Title	Application of Path Independent Integral to Welded Structural Members (Report I) : Fundamental Theory of Path Independent Integral to Heterogeneous Materials(Mechanics, Strength & Structural Design)
Author(s)	Horikawa, Kohsuke; Hayashi, Kenji
Citation	Transactions of JWRI. 1989, 18(2), p. 281-285
Version Type	VoR
URL	https://doi.org/10.18910/8473
rights	
Note	

Application of Path Independent Integral to Welded Structural Members (Report Ⅰ)†

— Fundamental Theory of Path Independent Integral to Heterogeneous Materials —

Kohsuke HORIKAWA* **and Kenji HAYASHI****

Abstract

The J-integral has been applied mainly to homogeneous cracked bodies, but not to heterogeneous cracked ones. However, it is important to develop the J-integral estimation procedures to heterogeneous materials with cracks such as welded structural members, because they are much used as the elements of welded structures.

In this paper, the new contour integral to heterogeneous cracks bodies was proposed through physical considerations and the theoretical considerations proved the path independency of that to be valid.

KEY WORDS : (Path Independent Integral) (Structural Member) (Heterogeneous Material) (Fracture Mechanics)

1. Introduction

The J-integral proposed by Rice[1)] and many other researchers is applied only to homogeneous materials but not applied to heterogeneous ones such as welded joints built up thin walled plates. As for the J-integral to heterogeneous materials, Smelser et al[2)] proposed the J-integral estimation procedures in physical consideration of bodies with a crack on phase boundary in non-linear elastic materials. However, the J-integral estimation procedures are not offered proposals for heterogeneous cracked bodies in elasto-plastic materials based on incremental plasticity theory which is well reflected the plastic characteristics of metallic materials. Therefore, from practical standpoints, it is of great significance to develop such estimation procedures in order to estimate the fracture resistant performance of thin walled structural members with cracks.

In case of welded structural members with cracks on weld toe as shown in **Fig.1**, the mechanical heterogeneity and the stress concentration due to structural discontinuity remain in them as if the stress relief of residual stress is performed. On that account, it is necessary to establish the J-integral to heterogeneous materials as a estimation technique of assessing the stability of cracks which quantifies the effects of the mechanical heterogeneity and the stress concentration on the structural performance of welded members.

The main aim of this paper is to develop the path independent integral to heterogeneous cracked bodies and to verify the path independency of that contour integral to be valid.

2. Path independent J_k-integral

The fracture of materials and structural members with cracks originates in the initiation, growth and incorporation of a propagating void and infinitesimal crack within fracture process zone in the vicinity of the crack tip. The ordinary continuum mechanics is not apply to the analysis of such infinitesimal behaviors within fracture process zone, but conversation laws are still composed throughout bodies including that zone.

Kishimoto et al[3)] have derived a path independent integral J, for spatially fixed paths, which has the meaning

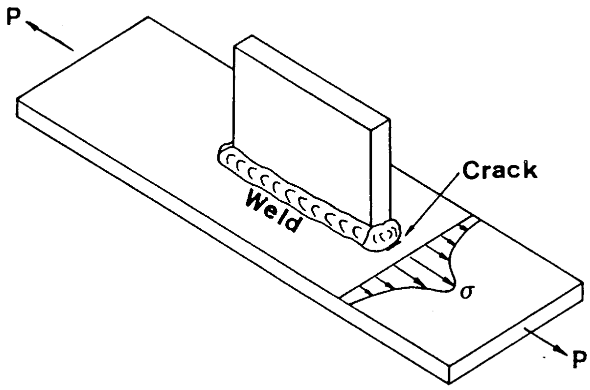


Fig. 1 A crack initiating into the weld toe of welded structural members.

† Received on October31, 1989
* Professor
** Topy Industries Ltd.

Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan

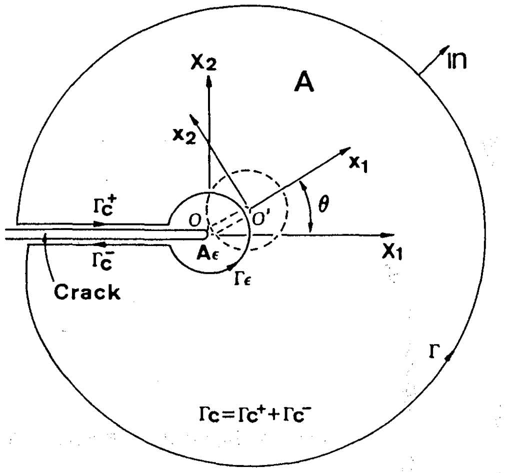


Fig. 2 Coordinate system in the vicinity of crack tip and the integrated path surrounding a crack.

of energy release rate for a stationary crack in a solid in elasto-plastic and dynamic equilibrium. In this section, the path independent integral proposed by Kishimoto et al is explained under mixed mode conditions.

Consider a stationary crack in elasto-plastic and dynamic solid, as shown in **Fig.2**. For a crack propagating at an angle θ measured from the global X_1 axis, the J-integral equivalent to the energy release rate can be expressed as

$$J=J_1\cos\theta+J_2\sin\theta \quad (1)$$

and

$$J_k=-\int_{\Gamma+\Gamma_c}T_i\frac{\partial u_i}{\partial X_k}d\Gamma + \int_A \{(\rho\ddot{u}_i-F_i)\frac{\partial u_i}{\partial X_k}+\sigma_{ij}\frac{\partial \varepsilon_{ij}}{\partial X_k}\}\, dA \quad (2)$$

where J_k are the components of the J-integral with respect to the global coordinates X_k, Γ is an arbitrary path of the integral from bottom to upper surface of the crack, Γ_c are the crack surfaces, T_i are the components of traction vectors and u_i are displacement components. A is the domain enclosed by Γ, Γ_c and Γ_ε except for fracture process zone A_ε, $\rho\ddot{u}_i$ are inertia forces, F_i are body forces and σ_{ij} and ε_{ij} are stress and strain tensors, respectively.

From eqn(1), the crack extension angle θ is evaluated by

$$\theta=\tan^{-1}(J_2/J_1) \quad (3)$$

Substituting the angle θ given by eqn(3) into eqn(1), the J-value in the direction of crack extension, that is, the norm of the J_k-integral given by eqn(2) can be expressed as

$$J=(J_1^2+J_2^2)^{1/2} \quad (4)$$

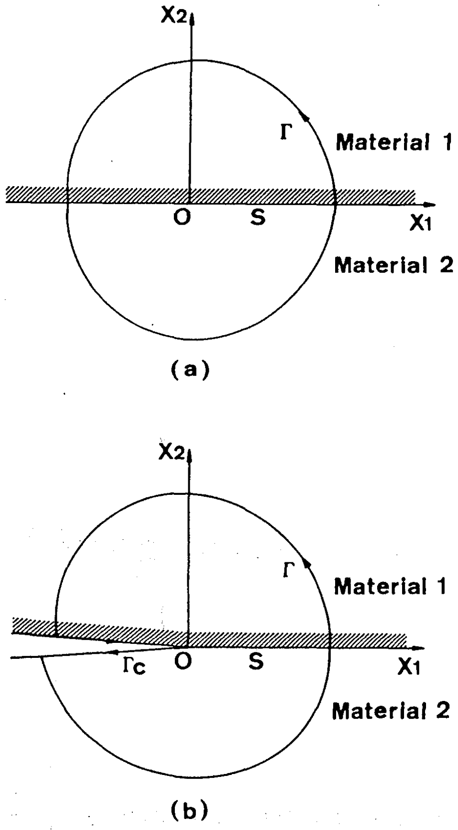


Fig. 3 A crack on phase boundary and contour path.

Especially assuming a body to be a nonlinear elastic material and ignoring the effects of thermal strain, inertia forces and body forces on its deformation characteristics, the J_k-integral can be modified as

$$J_k=\int_{\Gamma+\Gamma_c}(Wn_k-T_i\frac{\partial u_i}{\partial X_k})d\Gamma \quad (5)$$

in which n_k are the outward normal direction cosines and W is the strain energy density defined by

$$W=\int^{\varepsilon_{ij}}\sigma_{ij}d\varepsilon_{ij} \quad (6)$$

Knowles et al[4] have derived the J_k-integral given by eqn(5) and proved the path independency to be valid. The J_1-integral given by eqn(5) corresponds to the path independent J-integral proposed by Rice[1]. And further, Eshelby[5] have first defined the path independent integral given by eqn(5) as forces, energy momentum tensors, which move the singular point in nonlinear elastic materials to an arbitrary direction.

