m$ utilizing by our company, including the amount $\sum_i \sum_j y_{ikmjt}$ that transfers from other $i-j$ pairs, and the amount $\sum_{o\in N}\sum_{d\in N}(y_{kmodt}+y_{odkmt})$ that is transferred to or from other node pairs, these pairs are shown by o and d. The right-hand side is the total capacity of the carriers employed to link $k-m$ utilizing by our company. Constraints (5.18) state that the total flow for the rivals' hubs could at most be equal to the situation it had since past. In addition, constraints (5.19) state that the amount of flow carried through the rivals' hub should be less that its market share of the demand. Constraints (5.20) and (5.21) make sure that the total capacity of established hubs should not be violated. It worths noting that all variables in upper-level problem take discrete values while the variables in lower-level problem has continuous variables. Constraints (5.22) enforce the decision variables to be non-negative. The nature of the model variables plays an important role in reformulation techniques of bi-level problems. In the bi-level reformulation literature, there are many papers studying models having different kind of variables in upper-level and lower-level problem theoretically and practically.

$$\max \sum_{i}\sum_{j}\sum_{k\in U}\sum_{m\in U}\sum_{t} rev_{ijt} g_{ikmj} y_{ikmjt} - \sum_{k\in U} E_k H_k - \sum_{k\in U}\sum_{t} F_{kt} v_{kt} - \sum_{k\in U}\sum_{t} F'_{kt} h_{kt} - \sum_{k\in U}\sum_{m\in U}\sum_{l}\sum_{t} A_{kmlt}\, c_{km} x_{kmlt} \tag{5.1}$$

$$\sum_{k\in U} H_k = p \tag{5.2}$$

$$\sum_{k\in U}\sum_{t} v_{kt} = q \tag{5.3}$$

$$\sum_{t} v_{kt} \le 1 \qquad \forall k \in U \tag{5.4}$$

$$h_{kt} \geq H_k + v_{kt} \qquad \forall k \in U, t \qquad (5.5)$$

$$h_{kt} \geq h_{kt-1} \qquad \forall k \in U, t \qquad (5.6)$$

$$x_{kmlt} \leq M(h_{kt} + h_{mt}) \qquad \forall k \in U, m \in U, t \qquad (5.7)$$

$$x_{kml_0t} = 0 \qquad \forall k \in U, m \in U, l_0, where\ o_{km}{=}0 \qquad (5.8)$$

$$x_{kmlt} = x_{mklt} \qquad \forall\ k \in U, m \in U, l \neq l_0, t \qquad (5.9)$$

$$\sum_{k\in U}\sum_{m\in U}\sum_{l} cost_l x_{kmlt} \leq budg_t \qquad \forall t \qquad (5.10)$$

$$H_k, v_{kt}, h_{kt} \in \{0,1\} \qquad (5.11)$$

$$x_{kmlt} \in \mathbb{Z}^+ \qquad (5.12)$$

$$\min \sum_{i}\sum_{k\in U\cup R}\sum_{m\in U\cup R}\sum_{j}\sum_{t} rev_{ijt} y_{ikmjt}\, g_{ikmj} \qquad (5.13)$$

$$y_{ikmjt} \leq M\ h_{kt} \qquad \forall i, j, k \in U, m \in U, t \qquad (5.14)$$

$$y_{ikmjt} \leq M\ h_{mt} \qquad \forall i, j, k \in U, m \in U, t \qquad (5.15)$$

$$\sum_{k\in U\cup R}\sum_{m\in U\cup R} y_{ikmjt} = d_{ijt} \qquad \forall i, j \in U \cup R, t \qquad (5.16)$$

$$\sum_{i}\sum_{j} y_{ikmjt} + \sum_{o\in N}\sum_{d\in N}(y_{kmodt} + y_{odkmt}) \leq \sum_{l} cap_l x_{kmlt}$$

$$\forall k \in U, m \in U, t \qquad (5.17)$$

$$\sum_{i}\sum_{j} y_{ikmjt} + \sum_{o\in N}\sum_{d\in N}(y_{kmodt} + y_{odkmt}) \leq MSh_r \sum_{l} cap_l xr_{kml}$$

$$\forall k \in R, m \in R, t \qquad (5.18)$$

$$\sum_{k\in R}\sum_{m\in R} y_{ikmjt} \le MSh_r d_{ijt} \qquad \forall i,j \in U \cup R, t \tag{5.19}$$

$$\sum_{i}\sum_{m\in U}\sum_{j} y_{ikmjt} \le capc_k h_{kt} \qquad \forall k \in U, t \tag{5.20}$$

$$\sum_{i}\sum_{k\in U}\sum_{j} y_{ikmjt} \le capc_m h_{mt} \qquad \forall m \in U, t \tag{5.21}$$

$$y_{ikmjt} \in \mathbb{R}^{+} \qquad \forall i,j,k \text{ and } m \in U \cup R, t \tag{5.22}$$

