Risk Aversion and Longevity in an Overlapping Generations Model

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Abstract

We analyze how increasing longevity affects economic development based on differences in the risk attitudes of young and old individuals. We construct an overlapping generations model given an economy grows with the help of the capital and intermediate goods produced by individual activities. The outcomes of these activities are stochastically determined. We analytically and numerically show that increasing longevity hinders capital accumulation in the economy when old individuals are more risk-averse than young individuals. Thus, if old individuals are less willing to take risks in the economy, population aging will consequently slow economic growth.

Keywords: Risk Aversion, Longevity, Overlapping Generations Model, Population Aging

JEL classification MSC: O40, E20

1. Introduction

The United Nations Population Division (2019, 2020)[1][2] states that the number of people aged 65 years and older is projected to more than double by 2050. Increasing longevity is a key driver of population aging, and most

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countries have experienced substantial increases in life expectancy since 1950. In particular, Japan, Italy, Germany, and Korea face significant challenges from aging. Although these facts and forecasts could threaten sustainable economic growth, the extent to which the behavior of young and old individuals affects this issue remains underexplored. We bridge this gap in the literature in this study by focusing on the difference between the risk attitudes of young and old individuals to analyze how increasing longevity affects the economy.

Capital accumulation is important for economic growth. However, the activities that promote capital accumulation are associated with various risks and individuals' responses to those risks change throughout their lives. Moreover, the risk attitudes of individuals in a country experiencing increasing longevity differ from those in a vigorous economy comprising many young individuals. For example, Rolison et al. (2013)[3], Schurer (2015)[4], and Dohmen et al. (2017)[5] show that old individuals are more risk-averse than young ones.[1] These empirical results imply that aging populations face changes in society's response to risk.

To examine the relationships among increasing longevity, the risk attitudes of the young and old, and economic growth, we construct an overlapping generations model of an economy that grows by capital accumulation in which individuals' production activities are accompanied by risks. In this model, final goods are produced by capital and intermediate goods. The outcomes of intermediate goods production are determined stochastically. Both young and old individuals produce intermediate goods that are accompanied by risks. In-

[1] Using 2004–2011 panel data from Germany, Schurer (2015)[4] and Dohmen et al. (2017)[5] show that individuals become more risk-averse as they age. Moreover, using large representative panel datasets obtained from the Netherlands and Germany from 1990 to 2011, Dohmen et al. (2017)[5] show that an individual's willingness to take risks declines throughout their life. Some psychology studies also support the finding that individuals become more risk-averse as they age. For instance, Rolison et al. (2013)[3] show that this fact holds true for various domain sets such as financial and health domains using cross-sectional data from online advertising.

dividuals can allocate resources to consumption spare before they obtain the return on these goods. Further, we consider an economy in which old individuals are supposed to be more risk-averse than young individuals. The model shows that old individuals increase their consumption spare as they become more risk-averse. We examine how the risk attitudes of old individuals affect capital accumulation in an economy considering increasing longevity.

The main results are as follows. The effects of increasing longevity on an economy depend on the degree of the risk aversion of old individuals. We numerically and analytically show that increasing longevity hinders capital accumulation in an economy in which old individuals are much more risk-averse than young individuals. The intuition behind these results is as follows. In this model, the outcomes of the production of intermediate goods and thus their revenue are stochastically determined. When old individuals are significantly more risk-averse than young individuals, they allocate most of their resources to consumption spare, which they can obtain without any risk, whereas young individuals increase the inputs required for the production of intermediate goods associated with risks instead of saving. Therefore, capital stock per capita in such economies decreases due to old individuals' consumption spare. This result implies that the risk-averse behavior of old individuals hinders capital accumulation in an economy in which they are significantly more risk-averse than young individuals.

Early studies of increasing longevity, capital accumulation, and economic growth include Pecchenino and Pollard (1997)[6] and Futagami and Nakajima (2001)[7].[2] Using endogenous growth models, they show that increasing longevity

[2] Many studies analyze how increasing longevity affects human capital investment and thereby economic growth. De La Croix and Licandro (1999)[8], Kalemli-Ozcan et al. (2000)[9], Boucekkune et al. (2002)[10], Soares (2005)[11] and Cervellati and Sunde (2005)[12] show that increases in life expectancy promote human capital investment and economic growth based on the life-cycle model of Ben-Porath (1967)[13]. Hazan (2009)[14], Hansen and Lønstrup (2012)[15], and Yasui (2016)[16] also show the positive effects of increases in life expectancy on human capital investment, focusing on retirement decisions in line with Ben-Porath (1967)[13].

raises the savings rate and promotes economic growth.[3] Recently, several studies such as Acemoglu and Johnson (2007)[19] have shown the negative effects of increasing longevity on economic growth.[4] Hansen and Lønstrup (2015)[22] show that some developed countries have experienced the negative effects of increasing longevity on GDP growth. Bloom et al. (2011)[23] suggest that OECD countries are likely to see modest declines in the rate of economic growth with population aging during 2005– 2050. Maestas et al. (2016)[24] use U.S. data over 1980–2010 and show that annual growth in GDP slows as the older population increases. Using a growth theory with the endogenous replacement of physical capital accumulation by human capital accumulation, Minamimura and Yasui (2019)[25] show that life expectancy in some countries lowers human capital investment and economic growth in line with the empirical results of Acemoglu and Johnson (2007)[19]. Our study also shows the negative effects of increasing longevity on economic growth; however, we focus on the risk-averse attitude of old individuals. Thus, our study sheds new light on the relationship between increasing longevity and economic growth. Cervellati and Sunde (2011)[26] show a non-monotonic or negative relationship between life expectancy and growth in per capita income in countries before the onset of the demographic transition, whereas there are positive effects thereafter. Lee and Shin (2019)[27] also show that some countries have experienced a negative effect of increasing life expectancy on economic growth, while others have experienced positive effects. Kuhn and Prettner (2018)[28] theoretically and empirically show that

Our study provides a new insight into the effects of increasing longevity on economic growth, focusing on the differences between the risk attitudes of young and old individuals.

[3]Several empirical studies such as Horioka (1997)[17] and Bloom et al. (2003)[18] investigate the relationships among population aging, saving, and economic growth based on the life-cycle hypothesis. Horioka (1997)[17] shows that a decrease in the working-age population lowers the household savings rate and insists that a life-cycle hypothesis applies to countries such as Japan. Bloom et al. (2003)[18] show that increases in life expectancy lead to higher savings rates.

[4]In response to the criticism of Bloom et al. (2014)[20], Acemoglu and Johnson (2014)[21] revalidate the robustness of the results presented in their 2007 paper.

population aging, including increasing longevity, decreases the growth rate of consumption. Thus, consensus on how increasing longevity affects capital accumulation and economic growth is lacking. Our study contributes to addressing this gap in the literature by showing that increasing longevity reduces capital accumulation in an economy in which old individuals are much more risk-averse than young ones. The results of this study shed light on the problem that aging societies face: Can they accumulate capital as much as a vigorous economy with many young individuals can? Our results indicate that aging societies can accumulate capital if old individuals take more risks.

The remainder of this paper is organized as follows. Section 2 presents the basic structure of the proposed model. Section 3 describes its dynamic system. Section 4 describes the steady states of the model and analyzes how increasing longevity affects the economy in the steady state. This section also presents the numerical analyses of the comparative statics with respect to the steady state. Concluding remarks are presented in Section 5.

2. Model

2.1. Basic structure of the model

We use an overlapping generations model in which each generation lives at most for two periods. A new generation is born at a constant rate n in each period. From the young to the old period, an individual dies with probability $1 - \lambda \in (0, 1)$, which remains constant over time. Time is discrete. We assume a closed economy that produces final goods. Firms in the final goods sector produce final goods using intermediate goods and capital. Each individual produces intermediate goods whose output is determined stochastically, and they are then supplied to the final goods sector. We describe production in this economy and the behaviors of individuals in Sections 2.2 and 2.3, respectively.