3. Proposal of modified J_k-integral

3.1 Dissolution of singularity within integral paths

The J_k-integral given by eqn(2) or eqn(5) is derived

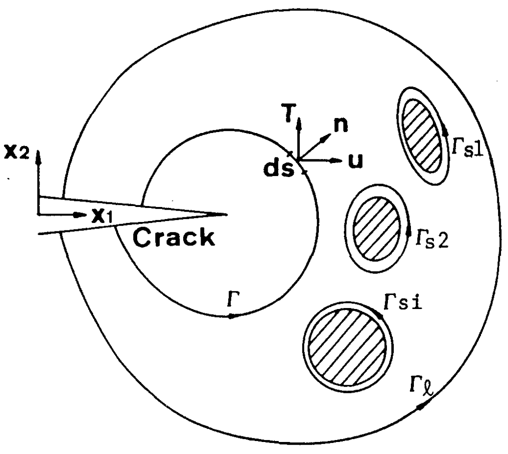


Fig. 4 Definition of J_k-integral for heterogeneous cracked materials.

from using conservation laws and Gauss's divergence theorem. In case of applying such theorem to verify the path independency of J_k, stress and strain must be single-valued and analytic within a closed path $\Gamma + \Gamma_c$ as shown in Fig.2, in which ignores the path $\Gamma\varepsilon$ surrounding fracture process zone. If a body have an another singularity except a crack, for example, crack, void, inclusion and phase boundary within the limits of a domain A, the path independency of J_k comes into existence no longer. Therefore, in case that may not disregard the effects of weld defects and bonded dissimilar on the J_k-value which exist in composite materials such as welded structural members, it is necessary to dissolve the singularity according to the existence of them.

Smelser et al[2] extended the J_k-integral given by eqn(5) to bi-material bodies. Consider a body composed of two homogeneous elastic materials bonded along X_1 axis, as shown in **Fig.3(a)**. The boundary conditions at the bond line are the balance of forces and the continuity of displacement, and these require that

$$[\sigma_{j2}]=0, \quad [u_j]=0 \tag{7}$$

where $[f](X_1)$ denotes the jump in a function $f(X_1, X_2)$ across the line $X_2 = 0$. Adding the contour integral for material 1 to that for material 2 with applying eqn(5) and eqn(7), the following expression can be obtained

$$\int_{\Gamma+\Gamma_c}(Wn_k - T_i\frac{\partial u_i}{\partial X_k})d\Gamma - \int_s([W]\delta_{k2} - \sigma_{i2}[\frac{\partial u_i}{\partial X_k}])\,d\Gamma = 0 \tag{8}$$

in which S is a integral path along the bond line. The above equation expresses the generalized conservation law to bi-material bodies, and the second term in the right side is omitted in case of k=1. Therefore, if a crack exists along the bond line as shown in **Fig.3(b)**, eqn(8)

