



Title	Multiple reflections of Lamb waves at a delamination
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- (c) Title: Multiple reflections of Lamb waves at a delamination
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- (f) Abstract: Wave propagation in laminated plates with delaminations was calculated using the semi-analytical finite element method. The visualization results and deeper numerical analyses revealed the following phenomena on the fundamental Lamb modes at delamination regions of laminated plates. First, Lamb wave propagates toward the delamination, and then splits into two independent waves at the “Entrance” of the delamination with no significant reflections. These two waves reach the “Exit” of the delamination with the different phases and arrival time. Thus reflected and transmitted waves are excited at the “Exit”. The repetition of such reflections at the “Exit” causes the multiple reflections at regular intervals corresponding to the delamination size.
- (g) Keywords: Multiple reflections, Lamb wave, Delamination, Numerical analysis
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## 1. Introduction

Nondestructive evaluations using Lamb waves are suitable for large plate-like structures due to their prominent characteristic of the long-range propagation. However, it is very difficult to understand the interaction of Lamb waves with defects or inhomogeneous regions owing to the mode conversions, dispersivities and the presence of many modes in received signals.

Computer simulation and visualization of Lamb waves are effective tools to clarify such complex ultrasonic wave motions. General-purpose computational codes such as FDM, FEM and BEM are also widely used for calculating wave propagation, but computational time and memory become fatal problems when applying these techniques to Lamb waves propagating in long range. On the other hand, analytical solutions, having been studied over 100 years, become more difficult to be derived for plates with complex geometries, and their solutions cannot be generally utilized.

Therefore, hybrid methods, combination techniques of analytical solutions and FEM or BEM, were developed for Lamb waves <sup>[1], [2]</sup>. In these hybrid methods, Lamb wave calculations became feasible with short calculation time and small computational memories, as arbitrary shaped regions like crack and/or inhomogeneous regions were described by the FEM or BEM and semi-infinite intact plate regions were calculated with analytical solutions. Numerical analyses for Lamb waves in laminated materials such as FRP or surface treated plates have been carried out using the Strip Element Method (SEM) proposed by G. R. Liu et al. <sup>[3], [4]</sup> that is the semi-analytical FEM based on the dynamic finite strip method developed by Y. K. Cheung <sup>[5]</sup>. Since the SEM uses layered elements that are discretized only in the thickness direction, its calculations require much less computational time and memories than the FEM. Moreover, in the

SEM, modal analysis is easily carried out because the solutions in the SEM are obtained as the sum of Lamb modes.

In this paper, Lamb wave propagation in laminated plates with a delamination is calculated using the SEM. Multiple reflections of Lamb waves at the zero volume delaminations are found in the visualization results. Moreover, detailed explanations for these multiple reflections are given by the dispersion curves of two regions divided by a delamination.

## 2. Simulation of Lamb wave propagation

To model a zero volume delamination, the regions I-IV should be combined in the SEM calculations as shown in Fig.1. The A0 mode or the S0 mode is excited by applying the transient point loads at  $(x, y)=(0,0)$  and  $(10d,d)$  on both surfaces of the plate in the same direction or opposite direction, respectively. The transient point loads are depicted by

$$F(\tilde{t}) = \exp \left\{ -2\pi i \tilde{f}_c (\tilde{t} - \tilde{t}_0) - \frac{(\tilde{t} - \tilde{t}_0)^2}{\tilde{\sigma}^2} \right\}, \quad (1)$$

where  $\tilde{t} = t/d$  [ $\mu\text{s}/\text{mm}$ ],  $\tilde{t}_0 = t_0/d = 7.0$  [ $\mu\text{s}/\text{mm}$ ],  $\tilde{f}_c = f_c d = 0.5$  [ $\text{MHz mm}$ ],

$\tilde{\sigma}^2 = \sigma^2/d^2 = 10.0$  [ $(\mu\text{s}/\text{mm})^2$ ],  $t$  is time,  $f_c$  is center frequency,  $\sigma^2$  is a variance and  $t_0$  is time delay. This dynamic load denotes ultrasonic waves in the frequency ( $f$ ) thickness ( $d$ ) region of  $fd=0.2\text{MHz mm}-0.8\text{MHz mm}$ .

