Title	Prediction of Fatigue Crack Path by Finite Element Method
Author(s)	Fukuda, Shuichi; Miyamoto, Hiroshi; Kujirai, Yoichi et al.
Citation	Transactions of JWRI. 1977, 6(1), p. 47-54
Version Type	VoR
URL	https://doi.org/10.18910/8310
rights	
Note	

Prediction of Fatigue Crack Path by Finite Element Method†

Shuichi FUKUDA*, Hiroshi MIYAMOTO, Yoichi KUJIRAI*** and Koji SUMIKAWA*****

Abstract

An attempt is made to predict the fatigue crack path in combined mode I and mode II, using the finite element method, with the purpose of serving for an optimum design of a structure against fatigue failures. Most crack analyses up to now have treated the problem of a straight extending crack. But cracks found in structures more or less deviate or extend in a zigzag manner due to the mixed mode stress state produced, for example, by neighbouring flaws or combined mode loadings. Especially in a high cycle fatigue where crack growth rate is quite slow, the change in stress state caused by the deviation of a crack affect the succeeding crack path and crack growth rate. Therefore, the prediction of a crack path is important from the standpoint of fatigue life evaluation as well. The adoption of the finite element method enabled the successive pursuit of deflecting crack extension. The predicted fatigue crack path emanating from an oblique crack agrees well with the experimental result.

1. Introduction

This paper describes the attempt to predict the fatigue crack path in combined mode I and mode II, using the finite element method, with the purpose of serving for an optimum design of a structure against fatigue failures. Most crack analyses up to now have treated the problem of a straight extending crack. But the majority of cracks found in structures are known to be in the mixed mode stress states, such as K_I-K_{II} or K_I-K_{III}. This is due to such facts as (1) other cracks or flaws are present in the neighbourhood, (2) the loading itself is a combined loading such as tension-shear, and (3) the stress at the point of interest is in mixed mode stress state due to the structural configuration although loading itself is uni-axial. Therefore, cracks in structures more or less deviate or extend in a zigzag manner.

The change in stress state caused by the deviation of a crack affects the succeeding crack path and consequently crack growth rate. This effect is especially predominant in a high cycle fatigue where crack growth rate is quite slow. This implies that the prediction of a crack path is important from the standpoint of fatigue life evaluation as well.

If we adopt the finite element method for analysis, we can pursue successively each stage of deflecting crack extension. This paper analyzed fatigue cracks emanating from an oblique crack as one example of fatigue cracks in K_I-K_{II} mixed mode, using finite element method and attempted to predict its crack path on the basis of linear fracture mechanics. The predicted crack path agreed quite well with the experimental crack path.

Nomenclature

$2c$=actual crack length, $2a$=projected crack length, σ_0=stress amplitude, $2W$=plate width, T=plate thickness, E=Young's modulus, ν=Poisson's ratio, θ=direction of crack extension, K_I, K_{II}, K_{III}=stress intensity factor for mode I, mode II and mode III respectively, X=symmetric axis of a test specimen in the horizontal direction, Y=symmetric axis of a test specimen in the vertical direction, x,y=crack tip coordinates parallel to X and Y. x',y'=coordinates which rotate the x-y coordinates by the angle θ.

2. Crack Path in Combined Mode I and Mode II

Considerable number of works are available on the crack path in mixed mode[1),2),3),4),5)]. But all of these works are about the branching crack immediately after initiation from the initial crack. There is, to the best of the authors' knowledge, no investigation which made the successive pursuit of the deflecting crack path on a computer. If we introduce the finite element method into the analysis, it is possible to pursue successively the deflecting crack path if (1) the condition for a fatigue crack to grow, (2) the condition as to the direction of crack extension and (3) the condition as to the increment of crack extension are given. As it seems that the condition (1) in mixed mode has not yet been made fully clear by the experiments, we concentrated our chief attention on the crack path predic-

† Received on March 31, 1977

* Research Instructor
** Professor, University of Tokyo
*** Postgraduate Students, University of Tokyo

tion and adopted the following Erdogan and Sih's criterion, i.e., maximum stress criterion for condition (2)[6].

$$K_I \sin\theta + K_{II} (3\cos\theta - 1) = 0 \qquad (1)$$

where θ is taken as shown in **Fig. 1**.

