Effects of Welding Residual Stresses and Initial Deflection on Rigidity and Strength of Square Plates Subjected to Compression (Report II)†

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Abstract

When welding is applied for construction of structures, shrinkage of welded portion may produce initial imperfections, such as residual stresses and distortion, in structural elements. This distortion is accompanied by local bending stresses which correspond to the curveture of the distortion.

In this 2nd report, influences of residual stresses, geometrical changes and local bending stresses of the distortion upon the behavior of a square plates under thrust were studied. A series of theoretical analysis was carried out using the finite element method. From the results of analysis, it was found that these factors reduce ultimate strength of the plate under thrust, and welding residual stresses reduce buckling strength remarkably, but not so much ultimate strength. Finally, a relation between the ultimate strength and the allowable initial deflection specified by the Japan Shipbuilding Quality Standards was discussed.

1. Introduction

It is usually very difficult to construct structures by welding without being accompanied by initial imperfections. For example, plates may be initially deformed as they are supplied from factories. Furthermore, when stiffeners are attatched to plate elements by fillet welding, initial deflection of plate elements is induced by angular distortion of the welded portion. At the same time, welding residual stresses promote initial deflection. These initial deflection is accompanied by local bending stresses which correspond to the curveture of the deflection. When a plate element is subjected to thrust, initial deflection and local bending stresses due to initial deflection reduce the rigidity and the ultimate strength of the plate.[1)3)] In addition to this, welding residual stresses also reduce the rigidity and the ultimate strength,[1)3)] and even when initial deflection is not existing, these stresses decrease the buckling strength.

In this report, a series of theoretical analyses is performed on the behavior of square plates with initial imperfection under thrust with the aid of the finite element method. First, the effect of initial deflection on ultimate strength is studied, and an empirical formula is proposed to predict the ultimate strength of a square plate with initial deflection. This formula is compared with those proposed by other investigaters.[6)7)] Then, effects of local bending stresses and of the aspect ratio of the plate on ultimate strength are investigated. Finally, a relation between the ultimate strength and the allowable initial deflection specified by the Japan Shipbuilding Quality Standers[4)] is discussed.

2. Effect of initial deflection on compressive ultimate strength

2.1 Effect of the magnitude of initial deflection

The theoretical ultimate strength of a square plate under compression (uniform compressive displacement) is plotted by ○(1st report[1)3)]) and △(2nd report[2)]) against $b/t\sqrt{\sigma_Y/E}$ in **Fig. 1**, where b/t, σ_Y and E are the breadth to thickness ratio, yield stress and Young's modulus of the material, respectively. The plate is assumed to have initial deflection of the form

$$w = w_0 \cos\frac{\pi x}{b}\cos\frac{\pi y}{b} \qquad (1)$$

where w_0 is the magnitude of initial deflection at the center of the plate, and w_0/t is chosen as 0.01, 0.25, 0.50 and 1.00. The origin of x and y-coordinates is taken at the center of the plate. The computed ultimate strength indicated by ○ and △ are smoothly connected by solid lines. These curves can be expressed in the following form applying the method of least squares,

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$$\frac{\sigma_u}{\sigma_Y}=\frac{1.338\eta^2+4.380\eta+2.647}{\xi+6.130\eta+0.720}-0.271\eta-0.088 \quad (2)$$

where

$\xi=b/t\sqrt{\sigma_Y/E}$ and $\eta=w_0/t$

The ultimate strength curves given by Eq. (2) are represented by chain lines in **Fig. 1**, and in good agreement with the solid lines especially for the range of $b/t\sqrt{\sigma_Y/E}>1.5$.

