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Theoretical Analysis of Weld Penetration Due to High Energy Density Beam†

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Abstract

Heat flow due to band and rectangular sources moving in direction perpendicular to the source plane is mathematically analysed. New mathematical formulations which give bead depth in high energy density beam welding such as electron and laser beam weldings as a function of beam parameters, physical constants of the material and welding speed are proposed.

The theoretical weld depth (bead penetration depth) agrees well with the published data obtained not only from electron beam welding but from laser welding.

1. Introduction

Many works have been reported in order to correlate the weld depth with the parameters in the electron beam welding. The theoretical treatment[1], however, to explane the data on a physical basis has not been sufficient, though it is important not only to predict the weld depth but also to clarify the welding mechanism.

In the typical electron beam welding, it is well known that the beam penetrates into the workpiece so as to sustain a deep hole which moves along the intended joint producing the straight sided narrow weld zone. It has been shown that the similar weld also may be obtained by means of recently developed CW CO_2 laser with high power level. In these weldings, the incident beam energy to the workpiece is consumed mainly for melting, evaporation and surface-reflecting which is important only in laser welding. In order to estimate the energy for melting the workpiece, which is the most predominant of all[2], a moving line source model has been used in the heat conduction treatment. However, this model which provides an infinite temperature at location of the source leads to noticeable error in temperature estimation at distant point from the beam axis compared with the beam diameter. In this model the size of the beam spot cannot be taken into account in calculating the temperature rise in spite of the fact that the weld depth is considerably affected by the beam diameter.

In the present paper, the authors propose to introduce a new model, band or rectangular plane heat source of uniform intensity, which moves in the direction perpendicular to the source plane, and of which width corresponds to the hole diameter. The temperature and bead penetration depth are described by a set of non-dimensional variables in order to establish an universal relationship independent of parameters such as physical constants of material, welding speed, source diameter, beam current and acceleration voltage. The calculated values of the penetration depth are compared with the published data which have been obtained in electron beam welding[3),4)] as well as laser welding[5)]. In these treatments variation of the phyisical constants of material, radiation heat loss and convection in the molten pool are neglected. Results obtained from the band source model may be also applied to the analysis of the gas cutting.

In sections 2-4 the theory of rectangular and band plane sources with uniform intensity moving with constant velocity in the direction perpendicular to the source plane is dealt with. Section 5 gives the theoretical weld depth due to these sources. In section 6 the theoretical weld depth is compared with the published data obtained from electron and laser beam weldings. The final section gives the summery of this study.

2. Fundamental solution

The evaporation of the material seems to play an important role to sustain a deep hole drilled in the electron beam welding from the view point of the dynamics. However the energy needed for evaporating the material is considerably smaller[2)]. Therefore it is assumed that the heat deliveres to the workpiece is consumed only for melting.

Due to the drilling action of the high energy

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density beam, heat delivered to the workpiece shows three-dimentional distribution profile, which varies with the physical properties of metal and the welding variables. But here in order to simplify the problem it is assumed that heat distributes two-dimensionally in a plane S of a certain shape perpendicular to the moving direction and steady temperature distribution is formed.

The temperature distributions due to such a source are obtained by integration of the solution of the moving point source in steady condition[6]. Assuming the moving direction is paralell to the x-axis and heat is liberated at the rate of q per unit time per unit area at (x', y', z'), the steady temperature at the point (x, y, z) at time considered in an infinite solid, initially zero temperature is

$$\theta = \frac{1}{4\pi k} \exp\left\{ -\frac{v}{4k_D}(x-x') \right\} \iint_S \frac{q}{R} \times \exp\left(-\frac{vR}{4k_D} \right) dy'\, dz' \quad (1)$$

where $R = \left\{ (x-x')^2 + (y-y')^2 + (z-z')^2 \right\}^{1/2}$, k = the heat conductivity of the metal, k_D = the heat diffusivity of the metal and v = moving velocity of the heat source. In the following sections the temperature rises due to a "rectangular heat source" and a "band heat source" with an infinite length are considered.