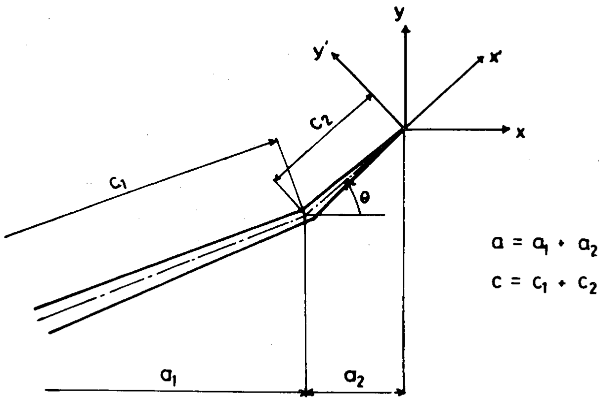

Fig. 1 Coordinates at the crack tip

The difficulty arises as to condition (3). In actual material, the amount of crack growth extension is expected to be determined by such metallurgical factors as the size of crystal grains or the orientation of crystals. But in this analysis, isotropicity and homogeneity are assumed, so that the crack once started does not stop. Since the condition (1) has not been made fully clear either as previously mentioned, the crack is extended to the direction determined by Erdogan and Sih's condition after an arbitrarily chosen increment of crack extension.

3. Finite Element Analysis Using Quadratic Shape Function

3.1 6 Node 12 Degree of Freedom Triangular Element

In the ordinary finite element analyses, constant stress elements are used. In the case of constant stress elements, the stress is constant and does not change within an element so that the difficulty arises as to which point should be selected as the stress representative point of the element. Some researchers recommend to take the center of gravity of the element for the stress representative point and others recommend to adopt the nodal stress which is obtained by averaging the stresses of all the elements related to its node as the stress at node point. No matter which method might be used in the analysis, extreme fine meshing is required to analyze the stress field at the tip of a deflecting crack with accuracy if constant stress elements are used. Especially to obtain K_I or K_{II} which represents the stress singularity, it is necessary to divide the elements into extreme fine meshes at the crack tip. This means that if constant stress elements are to be used in the crack path analysis, remeshing is required after a certain amount of crack extension. Consequently, a considerable amount of computational effort is necessary. As coarser meshing is possible without the loss of accuracy with increasing degree of shape functions[7], 6 node 12 degree of freedom triangular element derived from a complete quadratic polynomial function which was developed by Veubeke[8] and Argyris[9] was used in this analysis. Since this 6 node 12 degree of freedom element as shown in **Fig. 2** has the shape function of the following com-

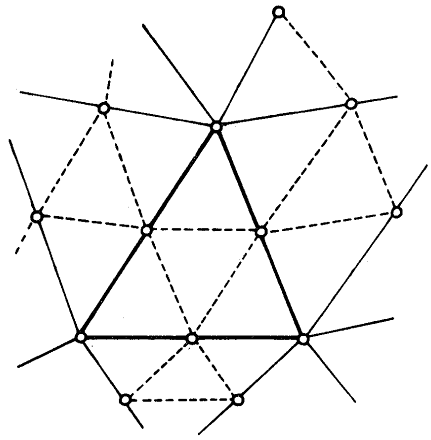

Fig. 2 6 node 12 degree of freedom triangular element

plete quadratic polynomial form,

$$\begin{aligned} u &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 \\ v &= \alpha_7 + \alpha_8 x + \alpha_9 y + \alpha_{10} x^2 + \alpha_{11} xy + \alpha_{12} y^2 \end{aligned} \qquad (2)$$

stress and strain change linearly within an element: i.e., the stress at an arbitrary point within an element can be calculated. But as the shape function is of quadratic form, the continuity of stress and strain between elements does not hold, so that the nodal stresses differ from element to element. In the following computation, the stress obtained by averaging all the nodal stresses of the elements related to its node is used as 'nodal stress'.

3.2 Application to 45° Oblique Crack

Iida and Kobayashi[10] studied experimentally fatigue crack growth from a central oblique crack in combined mode in a 7075-T6 aluminum alloy sheet. The solid line in **Fig. 3** shows the K_I and K_{II} of the experiment at each stage of crack extension calculated by the finite element method using constant stress elements and a step solution (zooming solution) technique. The plots of crack paths themselves are not presented