5.5 Reformulation as a single-level optimization Problem

A bi-level program with a convex and regular lower-level can be transformed into a single-level optimization problem using its optimality conditions. Two single-level reformulations for the modified problem are derived to be able to solve them and compare the results of each reformulation. First one is reformulation based on duality theory and the second one is obtained by Karush-Kuhn-Tucker (KKT) conditions.

5.5.1 Dual-based reformulation

To conduct this reformulation the upper-level problem remains unchanged and the reformulation is performed on lower-level problem. For this reformulation the equivalent dual problem of the lower-level problem has to be obtained. The dual constraints will be added to the constraints of the lower problem and the objective functions of primal and dual problem should be equated.

$v1_{ikmjt}, v2_{ikmjt}, v3_{ijt}, v4_{kmt}, \mathrm{v}5_{kmt}, v6_{ijt}, v7_{kt}$, and $v8_{mt}$ are the dual variables of the lower-level constraints presented in Eqns. (5.14) – (5.21), respectively. In this reformulation, the constraints of the upper-level problem (Eqs. (5.1) – (5.12)) remain unchanged.

(5.1) – (5.12), (5.14) – (5.22)

$$\sum_{i}\sum_{k\in U\cup R}\sum_{m\in U\cup R}\sum_{j}\sum_{t} rev_{ijt} y_{ikmjt}\, g_{ikmj} \tag{5.23}$$

$$= \sum_{i}\sum_{j}\sum_{t} d_{ijt} v3_{ijt} - \sum_{i}\sum_{j}\sum_{t} MSh_r d_{ijt} v6_{ijt}$$

$$- \sum_{i}\sum_{k\in U}\sum_{m\in U}\sum_{j}\sum_{t} M\, v1_{ikmjt} h_{kt}$$

$$- \sum_{i}\sum_{k\in U}\sum_{m\in U}\sum_{j}\sum_{t} M\, v2_{ikmjt} h_{mt}$$

$$- \sum_{k\in U}\sum_{m\in U}\sum_{l}\sum_{t} cap_l\, v4_{kmt} x_{kmlt}$$

$$- \sum_{k\in R}\sum_{m\in R}\sum_{l}\sum_{t} cap_l\, \mathrm{v}5_{kmt} xr_{kml} - \sum_{k\in U}\sum_{t} v7_{kt} h_{kt} capc_k$$

$$- \sum_{m\in U}\sum_{t} v8_{mt} h_{mt} capc_m$$

$$v1_{ikmjt} + v2_{ikmjt} - v3_{ijt} + v4_{kmt} + \mathrm{v}5_{kmt} + v6_{ijt} + v7_{kt} + v8_{mt} \geq -rev_{ijt} g_{ikmj}$$

$$\forall i, k \in U \cup R, m \in U \cup R, j, t \tag{5.24}$$

$$v1_{ikmjt}, v2_{ikmjt}, v4_{kmt}, \mathrm{v}5_{kmt}, v6_{ijt}, v7_{kt}, v8_{mt} \in \mathbb{R}^{+} \quad v3_{ijt} \in \mathbb{R}. \tag{5.25}$$

By equating the primal and dual objective functions of the lower-level problem in (5.23) the strong duality is enforced. The feasibility of primal and dual problems should be guaranteed in the reformulation. This important fact for primal is ensured by keeping constraints (5.14) – (5.22) and for the dual problem is ensured by adding constraints (5.24).

By the reformulation, it is obvious that the equation (5.23) is nonlinear that yields a mixed-integer nonlinear program (MINLP). Fortunately, we can linearize it by popular techniques known as linearizing the product of binary and continues variables, in which before that another reformulation should be applied to convert the integer variables to an expression, which is summation of binary variables. Both conversion techniques are available in operations research related material. By linearizing the nonlinear expressions in the (5.23), the bi-level problem is converted into a single-level problem and solution procedures can be applied to solve the problem.

