Energy trapping of circumferential resonant modes at a thin-walled groove in a hollow cylinder

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Running title: Energy trapping in a hollow cylinder

Abstract

Energy trapping of circumferential resonant modes at a thin-walled groove in a hollow cylinder is discussed using theoretical dispersion curves of guided waves, semi-analytical finite element (SAFE) calculations, and experiments. The analysis of the dispersion curves predicts the energy trapping at a thin-walled groove using the analogy of energy trapping of Lamb waves in a stepped plate. The energy trapping frequencies and displacement distributions at these frequencies were independently obtained with SAFE calculations and experiments. They agreed well with each other and proved that the energy trapping modes exist at the thin-walled groove in the hollow cylinder.

Keywords: Energy trapping, Guided waves, Hollow cylinder, circumferential resonance

1. Introduction

Energy trapping of elastic waves is a well-known phenomenon, as evidenced by its representative application in a Quartz Crystal Microbalance (QCM)[1-6]. In a QCM, which is generally a circular quartz plate with gold electrodes at the center of both surfaces, the vibration energy is confined in a thicker area in the electrodes. Because the resonance is not affected by the geometry and supports located away from the electrodes, it becomes very stable. Using this prominent characteristic, the QCM is used as a highly sensitive sensor for detecting small variations in mass caused by oxide growth, adhering viruses and proteins, polymers, and molecules. Johnson et al[7]. presented energy trapping of a circumferential surface wave in a stepped region in a circular bar, and Ogi et al[8]. utilized energy trapping of a surface wave to establish an extremely sensitive biosensor as well as wireless and electrodeless device technologies.

The author has previously conducted theoretical and experimental studies on guided wave propagation in a hollow cylinder for the purpose of efficient pipework inspection[9-11]. Using the calculation technique and experimental system presented in these previous studies, this work discusses the energy trapping induced by circumferential guided waves due to variations in wall thickness in a hollow cylinder.

2. Prediction of energy trapping by dispersion curves

First, the energy trapping is qualitatively investigated using dispersion curves of guided waves in a hollow cylinder of uniform thickness. The displacement field of a harmonic wave at the angular frequency ω is expressed as a summation of guided wave modes propagating in the longitudinal direction in the hollow cylinder as[9, 10, 12]

$$\mathbf{u}(r,\theta,z)=\sum_{n=-\infty}^{+\infty}\sum_{m=1}^{+\infty}\alpha_{nm}\bar{\mathbf{u}}_{nm}(r)\exp(in\theta)\exp(ik_{nm}z), \tag{1}$$

where $\mathbf{u}$ is the displacement vector consisting of displacements in the radial (r), circumferential (θ), and longitudinal (z) directions of a cylindrical coordinate system, α_{nm} is the complex amplitude for the (n, m) mode, $\bar{\mathbf{u}}_{nm}$ represents the wave structure vector that is the function of radius r, and k_{nm} denotes the wavenumber of the guided wave propagating in the z direction. A time harmonic function $\exp(-i\omega t)$ is omitted in the equation. When a point source is located at $\theta = 0$, the symmetry with respect to $\theta = 0$ provides the relationship $\alpha_{nm} = \alpha_{-nm}$. Then, Eq. (1) becomes

$$\mathbf{u}(r,\theta,z)=\sum_{n=0}^{+\infty}\sum_{m=1}^{+\infty}\alpha'_{nm}\bar{\mathbf{u}}_{nm}(r)\cos(n\theta)\exp(ik_{nm}z), \\ \alpha'_{nm}=\begin{cases}2\alpha_{nm} & at\ n\neq 0\\ \alpha_{nm} & at\ n=0\end{cases} \tag{2}$$