Cross-ply laminates of 8 layers with thickness  $d$  (thickness of one layer is  $d/8$ : Fig.1) are used here as a typical laminate. Material constants are shown in Table 1, where the suffixes denote the fiber direction (1) and its perpendicular directions (2,3). The zero-volume delamination with length  $12d$  was located from  $x=44d$  to  $x=56d$ . The

interaction of Lamb waves with the delaminations at different interfaces will be compared in the next section. Only 16 layered elements were required for these SEM calculations with a sufficient accuracy. Calculations were carried out in every frequency step and then resulting data series were converted into those in time domain by inverse FFT. As a result, calculations were completed within one hour using a Pentium II 450MHz 256MbRAM PC.

### 3. Calculation results and Discussions

Calculation cases and results shown in Figs.2 and 3 are listed in Table 2. Figs.2 (a) and (b) shows the transient responses of out-of-plane and in-plane displacements at  $(10d, d)$  for A0 and S0 mode incidence, respectively. Waveforms below  $20\mu\text{s/mm}$  are incident waves excited at the source point  $x=0$  and propagating into the right, while waveforms over  $20\mu\text{s/mm}$  are reflected waves from the delamination. For A0 incidence, large reflected waves are seen when the delamination is located in the 1-2, 2-3 and 3-4 interfaces (cases A, B, C), while for S0 incidence, in 1-2 and 3-4 interfaces (cases E, G). These results for S0 incidence were also obtained by the FEM model in Ref. [6].

Contrarily, for A0 mode incidence, reflected waves are small for 4-5 interface (the midplane, D) , and for S0 mode incidence, no reflection is observed for 2-3 (F) and 4-5 interface delamination (the midplane, H). In the cases A-C where reflected waves were observed for A0 incidence, some wave packets can be seen at regular intervals.

A visualization result of the case C (A0 mode incidence and 3-4 interface delamination) is shown in Fig.3 as an example in which large reflected waves were observed. Shadings represent intensities of the stress  $\sigma_{xy}$ , dominant to A0 mode. The incident A0 wave propagates into the “Entrance” of the delamination (Junction 1,  $x=44d$ ) and then enters the region II and III. In this moment, the reflected waves at the

“Entrance” are extremely small compared to transmitted waves. (Fig.3, at  $\tilde{t} = 36.0$ ) After propagating in region II and III independently as in Fig.3 at  $\tilde{t} = 42.0$ , these wave packets arrive at the “Exit” of the left edge of delamination (Junction 2:  $x=56d$ ), and then the reflected and transmitted waves can be seen in Fig.3, at  $\tilde{t} = 48.0$ . When the reflected waves reach the left “Exit” of delamination (Junction 1) again, the reflected waves going toward the right and the transmitted waves toward the left are observed in Fig.3,  $\tilde{t} = 54.0$ . This reflected waves at the “Exit” cause multiple reflections shown in received signals of Fig.2.

A visualization result of F is shown in Fig.4 as an example in which no reflected waves were observed. Shadings are intensities of the stress  $\sigma_{xx}$ , dominant to S0 mode. Similarly, reflections cannot be seen at the “Entrance”, and then S0 mode waves individually propagate in the divided two regions II and III. However, Lamb waves traveling in these two regions arrive at the right “Exit” at the same time and with the same phases. After that, reflected waves are very small even at the “Exit”, and most of energy is transmitted to the right.

In the cases A, B and C where large reflected waves were observed for A0 mode incidence, most of reflected waves were A0 mode, whereas mode conversions occurred at the “Exit” for S0 mode incidence (E, G) and thus reflected waves became mixed waves of A0 and S0. As a result, reflected waveforms were distorted as seen in the received signals in Fig.2(b), E and G.

From these results, it is concluded that reflected waves from delaminations are excited due to the differences of arrival time and phase at the “Exit”. In other words, reflections are strongly affected by the phase and group velocities in the two regions II and III divided by the delamination. So these reflections should be discussed by the

dispersion curves of phase and group velocities in both regions, shown in Figs.5 (a)-(d).