5.5.2 Reformulation using KKT condition

An appealing way to deal with general bi-level problems is the so-called KKT approach, where the lower-level constraints, that its variables are a global minimizer of the lower-level program, are firstly relaxed to the conditions that the optimal variables of the lower-level problem are a local minimizer for that lower-level problem. The main purpose of the KKT approach is to find (local) minimizers of the original bi-level program by computing (local) minimizers of the relaxation obtained from KKT equivalent. This is the original reformulation for bi-level problems that the lower-level is a LP to replace the lower-level problem with its KKT conditions. The resulting reformulation is presented in the following:

(5.1) – (5.12), (5.14) – (5.22), (5.25)

$$rev_{ijt}g_{ikmj}+v1_{ikmjt}+v2_{ikmjt}+v3_{ijt}+v4_{kmt}+\mathrm{v}5_{kmt}+v6_{ijt}+v7_{kt}+v8_{mt}-v9_{ikmjt}=0$$

$$\forall i,k\in U\cup R, m\in U\cup R, j, t \quad (5.26)$$

$$v1_{ikmjt}(y_{ikmjt}-M\,h_{kt})=0 \quad \forall i,k\in U, m\in U, j, t \quad (5.27)$$

$$v2_{ikmjt}\,(y_{ikmjt}-M\,h_{mt})=0 \quad \forall i,k\in U, m\in U, j, t \quad (5.28)$$

$$\mathrm{v}4_{kmt}\left(\sum_i\sum_j y_{ikmjt}+\sum_{o\in N}\sum_{d\in N}(y_{kmodt}+y_{odkmt})-\sum_l cap_l x_{kmlt}\right)=0$$

$$\forall k\in U, m\in U, t \quad (5.29)$$

$$\mathrm{v}5_{kmt}\left(\sum_i\sum_j y_{ikmjt}+\sum_{o\in N}\sum_{d\in N}(y_{kmodt}+y_{odkmt})-MSh_r\sum_l cap_l xr_{kml}\right)=0$$

$$\forall k\in R, m\in R, t \quad (5.30)$$

$$v6_{ijt}\left(\sum_{k\in R}\sum_{m\in R} y_{ikmjt}-MSh_r d_{ijt}\right)=0 \quad \forall i,j\in U\cup R, t \quad (5.31)$$

$$v7_{kt}(\sum_i\sum_{m\in U}\sum_j y_{ikmjt}-capc_k h_{kt})=0 \quad \forall k\in U, t \quad (5.32)$$

$$v8_{mt}(\sum_{i}\sum_{k\in U}\sum_{j} y_{ikmjt} - capc_m h_{mt}) = 0 \qquad \forall m \in U, t \qquad (5.33)$$

$$v9_{ikmjt}\, y_{ikmjt} = 0 \qquad \forall i, j, k \text{ and } m \in U \cup R, t \qquad (5.34)$$

$$v9_{ikmjt} \in \mathbb{R}^{+} \qquad (5.35)$$

The upper-level problem is kept unchanged as shown in (5.1) – (5.12). In KKT approach, there are two main conditions as stationary conditions and complementary conditions. Constraints (5.25) are the stationary conditions for the lower-level, where the constraints (5.27) – (5.34) represent the complementary conditions corresponding to inequalities (5.14), (5.15), (5.17) – (5.22). Note that in KKT conditions, the constraints representing the domain of the continuous variables are treated as constraints and complementary conditions must be written for these constraints either.

To avoid the non-linear complementary constraints, we rewrite Eqns. (5.14) – (5.15), (15.7) – (5.21) as equality constraints by using the slack variables and then use the big-M method to express either constraints (5.14), (5.15), (5.17) – (5.21) are active or the corresponding multipliers are zero. Similarly, the same big-M reformulation should be done for constraints (5.34).

This process has to be fulfilled for all complementary conditions and the necessary constraints for all of them should be included in the model. By this procedure, we can get rid of nonlinear complementary conditions. The result of this replacement for complementary constraints is a single-level mixed integer linear problem that can be solved by solvers.