2.2. Production

The production function of final goods exhibits constant returns to scale as follows:

$$Y_t = \chi(K_t)^{\alpha}(X_t)^{1-\alpha}, \tag{1}$$

where $\alpha \in (0, 1)$ and $\chi > 0$ denote the constant productivity parameters. Y_t is the output, K_t represents the amount of input of capital, and X_t is the amount of input of intermediate goods in period t. q_t and p_t denote the rental price of capital and price of intermediate goods in period t, respectively. Final goods are used as the numeraire. Since perfect competition prevails in this sector, profit-maximizing conditions give us

$$q_t = \alpha \frac{Y_t}{K_t}; \tag{2}$$

$$p_t = (1-\alpha)\frac{Y_t}{X_t}. \tag{3}$$

In this model, both young and old individuals can produce intermediate goods. Let $x_t^{y(o)}$ be the input for the production of intermediate goods when young (old) in period t. The production technology for intermediate goods is as follows:

$$G\left(x_t^i\right) = A^j x_t^i, \tag{4}$$

with $i \in \{y, o\}$ and $j \in \{H, L\}$. $A^{H(L)}$ denotes the productivity parameter of the production of intermediate goods, where $0 < A^L < A^H$. Each young and old individual is assigned to high productivity A^H with probability $\mu \in (0, 1)$ and low productivity A^L with probability $1-\mu$, where μ is the same for all individuals; whether an individual's productivity is high or low cannot be observed by all individuals ex ante. Thus, the output of intermediate goods is stochastically determined. This conversion is completed in period t; then, each individual supplies the output $A^j x_t^i$ to the intermediate goods market and obtains return $p_t A^j x_t^i$ in period t, where $j \in \{H, L\}$, $i = y$ when young and $i = o$ when old. Each individual uses its return to purchase consumption goods. Since the income earned from the production of intermediate goods is accompanied by risks, each individual spares some income (endowment when young or capital income when old) to purchase consumption goods before the return on the intermediate goods is realized. This is explained in the next subsection.

2.3. Individuals

Each young individual is endowed with $\overline{w}$ units of final goods. Let s_t be the amount of savings in period t. The return on capital investment is paid in the next period, $t+1$, when the individual becomes old. Since the income earned from intermediate goods production is accompanied by risks, each young individual saves s_t as well as spares γ_t^y to purchase consumption goods. Endowment is also used for the inputs for the production of intermediate goods in period t, x_t^y. Then, the following constraint holds: $\overline{w} = s_t + \gamma_t^y + x_t^y$.

Let Q_t denote the gross interest rate such that $Q_t \equiv 1 + q_t$. From the young to the old period, individuals die with probability $1 - \lambda \in (0, 1)$. Each young individual deposits their savings with an insurance company that operates under perfect competition. The insurance company invests in savings and repays its returns to the surviving old individuals. The no-arbitrage condition in period t entails that the returns on annuities become $\frac{Q_t}{\lambda}$ in period t, as in Blanchard (1985)[29] and Yaari (1965)[30]. Each old individual uses the return on savings when young to spare γ_t^o to purchase consumption goods because of the risky outcomes accompanied by the production of intermediate goods. They also use the return on savings when young as the inputs to produce intermediate goods in period t, x_t^o. Thus, the following constraint holds: $\frac{Q_t}{\lambda} s_{t-1} = \gamma_t^o + x_t^o$. The result of the production of intermediate goods is private information; that is, an individual cannot observe the other individuals' results. Thus, γ_t^o acts as self-insurance in this model.[5]

Although savings produce interest, neither the inputs for intermediate goods production x_t^y and x_t^o nor the spares for consumption γ_t^y and γ_t^o produce interest.

[5] No complete contracts are supplied to individuals by any insurance companies, which lack complete information about the types of insured individuals.

After the outputs of intermediate goods $A^j x_t^y$ and $A^i x_t^o$ with $i, j \in \{H, L\}$ are realized, each young and old individual obtains the returns on the production of these goods and uses their returns only for consumption. c_t^{yi} and c_t^{oj}, where $i, j \in \{H, L\}$ denotes the amount of consumption in period t.[6] We assume that the returns on the production of intermediate goods cannot be saved. Thus, we describe the budget constraint when young with productivity $i \in \{H, L\}$ in period t as follows:

$$c_t^{yi} = p_t A^i x_t^y = p_t A^i \left(\overline{w} - s_t - \gamma_t^y\right). \tag{5}$$

Next, we describe the budget constraint when old in period t as follows:

$$c_t^{oj} = p_t A^j x_t^o = p_t A^j \left(\frac{Q_t}{\lambda} s_{t-1} - \gamma_t^o\right). \tag{6}$$

The right-hand sides of equations (5) and (6) show that individuals face an income risk due to intermediate goods production whose output is determined stochastically. Thus, each individual decides how they consume, save, and spare for consumption goods ex ante to avoid such an income risk.

Assume that young individuals are risk-neutral and old individuals are risk-averse, as stated in the Introduction. We define average productivity A as $\overline{A} \equiv \mu A^H + (1-\mu) A^L$. For analytical simplicity, we focus on the economy in which the expected return on the production of intermediate goods is larger than one: $\overline{A} p_t > 1$. In such an economy, risk-neutral young individuals do not spare their endowment to purchase consumption goods ex ante and thus $\gamma_t^y = 0$. Then, the expected utility function of each young individual in period t is as follows:

$$u_t^y = \mu c_t^{yH} + (1-\mu)\, c_t^{yL}.$$

Each old individual's expected utility function in period t is as follows:

$$u_t^o = \mu \frac{\left(\gamma_t^o + c_t^{oH}\right)^{1-\theta}}{1-\theta} + (1-\mu) \frac{\left(\gamma_t^o + c_t^{oL}\right)^{1-\theta}}{1-\theta},$$

[6] Since we consider the quasi-linear utility function of an individual, the present model does not exhibit any income effect on expenditure when young. These properties enable us to analyze old individuals' risk-averse behavior.

where $\theta > 0$ denotes the degree of relative risk aversion (RRA) of old individuals. A smaller θ implies that old individuals take more risk, while a larger θ implies that they avoid further risk.

U_t is the expected utility of an individual born in period t and is given by

$$U_t = u_t^y + \beta\lambda u_{t+1}^o, \tag{7}$$

where $\beta \in (0,1)$ denotes the discount factor of an individual.

Each individual born in period t maximizes their expected utility (7) subject to the budget constraints when young (5) and when old (6). Combining equations (5) and (6) with (7), expected utility U_t is expressed as the following function of s_t and γ_{t+1}^o:

$$\begin{aligned} &U_t\left(s_t, \gamma_{t+1}^o\right) \\ &= \overline{A} p_t \left(\overline{w} - s_t\right) \\ &+ \beta\lambda \left\{ \mu \frac{\left[-\left(p_{t+1}A^H - 1\right)\gamma_{t+1}^o + p_{t+1}A^H \frac{Q_{t+1}}{\lambda} s_t\right]^{1-\theta}}{1-\theta} + (1-\mu) \frac{\left[\left(1 - p_{t+1}A^L\right)\gamma_{t+1}^o + p_{t+1}A^L \frac{Q_{t+1}}{\lambda} s_t\right]^{1-\theta}}{1-\theta} \right\}. \end{aligned} \tag{7'}$$

Then, the utility maximization problem for each individual born in period t is as follows: $\max_{s_t, \gamma_{t+1}^o} U_t\left(s_t, \gamma_{t+1}^o\right)$. Because we introduce a linear structure for the expected utility optimization problem, there are two cases: corner and interior solutions of γ_{t+1}^o. Let $\overline{\overline{A}}$ be the weighted average of a set of productivities for the production of intermediate goods $A \in \{A^H, A^L\}$, such that $\overline{\overline{A}} \equiv \frac{\mu A^H \left(A^L\right)^\theta + (1-\mu) A^L \left(A^H\right)^\theta}{\mu \left(A^L\right)^\theta + (1-\mu)\left(A^H\right)^\theta}$.[7] We consider an economy in which $\frac{1}{\overline{A}} < p_{t+1} < \frac{1}{\overline{\overline{A}}}$ for all t to focus on the interior solution of γ_{t+1}^o for all t in the following analyses. Focusing on such an economy allows us to analyze how old individuals' risk attitudes affect the economy by considering increasing longevity. Then, we obtain the following expected utility-maximizing conditions:

$$\overline{A} p_t = \beta p_{t+1} Q_{t+1} \left[\mu A^H \left(\gamma_{t+1}^o + c_{t+1}^{oH}\right)^{-\theta} + (1-\mu) A^L \left(\gamma_{t+1}^o + c_{t+1}^{oL}\right)^{-\theta}\right]; \tag{8}$$