3. The steady temperature due to the moving rectangular source

Here the temperature distribution of infinite body due to the moving rectangular source as shown in **Fig. 1** is treated; the heat is liberated uniformly at the rate q per unit time per unit area over the rectangle of sides $2a$ parallel to the y-axis and $2b$ to the z-axis, which moves with velocity v along the x-axis, and the center of the rectangle is at origin. In order to derive the universal equation independent of physical properries of the material, moving velocity and the beam parameters, dimensionless parameters are introduced as follows:

$$\left.\begin{aligned} l^* &= l/a, \\ v^* &= av/2k_D, \\ \theta^* &= \theta/\theta_m = 2k\theta/aq = 4k\theta b/Q \text{ and} \\ \theta_m &= aq/2k \end{aligned}\right\} \quad (2)$$

where l is a quantity with the dimension of length, and Q is total beam power per unit time. At the point (x, y) in $z=0$ plane, Eq (1) gives rise to

$$\left.\begin{aligned} \theta^* &= \theta_1^* + \theta_2^*, \quad \text{for } |y^*| \leqq 1, \\ \theta^* &= \theta_1^* - \theta_2^*, \quad \text{for } |y^*| > 1, \end{aligned}\right\} \quad (3)$$

where

$$\theta^*_{1,2} = \frac{\exp(-v^* x^*)}{\pi v^*} \left\{ \exp(-v^* |x^*|)(\alpha_{1,2} + \beta_{1,2}) - \int_0^{\alpha_{1,2}} \exp(-v^* R^*_{\alpha_{1,2}})\, d\varphi - \int_0^{\beta_{1,2}} \exp(-v^* R^*_\beta)\, d\varphi \right\}$$

$$\alpha_1 = \tan^{-1}\frac{b^*}{1+y^*}, \qquad \alpha_2 = \tan^{-1}\frac{b^*}{|1-y^*|}$$

$$\beta_1 = \tan^{-1}\frac{1+y^*}{b^*}, \qquad \beta_2 = \tan^{-1}\frac{|1-y^*|}{b^*}$$

$$R^*_{\alpha_1} = \left\{ x^{*2} + \frac{(1+y^*)^2}{\cos^2\varphi} \right\}^{1/2}, \quad R^*_{\alpha_2} = \left\{ x^{*2} + \frac{(1-y^*)^2}{\cos^2\varphi} \right\}^{1/2}$$

$$\text{and} \quad R^*_\beta = \left\{ x^{*2} + \frac{b^{*2}}{\cos^2\varphi} \right\}^{1/2} \quad (4)$$

For points in $x^*=0$ plane, Eq. (4) gives

$$\theta^*_1 = \frac{1}{\pi v^*}\left[(\alpha_1 + \beta_1) - \int_0^{\alpha_1} \exp\left\{ -\frac{v^*(1+y^*)}{\cos\varphi} \right\} d\varphi - \int_0^{\beta_1} \exp\left(-\frac{v^* b^*}{\cos\varphi} \right) d\varphi \right]$$

$$\theta^*_2 = \frac{1}{\pi v^*}\left[(\alpha_2 + \beta_2) - \int_0^{\alpha_2} \exp\left\{ -\frac{v^* |1-y^*|}{\cos\varphi} \right\} d\varphi - \int_0^{\beta_2} \exp\left(-\frac{v^* b^*}{\cos\varphi} \right) d\varphi \right] \quad (5)$$

At the edge of the rectangle P, $x^*=0$ and $y^*=1$, it becomes

$$\theta^* = \frac{1}{\pi v^*}\left\{ \tan^{-1}\frac{b^*}{2} + \tan^{-1}\frac{2}{b^*} - \int_0^{\tan^{-1}\frac{b^*}{2}} \exp\left(-\frac{2v^*}{\cos\varphi} \right) d\varphi - \int_0^{\tan^{-1}\frac{2}{b^*}} \exp\left(-\frac{v^* b^*}{\cos\varphi} \right) d\varphi \right\} \quad (6)$$

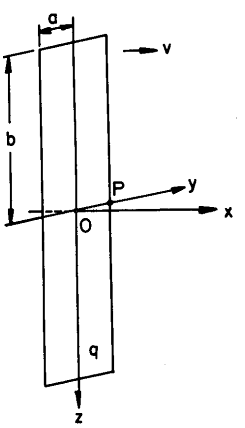

Fig. 1. Rectangular source with uniform intensity in the direction perpendicular to the source plane.