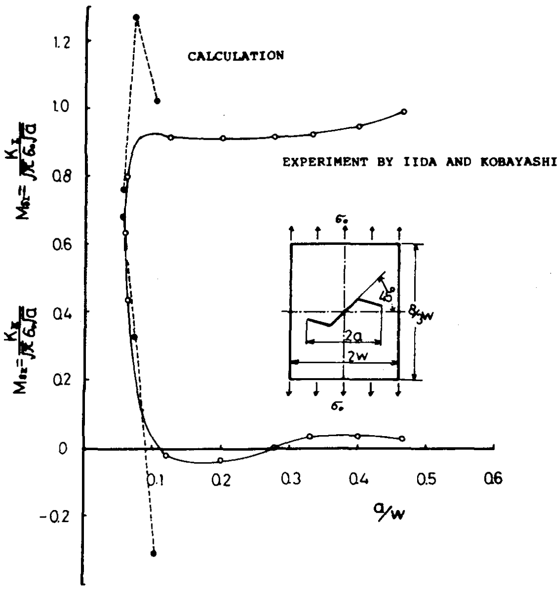

Fig. 3 Relation of K_I and K_{II} against half crack length a (45° oblique crack)

in their paper. Therefore, in order to examine whether it is possible to predict the crack path with coarser division if the quadratic shape function is used, crack path simulation is conducted and the K_I and K_{II} values are calculated at each stage of crack extension and are compared with those values experimentally obtained. The mesh division around the crack at stage 0 is shown in **Fig. 4**. The dotted line in Fig. 3 shows

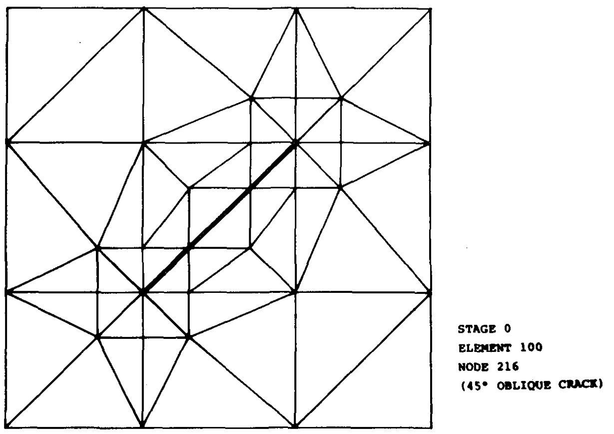

Fig. 4 Mesh division around crack

the simulation result. It is observed from the figure that although the K_I and K_{II} of the initial crack before propagation can be obtained accurately using a coarse mesh division if the quadratic shape function is adopted, sharp differences arise in the K_I and K_{II} values after propagation. So, even if the shape function is quadratic, the crack tip region must be divided into fine meshes repeatedly after every extension of a crack in order to obtain the accurate crack path and the reduction of computational efforts is not as much as expected. Therefore, as will be mentioned in the next chapter, further attempt is made in this investigation to extend the newly proposed Murakami's method to the quadratic shape function. Murakami's method determines K in a simplified manner using constant stress elements.

4. Extension of Murakami's Method to Quadratic Shape Function

4.1 Murakami's Method and its Extension

Murakami[11] recently proposed the simplified method of determining K with high accuracy using the conventional constant stress elements. His method, in essence, is based on the fact that the stress value of the crack tip element contains the stress when there is no crack and he obtained highly accurate K value by making corrections for this effect. As the procedure of the crack path determination involves re-divisioning, there is a contradictory demand that the division be as coarse as possible and that K be determined with high accuracy because the accuracies of the K_I and K_{II} values affect the accuracy of the direction θ of crack extension[12]. Murakami's method can contain the maximum error in K within 6%, whatever the element division may be. Therefore, the authors extended Murakamis' method to the quadratic shape function in order to use the coarser division and to reduce the computational effort. **Figure 5** shows the