[7] Since $\overline{A}$ and $\overline{\overline{A}}$ are average and weighted average productivity A, respectively, we obtain $\frac{1}{\overline{A}} < \frac{1}{\overline{\overline{A}}}$ from the definitions of $\overline{A}$ and $\overline{\overline{A}}$.

$$\mu\left(p_{t+1}A^H-1\right)\left(\gamma_{t+1}^o+c_{t+1}^{oH}\right)^{-\theta}=(1-\mu)\left(1-p_{t+1}A^L\right)\left(\gamma_{t+1}^o+c_{t+1}^{oL}\right)^{-\theta}. \quad (9)$$

Because $\frac{1}{A^H}<\frac{1}{\overline{A}}<p_{t+1}<\frac{1}{\overline{\overline{A}}}\leq\frac{1}{A^L}$ from the assumptions that $\frac{1}{\overline{A}}<p_{t+1}<\frac{1}{\overline{\overline{A}}}$ and $A^L<A^H$, both $p_{t+1}A^H-1$ and $1-p_{t+1}A^L$ in equation (9) take positive values. Condition (8) represents the inter-temporal trade-off, while condition (9) implies the intra-temporal condition of the utility maximization problem. Using the budget constraint when old (6) and first-order conditions above, we obtain the optimal amounts of resources to consume to avoid the risk γ_{t+1}^o as follows:[8]

$$\gamma_{t+1}^o=\left(A^H-A^L\right)^{\frac{1-\theta}{\theta}}\left(\frac{\beta p_{t+1}Q_{t+1}}{\overline{A}p_t}\right)^{\frac{1}{\theta}}\left[(1-\mu)^{\frac{1}{\theta}}A^H\left(A^Hp_{t+1}-1\right)^{\frac{-1}{\theta}}-\mu^{\frac{1}{\theta}}A^L\left(1-A^Lp_{t+1}\right)^{\frac{-1}{\theta}}\right]. \quad (10)$$

Equation (10) implies that the behavior of γ_{t+1}^o depends on the degree of risk aversion of old individuals θ; however, γ_{t+1}^o is not directly affected by changes in longevity λ.[9] Similarly, using the budget constraint when old (6) and first-order conditions above, we obtain the optimal amount of savings s_t as follows:[10]

$$s_t=\lambda\left[(A^H-A^L)p_{t+1}Q_{t+1}\right]^{\frac{1-\theta}{\theta}}\left(\frac{\beta}{\overline{A}p_t}\right)^{\frac{1}{\theta}}\left[(1-\mu)^{\frac{1}{\theta}}\left(A^Hp_{t+1}-1\right)^{\frac{\theta-1}{\theta}}+\mu^{\frac{1}{\theta}}\left(1-A^Lp_{t+1}\right)^{\frac{\theta-1}{\theta}}\right]. \quad (11)$$

Increasing longevity directly raises savings and promotes capital accumulation. However, as old individuals spare more for consumption as longevity rises, such an increase in the consumption spare γ_{t+1}^o reduces savings by changing the gross interest rate and prices of intermediate goods in the economy. Thus, whether increasing longevity promotes capital accumulation depends on both young and old individuals' behaviors, especially old individuals' risk aversion.

[8]We show the derivation of equation (10) in Appendix A.

[9]In the equilibrium, changes in λ affect the prices p_t, p_{t+1}, and Q_{t+1}, and these changes indirectly affect γ_{t+1}^o.

[10]We show the derivation of equation (11) in Appendix A.

2.4. Market equilibrium

We now consider the market equilibrium conditions. Let L_t be the number of young individuals in the economy at time t. Then, λL_{t-1} is the number of old individuals in the economy at time t. The equilibrium condition of the intermediate goods market in period t is

$$\overline{A} x_t^y L_t + \overline{A} x_t^o \lambda L_{t-1} = X_t. \tag{12}$$

The left-hand side of equation (12) denotes the amount of intermediate goods supplied by young and old individuals. The right-hand side of equation (12) denotes those demanded by firms in the final goods sector.

Because perfect competition prevails in the final goods sector, we obtain $Y_t = q_t K_t + p_t X_t$. At the beginning of period t, each young individual is endowed with $\overline{w}$ in terms of final goods and capital income is distributed to old individuals. They use this income for capital investment, inputs for intermediate goods production, and the purchase of goods for consumption spare. After the income from the production of intermediate goods $p_t X_t$ is realized, young and old individuals use it for consumption. Let $\overline{c}_t^y L_t$ and $\overline{c}_t^o \lambda L_{t-1}$ denote the average consumption in period t of the young and old, respectively; that is, $\overline{c}_t^y \equiv \mu c_t^{yH} + (1-\mu) c_t^{yL}$ and $\overline{c}_t^o \equiv \gamma_t^o + \mu c_t^{oH} + (1-\mu) c_t^{oL}$. We assume that capital does not depreciate. In the equilibrium, the following equation holds:

$$\overline{w} L_t + Y_t = K_{t+1} - K_t + x_t^y L_t + x_t^o \lambda L_{t-1} + \overline{c}_t^y L_t + \overline{c}_t^o \lambda L_{t-1}. \tag{13}$$

The equilibrium condition of the capital market in period t is

$$s_{t-1} L_{t-1} = K_t. \tag{14}$$

The left-hand side of equation (14) denotes the resources supplied by the old. The right-hand side of equation (14) is the amount of capital in the economy in period t.[11]

[11]Equation (14) can be derived from the final and intermediate goods market conditions (13) and (12), the production function and profit-maximizing conditions in the final goods sector (1)–(3), and the budget constraints (5) and (6) with $\gamma_t^y = 0$.

3. Dynamics

Next, we describe the dynamic system of two variables: capital stock per young individual k_t, where $k_t \equiv \frac{K_t}{L_t}$, and the gross interest rate Q_t. Let n be the fertility rate, where $n = \frac{L_t - L_{t-1}}{L_{t-1}}$ in each period t.[12] We assume that n is constant. Thus, the capital market equilibrium condition (14) can be rewritten as follows:[13]

$$k_{t+1} = \frac{s_t}{1+n}, \tag{15}$$

where s_t is the saving function of Q_t, p_t, and p_{t+1}, given by equation (11).

Using the profit-maximizing conditions of the final goods sector (2) and (3), the production function of final goods (1), and the definitions of k_t and q_t, p_t becomes a function of Q_t as follows:[14]

$$p_t = \chi^{\frac{1}{1-\alpha}} (1-\alpha) (\alpha)^{\frac{\alpha}{1-\alpha}} (Q_t - 1)^{-\frac{\alpha}{1-\alpha}}, \tag{16}$$

for $p_t \in \left(\frac{1}{\overline{A}}, \frac{1}{\overline{\overline{A}}} \right)$. Let $p_t(Q_t)$ be the right-hand side of equation (16). We summarize the properties of the function $p(Q_t)$ in Appendix B.

First, we derive the dynamics of capital stock per young individual k_t from the intermediate goods market-clearing condition (12) and capital market-clearing condition (15) as follows:[15]

$$k_{t+1} = \frac{\overline{w}}{1+n} + \frac{k_t}{1+n} \left[(Q_t - \Gamma(Q_t)) - \frac{1}{\overline{A}} \left(\frac{Q_t - 1}{\alpha\chi} \right)^{\frac{1}{1-\alpha}} \right], \tag{17}$$

where

$$\Gamma(Q_t) \equiv p(Q_t) Q_t \left[\frac{(1-\mu)^{\frac{1}{\theta}} A^H \left(A^H p(Q_t) - 1 \right)^{\frac{-1}{\theta}} - \mu^{\frac{1}{\theta}} A^L \left(1 - A^L p(Q_t) \right)^{\frac{-1}{\theta}}}{(1-\mu)^{\frac{1}{\theta}} \left(A^H p(Q_t) - 1 \right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}} \left(1 - A^L p(Q_t) \right)^{\frac{\theta-1}{\theta}}} \right].$$

Because the right-hand side of equation (17) is a function of Q_t and k_t, equation (17) determines the dynamics of k_t.

[12] In this model, the fertility rate is the same as the population growth rate.

[13] Using the definitions of k_t and n, capital stock per capita $\tilde{k}_t$ is defined as $\tilde{k}_t \equiv \frac{k_t(1+n)}{1+n+\lambda}$.

[14] The derivation of equation (16) is given in Appendix B.

[15] See Appendix C.