The traction-free boundary condition on both surfaces of the hollow cylinder for the wave field represented as Eq. (2) yields dispersion curves for wavenumber k_{nm}. Figure 1 shows the dispersion curves for aluminum alloy pipes (used in the experiments described later) with a longitudinal wave velocity $c_{\mathrm{L}} = 6300\ \mathrm{m/s}$ and a transverse wave velocity $c_{\mathrm{T}} = 3100\ \mathrm{m/s}$. The figure shows only the modes with the circumferential order $n = 5$ and thickness order $m = 1$. Following previous works discussing energy trapping using dispersion curves of Lamb waves[1-3], the vertical axis represents frequency, and the horizontal axes on the right- and lefthand sides are the real part and the imaginary part of wavenumber k_{nm}, respectively. The blue lines are the dispersion curves for a pipe with an outer diameter of 40 mm and a thickness of 2.0 mm, whereas the black lines are the dispersion curves for a pipe with an outer diameter of 39 mm and a thickness of 1.5 mm. The solid and dashed lines denote the real part and imaginary part of the wavenumbers $\mathrm{Re}(k_{nm})$ and $\mathrm{Im}(k_{nm})$, respectively. At the cut-off frequency for $k_{nm} = 0$, the mode does not propagate in the longitudinal (z) direction and has the distribution of $\cos 5\theta$ with the circumferential resonance. The cut-off frequency for the pipe with the diameter of 39 mm is $f_c = 24.5$ kHz, and the wavenumber at the frequency for the pipe with the diameter of 40 mm becomes a complex value, $k_{nm} = 40 + 127i$. This indicates that energy trapping can be observed at a groove on the pipe surface, which is predicted from the analogy of energy trapping in a stepped plate (such as QCMs). Namely, when a thin-walled groove of depth 0.5 mm is engraved around the circumference of an aluminum alloy pipe of diameter 40 mm and thickness 2.0 mm, and the harmonic vibration at the cut-off frequency is applied to the thin-walled area, a standing wave with the distribution $\cos 5\theta$ in the circumferential direction is formed without propagation in the longitudinal direction as $k_{nm} = 0$ at the thin-walled groove. Simultaneously, because the wavenumber becomes $k_{nm} = 40 + 127i$ outside the groove, the displacement decreases exponentially depending on the distance from the edge of the groove at $\exp(-127z)$ in the longitudinal direction. Thus, the vibration energy is confined at the thin-walled groove, with no leakage to the outside.

3. Calculations for energy trapping frequencies and wave structures

In the above-mentioned analysis using dispersion curves for a hollow cylinder of an infinite length, the energy trapping frequencies at a circumferential groove with a finite length cannot be obtained precisely. Here, the energy trapping frequencies are derived using a semi-analytical finite element (SAFE) calculation that was developed for analyzing guided wave propagation[13, 14]. In the SAFE calculation for circumferential guided waves in a hollow cylinder of finite length, an *r*-*z* cross-section is divided into small area elements, as shown in Fig. 2, and the displacement vector $\mathbf{U}$ that collects the displacement components along the nodal line stretching in the θ direction is described as

$$\mathbf{U}(\theta) = \sum_{n=-\infty}^{+\infty} \sum_{m=1}^{+\infty} a_{nm} \, \boldsymbol{\phi}_{nm} \exp(in\theta), \tag{3}$$

where a_{nm} is a complex amplitude, and $\boldsymbol{\phi}_{nm}$ is a displacement distribution vector that collects displacement components at nodal points on the *r*-*z* cross-section. Substituting Eq. (3) into the governing equation of the SAFE calculation yields a linear eigenvalue problem for a certain integer *n* with respect to the angular frequency ω. The eigenvalues and eigenvectors correspond to the resonant frequencies for a circumferential order of *n* and their displacement distributions on the *r*-*z* cross-section, respectively. Among the eigenvalues and eigenvectors for $n = 4, 5,$ and 6, the modes vibrating dominantly at the thin-walled groove are shown in Fig. 3. Figures 3 (a)–(c) are the energy trapping modes of flexural vibration with dominant out-of-plane displacement, and a displacement in the *r* direction is denoted by the surface color. The resonant frequencies were calculated as (a) 16.97 kHz, (b) 26.25 kHz, and (c) 37.19 kHz, respectively. Figures 3 (d)–(f) show the energy trapping modes of shear horizontal vibration in which in-plane displacement in the *z* direction is dominant, and the surface color denotes the displacement in the *z* direction. The resonant frequencies were given as (d) 107.41 kHz, (e) 131.95 kHz, and (f) 157.08 kHz, respectively. The mode shown in Fig. 3 (b) corresponds to the mode discussed using the dispersion curves of Fig. 1, and the resonant frequency of 26.25 kHz obtained with the SAFE calculation was just between the cut-off frequencies of the two hollow cylinders shown in Fig. 1. Moreover, these calculation results show that the energy trapping at a groove can be observed even for various values of circumferential order *n* and different vibration directions. The fact that the energy trapping can be obtained under a wide variety of conditions implies that this phenomenon offers promise for a wide variety of applications such as sensors and filters.