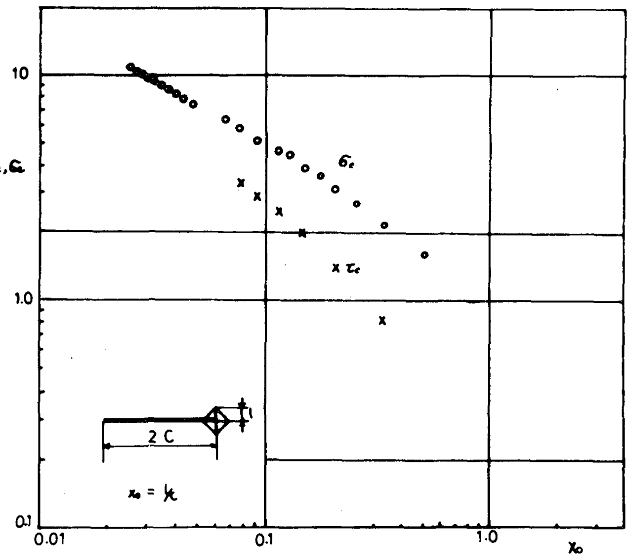

Fig. 5 Correction factors for extended Murakami's method

correction factors σ_c, τ_c for the quadratic shape function obtained by solving the same problems as Murakami did[11]. The only requirement in applying Murakami's method is to divide the crack tip region in the same manner as shown in Fig. 5. Therefore, when the crack path can be roughly estimated beforehand, the enormous reduction of efforts is possible if elements are divided in the same manner in the expected direction of crack extension because the simulation can be carried out by mere rotation of coordinates at the crack tip at each stage of crack extension.

4.2 Analysis of 45° Oblique Crack Using Extended Murakami's Method

Figure 6 shows the result of the re-analysis of the

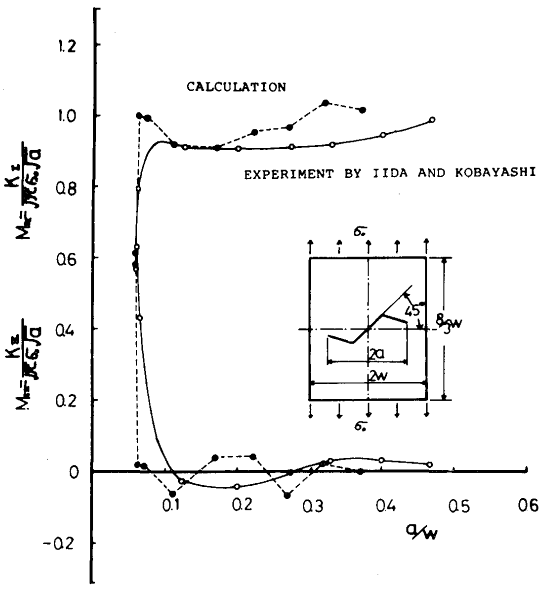

Fig. 6 Relation of K_I and K_{II} against half crack length a (45° oblique crack)

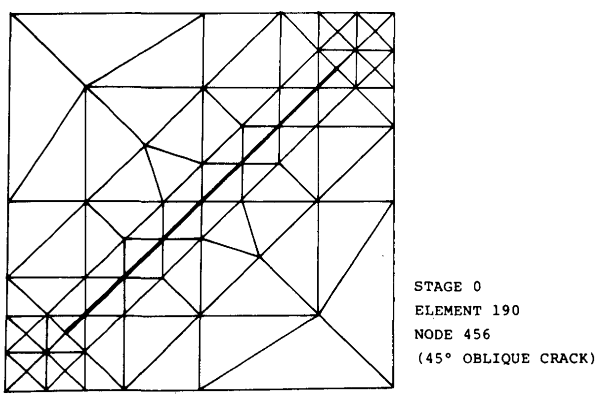

Fig. 7 Mesh division around crack

same 45° oblique crack case as analyzed in 3.2, using the extended Murakami's method. The mesh division around the crack at stage 0 is shown in **Fig. 7**. As can be seen from Fig. 6, the K values obtained are quite accurate, although these K values are obtained using almost the same size element as the one at the crack tip at stage 0 as in Fig. 7. But Fig. 6 indicates only the relation and accuracy of K against the projected crack length a. As previously mentioned, there is no experimental data to be compared with the numerically obtained crack path such as shown in **Fig. 8**. The

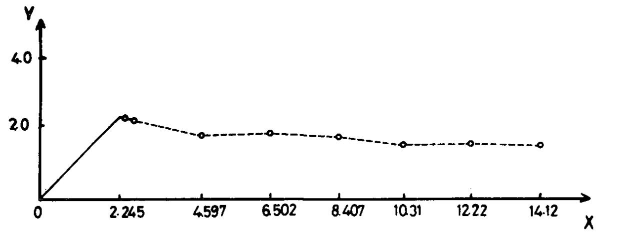

Fig. 8 Predicted crack path from 45° oblique crack

test was conducted, therefore, in which a 60° oblique crack was introduced into a commercial-grade anti-corrosion aluminum alloy sheet and the experimentally obtained crack path was compared with the numerically obtained path, as will be mentioned in the next chapter.