Next, we derive the dynamics of the gross interest rate from the above dynamics (17) and saving function (11) by considering equation (16) as follows:[16]

$$\lambda\left[(A^H - A^L)p\left(Q_{t+1}\right)Q_{t+1}\right]^{\frac{1-\theta}{\theta}}\left[(1-\mu)^{\frac{1}{\theta}}\left(A^H p\left(Q_{t+1}\right)-1\right)^{\frac{\theta-1}{\theta}}+\mu^{\frac{1}{\theta}}\left(1-A^L p\left(Q_{t+1}\right)\right)^{\frac{\theta-1}{\theta}}\right]$$
$$=\left(\frac{\beta}{\overline{A}p(Q_t)}\right)^{\frac{-1}{\theta}}\left\{\overline{w}+k_t\left[\left(Q_t-\Gamma(Q_t)\right)-\frac{1}{\overline{A}}\left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right]\right\}. \tag{18}$$

Because the left-hand and right-hand sides of equation (18) are functions of Q_{t+1} and of Q_t and k_t, respectively, equation (18) determines the dynamics of Q_t.

4. Steady-state analysis

We focus on the steady-state economy, where $k_{t+1} = k_t = k$ and $Q_{t+1} = Q_t = Q$ for all t. In the next subsection, we first describe how the steady-state values k and Q are determined and then examine how increasing longevity affects capital stock per young individual.

4.1. Steady-state economy

Hereafter, the variables without subscripts denote steady-state values. Evaluating equation (16) at the steady state gives us the following equation:

$$p = p(Q) \equiv \chi^{\frac{1}{1-\alpha}}\left(1-\alpha\right)\left(\alpha\right)^{\frac{\alpha}{1-\alpha}}\left(Q-1\right)^{-\frac{\alpha}{1-\alpha}}. \tag{19}$$

From equations (10) and (11), we can define the steady-state consumption spare and steady-state savings as the following functions of Q:

$$\gamma^o = \gamma^o(Q) \equiv \left(A^H - A^L\right)^{\frac{1-\theta}{\theta}}\left(\frac{\beta Q}{\overline{A}}\right)^{\frac{1}{\theta}}\left[(1-\mu)^{\frac{1}{\theta}}A^H\left(A^H p(Q)-1\right)^{\frac{-1}{\theta}}-\mu^{\frac{1}{\theta}}A^L\left(1-A^L p(Q)\right)^{\frac{-1}{\theta}}\right], \tag{20}$$

and

$$s = \lambda\tilde{s}(Q) \equiv \lambda\left[(A^H - A^L)Q\right]^{\frac{1-\theta}{\theta}}\left(p(Q)\right)^{-1}\left(\frac{\beta}{\overline{A}}\right)^{\frac{1}{\theta}}\left[(1-\mu)^{\frac{1}{\theta}}\left(A^H p(Q)-1\right)^{\frac{\theta-1}{\theta}}+\mu^{\frac{1}{\theta}}\left(1-A^L p(Q)\right)^{\frac{\theta-1}{\theta}}\right]. \tag{21}$$

[16] See Appendix C.

We consider a steady-state economy in which $\frac{1}{\overline{A}} < p < \frac{1}{\overline{\overline{A}}}$ to focus on an interior solution of the consumption spare γ^o. Let $\overline{Q}$ and $\overline{\overline{Q}}$ be such that $p(\overline{Q}) = \frac{1}{\overline{A}}$ and $p\left(\overline{\overline{Q}}\right) = \frac{1}{\overline{\overline{A}}}$. Then, from the definition of $p(Q)$, $\overline{Q}$ and $\overline{\overline{Q}}$ become as follows:

$$\overline{Q} \equiv 1 + \left(\overline{A}(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}} \alpha\chi^{\frac{1}{\alpha}},$$

and

$$\overline{\overline{Q}} \equiv 1 + \left(\overline{\overline{A}}(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}} \alpha\chi^{\frac{1}{\alpha}}.$$

Using the above definitions and $\overline{A} > \overline{\overline{A}}$, the condition of the interior solution can be rewritten as follows: $\overline{\overline{Q}} < Q < \overline{Q}$.[17]

In the steady-state economy, the intermediate goods price p and gross interest rate Q are determined at the equilibrium in both the intermediate goods and the capital markets. Then, capital stock per young individual k is determined at the capital market equilibrium. In both markets, each young and old individual supplies intermediate goods and capital, while the firms in the final goods sector demand these factors to produce final goods. In the previous section, we obtained equation (17) from the intermediate goods and capital market-clearing conditions, while we obtained equation (18) by substituting the saving function with the intermediate goods price function into equation (17). Evaluating them at the steady states gives us the following equations:[18]

$$\overline{A}\left(\overline{w} - \lambda\tilde{s}\left(Q\right)\right) + \overline{A}\frac{\lambda}{1+n}\left(Q\tilde{s}\left(Q\right) - \gamma^o(Q)\right) = \left(\frac{\lambda\tilde{s}\left(Q\right)}{1+n}\right)\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}, \quad (22)$$

and

$$k = \frac{\lambda\tilde{s}(Q)}{1+n}. \quad (23)$$

Equations (22) and (23) represent the intermediate goods and capital market equilibrium conditions at the steady state, respectively. Let $\Psi^S(Q;\lambda)$ and $\Psi^D(Q;\lambda)$ be the left- and right-hand sides of equation (22). $\Psi^S(Q;\lambda)$ and

[17] Average productivity $\overline{A}$ does not depend on the risk attitudes of old individuals θ, while weighted average productivity $\overline{\overline{A}}$ depends on θ.

[18] The derivations of equations (22) and (23) are in Appendix D.

$\Psi^D(Q;\lambda)$ represent the supply of intermediate goods and demand for intermediate goods per young individual, respectively. Q is determined by equation (22), given longevity λ. Thus, the steady-state level of capital stock per young individual is determined by equation (23).

Next, to examine how changes in longevity λ affect capital stock per young individual k, we set the following two assumptions on the productivity parameters α, χ, and $\overline{A}$.

Assumption 1. $\frac{1}{2} > \alpha$.

Assumption 2. $\chi \leq \alpha^{-2\alpha}(1-\alpha)^{-(1-2\alpha)}\left(\overline{A}\right)^{-(1-\alpha)}$.

Hereafter, we analyze the following two cases: (a) $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ and (b) $0 < \theta \leq 1$.[19] Case (a) means that old individuals are much more risk-averse than young individuals, while case (b) means that old individuals are more risk-averse than young ones, but to less extent. Let $\hat{\theta}$ be such that $\hat{\theta} \equiv \frac{\overline{Q}-1-\alpha n}{\overline{Q}-1-\alpha n-\alpha\overline{Q}}$.[20] We assume the following three assumptions regarding the risk attitudes of old individuals θ:

Assumption 3. $\overline{\overline{Q}} \geq \frac{\frac{\theta-1}{\theta}}{\frac{\theta-1}{\theta}-\frac{\alpha}{1-\alpha}}$.[21]

Assumption 4. $\hat{\theta} \geq \theta$.[22]

Assumption 5. $\frac{\overline{A}}{\overline{\overline{A}}} < \frac{1-\alpha}{\alpha}$.

For a demographic structure (n), we assume that following two assumptions:

Assumption 6. $\overline{Q} - (1+n) < \frac{\gamma^o(\overline{Q})}{\tilde{s}(\overline{Q})}$.

Assumption 7. $\frac{n}{\alpha} + 1 \leq \overline{\overline{Q}}$.

---

[19] Assumption 1 and $\alpha \in (0,1)$ guarantee $\frac{1-\alpha}{1-2\alpha} > 0$.

[20] $\overline{Q}$ does not depend on θ from the definition of $\overline{Q}$.

[21] When $\theta > \frac{1-\alpha}{1-2\alpha}$, $\frac{\theta-1}{\theta} - \frac{\alpha}{1-\alpha} > 0$ holds.

[22] Assumption 2 means that $\alpha\left(\overline{Q}-1\right) - (1-\alpha) \leq 0$ holds. $\hat{\theta} > \frac{1-\alpha}{1-2\alpha}$ corresponds to $\alpha n > \alpha\left(\overline{Q}-1\right) - (1-\alpha)$. Thus, since $n > 0$ and $\alpha \in \left(0, \frac{1}{2}\right)$ from Assumption 1, Assumptions 1 and 2 guarantee $\hat{\theta} > \frac{1-\alpha}{1-2\alpha} > 1$.