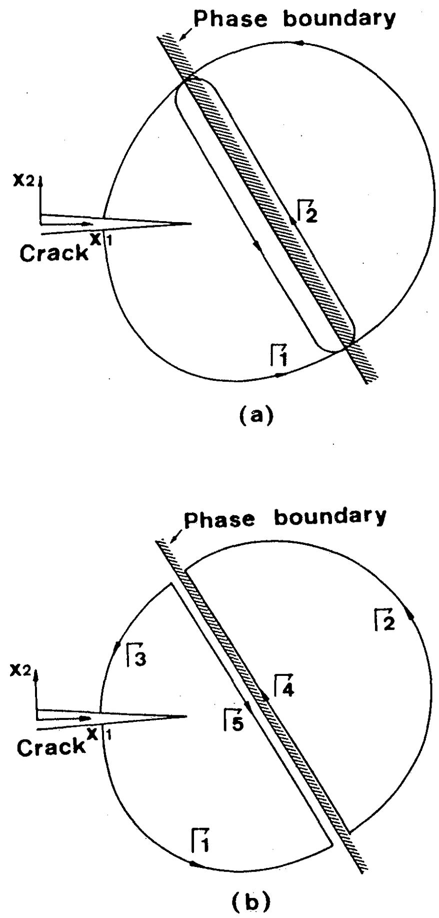


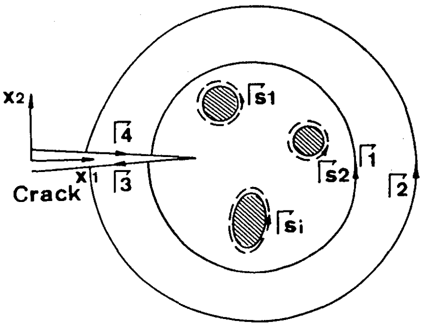
Fig. 5 A phase boundary near a crack tip and contour path surrounding its boundary and crack.

corresponds with the well-known Rice's J_1-integral. On the other hand, the J_2-integral can be estimated by adding a contour integral, which removes the effects of the bond line, to the J_2-integral given by eqn(5).

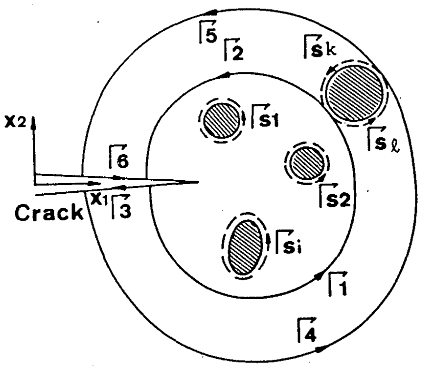
Yaw et al[6] have been analyzed the problem of interface crack, availing themselves of the above method and the technics proposed by Chen et al[7], which divided the J-integral into values to two equilibrium conditions. Miyamoto et al[8] proposed an estimation of the J_k-integral for cracked bodies which a phase boundary existed an arbitrary position. Further, Chen[9] has been analyzed cracked bodies with inclusions of different material properties.

3.2 Modified J_k-integral

The above methods are any estimation technique the J_k-integral extended to heterogeneous cracked bodies with phase boundary and inclusions, and are based on eqn(5). Therefore, it is important to develop the more general and practical methods which are based on eqn(2) as well as eqn(5) and remove the various singularities within integral paths. In this paragraph, the modified J_k-integral is explained which is developed by the authors for that



(a)



(b)

Fig. 6 Closed contour for proof of the path independent of J_k-integral to heterogeneous cracked bodies.

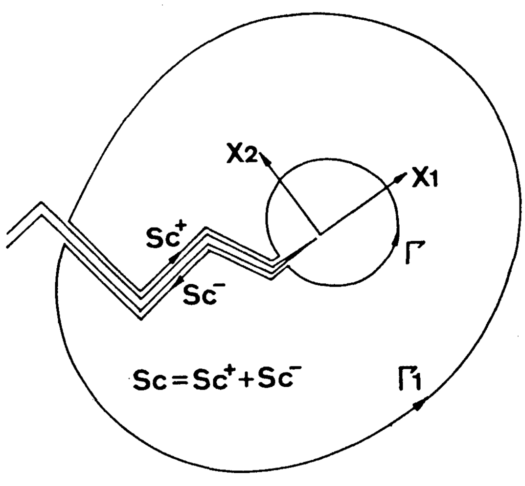


Fig. 7 A crack propagating zigzag.

purpose.

Consider bonded dissimilar materials as the model which have singularities within integral paths. The J_k-integral to heterogeneous elasto-plastic materials as shown in **Fig.4** may be defined as

$$J_k = J_{k\Gamma} = J_{k\Gamma 1} - \Sigma J_{k\Gamma si} \quad (9)$$

where $J_{k\Gamma}$, $J_{k\Gamma 1}$, $J_{k\Gamma si}$ are path integrals defined by the path Γ, $\Gamma 1$, Γsi, respectively, and denotes the summation. These integrals can be evaluated by using eqn(2) or eqn(5).