5. Crack Path Analysis of 60° Oblique Crack

5.1 Experiment

The geometry and the dimensions of a 60° oblique crack test specimen used for obtaining a crack path experimentally are shown in **Fig. 9**. An initial crack was introduced by wire-cutting. The material is commercial-grade anti-corrosion aluminum alloy sheet (equivalent to 3003–1/4 H). Tension-tension load with the load range of 3.30 kg/mm^2 was applied at 60 Hz. The 135 mm part as shown in Fig. 9 is the specimen length at testing and both ends are used for chucking.

5.2 Simulation

The crack path is simulated by the extended Murakami's method using coarse mesh (I) and fine mesh (II) to examine the effect of mesh divisioning. The conventional analysis using the quadratic shape function as described in 3.2 (hereafter called "direct method") was also carried out for comparison. **Figure 10** and **Fig. 11** show the mesh division around the crack of the division (I) of the extended Murakami's method at stage 0 and at stage 2 respectively. **Figure 12** and **Fig. 13** show the mesh division around the

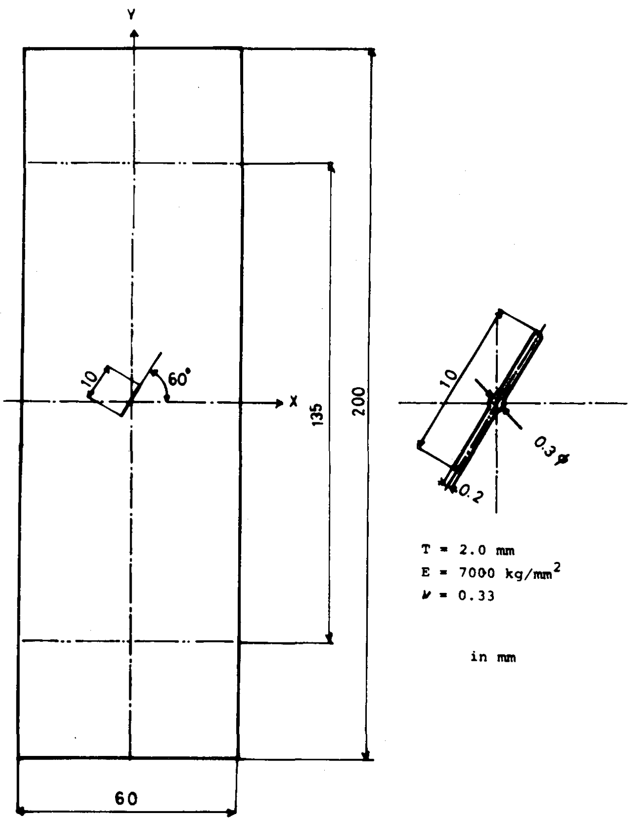

Fig. 9 Geometry and size of the specimen used for experiment

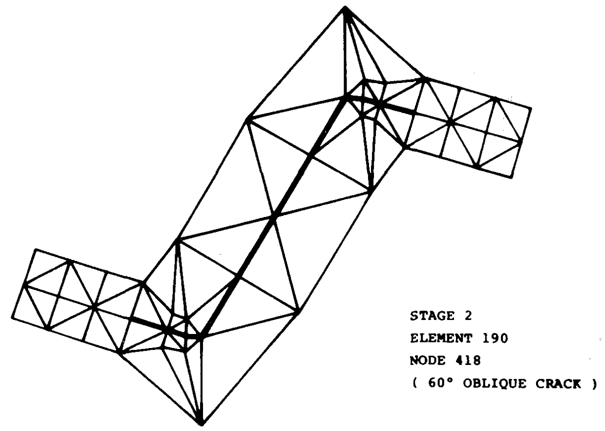

Fig. 11 Mesh division at stage 2 for extended Murakami's method (division I···coarse mesh)