We first examine how changes in the gross interest rate Q affect savings when young s and the consumption spare when old γ^o. Differentiating both sides of equation (21) with respect to the gross interest rate Q, we obtain the following lemma.

Lemma 1. *(a) Suppose that $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ with Assumptions 1 and 3. Then, increases in the steady-state gross interest rate Q decrease savings per young individual: $\frac{ds}{dQ} < 0$.*
(b) Suppose that $0 < \theta \leq 1$. Then, an increase in the steady-state gross interest rate Q increases savings per young individual: $\frac{ds}{dQ} > 0$.

Proof of Lemma 1. See Appendix E.

Lemma 1 shows that increases in the gross interest rate affect the steady-state level of savings per young individual depending on the risk attitudes of old individuals θ.

Differentiating equation (20) with respect to the gross interest rate Q, we obtain the following lemma.

Lemma 2. *Increases in the steady-state gross interest rate Q increase the consumption spare of an old individual: $\frac{d\gamma^o(Q)}{dQ} > 0$.*

Proof of Lemma 2. See Appendix F.

To examine how increasing longevity affects capital stock per young individual, differentiating equation (23) with respect to λ, we obtain

$$\frac{dk}{d\lambda} = \frac{1}{1+n}\left[\tilde{s}(Q) - \lambda\left(\frac{-d\tilde{s}(Q)}{dQ}\right)\frac{dQ}{d\lambda}\right]. \tag{24}$$

Equation (24) indicates when increasing longevity decreases capital stock per young individual.[23] Because Lemma 1 summarizes the properties of the saving function of the gross interest rate, we next examine how changes in longevity λ affect the gross interest rate Q through the intermediate goods market. To do

[23] We summarize the conditions in Lemma 4 in Appendix G.

this, we graph the supply and demand functions of intermediate goods. To do this, we first examine the properties of the supply and demand functions of this market.

In the steady-state economy, savings when young are less elastic with respect to the gross interest rate; that is $1 > \epsilon_{sQ}$ from Lemma 5 in Appendix H. This condition is equivalent to the demand function of intermediate goods being an increasing function of the gross interest rate; that is, $\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} > 0$.[24] Because increases in the gross interest rate imply decreases in the price of intermediate goods (see Appendix B), demand for intermediate goods per young individual decreases as the prices of these goods increase.

On the contrary, the supply function of intermediate goods per young individual $\Psi^S(Q;\lambda)$ has the two cases: $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} > 0$ and $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} < 0$. Figure 1 shows the former case, while Figure 2 shows the latter case. How increases in the gross interest rate affect the supply of intermediate goods depends on the risk attitudes of old individuals θ.[25] When θ takes sufficiently high values, an increase in the gross interest rate raises the supply of intermediate goods; that is, $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} > 0$. This is because the changes in the consumption spare of old individuals become small. By contrast, when θ takes sufficiently low values, an increase in the gross interest rate decreases the supply of intermediate goods; that is, $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} < 0$. This is because the changes in the consumption spare of old individuals become large.

We next examine how changes in longevity affect intermediate goods demand per young individual and intermediate goods supply per young individual. From the definition of $\Psi^D(Q;\lambda)$ in equation (22), differentiating $\Psi^D(Q;\lambda)$ with respect to λ, we obtain

$$\frac{\partial \Psi^D(Q;\lambda)}{\partial \lambda} = \frac{\Psi^D(Q;\lambda)}{\lambda} > 0. \tag{25}$$

[24]See Appendix H.

[25]See Appendix I. The properties of $\Psi^S(Q;\lambda)$ are mainly determined by savings when young and the consumption spare when old. These properties are summarized in Lemmas 1 and 2 and Appendices E and F.

From the definition of $\Psi^S(Q;\lambda)$ in equation (22) and Assumptions 1–4 and 6, differentiating $\Psi^S(Q;\lambda)$ with respect to λ, we obtain the following lemma.

Lemma 3. *(a) Suppose that* $\overline{\overline{Q}} < Q < \overline{Q}$ *and* $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ *with Assumptions 1–4. Then, increasing longevity decreases the supply of intermediate goods per young individual; that is,*

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} = \frac{\overline{A}}{1+n} \left\{ [Q-(1+n)]\, \tilde{s}(Q) - \gamma^o(Q) \right\} < 0.$$

(b) Let $\hat{Q}$ *be such that* $[\hat{Q}-(1+n)]\tilde{s}(\hat{Q}) = \gamma^o(\hat{Q})$ *and* $\overline{\overline{Q}} < \hat{Q} < \overline{Q}$*. Suppose that* $0 < \theta \leq 1$ *and* $\hat{Q} < Q < \overline{Q}$ *with Assumption 6. Then, increasing longevity decreases the supply of intermediate goods per young individual; that is,*

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} = \frac{\overline{A}}{1+n} \left\{ [Q-(1+n)]\, \tilde{s}(Q) - \gamma^o(Q) \right\} < 0.$$

Proof of Lemma 3. See Appendix J.

From Lemma 3, $\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} < 0$ is equivalent to $[Q-(1+n)]\, \tilde{s}(Q) < \gamma^o(Q)$. Thus, we focus on an economy with a large consumption spare to analyze the large effect of the risk-averse behavior of old individuals.

We finally analyze how increasing longevity affects the gross interest rate and then capital stock per young individual in the following two cases: (a) old individuals are much more risk-averse than young ones; that is, $\theta > \frac{1-\alpha}{1-2\alpha} > 1$, and (b) old individuals are more risk-averse than young ones, but to less extent; that is, $0 < \theta \leq 1$. First, we analyze case (a). Let longevity λ rise to λ'. Figure 1 shows the case in which increasing longevity raises the gross interest rate; that is, the gross interest rate increases from Q^{E*} to Q^{E**} and thus the equilibrium amount of intermediate goods increases. We can intuitively describe these changes as follows. When old individuals are highly risk-averse ($\theta > \frac{1-\alpha}{1-2\alpha}$), $\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} < 0$ holds (see Lemma 3). This is because they increase their consumption spare to avoid the risks associated with intermediate goods production. Therefore, increasing longevity decreases the supply of intermediate goods given the gross interest rate Q (from E^* to D in Figure 1). Moreover,

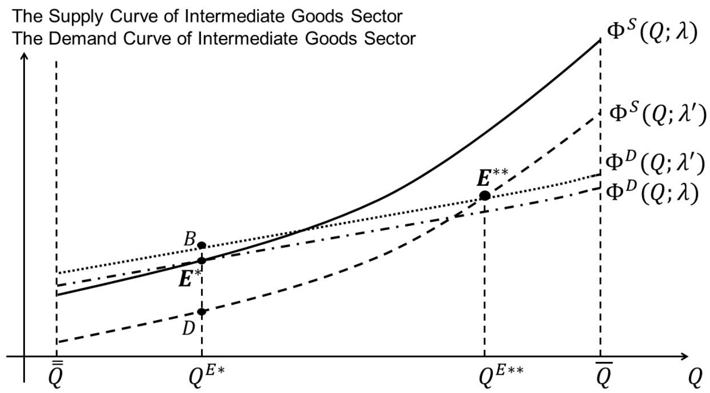

Figure 1: The effects of increasing longevity on the gross interest rate in the steady state in which old individuals are much more risk-averse than young ones ($\theta > \frac{1-\alpha}{1-2\alpha} > 1$)

as shown in inequality (25), increasing longevity increases demand for intermediate goods (from E^* to B in Figure 1). To attain the equilibrium, the gross interest rate must increase. When the gross interest rate rises, young individuals decrease their savings and raise inputs to produce intermediate goods. By contrast, old individuals use most of the increase in the returns on savings due to the increase in the gross interest rate to raise their consumption spare. However, as shown in Appendix F, when θ takes sufficiently high values, the change in the consumption spare of old individuals becomes small. Thus, an increase in the gross interest rate raises the supply of intermediate goods. Therefore, the gap between the supply of intermediate goods and their demand due to increasing longevity is resolved by increasing the gross interest rate.

The following proposition summarizes the effects of increasing longevity on capital stock per young individual in an economy when $\theta > \frac{1-\alpha}{1-2\alpha}$, as shown in Figure 1.