5.6 Efficient solution procedure

To solve the single-level problem, which is obtained in the previous section the general-purpose MIP solvers, can solve small problems up to 10 nodes. This fact shows the necessity to propose or apply other available solving methods. As the decision variables can easily be classified into two categories; integer and continuous variables, a solving method based on Benders decomposition is possible to be applied to solve the problem.

There are some other decomposition techniques in literature that are proposed based on the Benders decomposition. One of these methods are proposed by Saharidis and Ierapertiton [84] for mixed integer bi-level linear problems. In this method after dividing the problem into sub-problem and master problem, Saharidis and Ierapertiton [83] convert the bi-level sub-problem into single-level using KKT conditions. Then similar to Benders decomposition method the master problem and sub-problem is solved and they are synced in each iteration.

For the conversion process in the sub problem both duality-based method and KKT method are used. The algorithm is as follows:

Step 1. Set $UB = +\infty$ and $LB = -\infty$.

Step 2. Generate initial solutions for decision variables of master problem. Solve the sub problem for the given $\bar{H}_k, \bar{h}_{kt}, \bar{v}_{kt}$, use duality theory or KKT conditions to convert it to a single-level problem. Solve the problem and identify the active constraints.

Step 3. Create a linear programing (LP) model using the active constraints plus the rest of the constraint of the original sub problem (having fixed values for the variables of master problem). The solution of this LP (Z') is one of the following:

- *The LP is unbounded*, add a feasibility cut $\bar{u}(b - Ah) \leq 0$ to the master problem.
- *The LP is bounded*. Update $= \max(LB, Z')$. If adding the $\bar{u}(b - A\bar{h}) - \varphi \leq 0$ does not restrict master problem then algorithm continues by excluding the current integer solution using the following cut:

$$\sum_{i \in \rho} h_i - \sum_{j \in \rho'} h_j \leq |\rho| - 1$$

where ρ is the set of indices of variables that have the value 1, $\rho = \{i | h_i{}^* = 1\}$. Similarly ρ' is the set of indices where the corresponding variables have the value 0, $\rho' = \{j | h_j{}^* = 0\}$. By $|\rho|$ we denote the number of variables $h_i{}^*$that are equal to one. Also u is the dual variables of constraints and $(b - Ah)$ refers the right-hand side of constraints. With h we mean all binary variables.

- *The LP is bounded*, update $LB = \max(LB, Z')$. However, if $\bar{u}(b - A\bar{h}) - \varphi > 0$, to satisfy the $\bar{u}(b - A\bar{h}) - \varphi \leq 0$ add $\bar{u}(b - Ah) - \varphi \leq 0$ as optimality cut to the master problem.

Step 4. After adding the appropriate cut, solve the master problem and derive an optimal solution Z and update $UB = \max(UB, Z)$.

Step 5. If $UB - LB \leq \varepsilon$ then stop, else continue with step 2.

Subproblem:

The sub problem is derived by fixing decision variables H_k, h_{kt}, v_{kt} with $\bar{H}_k, \bar{h}_{kt}, \bar{v}_{kt}$ in inner problem by the objective function which contains the expressions related to the variables y_{ikmjt} and x_{kmlt}. As the sub problem would be a bi-level problem, to convert it to a single-level problem both the dual based and KKT condition reformulation will be used and the obtained results would be compared. Therefore, the sub problem is:

$$\max \sum_{i} \sum_{j} \sum_{k \in U} \sum_{m \in U} \sum_{t} rev_{ijt} y_{ikmjt} g_{ikmj} - \sum_{k \in U} \bar{H}_k E_k - \sum_{k \in U} \sum_{t} F_{kt} \bar{v}_{kt} - \sum_{k \in U} \sum_{t} F'_{kt} \bar{h}_{kt} - \sum_{k \in U} \sum_{m \in U} \sum_{l} \sum_{t} A_{kmlt}\, c_{km} x_{kmlt} \quad (5.36)$$

s.t: (5.7) – (5.10), (5.12) – (5.22).

To be able to solve the sub problem by decomposition method, the integer variables x_{kmlt} are relaxed to be positive continues ones.

Master problem:

The master problem is the upper-level problem, which optimizes variables$H_k, h_{kt}, w_{kt}, v_{kt}$. Note that in solving process the necessary cuts also should be added to this problem.

Min

$$\sum_{k\in U} H_k E_k + \sum_{k\in U}\sum_{t} F_{kt} v_{kt} - \sum_{k\in U}\sum_{t} F'_{kt} h_{kt} - \varphi \tag{5.37}$$

s.t: (5.2) – (5.6), (5.11).