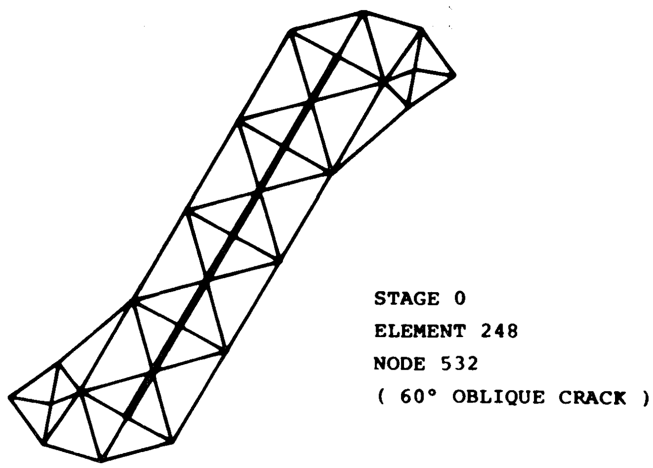

Fig. 12 Mesh division at stage 0 for extended Murakami's method (division II···fine mesh)

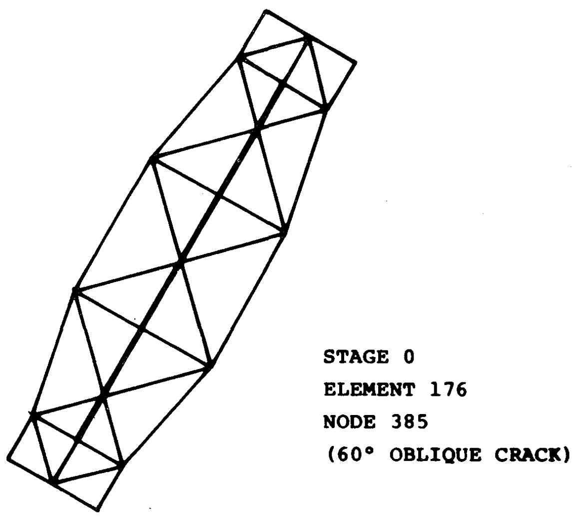

Fig. 10 Mesh division at stage 0 for extended Murakami's method (division I···coarse mesh)

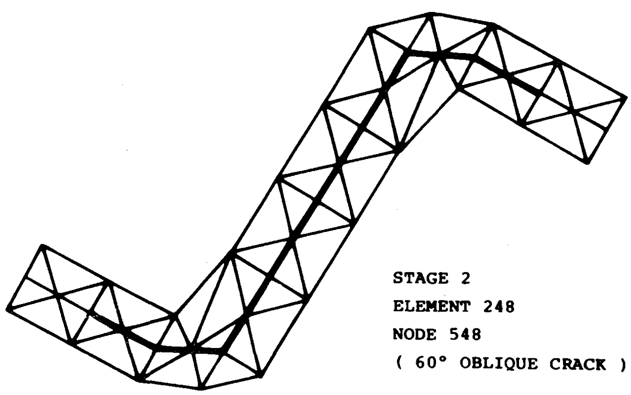

Fig. 13 Mesh division at stage 2 for extended Murakami's method (division II···fine mesh)

crack of the division (II) at stage 0 and at stage 2 respectively. To facilitate the comparison, elements of the same size and configuration as used in the extended Murakami's method was also used at the crack tip in the direct method, and the stress value at the 'mid-side' node as pointed by an arrow in **Fig. 14** was

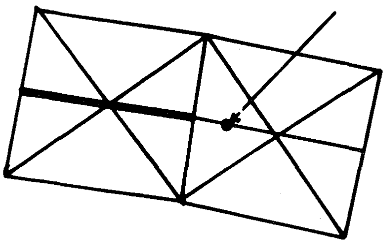

Fig. 14 'Midside' node used for obtaining K in conventional analysis

used in calculating K by the direct method. In the extended Murakami's method, the nodal stress value at the crack tip was used for obtaining K.

