Analysis of Welding Stress Relieving by Annealing based on Finite Element Method†

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Abstract

In order to analyze theoretically creep behaviors accompanied by stress relief annealing, the authors extended their theory of thermal elastic-plastic analysis based on the finite element method. In the developed theory, the temperature dependency of the mechanical properties of metal is taken account.

Examples were analyzed for two kinds of creep laws, the power and the exponential laws, which are observed in such material as quenched and tempered H. T. 60. The solution by the method was assured its convergence for a comparatively large time increment if changes of stresses and creep properties in one increment of calculation steps are taken into consideration. The results of analysis show good coincident with the experiment. By this method, the behavior of welded joints revealed during the entire thermal cycle, from welding to stress relief annealing, can be analyzed consistently.

1. Introduction

Of mechanical behaviors during welding, the authors have extensively conducted thermal elastic-plastic analysis [1), 2)] by means of the finite element method (F. E. M.) with consideration of temperature dependency of mechanical properties (yield stress, Young's modulus, coefficient of thermal expansion etc.) of metals based on their developed theory[3)].

In order to research stress annealing behaviors of welded joints, the effect of creep strain should be included in the previous theory. There are several researches to this problem based on the finite element method. Yamada et al.[4)] developed the method of thermal visco-elastic analysis to mechanical models composed of springs and dashpots, and showed possibility to include temperature dependency of mechanical properties into the analysis for such materials as thermalrheologically simple. Fujita et al.[5)] extended this method to thermal visco-elastic-plastic ploblem with consideration of temperature dependency of yield stress.

However, if materials were not thermalrheologically simple, it would be difficult to take account of temperature dependency of Young's modulus and creep properties, since total strains and stresses produced are evaluated by the hereditary integral on this kind of mechanical models. On the other hand, according to researches to welded plates of structural steels by Tanaka and Obata[6)], welding stresses reached at the yield stress are easily relieved and dropped into the elastic range during the heating stage for annealing. And at the cooling stage after stresses relief heat treatment, Young's modulus is closely connected with the resulting residual stress. Therefore, in the analysis of such a problem, it is important to take temperature dependency of Young's modulus into consideration. To solve this difficulty, N. A. Cyr and R. D. Teter[7)] propose separation of creep strain from elastic one for the formulation of stress-strain relation.

In this paper, the authors adopt this way of treatment in the developed analytical method of thermal elastic-plastic problems, and conduct an analysis of stress relief annealing of welded joints employing two different creep laws at two indivisual temperature ranges. The accuracy of analytical result is examined in comparison with the experiments[6)]. Finally, the mechanical phenomenon observed during not only welding but also stress relief annealing is consistently analyzed by the method.

2. Theoretical analysis

2.1 Stress-strain relation

2.1.1 Creep law

There are many researches on the methods to describe the creep property by mathematical expression. In this paper, the analytical theory is developed to two kinds of creep laws. For example, quenched and tempered H. T. 60 is said that this material shows different creep properties at a comparatively high temperature range and low, namely a power creep law for more than 550°C and an exponential creep law for less than 550°C.

In one dimensional stress state, these creep laws state,

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$\dot{\varepsilon}^c = \beta \sigma^n$: power creep law (T>550°C for H. T. 60) (1)

$\dot{\varepsilon}^c = A\exp(\beta \sigma)$: exponential creep law (T≦550°C for H. T. 60) (2)

where

σ : axial stress

$\dot{\varepsilon}^c$: creep strain rate

β, n, A, B : constants dependent on the creep properties of materials

In three dimensional stress state, assuming that the creep behavior is described by the theory of the incremental strain of plasticity, the component of the creep strain rate is expressed in the following form,

$$\{\dot{\varepsilon}^c\} = \frac{3}{2\bar{\sigma}} \dot{\bar{\varepsilon}}^c \{\sigma'\} \tag{3}$$

where σ' : deviatoric stress

$\dot{\bar{\varepsilon}}^c$: equivalent creep strain rate

$\bar{\sigma}$: equivalent stress

Taking account of the effect of variation in the stress state and creep properties, the creep strain increment $\{d\varepsilon^c\}$ during a time increment dt is given as the summation of two terms as follows,

$$\{d\varepsilon^c\} = [C_c]\{d\sigma\} + \{d\hat{\varepsilon}^c\} \tag{4}$$

Introducing the assumed two creep laws, the creep strain rate $\{\dot{\varepsilon}^c\}$ and the increment of creep strain $\{d\varepsilon^c\}$, which are Eqs. (3) and (4), are given by the following equations.