Proposition 1. *Suppose that $\theta > \frac{1-\alpha}{1-2\alpha}$ and $\overline{\overline{Q}} < Q < \overline{Q}$ with Assumptions 1–5 and 7. Then, increasing longevity decreases capital stock per young individual; that is, $\frac{dk}{d\lambda} < 0$.*

Proof of Proposition 1. See Appendix K.

We next analyze the second case in which old individuals are more risk-averse than young ones, but to less extent; this means that $0 < \theta \leq 1$. Hereafter, we assume that $\hat{Q} < Q < \overline{Q}$ to analyze the economy when increasing longevity lowers the supply of intermediate goods from Assumption 6.[26] Let longevity λ rise to λ'. Figure 2 shows that the equilibrium of the intermediate goods market changes from F^* to F^{**}; thus, increasing longevity decreases the gross interest rate Q from Q^{F*} to Q^{F**} and decreases intermediate goods per young individual.[27] This decrease in the gross interest rate is smaller than the case of $\theta > \frac{1-\alpha}{1-2\alpha}$. Consequently, increasing longevity increases capital stock per young individual; that is, $\frac{dk}{d\lambda} > 0$.[28]

The intuition is as follows. When $0 < \theta \leq 1$, increasing longevity from λ to λ' raises demand for intermediate goods (from F^* to H in Figure 2), but decreases the supply of intermediate goods (from F^* to J in Figure 2). This supply decrease is caused by the large consumption spare of old individuals (see Lemma 3 and Appendix J). These effects are the same as in the economy shown in Figure 1, but to less extent. Moreover, the supply of intermediate goods decreases as the gross interest rate increases when $0 < \theta \leq 1$ because the elasticity of savings when young with respect to the gross interest rate is small when $\theta \leq 1$.[29] Thus, to attain the new equilibrium F^{**}, the gross interest rate decreases. When the gross interest rate falls from Q^{F*} to Q^{F**}, each old individual decreases their consumption spare γ^o, while young individuals decrease their savings, but to less extent because there is no need for them to increase the inputs of the production of intermediate goods (see Lemmas 1 and 2 and Appendices E and F). Thus, in the new equilibrium F^{**}, the economy

[26]See Lemma 3.

[27]In Appendix L, we show that $\frac{dQ}{d\lambda} < 0$ and $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} < 0 < \frac{\partial \Psi^D(Q;\lambda)}{\partial Q}$ when $0 < \theta \leq 1$ and $\hat{Q} < Q < \overline{Q}$.

[28]Lemma 4 (ii) in Appendix G shows that $\frac{dk}{d\lambda} > 0$ when the elasticity of the gross interest rate with respect to longevity $\epsilon_{Q\lambda}$ is sufficiently small such that $\epsilon_{sQ}\epsilon_{Q\lambda} < 1$.

[29]See Lemma 4 (Appendix G) and Appendix L.

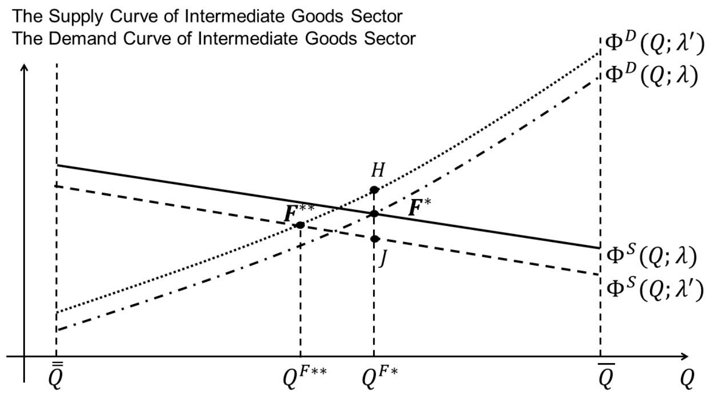

Figure 2: The effects of increasing longevity on the gross interest rate in the steady state in which old individuals are more risk-averse than young ones ($\theta \leq 1$)

attains smaller amounts of intermediate goods and larger amounts of capital stock per young individual than those in the old equilibrium F^*.

In the next subsection, we numerically show the above results.

4.2. Numerical analyses

We now present the numerical results of the comparative statics of the steady-state economy examined in the previous section. We choose parameters that explain the recent and future economies in Germany because Schurer (2015)[4] and Dohmen et al. (2017)[5] show that old individuals are more risk-averse than young individuals, as in our model, using German data. We set the steady-state economy before the rise in longevity as the economy in Germany in 2019. Thus, our numerical results show that increasing longevity affects the steady state in economies such as Germany and other OECD countries taking account of the difference in the risk attitudes of young and old individuals and risk-averse behavior of old individuals. Table 1 summarizes the parameter settings used in this subsection. Our model is a two-period overlapping generations model. Since the working-age population is defined as those aged 15 to 64 and average life expectancy at birth in 2019 in Germany was about 80

Table 1: Parameter values

Preference		
β	0.5	Discount Factor (Impatience)
θ	$\{0.7, 12\}$	Degree of RRA
λ	$\{0.82, 0.85\}$	Survival Rate from Young to Old
Demography		
n	0.067	Population Growth Rate
Productivity (Final Goods)		
α	0.402	Capital Share
$1-\alpha$	0.598	Intermediate Goods Share
χ	1.5	Multifactor Productivity
Productivity (Individual Activities)		
μ	0.4	Probability of High Productivity
A^H	10	High Productivity of the Production of Intermediate Goods
A^L	0.2	Low Productivity of the Production of Intermediate Goods
$\overline{A}$	4.12	Average Productivity of the Production of Intermediate Goods
Endowment when Young		
$\overline{w}$	0.55	Endowment when Young

years according to OECD and World Bank data, the young and old populations in our model are defined as those aged 15 to 47.5 and those aged 47.5 to 80, respectively. Moreover, one period takes 32.5 years in our model. Then, we set the discount rate to a reasonable value: $0.912^{32.5}$. Since the annual population growth rate in 2019 (year on year) in Germany was 0.2 percent according to OECD data, we calculate n (population growth rate over 32.5 years) such that $n = 1.002^{32.5} - 1$. The technology parameters in the final goods sector are set as follows. Using OECD data from 1987 to 2019, we choose α as the average non-ICT capital contribution to GDP in per capita growth in Germany and $1 - \alpha$ as the average hours worked and ICT capital contribution to GDP in per capita growth in Germany. χ is standardized to 1.5.[30] We consider the following probability of survival. We set $\lambda = 0.82$ as the baseline value and change the parameter to $\lambda = 0.85$ to consider the effects of increasing longevity. The baseline value is based on the average survival rate to age 65 in Germany from 1987 to 2019 from the World Bank.[31]

We compare the case in which the degree of RRA is $\theta = 12$ with the case in which it is $\theta = 0.7$. The former case represents the economy inhabited by old individuals, who are highly risk-averse, and the latter represents the economy inhabited by old individuals, who are slightly risk-averse. This setting is consistent with studies that estimate RRA such as Chetty (2006)[31].[32] We

[30]From 1987 to 2019, multifactor productivity contribution to GDP in per capital growth in Germany ranged from -4 to 2.9. This variable changes year on year. For example, it changed significantly during the years of the global financial crisis (2008–2010). Thus, we remove the values in these years and average them.

[31]We calculate this average survival rate using World Bank data, which provide the survival rates to age 65 for male and female as well as the sex ratio. The average survival rate to age 65 for female (male) in these years is about 0.87 (0.76) and the average sex ratio is about 0.50. Our findings in the previous section are robust to changes in λ.

[32]There is a large literature on the estimation of RRA. For instance, the following studies estimate that the mean ranges from 0.6 to 15.8. Using Health and Retirement Survey data, Barsky et al. (1997)[32] estimate that RRA takes a value from 0.7 to 15.8. Using the Michigan Health and Retirement Study, Halek and Eisenhauer (2001)[33] estimate it as 3.74 and insist that individuals become more risk-averse as they age. Chetty (2006)[31] summarizes studies

consider the economy when $\theta = 12$ in which old individuals are much more risk-averse to represent actual developed countries well. For individuals' technology, we set $\mu = 0.4$, $A^H = 10$, and $A^L = 0.2$ and obtain average productivity $\overline{A}$ of 4.12. In the calibration of these values, capital stock per young individual $k = 0.511$ when $\theta = 12$ is targeted.[33] We set the endowment when young $\overline{w}$ as the value that equals savings when young in an economy when $\theta = 12$.[34]

First, we investigate the effects of increasing longevity on the steady-state economy in which $\theta = 12$ based on the above parameters. This means that old individuals are much more risk-averse than young ones. Figure 3 shows the effect of increasing longevity on the gross interest rate and capital stock per young individual at the steady state when $\theta = 12$. It plots $\Psi^S(Q; \lambda)$ and $\Psi^D(Q; \lambda)$, which indicate the supply of and demand for intermediate goods per young individual given $\lambda = 0.82$ and $\lambda = 0.85$, respectively. Figure 3 shows the case in which increasing longevity changes the equilibrium from E_1 to E_2. This implies that it increases the steady-state gross interest rate as well as both demand for and the supply of intermediate goods.