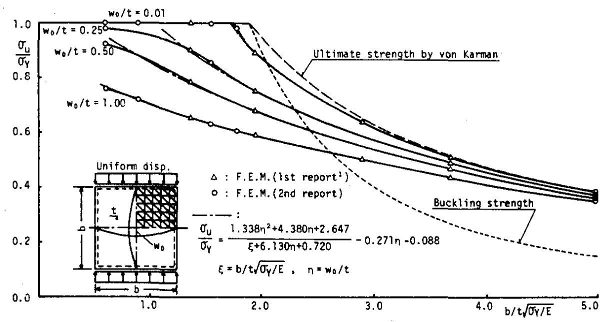

Fig. 1 Ultimate strength of square plate with initial deflection under compression

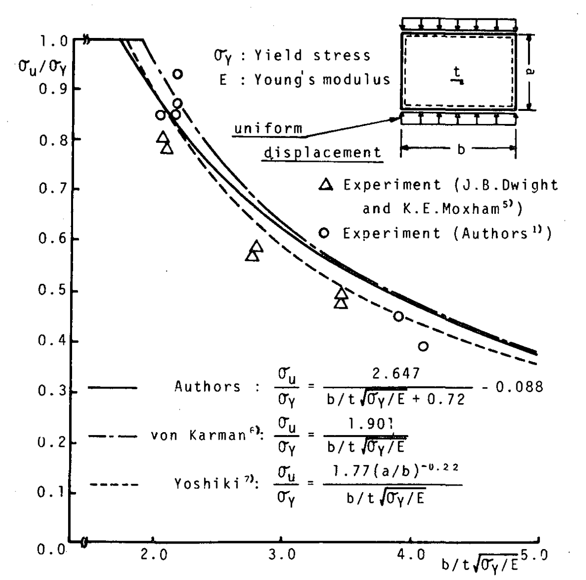

Fig. 2 Comparison of ultimate strengths predicted by approximate formula

2.2 Comparison of the proposed formula with those by other investigators

Putting $\eta=0$ in Eq. (2), the ultimate strength of a square plate witnout initial deflection under thrust can be expressed in the following form,

$$\frac{\sigma_u}{\sigma_Y}=\frac{2.647}{\xi+0.720}-0.088 \quad (3)$$

The ultimate strength obtained by Eq. (3) is compared with those obtained by the formulae proposed by von Kármán[6] and Yoshiki[7], respectively, that is,

(1) Von Kármán

$$\frac{\sigma_u}{\sigma_Y}=\frac{1.901}{\xi} \quad (4)$$

(2) Yoshiki

$$\frac{\sigma_u}{\sigma_Y}=\frac{1.77(a/b)^{-0.22}}{\xi} \quad (5)$$

where a/b in Eq. (5) is the aspect ratio of the plate. The ultimate strength curves obtained from these three formulae are illustrated in **Fig. 2.** The ultimate strength of a square plate obtained from Eq. (4) shows a good agreement with that from Eq. (3), when the plate is thin ($b/t\sqrt{\sigma_Y/E}>3$).

Equation (5) was derived by determining the values of the coefficient from the experimental results, considering the effects of the aspect ratio and initial deflection of the specimens. The ultimate strength evaluated by this formula is somewhat lower than that by Eq. (4) and rather correlated well to that by Eq. (2) when $w_0/t=0.0$ for thick plates and $w_0/t=0.3\sim0.5$ for thin plates. According to Yoshiki's paper[7], the specimens used in his experiment had initial deflection of which magnitude, w_0/t, is about 0.5 at maximum, and lower prediction of the ultimate strength by Eq. (5) may be due to the existence of initial deflection of the specimens on which Eq. (5) is based.