The value φ is the value for terms related to variables $\bar{y}_{ikmjt}$ and $\bar{x}_{kmlt}$ in the sub problem.

5.7 Computational results

In this section, we show some numerical examples to evaluate the model and solving methods' efficiency. Most of the main parameters of the problem such as travel time (distance), demand, hub capacities and costs, which are used as an example to solve, are from a real case in Turkey. Other parameters such as carrier capacities and hourly transportation cost of carriers are taken from [82]. Other parameters are included in Table 5.1. This should be marked that for parameters A_{kmlt} and rev_{ijt} to save space, we just include the minimum and maximum values using [$\min_{ij}$, $\max_{ij}$] format (there are four sets of ([$\min_{ij}$, $\max_{ij}$] values at each period t for A_{kmlt} , which refers to each l). We set the other parameters α=0.7, p=2, q=2, l=4, T=4, β=1300 for 10 nodes instance. Furthermore, MSh_r=0.3 and for the availability of outsource service parameter o_{km}, we have set the condition that this service is available for demands lower than 30000. All the problems are coded in GAMS 24.1 CPLEX 24.1.2 solver and run on a personal computer of Core i7, 3.40 GHz, 8 GB RAM.

The first problem to solve has 10 nodes in the network, the number of hubs at the start of planning horizon would be two, and it is allowed to have two more hubs after second period. The results for this problem are included in Table 5.2. In this table the first and second columns contain the results for single-level reformulations based on duality theory and KKT conditions, respectively, also, third and fourth columns contain the results obtained from the decomposition methods using the duality theory and KKT conditions in reformulating the sub problem, respectively. The first row illustrates the objective value and the other rows contain the different

Table 5.1: Parameters for 10 nodes instance.

Parameter	Values
E_k	{4790, 7748, 4738, 6716, 5653, 3104, 3265, 6521, 5497 ,3618}
F_{kt}	t=1 {6418, 10383, 6349, 8999, 7575, 4160, 4375, 8738, 7366, 4848} t=2 {7328, 11855, 7249, 10276, 8649, 4750, 4995, 9976, 8410, 5535} t=3 {7951, 12862, 7865, 11149, 9384, 5153, 5419, 10824, 9125, 6006}
F'_{kt}	t=1 {1284, 2077, 1270, 1800, 1515, 832, 875, 1747, 1473, 970} t=2 {1466, 2371, 1450, 2055, 1730, 950, 999, 1995, 1682, 1107} t=3 {1590, 2572, 1573, 2230, 1877, 1031, 1084, 2165, 1825, 1201} t=4 {1619, 2619, 1601, 2270, 1911, 1049, 1103, 2204, 1858, 1223}
A_{kmlt}	t=1 {[1.29, 1.47], [1.12, 1.29], [0.6, 0.77], 0.0001} t=2 {[1.32, 1.49], [1.14, 1.32], [0.61, 0.79], 0.0001} t=3 {[1.34, 1.52], [1,16, 1.34], [0.62, 0.80], 0.0001} t=4 {[1.38, 1.56], [1.19, 1.38], [0.64, 0.82], 0.0001}
cap_l	{40000, 28000, 14000,1}
$budg_t$	{31, 717, 470, 38, 432, 670, 35, 503, 750, 33, 011, 380}
rev_{ijt}	t=1 [3.9, 23.5] t=2 [4.02, 23.7] t=3 [4.1, 24.1] t=4 [4.2, 24.6]
xr_{kml}	$xr_{10,3,1} = 1, xr_{3,5,2} = 1, xr_{5,6,1} = 1$, others=0

Table 5.2: Results of an example for 10 nodes.

N=10, p=2	Sngl.lvl Dual	Sngl.lvl KKT	Deco. KKT	Deco. Dual
Obj	3,074,799	2,683,615	2,788,362	2,865,265
Income	3,170,522	2,788,483	2,877,493	2,950,721
Hub cost	55,337	60,411	53,736	53,736
Operational cost	11,067	12,082	10,747	10,747
Transportation cost	29,318	32,373	24,646	20,971
Time (s)	1,006	1,003	11	2

Table 5.3: Results of some sensitivity analysis.