For the power creep law,

$$\{\dot{\varepsilon}^c\} = \frac{3}{2} \beta \bar{\sigma}^{n-1} \{\sigma'\} \tag{5}$$

Assuming that β, $\bar{\sigma}$, σ' in Eq. (5) vary linearly in a small time interval dt and n is constant, the matrix $[C_c]$ and vector $\{d\hat{\varepsilon}^c\}$ appeared in Eq. (4) are expressed as

$$[C_c] = dt\left(\frac{\beta}{2} + \frac{d\beta}{3}\right)[C_{ij}] \tag{6}$$

where

$C_{11} = F\sigma_x'^2 + \bar{\sigma}^{n-1}$ $C_{21} = F\sigma_x'\sigma_y' - \bar{\sigma}^{n-1}/2$
$C_{22} = F\sigma_y'^2 + \bar{\sigma}^{n-1}$ $C_{31} = F\sigma_x'\sigma_z' - \bar{\sigma}^{n-1}/2$
$C_{32} = F\sigma_y'\sigma_z' - \bar{\sigma}^{n-1}/2$ $C_{33} = F\sigma_z'^2 + \bar{\sigma}^{n-1}$
$C_{41} = 2F\sigma_x'\tau_{yz}$ $C_{42} = 2F\sigma_y'\tau_{yz}$
$C_{43} = 2F\sigma_z'\tau_{yz}$ $C_{44} = 4F\tau_{yz}^2 + 3\bar{\sigma}^{n-1}$
$C_{51} = 2F\sigma_x'\tau_{zx}$ $C_{52} = 2F\sigma_y'\tau_{zx}$
$C_{53} = 2F\sigma_z'\tau_{zx}$ $C_{54} = 4F\tau_{yz}\tau_{zx}$
$C_{55} = 4F\tau_{zx}^2 + 3\bar{\sigma}^{n-1}$ $C_{61} = 2F\sigma_x'\tau_{xy}$
$C_{62} = 2F\sigma_y'\tau_{xy}$ $C_{63} = 2F\sigma_z'\tau_{xy}$
$C_{64} = 4F\tau_{yz}\tau_{xy}$ $C_{65} = 4F\tau_{zx}\tau_{xy}$
$C_{66} = 4F\tau_{xy}^2 + 3\bar{\sigma}^{n-1}$ $C_{ij} = C_{ji}$ (i, j = 1~6)

$$F = \frac{9}{4}(n-1)\bar{\sigma}^{n-3}$$

$$\{d\hat{\varepsilon}^c\} = \frac{3}{2} dt\left(\beta + \frac{d\beta}{2}\right) \bar{\sigma}^{n-1}\{\sigma'\} \tag{7}$$

where $d\beta$: change of β in a small time increment dt

For the exponential creep law, the creep strain rate is

$$\{\dot{\varepsilon}^c\} = \frac{3}{2\bar{\sigma}} A\exp(B\bar{\sigma})\{\sigma'\} \tag{5'}$$

In this case, it is difficult to include the effect of changes of the parameters, A and B, during the time interval dt. Consequently, the matrices of Eq. (4) are shown to be

$$[C_c] = 0 \tag{6'}$$

$$\{d\hat{\varepsilon}^c\} = \frac{3}{2\bar{\sigma}} dt\, A\exp(B\bar{\sigma})\{\sigma'\} \tag{7'}$$

When this creep law is adopted, the incremental analysis should be proceeded in much smaller time increment than in the case of the power creep law in order to assure the convergence of the solution.

2.1.2 Stress-strain relation in elastic range

In the analysis, the effect of temperature is considered in the lapse of time. When materials display elastic behavior, the total strain increment $\{d\varepsilon\}$ at a point is expressed as the summation of elastic, creep and thermal strains which are $\{d\varepsilon^e\}$, $\{d\varepsilon^c\}$ and $\{d\varepsilon^T\}$

$$\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^c\} + \{d\varepsilon^T\} \tag{8}$$