2.3 Effect of local bending stresses

Initial deflection in the plate elements of welded structures is produced by angular distortion of the welded portion and it is promoted by welding residual stresses in compression. However, in this section, it is assumed that initial deflection is produced only by the angular distortion. In the theoretical analysis, this angular distortion is replaced by the action of uniformly distributed bending moment along edges, and the resulting initial deflection is accompanied by local bending stresses. **Figure 3** shows the relation between the bending moment and the lateral deflection at the center of the plate of 500×500 mm, of which thicknesses are 4.5 mm, 9.0 mm and 12.7 mm. In the case of thick plate, plastification occures first at the surface of the four corners of the plate when the central deflection reaches $20\sim30\%$ of the plate thickness, and the plastic zones spread as the bending moment increases. When the plate is subjected to thrust, the behavior of the square plate with initial deflection accompanied by local bending stresses is illustrated in **Fig. 4**, together with that excluding the effect of local bending stresses. It is observed that the local bending stresses remarkably reduce the ultimate strength especially in the case of thick plate.

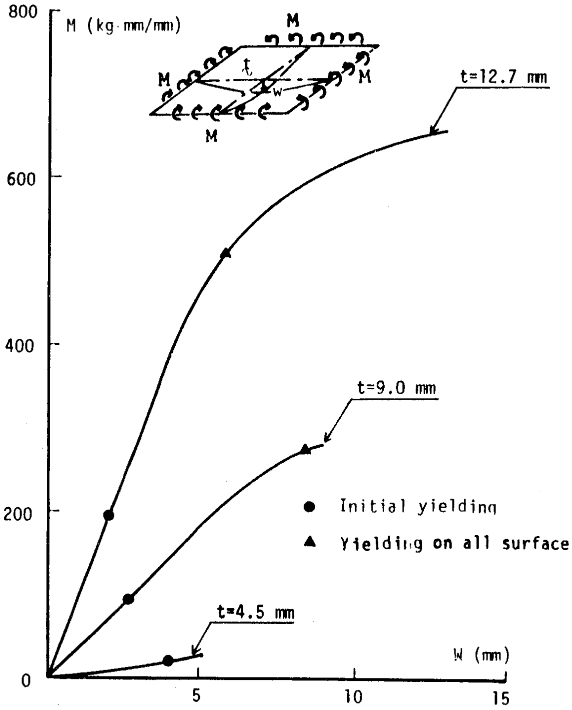

Fig. 3 Relation between uniformly distributed moment and central deflection

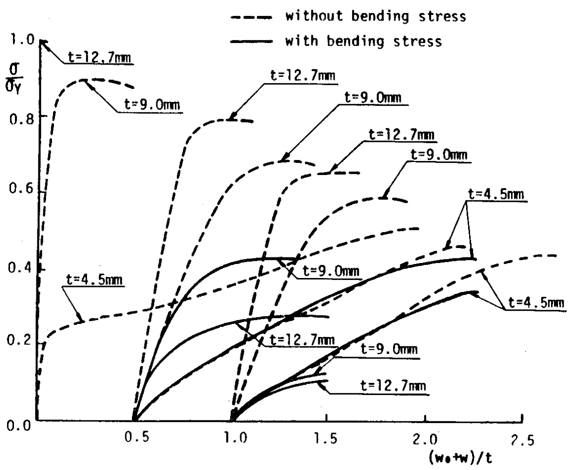

Fig. 4 Effect of local bending stresses induced by initial deflection on ultimate strength

2.4 Effect of the aspect ratio of the plate

In general, the aspect ratio of plate elements in actual structures is not necessary to be unity. When a rectangular plate of the aspect ratio being more than 2 is subjected to thrust, local buckling occures so as to take the minimum buckling strength. In this case, the aspect ratio of the locally buckled portion, that is one half wave length of the local buckling mode to the width, is close to unity, and the plate reaches to its ultimate strength usually keeping this local buckling mode. Similarly, in the case where the aspect ratio is less than unity, the deflection mode at the ultimate strength is same as that of local buckling. The above discussion is valid only when the plate has no or very small initial deflection. However, when there exists large initial deflection in the plate, the plate sometimes reaches to its ultimate strength without taking such a deflection mode as to give the minimum buckling strength. Therefore, it is also important to know the ultimate strength of a rectangular plate with initial deflection. In this section, as one of such cases, a rectangular plate of which aspect ratio is 0.5 is dealt with.