	N=10, p=2	Sngl.lvl Dual	Sngl.lvl KKT	Deco. KKT	Deco. Dual
1	Main	3,074,799	2,683,615	2,788,362	2,865,265
2	β=1500	3,439,385	2,854,862	3,355,062	3,362,556
3	C.cap -25%	2,402,696	1,649,895	2,135,054	2,158,608
4	C.cap +25%	3,584,927	2,838,918	3,123,215	3,388,515
5	H.cap -25%	3,004,058	2,639,804	2,780,162	2,858,275
6	H.cap +25%	3,089,888	2,719,115	2,793,311	2,869,459

terms in the objective function, to have a better insight of them. The last row illustrates the computational time to solve each problem in seconds.

According to the results in Table 5.2, it can be concluded that the dual based single-level reformulation has a better functionality, as its solution is more than the KKT reformulation and the solution value is closer to the solution obtained from decompositions. Between the two decomposition methods, the decomposition that has used duality theory has produced better solution in much less time. This problem is solved for some changes in parameters of the maximum allowed

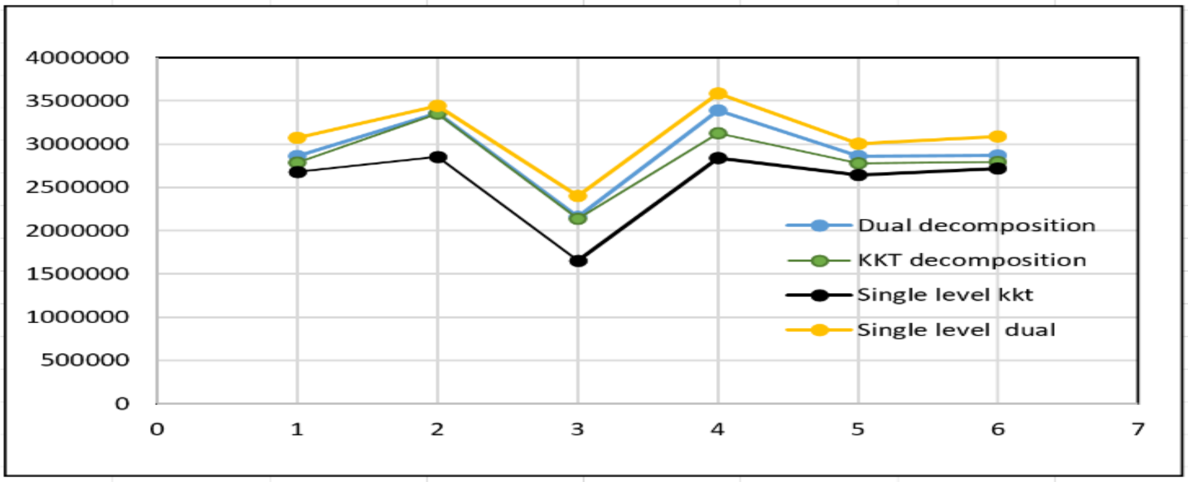

Figure 5.2: Results of some sensitivity examinations.

coverage distance in addition to 25% increase and decrease on hubs and carriers capacities. The results are illustrated in Table 5.3 and Figure 5.2.

The results in Table 5.3 and Figure 5.2 show the same trends in Table 5.2. Comparing single-level reformulations (third and fourth column) it is obvious that reformulation based on duality theory has apparent superiority to KKT reformulation method. The same conclusion can be set off from comparing decomposition methods using both KKT and Duality reformulation in which decomposition method using duality reformulation produces better objective values. For all test problems of sensitivity analysis, the dual reformulation approach has the maximum objective value and KKT reformulation has the minimum objective value for all instances.

However, the main purpose of this table is to evaluate the model performance by running some sensitivity analysis. All approaches have the same level of efficiency regarding objective function value, being able to find reasonable values for test problems. When the coverage distance, the capacity of hubs and carries are increased it is supposed that the objective value will increase and by reducing the capacities, the objective value is supposed to be reduced and the solutions in Table 5.3 confirms these assumptions.

According to Table 5.3 and regarding the increasing the coverage parameter from 1300 to 1500, all the results show increased values which they were assumed to, due the fact that by increasing the coverage parameter more nodes may be covered and consequently the profit will be increased. The same results are obvious for cases increasing capacity of carriers and hubs, and on the contrary reducing the capacity has resulted to less values of objective function. In addition, comparing the impact of differences in capacities, it can be implied that increasing the carriers' capacity has more impact on solutions rather than hub capacities. The same results apply to other approaches which shows if the company wants to choose between increasing the capacity of carriers or hubs the priority is with increasing the capacity of carriers.