The elastic strain is expressed by

$$\{\varepsilon^e\} = [D^e]^{-1}\{\sigma\} \equiv [C_e]\{\sigma\}$$

where $[D^e]$: elasticity matrix

With consideration of temperature dependency of the elasticity matrix $[D^e]$, the incremental relationship between stress and strain is

$$\{d\varepsilon^e\} = [D^e]^{-1}\{d\sigma\} + \frac{\partial [D^e]^{-1}}{\partial T}\{\sigma\} dT \tag{9}$$

and the thermal strain increment is

$$\{d\varepsilon^T\} = \{\alpha\} dT \tag{10}$$

where α: instantaneous coeff. of thermal expansion $\left(= \alpha_0 + \frac{\partial \alpha_0}{\partial T} T\right)$

α_0: coeff. of thermal expansion (usually defined)

Substitution of Eqs. (4), (9) and (10) into Eq. (8) results in

$$\{d\sigma\} = [\hat{D}^e]\{d\varepsilon\} - \{d\tau_0^e\} \tag{11}$$

where $[\hat{D}^e]=[C_e+C_c]^{-1}$

$$\{d\tau_0{}^e\}=[\hat{D}^e]\left[\left(\{\alpha\}+\frac{\partial[D^e]^{-1}}{\partial T}\{\sigma\}\right)dT+\{d\hat{\varepsilon}^c\}\right]$$

2.1.3 Stress-strain relation in plastic range

It is assumed that the plastic strains are completely independent of the creep strain.
Yielding function f is defined as;

$$f=\bar{\sigma}^2-\sigma_Y{}^2=0 \tag{12}$$

where σ_Y: yield stress in uniaxial tension

Based on the incremental strain theory of plasticity, the plastic strain increment $\{d\varepsilon^p\}$ is expressed in the following form with a positive scalar λ,

$$\{d\varepsilon^p\}=\lambda\left\{\frac{\partial f}{\partial\sigma}\right\} \tag{13}$$

With the additional component $\{d\varepsilon^p\}$ to those in the elastic range, which are the right hand of Eq. (8), the total strain increment $\{d\varepsilon\}$ in the plastic range is shown as,

$$\{d\varepsilon\}=\{d\varepsilon^e\}+\{d\varepsilon^p\}+\{d\varepsilon^c\}+\{d\varepsilon^T\} \tag{14}$$

When the material is under loading in the plastic range, the following equation must be satisfied.

$$0=df=\left\{\frac{\partial f}{\partial\sigma}\right\}^T\{d\sigma\}+\frac{\partial f}{\partial\bar{\varepsilon}^p}d\bar{\varepsilon}^p+\frac{\partial f}{\partial T}dT \tag{15}$$

where $\bar{\varepsilon}^p$: equivalent plastic strain
$\{\ \}^T$: transposed vector

From Eqs. (4), (9), (10), (13), (14), and (15), λ is derived as;

$$\lambda=\left(\left\{\frac{\partial f}{\partial\sigma}\right\}^T[\hat{D}^e]\left\{\frac{\partial f}{\partial\sigma}\right\}^T-\left\{\frac{\partial f}{\partial\sigma}\right\}^T[\hat{D}^e]\left[\left(\{\alpha\}+\frac{\partial[D^e]^{-1}}{\partial T}\times\{\sigma\}\right)dT+\{d\hat{\varepsilon}\}\right]+\frac{\partial f}{\partial T}dT\right)/S_0. \tag{16}$$

where H′ : rate of strain hardening

$$S_0=\left\{\frac{\partial f}{\partial\sigma}\right\}^T[\hat{D}^e]\left\{\frac{\partial f}{\partial\sigma}\right\}+4\sigma_Y^2H'$$

With this λ, the stress and strain relation is obtained,

$$\{d\sigma\}=[\hat{D}^p]\{d\varepsilon\}-\{d\tau_0{}^p\} \tag{17}$$

where $[\hat{D}^p]=[\hat{D}^e]-[\hat{D}^e]\left\{\frac{\partial f}{\partial\sigma}\right\}\left\{\frac{\partial f}{\partial\sigma}\right\}^T[\hat{D}^e]/S_0.$

$$\{d\tau_0{}^p\}=[\hat{D}^p]\left[\left(\{\alpha\}+\frac{\partial[D^e]^{-1}}{\partial T}\{\sigma\}\right)dT+\{d\hat{\varepsilon}^c\}\right]+[\hat{D}^e]\left\{\frac{\partial f}{\partial\sigma}\right\}\left(\frac{\partial f}{\partial T}\right)dT/S_0$$