For A and E (1-2 interface delamination), regions II and III correspond to the laminates of  $[0^\circ]$  with thickness  $d/8$  and of  $[90^\circ / 0^\circ / 90^\circ / 90^\circ / 0^\circ / 90^\circ / 0^\circ]$  with thickness  $7d/8$ . The phase velocities in both regions and frequency spectrum of the incident wave are shown in Fig.5 (a). In Fig.5 (b), the dispersion curves are shown for 2-3 interface delamination (B, F;  $[0^\circ / 90^\circ]$  and  $[0^\circ / 90^\circ / 90^\circ / 0^\circ / 90^\circ / 0^\circ]$ ), in Fig. 5(c) for 3-4 interface delamination (C, G:  $[0^\circ / 90^\circ / 0^\circ]$  and  $[90^\circ / 90^\circ / 0^\circ / 90^\circ / 0^\circ]$ ), and in Fig.5 (d) for 4-5 interface delamination (D, H:  $[0^\circ / 90^\circ / 0^\circ / 90^\circ]$ ). The phase velocities of regions II and III are different in the cases A, B, C, E, G where large reflected waves were observed, whereas the phase velocities are about the same in F of no reflections. Since the phase velocities of the two regions are exactly the same for D and H where a delamination is located in the midplane, large reflections did not occur. For A0 mode incidence, however, small reflected waves were obtained as shown in Fig.2 (a) D, even when the phase velocities were the same. This is due to the discontinuity of shear stress, dominant stress of the A0 mode, at the delamination.

#### 4. Conclusions

Lamb wave propagation in laminated plates with delaminations were calculated using the Strip Element Method that required relatively short calculation time and small computational memories. The fundamental Lamb modes around delaminations in laminated plates were observed and the following results were obtained. Lamb wave reflections at delaminations occur not at the “Entrance” of delamination but at the “Exit”. Lamb waves propagate in the two regions divided by the delamination individually and then reach the “Exit”. Large reflections occur only when there are phase differences between the divided two regions. For A0 mode incidence, mode

conversions do not occur during reflections and transmissions, and reflections and transmissions at the “Exit” of delamination are repeated, which results in the multiple reflections. Since time intervals of the received multiple reflection signals correspond to the length of the delamination, analyzing these signals provides the information of length of delaminations. For S0 mode incidence, multiple reflections are not seen in received signals, as mode conversions occur during reflections and transmissions at the “Exit” and cause wave distortion.

### **Acknowledgements**

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Table 1 Material constants used in the calculations.

Material	E <sub>1</sub> (GPa)	E <sub>2</sub> (GPa)	G <sub>12</sub> (GPa)	$\nu_{13}$	$\nu_{23}$	$\rho$ (g/cm <sup>3</sup> )
Carbon/Epoxy	142.2	9.3	4.8	0.33	0.49	1.9

Table 2 Calculation cases and results in this study.

Case	Incident Wave	Location of delamination	Reflected wave
A	A0	1-2 interface	multiple reflections
B	A0	2-3	multiple reflections
C	A0	3-4	multiple reflections
D	A0	4-5: the midplane	small
E	S0	1-2	large
F	S0	2-3	none
G	S0	3-4	large
H	S0	4-5: the midplane	none

## Figure captions

Fig.1 SEM calculation for 8 layered plates with a delamination.

Fig.2 Comparison of displacements with different locations of delamination.

(a) A0 incidence., (b) S0 incidence.

Fig.3 Wave motions in the case where multiple reflections occurred. (Case C).

Fig.4 Wave motions in the case where no reflections occurred. (Case F).

Fig.5 Dispersion curves of regions II and III.

(a) 1-2 interface (A, E), (b) 2-3 interface (B, F),

(c) 3-4 interface (C, G), (d) 4-5 interface (D, H)