Single-level reformulations were not able to solve problems more than 10 nodes, so that no more comparison was possible to conduct, especially for comparing reformulation and decomposition both based on duality theory. That is the reason that the rest of the comparisons are performed on decompositions based on KKT and duality theory. In the similar way, some test problems are solved increasing the number of the nodes in the network and results are shown in the Table 5.4. The rows in this table refer to different problems solved. For example, the first row is related to the problem having 15 nodes in which $p = 3$ hubs have to be located in the first period. In all examples, the number of new hubs to be located as expansion plan is fixed to $q = 2$. Both reformulations were unable to solve the examples, so that just the solutions of two decompositions exist in the table. The decomposition which uses the KKT condition in the subproblem (first and second column) has solved the problem much longer than the dual based decomposition did (the third and fourth column). Considering the computational efforts, it is apparent that duality based decomposition has a much better performance than KKT based decomposition. Also, KKT based decomposition method could not find the solution to problems more than 20 nodes, however the dual based decomposition could obtain the solution for problems up to 30 nodes at an acceptable computational time. Comparing the objective value for 15 and 20 nodes of both decompositions, both of them have relatively acceptable values, although dual based decomposition was able to obtain more values. This superior performance of dual based decomposition method, being able to find solution for larger problem in less time, is because of having the smaller number of binary variables and constraints added to the model in the reformulation procedure of bi-level sub problem to a single-level one.

Table 5.4: Results for larger instances.

	Deco.KKT	Time (s)	Deco.Dual	Time (s)
N=15, p=3	4,489,356	438	4,500,202	10
N=20, p=4	5,291,338	796	5,298,723	27
N=25, p=5	Unsolved	-	5,190,791	211
N=30, p=6	Unsolved	-	7,708,827	653

5.8 Summary

In this chapter, a bi-level mixed integer formulation for a freight transportation planning problem which is performing in a hub covering system by maximizing the amount of covered demand of the market has been introduced. The model includes many possible conditions that exist in a related business environment such as dynamic demands of markets, the possibility to outsource the demand and considering different kind of carriers. Also, as the market power to participate in pricing decision has been studied in the model, it is a bi-level problem. To encounter the hierarchical structure, the reformulations based on two methods, KKT conditions and duality theory have developed for our problem. To solve the problem a decomposition method based on Benders decomposition has been used and some test problems are solved to investigate the efficiency of the solving procedures and model's accuracy. The results of the numerical examples approve the superiority of decomposition fulfilled by duality theory.

Chapter 6

6. Conclusion and Remarks

In this thesis, the covering facility location problems are investigated using two main approaches. In the first approach, a more improved formulation for using these problems in real-life applications is studied. The second approach is the non-cooperative game theory that is applied to covering facility location problem. One of the important endeavors in those approaches is the proposed solution techniques that enables to solve real-life problems of large sizes.

In Chapter 3, an extension of maximal covering location problem was developed for locating emergency facilities having three main real-life conditions in one model. Three improvements were multi-period planning horizon, modular and capacitated facilities and considering back-up coverage for demand points. A genetic algorithm was developed to solve the problem due to this metaheuristic's strength to solve binary optimization problems and other extension of facility location problems. In addition, a heuristic method was proposed to have approximation values for the objective function and to evaluate the results obtained from genetic algorithm.

In Chapter 4, a novel model combining two major covering location problems was developed that addresses the facility location problem in a disaster relief situation. The coverage concept of two major covering location problems, i.e., set covering location problem and maximal covering location problem, was utilized to develop the model of the hybrid covering location problem. In the developed hybrid model, the set covering location problem determines the location of the facilities and provides full coverage for the demand points. The limited number of modules that provide different services can be assigned to the facilities to provide services in tactical periods. To investigate the capability of the developed hybrid covering location problem, a case study using real data for demand points in Japan was used together with several randomly generated test problems. Furthermore, the other possible integrations of covering location problems were developed as the variants of the main hybrid covering location problem. To evaluate four developed models, some test problems were generated and solved for all proposed integrated models. The computational results approve that the main developed hybrid model can outperform the other three models in terms of coverage percentage, solution quality and feasibility of the solutions.