In the plastic range, unloading is detected by the sign of λ,

$$\lambda>0: \text{loading},\quad \lambda=0: \text{neutral loading},\quad \lambda<0: \text{unloading} \tag{18}$$

2.2 Stiffness matrix and equivalent nodal force

The displacements in an element $\{w\}$ are expressed by the nodal displacement $\{w\}^e$ as;

$$\{w\}=[A]\{w\}^e \tag{19}$$

where $[A]$: interpolation function

With appropriate differentiation of $[A]$ with respect to the ordinates, the strains are obtained

$$\{\varepsilon\}=[B]\{w\}^e \tag{20}$$

According to the principle of virtual work, a relation between increment of nodal force $\{dF\}^e$ and one of the nodal displacement is expressed as;

$$\{dF\}^e=[K]^e\{dw\}^e-\{dL\}^e \tag{21}$$

where $[K]^e=\int[B]^T[D][B]d(vol)$: stiffness matrix
$\{dL\}^e=\int[B]^T\{d\tau_0\}d(vol)$: equivalent nodal force

Without any external load, the equilibrium condition states at each nodal point;

$$0=\Sigma\{dF\}^e=\Sigma[K]^e\{dw\}^e-\Sigma\{dL\}^e$$
$$\therefore\ \Sigma\{dL\}^e=\Sigma[K]^e\{dw\}^e \tag{22}$$

The notation Σ in Eq. (22) implys the summation of $\{dL\}^e$ and $[K]^e$ of each element over the entire continuum under consideration.

To proceed the analysis, it is seen from Eq. (21) that the matrix $[K]^e$ and the vector $\{dL\}^e$ is necessary to evaluate. These contain the matrix $[\hat{D}]$ and the vector $\{d\tau_0\}$, respectively. They are explicitely given by $[\hat{D}^e]$ and $\{d\tau_0{}^e\}$ of Eq. (11) for the elastic range, and $[\hat{D}^p]$, $\{d\tau_0{}^p\}$ of Eq. (17) for the plastic one.

3. Example of analysis

The material furnished to study is quenched and tempered high strength steel H. T. 60 and its temperature dependency of the mechanical properties are illustrated in **Fig. 1.** The creep properties at the heating stage of annealing are considered to be equivalent

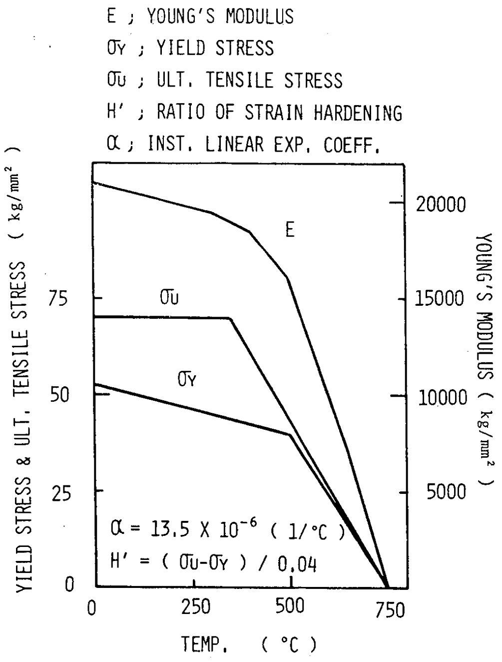

Fig. 1. Temperature dependency of mechanical properties.

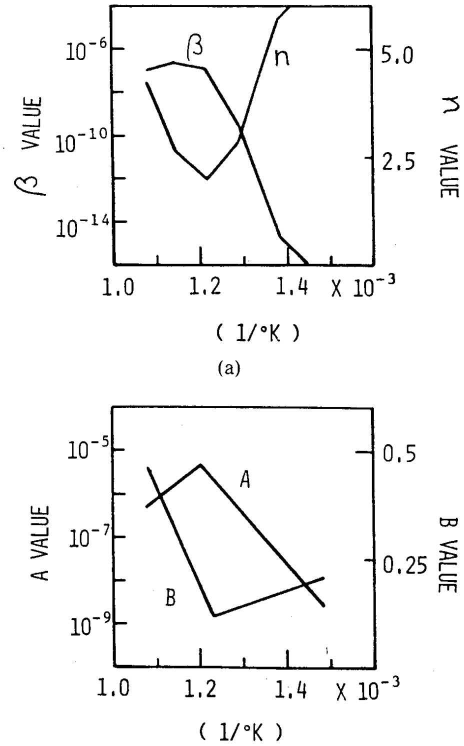

Fig. 2. Temperature dependency of creep properties (for heating stage).
(a) power creep law (b) exponential creep law