4. The steady temperature due to moving band source

In this section steady temperature distribution of infinite body due to moving band source in which heat is liberated uniformly at the rate q per unit time per unit area over an infinite strip parallel to the z-axis and of width $2a$ along the y-axis is presented.

Supposing that the center of the band which moves with the velocity v in the direction to the x-axis is at origin at the instance considered, the temperature rise at (x, y) is obtained by putting $b^* \rightarrow \infty$ in Eq. (3) and (4). For the present only the temperature in the plane of the source, $z=0$, will be discussed. Putting $x^*=0$ and $b^* \rightarrow \infty$, Eq. (3) becomes

$$\theta^*=\frac{1}{v^*}\left[1-\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left\{\exp\left(-\frac{v^*(1+y^*)}{\cos\varphi}\right)+\exp\left(-\frac{v^*(1-y^*)}{\cos\varphi}\right)\right\}d\varphi\right], \quad |y^*|\leq 1,$$
$$\theta^*=\frac{1}{v^*}\int_0^{\frac{\pi}{2}}\left\{\exp\left(-\frac{v^*(y^*-1)}{\cos\varphi}\right)-\exp\left(-\frac{v^*(y^*+1)}{\cos\varphi}\right)\right\}d\varphi, \quad |y^*|>1, \tag{7}$$

At the edge of the source, putting $y^*=1$ in Eq. (7) it becomes

$$\theta^*=\frac{1}{\pi v^*}\left\{\frac{\pi}{2}-\int_0^{\frac{\pi}{2}}\exp\left(-\frac{2v^*}{\cos\varphi}\right)d\varphi\right\} \tag{8}$$

Numerical values θ^*/θ_0^* in the source plane, that is, the ratio of θ^* to that at the center of the source, are shown in **Fig. 2** for various values of $v^*=av/2k_D$. For large v^*, the integral in Eq. (8) may be approximated by $(\pi/2)\exp(-2v^*)$, and then it becomes

$$\theta^*=\frac{1}{2v^*}\left\{1-\exp(-2v^*)\right\} \tag{9}$$

Eq. (8) also represents the temperature due to a semi-circular plane source at the center which moves in the direction perpendicular to the plane and of which center is at the location considered. For $v^*\gg 1$,

$$\theta^*=\frac{1}{2v^*} \tag{10}$$

The temperature due to the band source also may be obtained by integration of the solution of the moving line source[7]. Then Eq. (8) may be written in the another form:

$$\theta^*=\frac{1}{2\pi v^*}\int_0^{2v^*}K_0(u)du \tag{11}$$

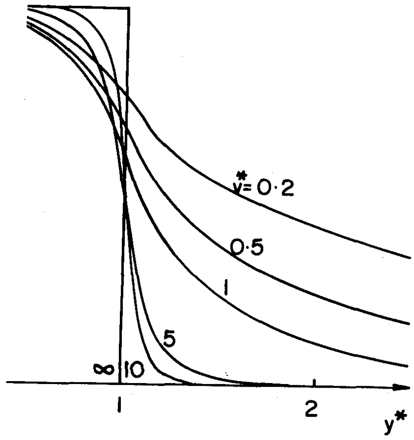

Fig. 2. Relation between θ^*/θ_0^* and y^* in the $x^*=0$ plane for band source

where K_0 is the modified Bessel function of the second kind of order zero. In the case $v^*\ll 1$, it becomes

$$\theta^*=\frac{2}{\pi}(1.1159-\log_e 2v^*) \tag{12}$$

5. Penetration depth

Since a liquid-enveloped deep hole moves along the intended joint in high power electron or laser beam welding, the temperature at the edge of the rectangular or band source may be somewhat higher than the fusion temperature of the material θ_f. But here this temperature is approximated by θ_f to simplify the problem.