Figure 5 shows the ultimate strength of the rectangular plate with the aspect ratio of 0.5, together with that of a square plate and those obtained from Eqs. (4) and (5). When initial deflection is small, the ultimate strength of a rectangular plate with the aspect ratio of 0.5 is higher than that of a square plate, but lower when initial deflection is large.

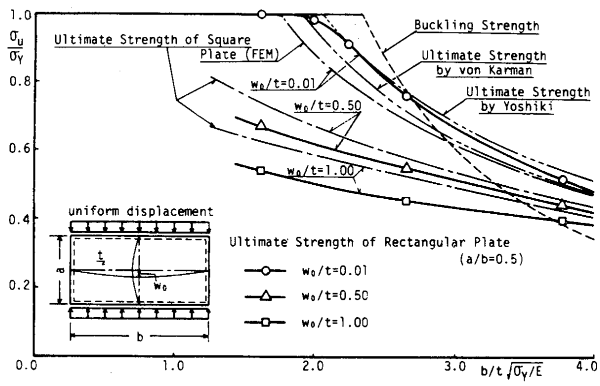

Fig. 5 Ultimate strength of rectangular plate with initial deflection under compression (a/b=0.5)

The ultimate strength obtained shows a good agreement with that computed from Eq. (4) when $b/t\sqrt{\sigma_Y/E}<4$, and that from Eq. (5) when $b/t\sqrt{\sigma_Y/E}>4$. Equation (4) was proposed under the assumption that the ultimate strength is not affected by the aspect ratio of the plate, and this assumption may be valid when the plate is thin ($b/t\sqrt{\sigma_Y/E}>4$).

3. Effect of welding residual stresses

3.1 Distribution of welding residual stresses

In this section, the deck plate in a ship structure is considered as a plate element. It may be assumed that welding residual stresses in the deck plate are produced only in the direction of the deck longitudinal members by attatchment of these members to the deck plate by fillet welding. Two simplified patterns are assumed as possible distributions of the welding residual stresses in the deck plate. These patterns are illustrated in **Fig. 6**, where the stresses are self-balanced. The notation of the pattern is read as follows; for example, taking Type A/5, the pattern of the distribution is type A, and the breadth in which the residual stresses are tensile is 1/5 times the plate breadth.

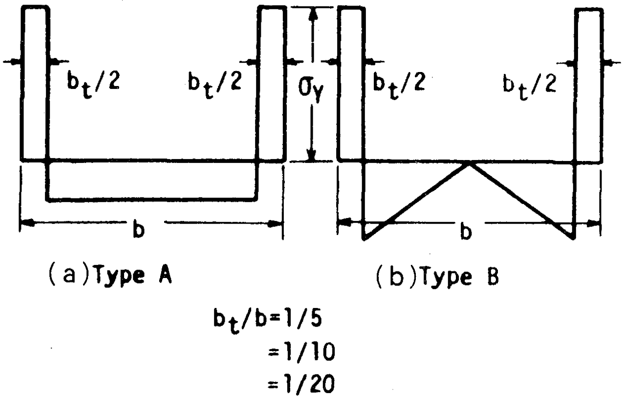

Fig. 6 Assumed distribution of welding residual stresses

3.2 Effect of welding residual stresses on buckling strength

In welded structures, welding residual stresses are always induced in plate elements, but initial deflection is not necessarily produced. In such a case when a plate is completely flat, buckling of the plate elements may occure under thrust, and the welding residual stresses affect the bukcling strength. The theoretical buckling strength of a square plate with welding residual stresses are represented in **Fig. 7**. The assumed distribution of the welding residual stresses shown in Fig. 6 are employed, taking $b_t/b=1/5$, 1/10 and 1/20 for each pattern. With the welding residual stresses, the buckling strength decreases, and more remarkable decrease is indicated when the distribution of Type A is employed of which residual stresses in compression are higher in the central portion of the plate. It can also be mentioned that the decrease of the elastic buckling strength due to the welding residual stresses is the same for a specified stress distribution, and that, the plate buckles by only residual stresses without any external load when the plate is thin and the magnitude of the residual stresses is great.[8]