to the transient creep, shown in **Figs. 2 (a)** and **(b).** And these properties during holding at a constant temperature are regarded as to the steady-state creep. In this latter case, the power creep law should be applied since the holding temperatures are above 550 °C and the constants in Eq. (1) were given from the experiment as follows,

$$
\begin{aligned}
n&=4.2, \ \beta=2.7\times10^{-9} \quad \text{at } T=600^{\circ}\mathrm{C} \\
n&=4.2, \ \beta=5.4\times10^{-8} \quad \text{at } T=650^{\circ}\mathrm{C}
\end{aligned} \tag{23}
$$

3.1 Accuracy of the present method

First, an analysis of annealing in one dimensional stress state at a constant temperature was conducted to investigate the convergence of solutions obtained by the present method using various time increments (**Fig. 3**). As seen in the figure, if changes of the creep properties and the effect of stress variation in a time increment are taken into account, that is, $[C_c]$ are adopted, the convergence of solutions is assured even for a comparatively large time increment. When the interval dt is taken less than one min., the solution by F. E. M. is regarded as to be completely coincided with the analytical solution given by Eq. (24),

$$t=\frac{1}{\beta E(n-1)}\left(\sigma^{-n+1}-\sigma_0^{-n+1}\right) \tag{24}$$

where σ_0: initial stress

Unless the above mentioned changes can be included, the convergence of solutions is attained only for a sufficiently small time increment and even so its accuracy is worse than one in the previous case. **Fig. 4** illustrates the processes of relaxation of the given stress in

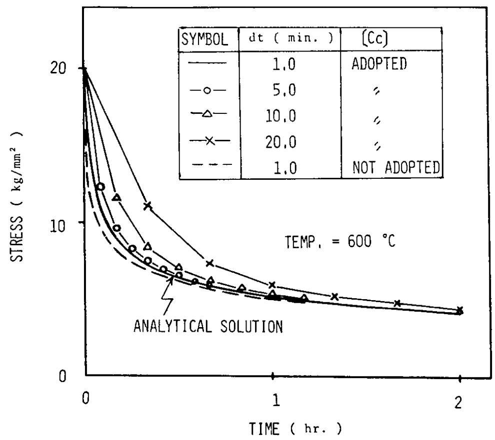

Fig. 3. Convergence of solution by the present method at a constant temperature.

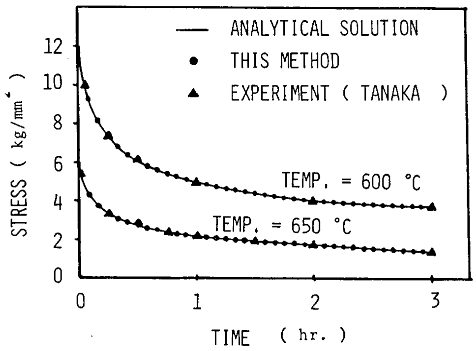

Fig. 4. Comparison of solution by the present method with analytical one.

one dimensional stress state at a constant temperature, which are observed in the experiment[6] and obtained by the present method. They show very good coincidence and it is noted that the mechanism of stress relief annealing at a constant comparatively high temperature follow the power creep law.

In usual stress relief annealing, it is a common process that the structure is gradually heated up to a holding temperature. So, the initial stress are decreasing during the heating stage by reduction of Young's modulus and the yield stress, and by increase of the creep strain. To study these effects, the following analysis was conducted: The initial stress was generated in the bar by tension and then it was heated up to a high temperature at the heating rate of 600°C/hr. and holded. At this stage, the bar is supported being free from any effect of thermal expansion. As shown in **Fig. 5,** this analytical result shows good coincidence with the stress variation observed in the experiment by Tanaka. At the range below 400°C, the stress decreases by reduction of Young's modulus and at the range above 400°C, the stress indicates lower values than one estimated from only the change of Young's modulus. In this case, the stress is in the elastic range even if holding at high temperature. Then, it decreases not by fall of the yield stress but production of the creep strain.