Putting $\theta=\theta_f$ in Eq. (2),

$$\theta_f^*=\frac{4k\theta_f b}{Q}=\frac{4ka\theta_f}{Q}b^* \tag{13}$$

The solusion of θ_f^* in the case of the rectangular source is obtained by equating Eq. (6) to Eq. (13), which gives the welding conditions. Relation between θ_f^* and b^* for the rectangular source is shown by the solid curves in **Fig. 3.** The values in the limit $b^*\rightarrow\infty$ in this figure correspond to that of the band source given by Eq. (8).

On the other hand θ_f^* in Eq. (13) is a linear function of b^* with gradient $4ka\theta_f/Q$, which is plotted as a function of Q for various materials for $a=0.5$ mm in **Fig. 4.** It may be easily found that the solution θ_f^* in case of the rectangular source, which is given by the coordinate of the intersection of Eq. (13) and Eq. (6), approaches to that of the band source as Q increases, or k, a or θ_f decreases. The dotted lines in **Fig. 3** represent the relation given by Eq. (13) for various materials in the case of $Q=5$ KW and $a=$

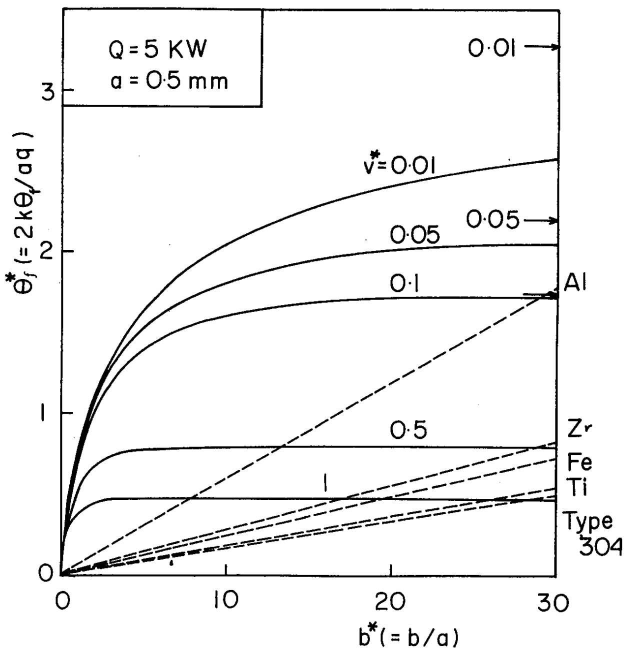

Fig. 3. Relation between θ_f^* and b^* for rectangular source.

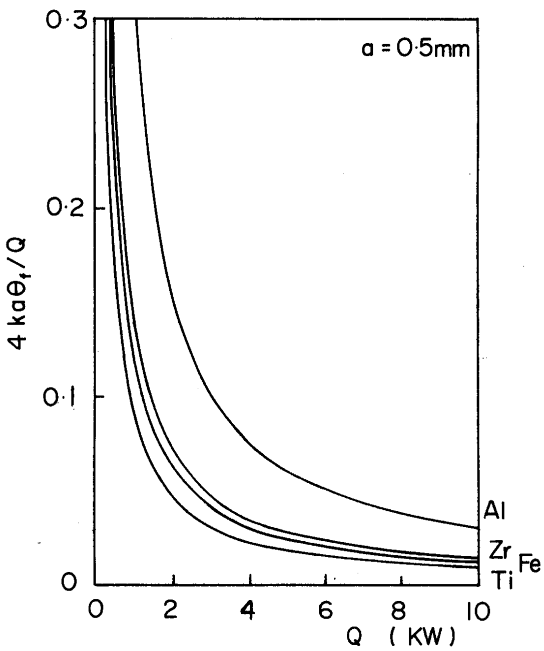

Fig. 4. Relation between Q and $4ka\,\theta_f/Q$.