5.3 Results and Discussion

Figure 15 shows the crack paths obtained from

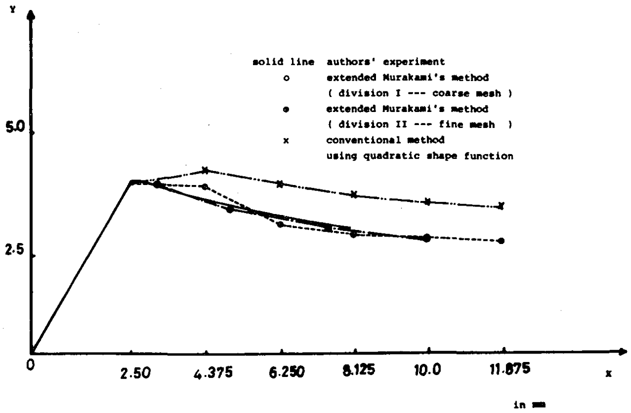

Fig. 15 Crack Path from 60° oblique crack

experiments and simulations. It should be noted that no matter what element subdivision is used, the extended Murakami's method predicts a crack path quite well as can be seen easily comparing the results of ○ and ●, i.e., the results of coarse mesh division and fine mesh division. This implies that a macroscopic crack path thus predicted is comparatively insensitive to the crack growth increment chosen. It is also observed that an accurate crack path cannot be predicted by the direct method, i.e., by the conventional analysis using quadratic shape function, unless the crack tip region is divided into finer meshes. In the direct method analysis, the predicted crack path differs from the actual one at the early stage of crack extension. But as the crack extends longer, the predicted crack path comes to run parallel with the actual one. This is considered due to the fact that as the crack length increases, the relative size of the crack tip element decreases in comparison with the crack length so that accuracy increases.

Figure 16 shows the relationship of the K_I and K_{II}

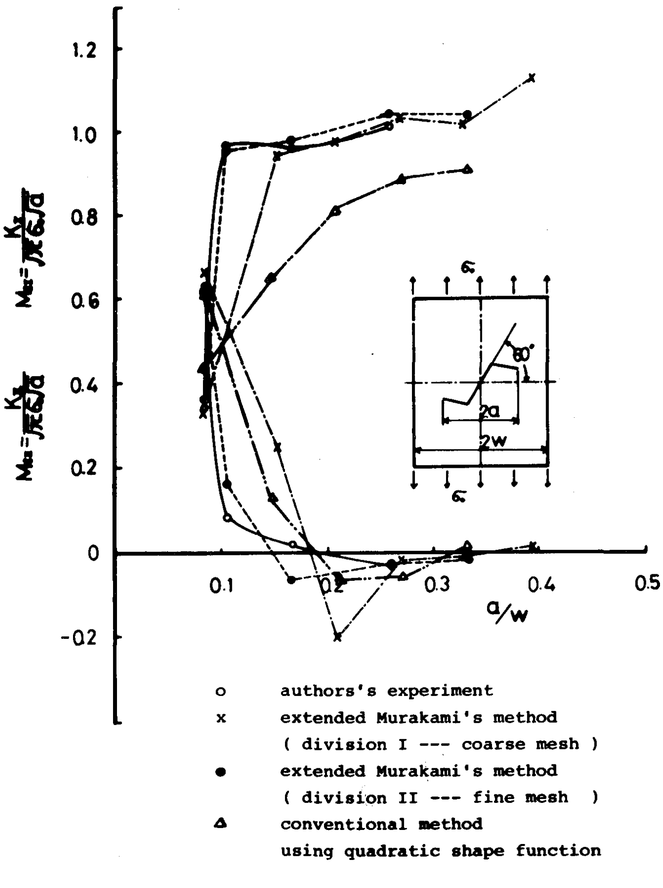

Fig. 16 Relation of K_I and K_{II} against half crack length a (60° oblique crack)

against the projected half crack length a. The result obtained by the extended Murakami's method naturally agrees well with that of the experiment. But as can be expected, the K_I value obtained by the direct method differs greatly. Although experimental works which studied fatigue crack growth in $K_I - K_{II}$ mixed mode are quite few, Iida and Kobayashi[13], and Kitagawa and Susuki[14] found that the linear relationship between the crack growth rate da/dN expressed in terms of the projected crack length a, and the K_I of the mixed mode is in parallel with that of $da/dN - K_I$ in mode I, and that the difference between these crack growth rates is 10% at the highest. Therefore, although the condition for a fatigue crack to grow in mixed mode is at present not fully clear, these experimental results indicate that if the crack path is accurately predicted and the relationship between a and K_I can be precisely calculated based on the predicted crack path, then an engineering optimum design against fatigue failures can be carried out. It is apparent, therefore, that the conventional analysis which can not easily evaluate the precise relationship between a and K_I is not suitable for studying the behavior of fatigue crack growth in combined mode.

This technique can be applied in a straightforward manner to such problems as those of deciding the optimum location for the staggered rivet holes or bolt holes in an airplane to prevent fatigue failures.