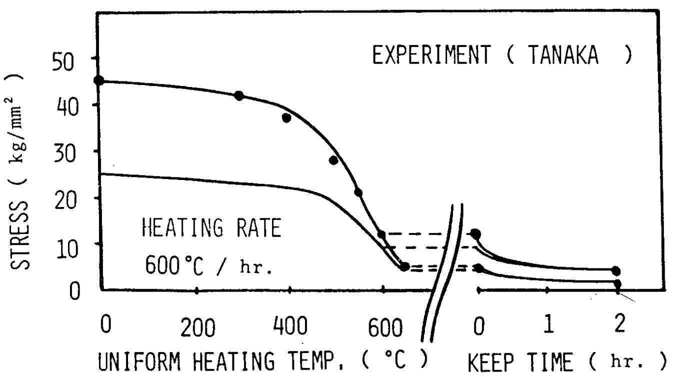

Fig. 5. Transient stresses during heating and holding stages in stress relief annealing.

Fig. 6 represents the effect of the heating rate at the heating stage on variation of the initial stress of the same problem. The curve indicated by Inst. Heat. in Fig. 6 shows decrease of the stress by instantaneous heating. This implys that the creep strain is not produced instantaneously and the curve shows only the temperature effect on reduction of Young's modulus. Accordingly, when the bar is cooling down at the room temperature, the stress is recovered. In view of this fact, the actual value of stress relief is evaluated as the difference between this curve and the

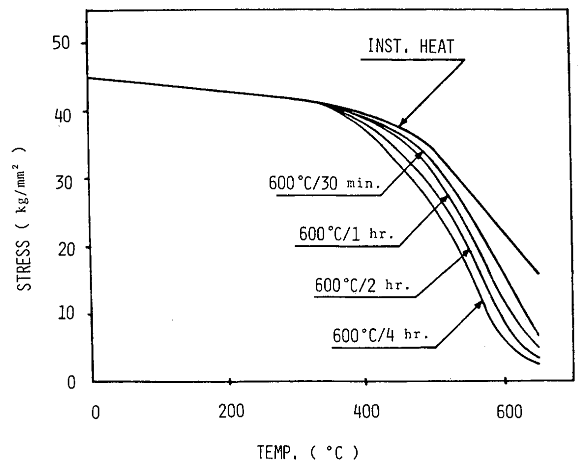

Fig. 6. Effect of heating rate in stress relief annealing.

corresponding curve to the real heating rate. For example, at 600°C, the amount of stress relief is about 16.0 kg/mm² for the heating rate of 600°C/4 hrs, and the final amount is 33.6 kg/mm² after the bar reaches to the room temperature.

3.2 Analysis of y-type weld cracking specimen

As an application to plane stress state, the mechanical behaviors of the y-type specimen exhibited during not only welding but also stress relief annealing is analyzed consistently by the present method. The authors have already conducted the thermal elastic-plastic analysis on the behaviors of this specimen revealed during welding and the following fact was pointed out that there is essentially no difference in the resulting residual stresses produced between by a moving heat source and by an instantaneous heat source. In order to make the problem clear, the welding stress analysis is conducted under instantaneous heat source, that is, the stress distribution is symmetry with respect to the two orthogonal axes. Material furnished to this example is H. T. 60 which is the same as the previous examples and the heat input is equal

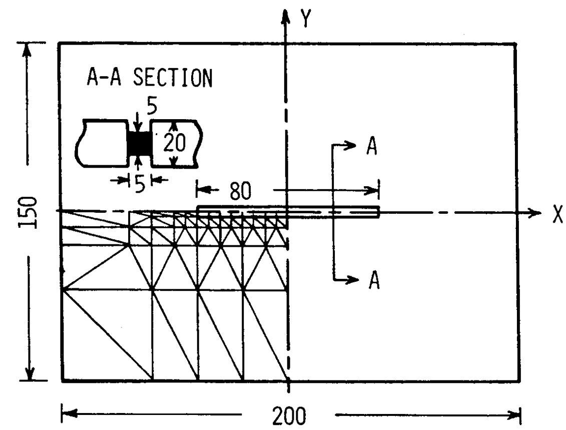

Fig. 7. y-type weld cracking specimen and its mesh division for analysis.