0.5 mm, for an example. For low heat conduction material such as titanium, zirconium and type 304 stainless steel, θ_f^* for the rectangular source agrees well with that of the band source. Even for aluminium with large k, it seems to be quite well to consider that θ_f^* for both sources agrees well for $v^*>0.05$. Thus Eq (8) is available for estimating the penetration depth in typical electron beam welding, in which the beam power is usually 5 KW or more, because the contribution of a spurious part of the band source from $z^*=b^*$ to ∞ may be neglegible. In order to expand the problem to the case of generalized high power beam involving laser and electron beam, and to make the estimation more accurate, taking the effects of the reflectance loss of the beam and latant heat of the fusion into consideration, the final result of θ^* is given by

$$\theta_f^* = \frac{Q}{AQ-2abHv}\cdot\frac{1}{\pi v^*}\left\{\frac{2}{\pi} - \int_0^{\frac{\pi}{2}} exp\left(-\frac{2v^*}{\cos\varphi}\right)d\varphi\right\} \tag{14}$$

where A is the beam absorptance of the material and H the latant heat of fusion in the form cal/cm^3. In the case of the electron beam A is equal to unit, but in the laser beam it is usually less than unit because the beam reflectance for normal incidence is higher. However, the absorptance approaches to unit as the drilled hole becomes deep, because the beam is refocused to the bottom of the hole through the self-focusing effect of its wall[9] trapping the beam within the hole effectively.

6. Comparison with experimental data

In the calculation of the bead penetration depth in electron or laser beam welding one of the most difficulty to be overcome is the estimation of the diameter of the beam-drilled hole, which corresponds to the width of the band or rectangular source. In general the diameter varies not only with the beam parameters which involve the beam current, the acceleration voltage, the current of the focusing coil and work distance but also with the physical properties of the workpiece and welding velocity. Furthermore, it is impossible or very difficult to measure the hole diameter during welding. Here the diameter of the hole during welding will be approximated by that of the fusion isotherm obtained from micro-examination for convenience. In general the electron beam welding produces a penetration profile with a certain deviation from the ideal rectangle as shown in **Fig. 5.** In well-controlled electron beam

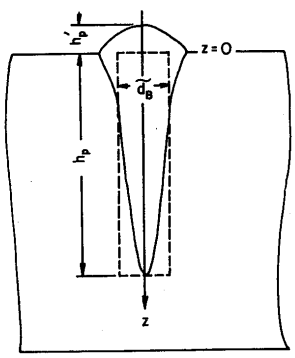

Fig. 5. Equivalent rectangle having same area and depth with actual bead.

welding the deviation is not appreciable, and then the penetration profile may be approximated by an equivalent rectangle having same depth and area with it. The width of the equivalent rectangle is given by

$$\tilde{d}_B = \frac{1}{h_p} \int_{-h_p'}^{h_p} d(z)\,dz \tag{15}$$

where d(z) is the diameter at depth z, h_p penetration depth. As the result putting $2a = \tilde{d}_B$ and $b = h_p$, v^* and θ_f^* are written in the forms

$$v^* = \frac{\tilde{d}_B v}{4k_D} \tag{16}$$

$$\theta_f^* = \frac{2k\theta_f \tilde{d}_B}{Q} b^* \tag{17}$$

The experimental data to be compared with the theory are obtained from references 3, 4 and 10 in which the photographs of the penetration profile are illustrated. Consequently, the data give the variety to the welding parameters involving beam current, accelerating voltage, focal current, work distance, welding speed and physical properties of material as shown in **Table 1.** Relation between $\theta_f{}^*$ and v^* reduced from the experimental data is shown in **Fig. 6.** The solid curve in this figure represents the theoretical value calculated from the band source model, which has been found to be reasonable for analysing the concerned problems from the study in section 5 because the beam power is higher or equal to 5 KW. Close agreement between the data and theoretical over a wide range of v^* from