4. Experiments using non-contact measurements by laser

Finally, non-contact measurements of elastic waves were carried out for an aluminum alloy (A1070) pipe, as shown in Fig. 4 (a), to confirm the energy trapping at a groove in a hollow cylinder experimentally. Modulating a continuous wave from a fiber laser (Fujikura FLC-0300S) allowed elastic wave generation and control, and burst waves of duration 50 ms were generated in these experiments. The elastic waves were detected by a laser Doppler vibrometer (Polytec OFV-5000)[11].

First, the resonant frequencies of the flexural vibration modes with the circumferential orders of *n* = 4, 5, and 6, as shown in Figs. 3 (a), (b), and (c) were experimentally identified. The source of the elastic waves and the receiving point were located at the groove, as shown in Fig. 4 (a). The modulation signals changed from 10 kHz to 45 kHz in increments of 0.02

kHz, and Fig. 4 (b) was plotted using the frequency spectrum peak of the detected waveform. The red arrows in the figure denote the resonant frequencies of 17.06 kHz, 26.32 kHz, and 37.22 kHz, which agree well with the energy trapping frequencies obtained by the calculations for the circumferential orders of n = 4, 5, and 6, (i.e., 16.97 kHz, 26.25 kHz, and 37.19 kHz), respectively.

Next, distributions of the frequency spectrum peak were created by generating the energy trapping frequencies and rastering the laser source, as shown in Figs. 4 (c) - (e). Although these figures were obtained by rastering the source point of the elastic wave, these results are identical to those obtained by rastering the detection point for a fixed source, considering the reciprocal theorem of elastic waves. Namely, these figures represent wave distributions for a fixed elastic wave source located at the groove. Stable measurements were realized by rastering the laser source instead of moving the receiving point that is largely affected by the surface condition[11]. Because the generation laser was emitted about 2.5 m away from the pipe and a collimated beam about the diameter of 5 mm was used, the beam diameter maintained an ellipse with the major axis that is sufficiently smaller than the wavelength at most of the rastering points and elastic waves generated appropriately over the pipe surface[11]. The images are shown in gray scale where the spectrum peaks are normalized by the maximum value in each figure and the maximum values are indicated in black and zero values are indicated in white. These figures agree well with the calculation results shown in Fig. 3, and thus, the energy trapping modes at the groove in the hollow cylinder were experimentally verified.

5. Discussions

The energy trapping modes will mainly contribute to improvement in the sensitivities and responsiveness of various sensors using vibrations, as in QCMs. In these experiments, flexural vibration modes below 100 kHz were discussed using the aluminum pipe with the thickness of 2.0 mm because the experimental system was originally designed for pipe inspection studies[11]. However, the energy trapping is feasible at higher frequencies by reducing the overall dimensions of a pipe. The use of a thinner pipe leads to a highly-sensitive vibration sensor. For example, a grooved small pipe with a one-hundredth scale can be easily machined, and a commercial ultrasonic pulser-receiver can generate and receive ultrasonic at 10 MHz, which is 100 times higher than the frequency used in this paper. In a QCM, the energy trapping occurs in the thick electrode area, which limits their sensitivity. On the contrary, because the energy trapping area can be very thin in a hollow cylinder with a groove, we can expect to realize highly sensitive sensors. Moreover, as the calculation results in Figs. 3 (d)–(f) show, energy trapping is possible in the in-plane modes, indicating a wide variety of applications. For example, consider that the molecules in a solution are the detection targets. The Q value reduces significantly due to energy leakage into the liquid if the flexural vibration modes shown in Figs. 3 (a)–(c) are used. In the in-plane vibration modes seen in Figs. 3 (d)–(f), the energy leakage is relatively small and high Q values are provided[8]. Moreover, the in-plane modes can be generated and detected without contact and without electrodes by using electromagnetic acoustic transducers, thereby resulting in sensors with extremely high sensitivities.

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Figure captions

Fig. 1 (Color Online) Dispersion curves of the mode n = 5 for hollow cylinders of different thicknesses. Blue: outer diameter = 40 mm, thickness = 2.0 mm. Black: outer diameter = 39 mm, thickness = 1.5 mm.

Fig. 2 Hollow cylinder model used in the semi-analytical finite element calculations.

Fig. 3 (Color Online) Wave structures of energy trapping modes. (a)–(c) show flexural vibration modes. Color denotes out-of-plane displacement. (d)–(f) show shear horizontal vibration modes. Color denotes in-plane displacement in the longitudinal direction.

Fig. 4 (Color Online) Experimental results.