Physical meanings of the above equation can be explained with Eshelby's concept. $J_{k\Gamma 1}$ is a force moving a crack and inclusions, as $J_{k\Gamma}$ defined by a path Γ is a crack extension force. Therefore, the force extending only a crack can be estimated subtracting the summation of forces moving only inclusions from $J_{k\Gamma 1}$.

The method removing the effects of the phase boundary in bonded dissimilar on the path integral can be shown by eqn(9). The J_k-integral to bi-material bodies as shown in **Fig.5** can be expressed as follows by Miyamoto[8)]

$$J_k = J_{k\Gamma 1} - J_{k\Gamma 2} \quad (10)$$

The above equation can be derived as follows by eqn(9). That is to say, using eqn(9)

$$J_k = J_{k\Gamma 1} + J_{k\Gamma 2} + J_{k\Gamma 3} - (J_{k\Gamma 2} + J_{k\Gamma 5})$$

As the body is composed of only one material, homogeneous, even if a contour integral along a closed path $\Gamma 2$- $\Gamma 4$ on the right side of phase boundary is carried out, $J_{k\Gamma 2} - J_{k\Gamma 4} = 0$. Substituting the second term on the right side of above equation into that, we have

$$J_k = J_{k\Gamma 1} + J_{k\Gamma 2} + J_{k\Gamma 3} - (J_{k\Gamma 4} + J_{k\Gamma 5})$$

This corresponds the results applied to eqn(10). Consequently, eqn(10) is supposed to be the expression modified eqn(9) to apply to bi-material bodies, and it is possible to extend eqn(10) to multi-material bodies easily by eqn(9). In a homogeneous body, as $J_{k\Gamma si}$ of eqn(9) is equal to zero which is a contour integral of a closed path, the J_k-integral of eqn(9) corresponds that defined by knowles et al[4)] in case of applying eqn(5) to estimations of each component of the J_k-integral. Hence, eqn(9) is the more generalized J_k-integral to apply to heterogeneous cracked bodies.

3.3 Path independency of modified J_k-integral

The path independency of the J_k-integral given by eqn(9) can be proved to be valid for elasto-plastic cracked bodies based on an arbitrary constitutive law by Gauss's divergence theorem. In this paragraph, the path independency of that is expressed using which a value of the closed path integral composed of only one material is zero.

Consider the path independency of J_k given by eqn(9) in case of a cracked body containing inclusions among a path $\Gamma 1$ and a path $\Gamma 2$ as shown in **Fig.6(b)**, as the proof of the path independency is easier in case of one not containing inclusions among paths in **Fig.6(a)**. Accordingly, deal with the case that these two paths are in contact with inclusions. Assuming the integrals of the inside path and the outside path to be J_{k1} and J_{k2}, respectively, they can be expressed as follows

$J_{k1}=J_{k\Gamma 1}+J_{k\Gamma 2}-\Sigma J_{k\Gamma si}$

$J_{k2}=J_{k\Gamma 3}+J_{k\Gamma 4}+J_{k\Gamma 5}+J_{\Gamma 6}-\Sigma J_{k\Gamma si}-(J_{k\Gamma sl}+J_{k\Gamma sk})$

Subtracting an upper expression from a lower expression, we have

$$J_{k2}-J_{k1}=(J_{k\Gamma 3}+J_{k\Gamma 4}-J_{k\Gamma sl}-J_{k\Gamma 1})+(J_{k\Gamma 5}+J_{k\Gamma 6}-J_{k\Gamma 2}-J_{k\Gamma sk})$$

The first and second term on the right side of above expression are zero, since they are also the contour integral of the closed paths to homogeneous cracked bodies. The summation of them also is zero. Therefore, it is verified that the path independency of the J_k-integral given by eqn(9) comes into existence. Moreover, such path independency can be proved as follows.