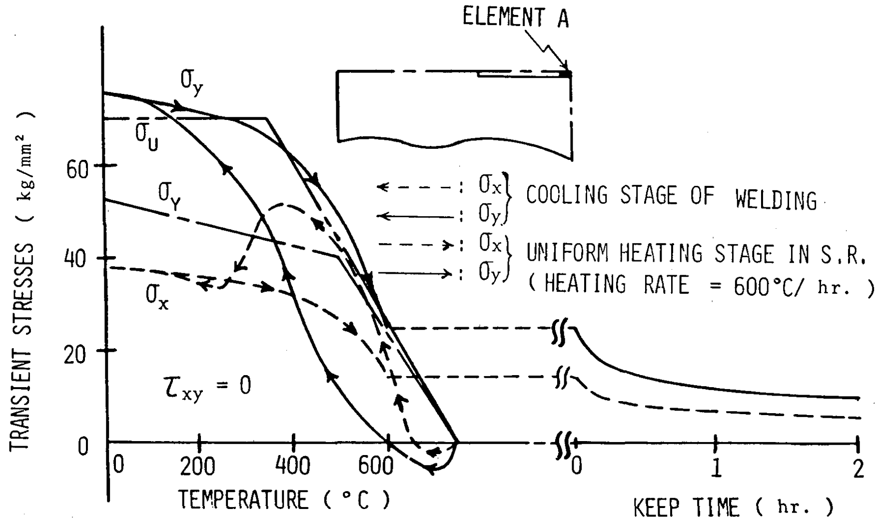

Fig. 8. Transient stresses during not only cooling stage of welding but also heating and holding stages in stress relief annealing.

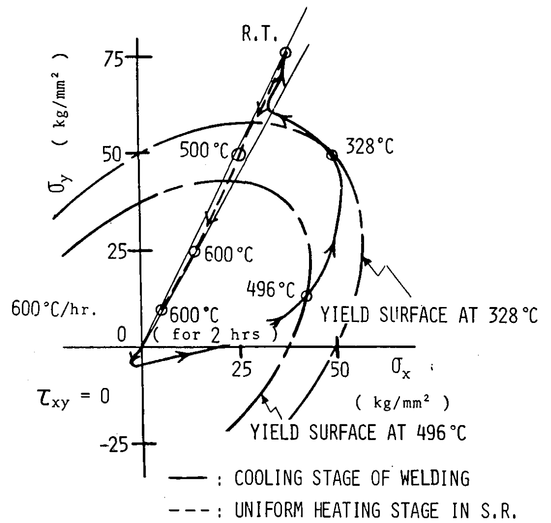

Fig. 9. Stress profile and yield surfaces during welding and stress relief annealing.

Figs. 8 and 9 represent the change of stress in the central element A of the specimen at the entire process from welding to stress annealing. It is shown that the stress ratio, σ_x and σ_y, greatly changes along the yield surface during the abrupt change of temperature after welding, but that the ratio hardly changes as the temperature difference in the specimen becomes small. As shown in Fig. 8, R. T. represents the amount of residual stress at the room temperature after welding. After then, for the stress relief annealing, the entire specimen is heated up to a holding temperature. At this stage, the ratio of stresses is almost constant and the existing stresses decrease pointing to the origin of the coordinates. This implys that the stresses decrease without change of their ratio, if there is no temperature difference observed in the plates, and this fact coincides with the above mentioned one.

to 3,000 cal/cm. For simplicity, it is assumed that the weld metal has the same creep properties as those of the base metal (see Eq. (23) and Fig. 2). The heating rate for stress relief annealing is 600°C/1 hr..

Figs. 10 (a) and **(b)** represent distributions of the welding residual stress and the stress varying the