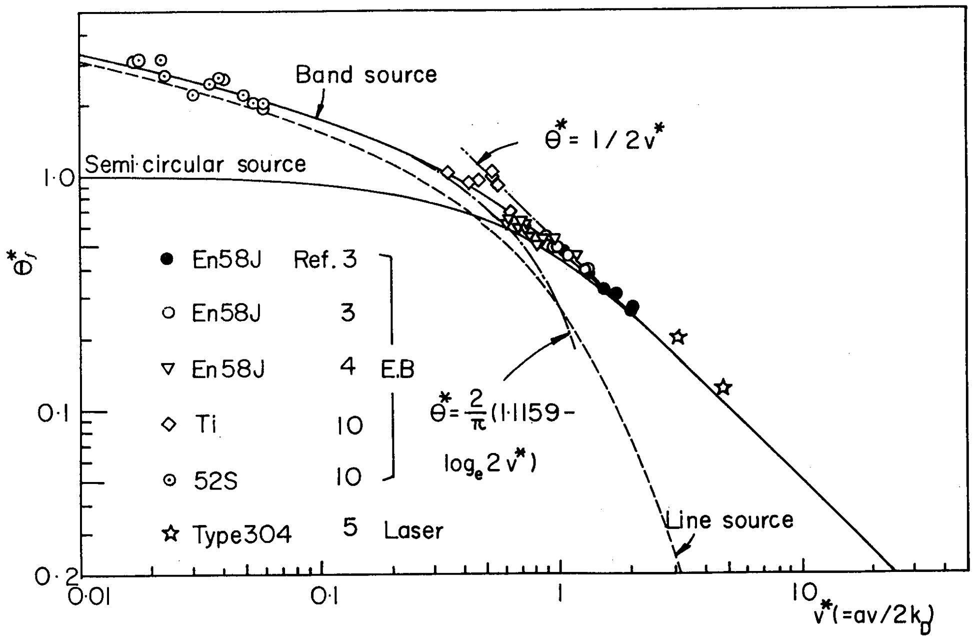

Fig. 6. A comparison of the theoretical equations with experimental data.

Table 1.

Reference	3		4	10		5
Material	En58J**		En58J	52S*	Ti	Type 304
k (cal/cm. sec. °C)	0.07		0.07	0.4	0.065	0.07
k_D(cm²/scc)	0.05		0.05	0.76	0.08	0.05
	Electron Beam					Laser
Beam	130 KV 40 mA	27～33 KV 150 mA	30 KV 100 mA	150 KV 30 mA		20 KW
v (cm/min)	120～300	66 const.	66 const.	50 const.		126～430
Parameter	Velocity	Beam focusing current	Beam focusing current and focus position	Work distance		Velocity

* 52S (Mg-2.4 %, Cu-0.04 %, Fe- 0.18 %, Si-0.1 %, Cr-0.17 %, Al-remainder)
** En58J (C-0.06 %, Ni-10.89 %, Cr-17.65 %, Mo-2.85 %, Fe-remainder)

about 0.01 to 2.0 is obtained.

Additional experimental data are obtained from the CO_2 laser welding[5]. The laser beam absorptance A is regarded as 100 % in the estimation of θ_f^*, because the extremely high laser power, 20 KW, provides deep penetration, about 15 mm, in which the incident laser beam may be trapped without any loss. The data agree well with the theory, though the data are not sufficient in number because only a few data of the CO_2 laser welding have yet been published.

7. Summary

Heat flow in electron and laser beam welding has been mathematically analysed by introducing rectangular and band heat source model, and new mathematical formulations avairable for estimating the weld depth as a function of beam parameters, physical constants of the material and welding speed have been derived. In the power level of several kilowatts or above, it has been found from the mathematical analysis that a value obtained from the rectangular source may be approximated by that from the band source. The theoretical values have been compared with published data obtained from laser and electron beam welds. The actual bead has been replaced by an equivalent rectangle with the width $\widetilde{d}_B$ having same depth and area with the actual, and then agreement between theoretical bead depth and experimental one has been obtained.

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