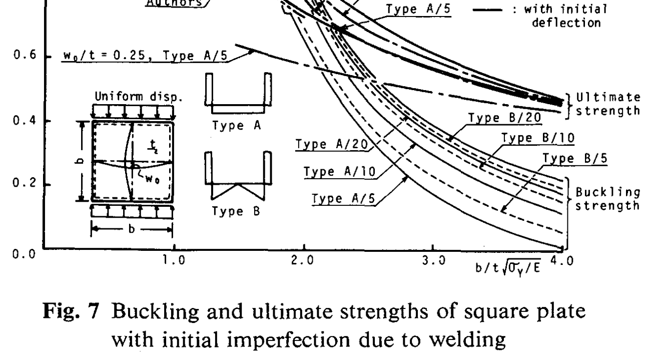

Fig. 7 Buckling and ultimate strengths of square plate with initial imperfection due to welding

3.3 Effect of initial imperfection on the ultimate strength

As mentioned in 2.3, welded structures are always accompanied by welding residual stresses, local bending stresses and initial deflection, and these components of the initial imperfection interact each other. Here, the problem is restricted to the deck plate of a ship structure, and simplified such that the first initial deflection accompanied by local bending stresses is produced by angular distortion of the fillet weld laid to fit the longitudinal members to the deck plate, and then, additional deflection is produced by the compressive residual stresses due to the shrinkage of the welded portion. In this section, a square plate is taken out from one longitudinal space, and the plate is assumed to be simply supported along all edges. The initial imperfection of the plate due to welding is assumed to be produced by giving angular distortion of the weld (uniformly distributed bending moment) along two parallel edges, and the welding residual stresses simultaneously. The magnitude of the initial deflection at the center of the plate is specified to 0.01 and 0.25 times the plate thickness. The employed distributions of the welding residual stresses are Type A/5 and Type B/20 with $w_0/t=0.01$, and Type A/5 with $w_0/t=0.25$. The ultimate strength for each case is illustrated in Fig. 7 by the chain lines with one dot, together with that with only initial deflection, which is represented by solid lines. As is known from the analytical results for the case of $w_0/t=0.01$, the welding residual stresses reduce the ultimate strength in the range of $b/t\sqrt{\sigma_Y/E}>1.8$, but not so much as in the case of the buckling strength. It can also be said that the welding residual stresses do not affect on the ultimate strength for $b/t\sqrt{\sigma_Y/E}<1.8$. In the case of $w_0/t=0.25$, the ultimate strength is remarkably reduced due to initial imperfection when the plate is thick. This reduction is mainly due to the existence of the local bending stresses, judging from the results obtained in the 1st report,[1)3)] since the 1st report furnished the following fact; when the plate has initial deflection and welding residual stresses together excluding the local bending stresses, the ultimate strength does not decrease more than that of the plate with initial deflection only, in the case of thick plate.

4. Discussion on the allowable initial deflection of deck plates specified by the Japan Shipbuilding Quality Standards[4]

As clarified in the preceeding discussion, initial imperfection due to welding reduce the ultimate strength of a plate element in welded structures. The ultimate strength of a square plate under uniform compressive displacement can be expressed by Eq. (2) including the effect of initial deflection free from any local bending stresses. When initial deflection is accompanied by local bending stresses which correspond to the curveture of the deflection, these stresses furthermore reduce the ultimate strength. This decrease is more remarkable especially when the plate is thick, which is due to the early plastification of the plate. Furthermore, the welding residual stresses reduce the ultimate strength, and this decrease is remarkable when $1.8 < b/t\sqrt{\sigma_Y/E} < 2.5$.