Chapter 5 addressed a bi-level mixed integer formulation for maximal hub covering location problem that maximizes the total covered demands. The model included many conditions that exist in a related business environment such as dynamic demands of markets, the possibility to outsource the fulfillments of demand requests and availability of different kind of carriers. Also, as the market power to participate in pricing decision has been studied in the model as a bi-level problem. To treat the bi-level structure, two reformulations based KKT conditions and duality theory have developed for the bi-level problem. To solve the obtained single level problems, a decomposition method based on Benders decomposition was applied and some test problems were solved to investigate the efficiency of the solving procedures and the accuracy of the solution procedure.

According to overall mathematical models, solution procedures, extensive numerical experiments and sensitivity analysis developed and conducted in different chapters of this thesis, the attainments of the research goals are described briefly as follows:

Research goal 1: develop covering location models to model the real-life conditions and take the advantage of two separate covering modes in an integrated model.

The developed models in Chapters 3 and 4 could propose a closer formulation of the real-life conditions and improve service quality. Especially, the developed hybrid covering location problem has a very novel formulation that can have applications in humanitarian logistics problems as studied in the case study. The superiority of the hybrid covering location problem became more apparent when it was compared with other possible way of combining set covering and maximal covering location problems.

Research goal 2: Apply the game theory to the covering facility location problems.

The maximal covering location problem was studied from non-cooperative in Chapters 5. The non-cooperative maximal covering location problem was studied in the hub locating framework and resulted in a bi-level mathematical model considering the leaders in the first level and the follower as the second level optimization problem. The obtained bi-level formulation is necessary for developing reformulation techniques enabling to conduct numerical experiments.

For future work and research directions, the followings are planned to develop:

- The modeling of uncertainty can be applied to the hybrid covering location problem and this problem can investigate using stochastic and robust optimization models.
- An efficient solution procedure can be developed for solving the hybrid covering location problem.

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List of Publications

Papers in Reviewed Journals

[1] R. Alizadeh and T. Nishi, “Dynamic p+q maximal hub location problem for freight transportation planning with rational markets,” *Adv. Mech. Eng.*, vol. 11, no. 2, 2019, doi: 10.1177/1687814018822934.

[2] R. Alizadeh and T. Nishi, “A genetic algorithm for multi-period location problem with modular emergency facilities and backup services,” *Trans. Inst. Syst. Control Inf. Eng.*, vol. 32, no. 10, pp. 370–377, 2019.

[3] R. Alizadeh and T. Nishi, “Hybrid set covering and dynamic modular covering location problem: Application to an emergency humanitarian logistics problem,” *Appl. Sci.*, vol. 10, no. 20, 2020, doi: 10.3390/app10207110.

[4] R. Alizadeh and T. Nishi, Jafar Bagherinejad, Mahdi Bashiri, “Multi-period maximal covering location problem with capacitated facilities and modules for natural disaster relief services,” *Appl. Sci.*, vol. *11*, no 1, 397, 2021; https://doi.org/10.3390/app11010397.

International Conferences

[1] R. Alizadeh and T. Nishi, "A bilevel dynamic maximal hub location problem for freight transportation," *Abstract of 29th European Conference on Operational Research*, 2018.

[2] R. Alizadeh and T. Nishi, "Mixed-integer bi-level dynamic hub location problem for freight transportation," *Proceedings of the International Symposium on Flexible Automation*, 2018, pp. 171–174.

[3] R. Alizadeh and T. Nishi, "Multi-period maximal covering location problem with modular facilities for locating emergency facilities with back-up services," *2018 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, 2018, pp. 76–79.

[4] R. Alizadeh and T. Nishi, "Core formation and profit allocation of horizontal cooperation in covering location problems: a game theoretical approach," *Abstract of 30th European Conference on Operational Research*, 2019.

[5] R. Alizadeh and T. Nishi, "Cooperative maximal covering location problem – A core based profit allocation approach," *Proceedings of the International Symposium on Scheduling*, 2019, pp. 70-74.

[6] R. Alizadeh and T. Nishi, "Hybrid covering location problem: set covering and modular maximal covering location problem," *2019 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, 2019, pp. 865–869.

[7] R. Alizadeh and T. Nishi, "Hybrid covering location problem: set covering and modular maximal covering location problem," *Proceedings of Scheduling Symposium,* 2020.

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