Figure 4 shows the case in which increasing longevity decreases capital stock per young individual (see equation (23)). Increasing longevity from $\lambda = 0.82$ to 0.85 shifts the supply of the capital market upward from the solid curve to the dotted curve. Thus, when $\theta = 12$, as shown in Figures 3 and 4, increasing longevity raises the amount of intermediate goods and decreases capital stock

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of RRA estimation from 1981 to 2004 and calibrates RRA from the date of this survey, finding mean RRA of 0.71. However, some studies such as Fullenkamp et al. (2003)[34] estimate that mean RRA is less than 1, whereas others such as Chiappori and Paiella (2011)[35] estimate that the mean is larger than 1. Many researchers have conducted empirical and experimental studies using recent data and methods. These studies assume a representative agent model, while our study adopts an overlapping generations economy in which each individual lives for two periods.

[33] We calculate $k = 0.511$ using capital stock in Germany in 2019 from FRED data and the working-age population in 2019 from World Bank data.

[34] Using a different value of $\overline{w}$ in an economy when $\theta = 0.7$ does not change the results. Thus, we set the same value of $\overline{w}$ when $\theta = 12$ and $\theta = 0.7$.

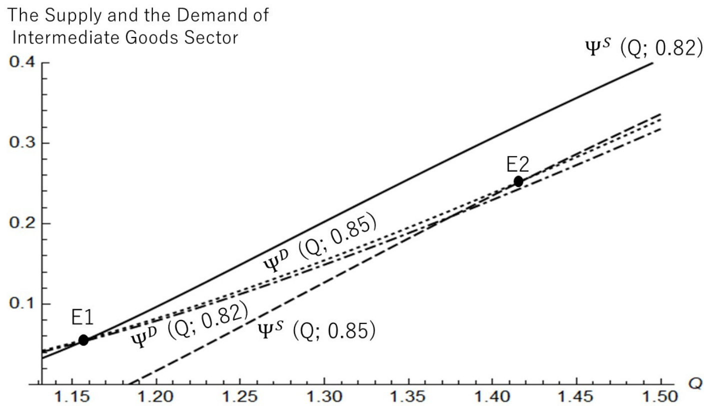

Figure 3: The effects of increasing longevity from $\lambda = 0.82$ to $\lambda = 0.85$ on the gross interest rate in a steady state in which old individuals are much more risk-averse than young ones ($\theta = 12$)

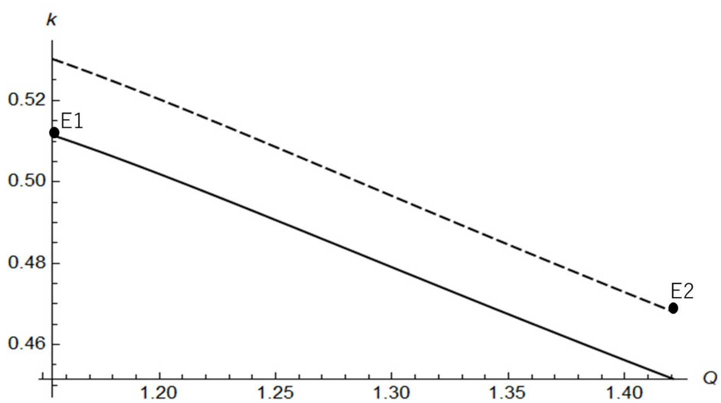

Figure 4: The effects of increasing longevity from $\lambda = 0.82$ to $\lambda = 0.85$ on capital stock per young individual in a steady state in which old individuals are much more risk-averse than young ones ($\theta = 12$)

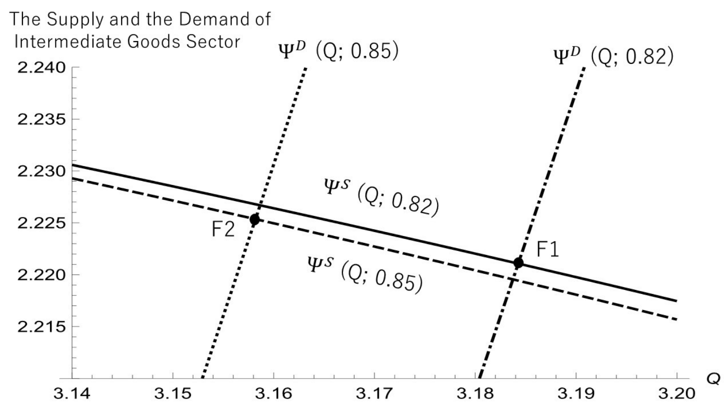

Figure 5: The effects of increasing longevity from $\lambda = 0.82$ to $\lambda = 0.85$ on the gross interest rate in a steady state in which old individuals are more risk-averse than young ones ($\theta = 0.7$)

per young individual in the equilibrium.

Figure 5 shows that increasing longevity affects the gross interest rate in the steady-state economy in which $\theta = 0.7$ by plotting $\Psi^S(Q;\lambda)$ and $\Psi^D(Q;\lambda)$. This figure shows the case in which increasing longevity changes the equilibrium from F_1 to F_2. This implies that increasing longevity decreases the steady-state gross interest rate, decreases demand for intermediate goods, and increases the supply of intermediate goods. Thus, when $\theta = 0.7$, the equilibrium amount of intermediate goods slightly changes with increasing longevity.

Figure 6 shows the case in which increasing longevity increases capital stock per young individual (see equation (23)). When $\theta = 0.7$, it decreases the gross interest rate and savings of young individuals; however, the rise in savings due to increasing longevity outweighs this decrease. Thus, increasing longevity raises capital stock per young individual.

Figure 7 shows the consumption spare of each old individual when $\theta = 12$ is larger than that in an economy when $\theta = 0.7$ (see equation (20) and Lemma 2). The consumption spare in an economy in which $\theta = 12$ gradually rises as the gross interest rate increases, whereas that when $\theta = 0.7$ significantly increases

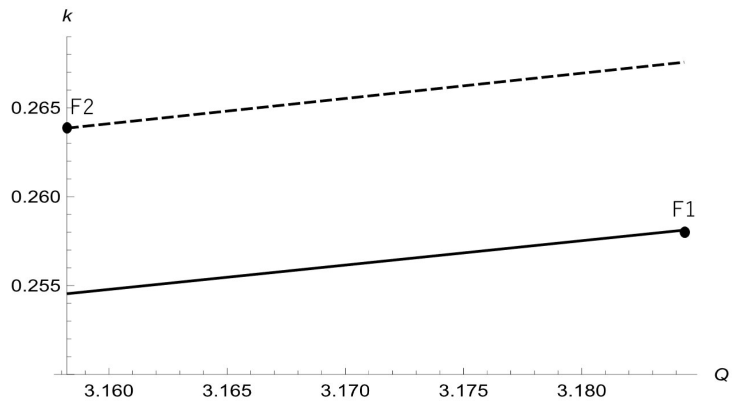

Figure 6: The effects of increasing longevity from $\lambda = 0.82$ to $\lambda = 0.85$ on capital stock per young individual in a steady state in which old individuals are more risk-averse than young ones ($\theta = 0.7$)

as the gross interest rate rises.[35]

Therefore, when old individuals are much more risk-averse than young ones ($\theta = 12$), increasing longevity causes a large gap between intermediate goods supply and demand because of the large consumption spare of old individuals. Then, it causes a large rise in the gross interest rate and increases the equilibrium amount of intermediate goods. When the gross interest rate increases, demand for consumption goods by old individuals, especially the consumption spare of old individuals to avoid the risks associated with the production of intermediate goods, increases. By contrast, young individuals significantly increase the supply of intermediate goods and significantly decrease their savings. Consequently, this risk-averse behavior of old individuals (i.e., their large consumption spare) depresses capital accumulation in this economy.