According to the Japan Shipbuilding Quality Standards, the allowable initial deflection of deck plates is specified as 6 mm regardless of the dimensions of the plate. Here, taking the yield stress, $\sigma_Y = 32$ kg/mm², and the longitudinal space, $b = 1000$ mm, the ultimate strength of a square plate taken out from one longitudinal space can be calculated by using Eq. (2), when the magnitude of initial deflection, w_0, is 6 mm. The calculated ultimate strength is represented by a chain line with two dots in Fig. 7, in the range of the plate thickness used in tankers of 50,000∼400,000 tons in dead weight ($1.0 < b/t\sqrt{\sigma_Y/E} < 2.0$). The results of computation indicates that the allowable initial deflection specified by the JSQS guarantees approximately the same ultimate strength as that of a square plate with initial deflection of $w_0/t = 0.25$, when the effect of the local bending stresses is not considered.

On the other hand, the chain line with three dots in Fig. 7 shows the ultimate strength obtained from the allowable initial deflection based on the method[9] proposed by the authors applying the probabilistic concepts. This allowable initial deflection is determined based on the result of a statistical investigation into initial deflection of the deck plate of newly constructed tankers and loads acting on the deck plates of the existing tankers, and examining the probability of the failure of the deck plate. The ultimate strength of a deck plate with such allowable initial deflection is 85% of the yield strength, regardless of the dimensions of the plate, and this standards may be more reasonable than the JSQS when the deck plate of tankers of 50,000∼400,000 tons in dead weight is considered ($1.0 < b/t\sqrt{\sigma_Y/E} < 2.0$).

In the above discussion, only the geometrical effect of initial deflection is considered. On the other hand, initial deflection due to welding is always accompanied by local bending stresses, and these stresses may furthermore reduce the ultimate strength of a deck plate. However, the aspect ratio of one panel of the deck plate is usually from 4 to 5, and the actual initial deflection is in a very complex form, which may be expressed in the following form,

$$w = w_{01}\cos\frac{\pi x}{b}\cos\frac{\pi y}{a} + w_{02}\cos\frac{\pi x}{b}\sin\frac{2\pi y}{a} + w_{03}\cos\frac{\pi x}{b}\cos\frac{3\pi y}{a} + \cos\frac{\pi x}{b}\sin\frac{4\pi y}{a} + \cdots\cdots\cdots \tag{6}$$

where a and b are the length and breadth of the plate, respectively, and the origin of the x and y-coordinates is taken at the center of the plate. The thermal angular distortion produces deflection expressed mainly by the first term in Eq. (6), and by the curvature of this deflection, smaller local bending stresses in the loading direction (that is, in y direction) is induced than those of a square plate. Therefore, the decrease in the ultimate strength of a deck plate under compression may be much smaller than that of a square plate. Furthermore, it is quite possible that a certain combination of the deflection modes increases the ultimate strength of a rectangular plate than that of a square plate.[10] On this matter, further investigations are expected.

5. Conclusions

The effects of the initial imperfections due to welding on the rigidity and ultimate strength are investigated based on the results of a series of theoretical analyses using the finite element method. The results obtained are as follows.

(1) Initial deflection reduces the rigidity and ultimate strength of a plate subjected to compression (uniform compressive displacement).

(2) The local bending stresses which is proportional to the curvature of the initial deflection furthermore reduce the rigidity and ultimate strength.

(3) Welding residual stresses reduce the buckling strength remarkably, but have a little effect on ultimate strength when the plate is thin. On the other hand, when the plate is thick, welding

residual stresses reduce ultimate strength remarkably if there exists initial deflection accompanied by the local bending stresses.

(4) The allowable initial deflection specified by the Japan Shipbuilding Quality Standards guarantees the same ultimate strength as that of a square plate with initial deflection of $w_0/t=0.25$. On the other hand, the allowable initial deflection based on the probabilistic concepts guarantees ultimate strength of 85% of the yield strength.

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