Fig.1

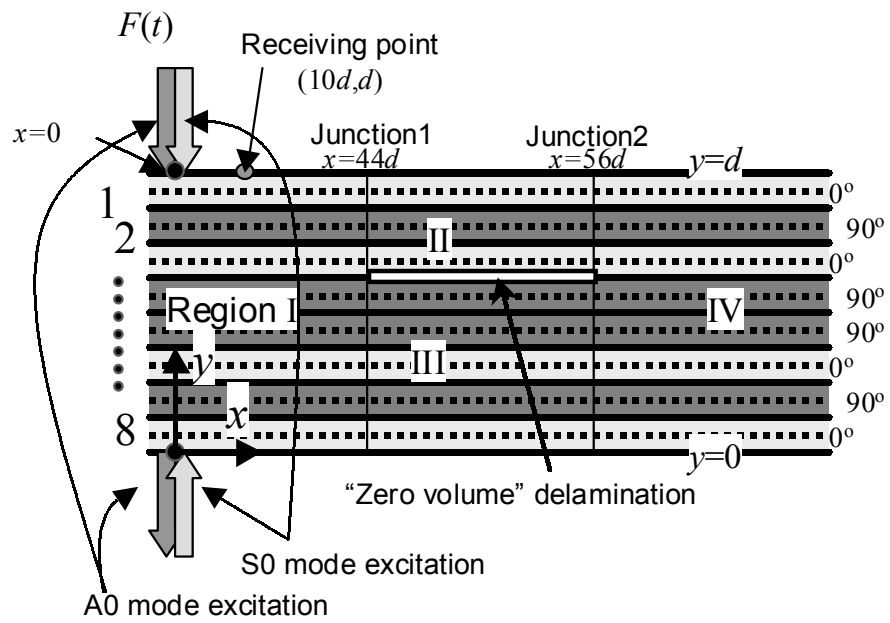
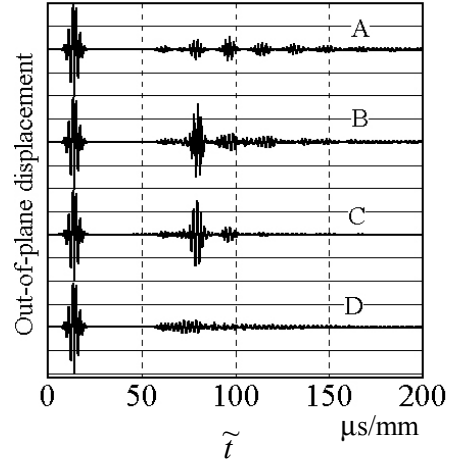
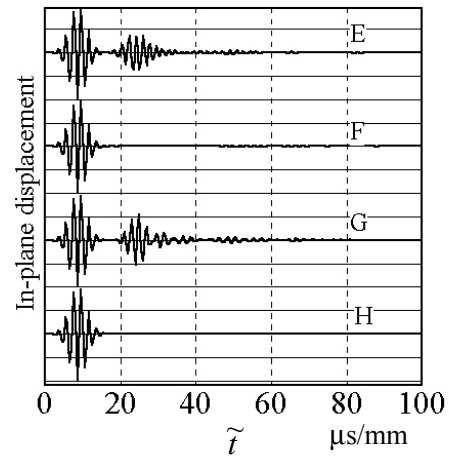


Fig.2(a)(b)



(a) A0 incidence.



(b) S0 incidence.