Now, assume A to be a domain surrounded by a finite number of Jordan's curves Γ_1, $\cdots$ Γ_{S1}, Γ_{Si}, which are piecewise smooth and do not intersect each other, and each Γ_{si} to exist within Γ_1. The following expression can be obtained by Cauchy's integral theorem provided that an arbitrary complex function is regular within the closed curves, which is defined a integrated value of this function as $J_k{}^*$.

$$J_{k\Gamma 1}{}^*=\Sigma J_{k\Gamma si}{}^* \qquad (11)$$

This is derived from the assumption that the multiply connected domain are divided into two simply connected domains and that a complex function is regular within each simply connected domain. Hence, using eqn(11), the path independency of the J_k-integral given by eqn(9) can be proved in accordance with that divide the multiply connected domain contained cracks, holes and inclusions into two simply connected domains properly, that is, by analogy.

3.4 Advantage of modified J_k-integral

As mentioned above, the advantages of modified J_k-integral can be expressed as follows

(1) removing various singularities within an objective domain,
(2) calculating a crack extension angle by eqn(3) and carrying out the analysis of crack propagation and
(3) performing an analysis based on an arbitrary constitutive law and examining the fatigue characteristics of materials and structural members.

Next, such advantages are described from a brief example.

As pointed out by Yatomi[10], using the J_k-integral given by eqn(5), generally it is difficult to carry out an analysis of the problem of a crack propagating zigzag as shown in **Fig.7.** This is on account of the path independency of the J_k-integral not being realized, since it also contains the forces moving the upper and lower crack surface which are zigzag. Hence, using the modified J_k-integral, the force extending only a crack can be estimated by subtracting the J_k-integral to a path from that to a path. Moreover, the characteristics of the crack propagating zigzag may be examined by using the modified J_k-integral and eqn(3).

4. Conclusion

In this paper, from practical standpoints, the attempt was made to extend the J_k-integral to heterogeneous and elasto-plastic cracked bodies. The results obtained are summarized as follows.

(1) A new J_k-integral extended to heterogeneous and elastoplastic bodies, the modified J_k-integral, is introduced and its path independency is theoretically proven through physical considerations.
(2) Using the modified J_k-integral, the analysis of crack propagation may be carried out, and also the problem of a crack propagating zigzag can be solved.

At the end, in the following paper, the authors would like to apply the modified J_k-integral to heterogeneous cracked bodies such as welded structural members, and examine the effects of the mechanical heterogeneity and stress concentration on the fracture resistance.

References

1) Rice,J.R. : A path independent integral and the approximate analysis of strain concentration by notches and cracks : Trans. ASME, Journ. Appl. Mech. 1968, pp.379-386.
2) Smelser,R.E. and M.E.Gurtin : On the J-integral for bi-material bodies : Journ. Fracture, Vol.13, 1977 pp.382-384.
3) Kishimoto,K., S.Aoki and M.Sakata : On fracture mechanics parameter describing crack extension behaviors : Trans. JSME, Vol.46, No.410, 1980, pp.1049-1061 (in Japanese).
4) Knowles,J.K. and E.Sternberg : On a class of conservation laws linearized and finite elastostatics : Arch. Rational Mech. Analy., Vol.44, 1972,pp.187-211.
5) Eshelby,J.D. : Energy relations and the energy-momentum tensor in continuum mechanics : in Inelastic behavior of solids, eds. M.K.Kanninen et al, McGraw-Hill, 1970, pp.77-115.
6) Yaw,J.F. and S.S.Wang : An analysis of interface cracks between dissimilar isotropic materials using conservation integrals in elasticity : Eng. Frac. Mech. Vol.20, No.3, 1984 pp.423-432.
7) Chen,H.K. and R.T.Shield : Conservation laws in elasticity of J-integral type : Journ. Appl. Math. Phys., Vol.28, 1977, pp.1-22.
8) Miyamoto,H. and M.Kikuchi : Evaluations of J_k integrals for a crack in two-phase materials : in Numerical methods in fracture mechanics, edited by D.R.J.Owen and A.R.Luxmoore, 1980, pp.359-370.
9) Chen,W.H. : On the J-integral for nonhomogeneous cracked composites : Trans. 5th Int. Conf. SMiRT, Vol.m9/4, 1979, pp.1-8.
10) Yatomi,C. : New expression for the energy release rate and its applications : Journ. Soc. Mate. Scie., Vol.35, No.394, 1986, pp.767-771 (in Japanese).