In the case of weldments, the assumption of homogeneity does not hold any more because material properties change by welding heat input so that the straightforward application of this technique is not possible. But if the fatigue tests are conducted using specimens with material properties corresponding to each region such as weld metal, HAZ, or base metal which can be obtained by welding thermal cycle tests, and once the relationship between fatigue crack growth rate and stress intensity factor can be established for each region, it is considered possible to incorporate inhomogeneity into the analysis at least from the standpoint of design. Another important problem in welding is the problem of residual stress. But now as welding residual stresses can be evaluated by the thermal elasto-plastic finite element method, the K value in the residual stress field can be obtained. In this manner, therefore, it is considered possible to apply the present technique to the optimum design of weldments against fatigue failures based on fracture mechanics. Further, to cite a practical example, this technique could be applied to such problems as the determination of, for example, optimum root gap or optimum leg length of a fillet joint with a root gap from the standpoint of prevention of fatigue failures.

6. Summary and Conclusions

The prediction of fatigue crack path in the combined mode by the finite element method was attempted, with the purpose of serving for an optimum design of structures against fatigue failures and the fatigue crack path emanating from an oblique initial crack was successfully predicted.

The following conlcusions are obtained;

(1) The fatigue crack path from an oblique crack in combined mode is successfully simulated by the finite element method.

(2) In the crack path prediction in combined mode, the reduction of computational efforts by the introduction of higher shape function is not so great as in other cases.

(3) Extension of Murakami's method to the quadratic shape function brings about the considerable amount of reduction of computational efforts in the crack path prediction without the loss of accuracy.

Acknowledgements

The authors would like to thank Mr. Seiji Tanaka of Fujitsu, Ltd. for his help with the wire-cutting of slits and Dr. Koji Koibuchi and Mr. Sumihisa Kotani of Hitachi, Ltd. for their permission to use the fatigue testing machine. The auhtors wish to thank Prof. Hiroyuki Okamura of the University of Tokyo and Prof. Takeshi Kunio of Keio University for their kind suggestions and discussions.

This work has been supported in part by the Government Grant, Ministry of Education.

References

1) (References (1)–(5) are only the typical part of the papers on crack path problem)
F. Erdogan and G. C. Sih, "On the Crack Extension in Plates under Plane Loading and Transverse Shear", Trans. ASME, Ser. D, Vol. 85, 1963, p. 519
2) G. C. Sih, "Some Basic Problems in Fracture Mechanics and New Concepts" Eng. Frac. Mech., Vol. 5, 1973, p. 365
3) I. Finnie and H. D. Weiss, "Some Observations on Sih's Strain Energy Density Approach for Fracture Prediction", Int. J. Frac., Vol. 10, No. 1, March 1974, p. 136
4) J. Coughlan and B. I. G. Barr, "Future Path Prediction" Int. J. Frac., Vol. 10, No. 4, Dec. 1974, p. 590
5) H. Kitagawa and R. Yuuki, "Stress Intensity Factors and Several Properties of a Deflecting Crack-Work on Crack Morphology-Report 2", Preprint of JSME, No. 750–1, 1975, p. 223 (in Japanese)
6) Ref. (1)
7) O. C. Zienkiewicz, The Finite Element Method in Engineering Science, 1975, McGraw-Hill
8) O. C. Zienkiewicz and G. S. Holister, Stress Analysis, John Wiley and Sons, 1965
9) J. H. Argyris, J. Roy. Aero. Soc. Vol. 69, 1965
10) S. Iida and A. S. Kobayashi, "Crack Propagation Rate in 7075–T6 Plates Under Cyclic Tensile and Transverse Shear Loadings", Trans. ASME, Ser. D, 1969, p. 764
11) Y. Murakami, "A Simple Procedure for the Accurate Determination of Stress Intensity Factors by Finite Element Method", Trans. JSME, Vol. 42, No. 360, 1976, p. 2305 (in Japanese)
12) B. Cotterell, "Notes on the Paths and Stability of Cracks" Int. J. Frac. Mech., Vol. 2, No. 3, Sep. 1966, p. 526
13) Ref. (10)

14) H. Kitagawa and I. Susuki, "Interaction of Cracks and their Growth by Fatigue (Work on Distributed Cracks-Report 1)" Preprint of JSME, No. 740–10, 1975, p. 227 (in Japanese)
15) S. Iida and Y. Kakuta, "Propagation Behavior of Staggered Fatigue Cracks", Preprint of JSME, No. 720–9, 1973, p. 93 (in Japanese)
16) The following paper is a comprehesive and detailed survey of the determination of stress intensity factors by the finite element method including the case of mixed mode.
H. Pian, "Crack Elements", Proc. World Congress on Finite Element Methods in Structural Mechanics, Oct. 1975 England