Fig.3

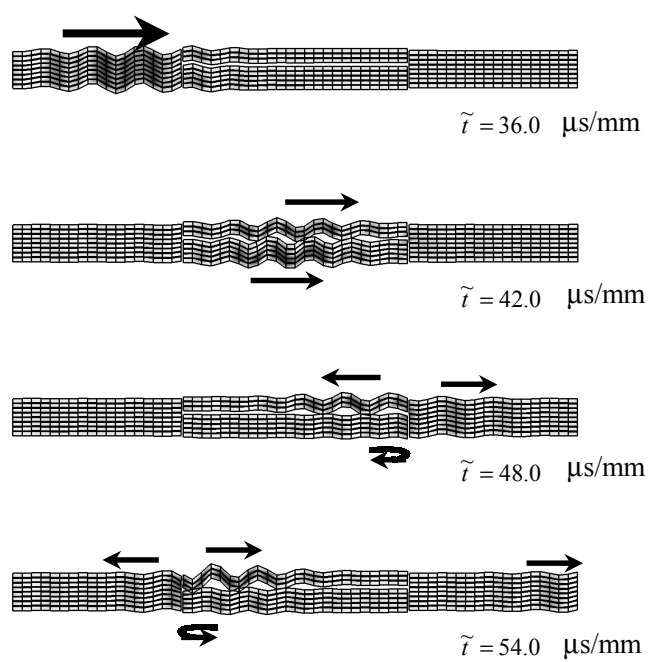


Fig.4

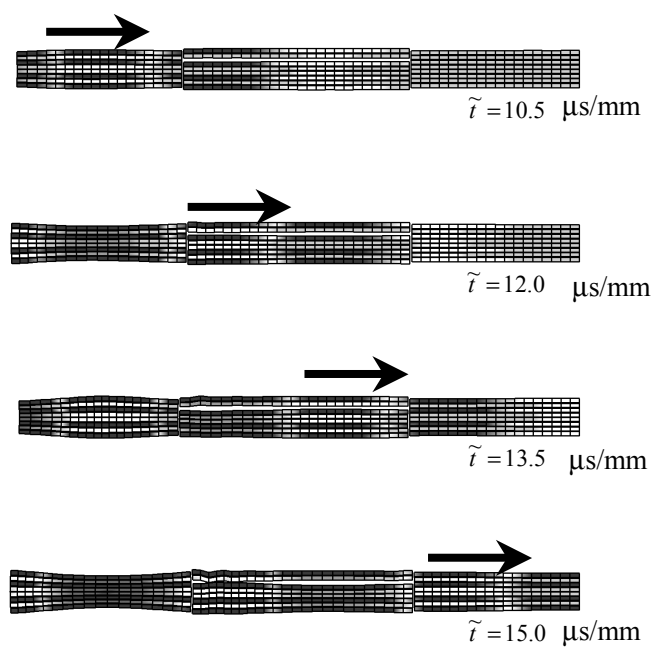
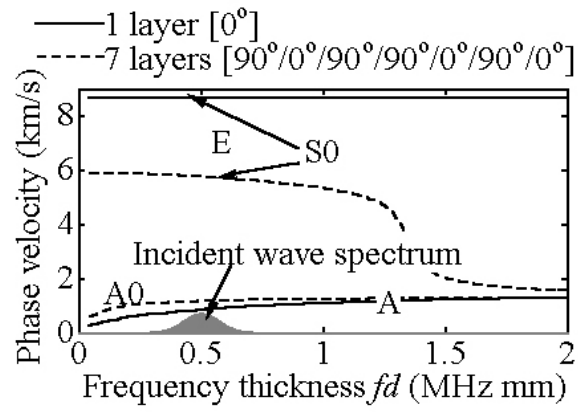
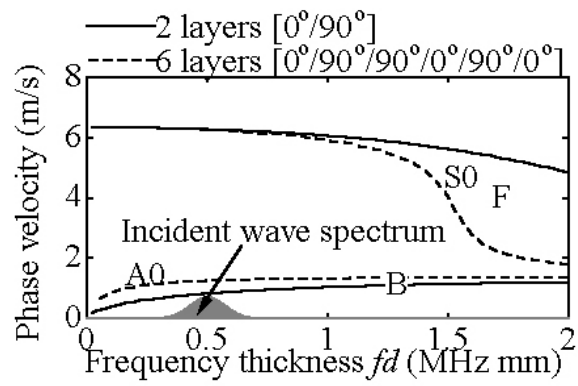


Fig.5 (a)(b)



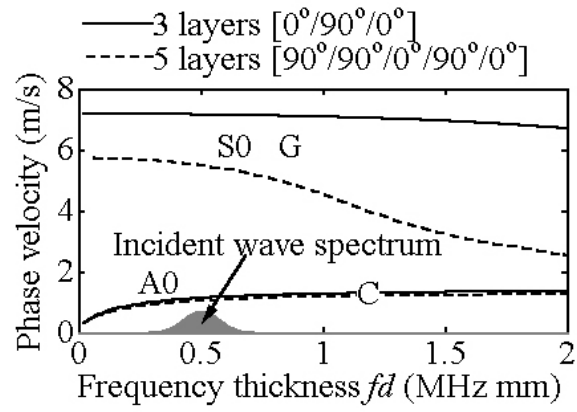
(a) 1-2 interface (A, E)



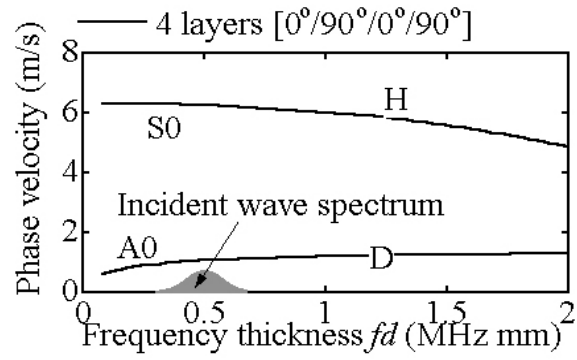
(b) 2-3 interface (B, F)



Fig.5 (c)(d)



(c) 3-4 interface (C, G)



(d) 4-5 interface (D, H)