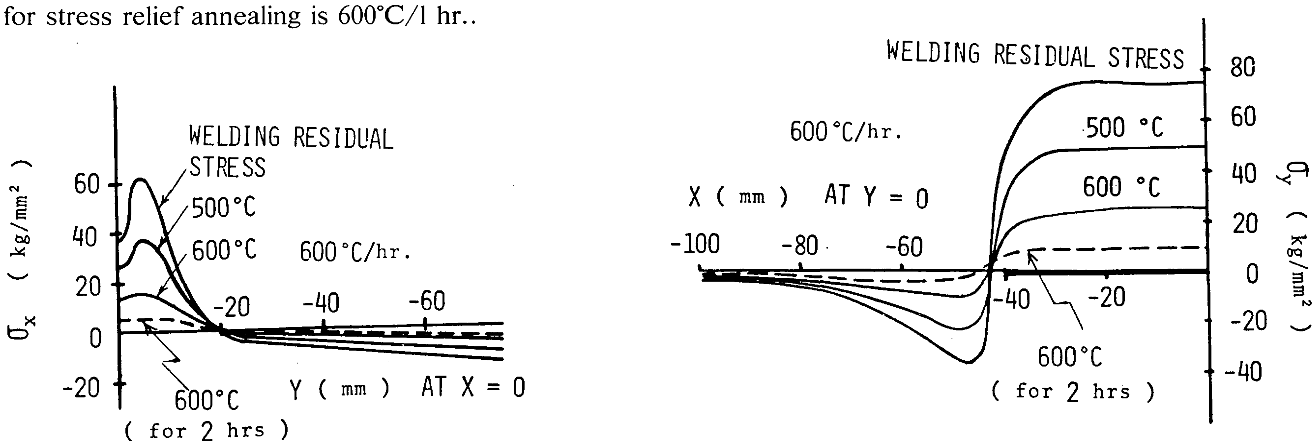

Fig. 10. Variation of welding residual stress distribution during annealing.
(a) σ_x at the section Y=0 (b) σ_y at the section X=0:

annealing, σ_x at the section X=0 and σ_y at Y=0, respectively. After finishing the annealing, the specimen cools down to the room temperature. If the creep strain has not been produced during this cooling stage, the residual stress after annealing is estimated from the value in the figure by multiplying the ratio of Young's moduli E_R/E_T(E_R : Young's modulus at the room temp., E_T: one at a holding temperature). Applying this method, the resulting residual stress is calculated for the stresses during annealing, shown in Fig. 8. These correlation is represented in **Fig. 11.** From the result of the analysis, it is seen that the creep strain is hardly produced for the first 30 min. until the temperature reaches 300°C and the stress decreases from 72 kg/mm^2 to 52 kg/mm^2 for the rest of 30 min. during a temperature range from 300°C to 600°C. The reduction of the stress for 1 hr. of holding at 600°C is almost same as one obtained for 1 hour in the heating process, but the stress scarcely decreases for another one hour of holding.

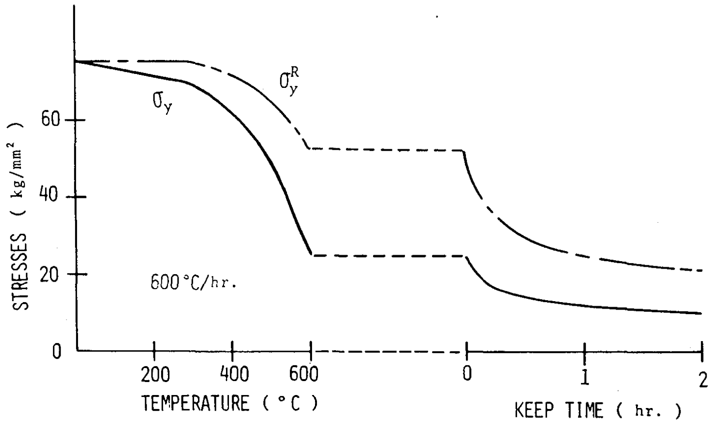

Fig. 11. Resulting residual stress produced at room temperature after annealing.

4. Conclusion

In this paper, the authors extended their theory of thermal elastic-plastic analysis based on the finite element method in order to include effects of creep strain. By the new theory, it becomes possible to analyze mechanical behavior produced during the whole thermal cycle due to welding and the stress relief annealing. In the example analyses, two different creep laws are used separately for two indivisual temperature ranges.

The following information is obtained:

(1) If changes of the creep properties and effect of stress variation in a time increment are considered, that is, $[C_c]$ are adopted, the convergence of solutions by the method is assured even for a comparatively large time increment. But unless the above mentioned changes can be included, the convergence of solutions is attained only for a sufficiently small time interval.

(2) With correct creep laws, the result of analysis of creep behavior shows good coincidence with that of experiment. This indicates that the present method is useful and can be applied for analysis.

(3) The entire behavior during thermal cycles, not only welding but also stress relief annealing, can be analyzed consistantly by this method.

(4) In plane stress state, assuming that the temperature increase uniformly over the specimen for annealing the stress ratio, σ_x and σ_y along the axes of symmetry, hardly changes the entire course of annealing.

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