[35] As shown by the difference in the shapes of $\gamma^o(Q)$, the supply curve of intermediate goods $\Psi^S(Q; \lambda)$ is an increasing (a decreasing) function when $\theta = 12$ ($\theta = 0.7$). See Appendix I.

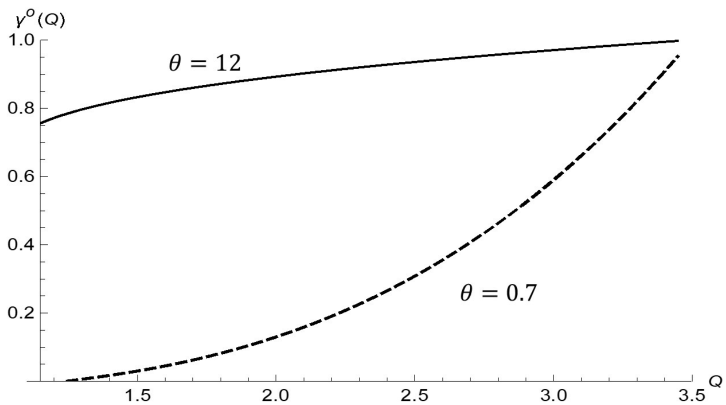

Figure 7: Consumption spare of an old individual

5. Concluding remarks

This study analyzes how increasing longevity affects an economy by considering the risk-averse behavior of old individuals. We construct an overlapping generations model considering that the economy grows based on the capital and intermediate goods produced by individual activities. The outcomes of the production of intermediate goods are stochastically determined. Moreover, older individuals are assumed to be more risk-averse than younger ones. We analytically and numerically show that increasing longevity affects an economy depending on the differences between the risk attitudes of young and old individuals. We show that increasing longevity hinders capital accumulation when old individuals are much more risk-averse than young individuals because they do not engage in risky behaviors. Our results indicate that aging societies can accumulate capital stock if old individuals take more risk.

There are two possible future research directions. Since we focus on self-insurance in this model, we do not consider insurance contracts and annuities. Future studies could consider insurance contracts in an incomplete insurance (annuity) market. This extension of the model might explain the risk-averse behavior of old agents in an aging population more in depth. Thus, considering insurance contracts and risk sharing under incomplete information among individuals in this model is left for future research. The other future research avenue would be to consider private and public annuities, including reforming social security. Such policy analyses may have positive effects on capital accumulation and economic growth.

Acknowledgments

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Appendix A. Derivation of equations (10) and (11) and the condition of the interior solution of γ_{t+1}^{o}

First, we derive equation (10). The first-order condition with respect to γ_{t+1}^{o} (9) can be rearranged as

$$\left(\gamma_{t+1}^{o}+c_{t+1}^{oH}\right)^{-\theta}=\frac{(1-\mu)\left(1-p_{t+1}A^{L}\right)\left(\gamma_{t+1}^{o}+c_{t+1}^{oL}\right)^{-\theta}}{\mu\left(p_{t+1}A^{H}-1\right)}. \tag{9'}$$

Substituting equation (9') into the first-order condition (8), we obtain

$$
\begin{aligned}
\overline{A}p_t &= \beta p_{t+1}Q_{t+1}\left\{\mu A^H\left[\frac{(1-\mu)\left(1-p_{t+1}A^L\right)\left(\gamma^o_{t+1}+c^{oL}_{t+1}\right)^{-\theta}}{\mu\left(p_{t+1}A^H-1\right)}\right]+(1-\mu)A^L\left(\gamma^o_{t+1}+c^{oL}_{t+1}\right)^{-\theta}\right\} \\
&= \frac{\beta(1-\mu)p_{t+1}Q_{t+1}\left(\gamma^o_{t+1}+\gamma^{oL}_{t+1}\right)^{-\theta}}{\left(p_{t+1}A^H-1\right)}\left[A^H\left(1-p_{t+1}A^L\right)+A^L\left(p_{t+1}A^H-1\right)\right] \\
&= \frac{\beta(1-\mu)\left(A^H-A^L\right)p_{t+1}Q_{t+1}\left(\gamma^o_{t+1}+c^{oL}_{t+1}\right)^{-\theta}}{\left(p_{t+1}A^H-1\right)}. \qquad \text{(A-1)}
\end{aligned}
$$

Multiplying both sides of equation (A-1) by $\frac{\left(\gamma^o_{t+1}+c^{oL}_{t+1}\right)^{\theta}}{\overline{A}p_t}$, we obtain

$$
\left(\gamma^o_{t+1}+c^{oL}_{t+1}\right)^{\theta}=\frac{\beta(1-\mu)\left(A^H-A^L\right)p_{t+1}Q_{t+1}}{\overline{A}p_t\left(p_{t+1}A^H-1\right)}>0. \qquad \text{(A-1')}
$$

The last inequality of equation (A-1') is guaranteed by $\beta\in(0,1)$, $\mu\in(0,1)$, $A^L<A^H$, $\overline{A}>0$, and $p_{t+1}A^H-1>0$ from the assumption that $\frac{1}{A^H}<\frac{1}{\overline{A}}<p_{t+1}$. Substituting equation (A-1') into equation (9'), we obtain

$$
\left(\gamma^o_{t+1}+c^{oH}_{t+1}\right)^{\theta}=\frac{\beta\mu\left(A^H-A^L\right)p_{t+1}Q_{t+1}}{\overline{A}p_t\left(1-p_{t+1}A^L\right)}>0. \qquad \text{(A-2)}
$$

The last inequality of equation (A-2) is guaranteed by $\beta\in(0,1)$, $\mu\in(0,1)$, $A^L<A^H$, $\overline{A}>0$, and $1-p_{t+1}A^L>0$ from the assumption that $p_{t+1}<\frac{1}{\overline{\overline{A}}}<\frac{1}{A^L}$.

From equation (6), we obtain

$$
\gamma^o_{t+1}+c^{oi}_{t+1}=\gamma^o_{t+1}+p_{t+1}A^i x^o_{t+1}, \qquad \text{(A-3)}
$$

for $i=\{H,L\}$. Combining equations (A-1') and (A-2) with (A-3) yields

$$
\begin{aligned}
\left(\gamma^o_{t+1}+p_{t+1}A^H x^o_{t+1}\right)-\left(\gamma^o_{t+1}+p_{t+1}A^L x^o_{t+1}\right) &= \left(\gamma^o_{t+1}+c^{oH}_{t+1}\right)-\left(\gamma^o_{t+1}+c^{oL}_{t+1}\right) \\
&= \left[\frac{\beta\mu\left(A^H-A^L\right)p_{t+1}Q_{t+1}}{\overline{A}p_t\left(1-A^L p_{t+1}\right)}\right]^{\frac{1}{\theta}}-\left[\frac{\beta(1-\mu)\left(A^H-A^L\right)p_{t+1}Q_{t+1}}{\overline{A}p_t\left(A^H p_{t+1}-1\right)}\right]^{\frac{1}{\theta}}.
\end{aligned}
\qquad \text{(A-4)}
$$

Equation (A-4) can be rewritten as

$$
p_{t+1}(A^H-A^L)x^o_{t+1}=\left[p_{t+1}(A^H-A^L)\right]^{\frac{1}{\theta}}\left(\frac{\beta Q_{t+1}}{\overline{A}p_t}\right)^{\frac{1}{\theta}}\left[\left(\frac{\mu}{1-A^L p_{t+1}}\right)^{\frac{1}{\theta}}-\left(\frac{1-\mu}{A^H p_{t+1}-1}\right)^{\frac{1}{\theta}}\right]
$$

$$\rightarrow x^o_{t+1} = \left[p_{t+1}(A^H - A^L)\right]^{\frac{1-\theta}{\theta}} \left(\frac{\beta Q_{t+1}}{\overline{A}p_t}\right)^{\frac{1}{\theta}} \left[\frac{\mu^{\frac{1}{\theta}}\left(A^H p_{t+1}-1\right)^{\frac{1}{\theta}} - (1-\mu)^{\frac{1}{\theta}}\left(1-A^L p_{t+1}\right)^{\frac{1}{\theta}}}{\left(A^H p_{t+1}-1\right)^{\frac{1}{\theta}}\left(1-A^L p_{t+1}\right)^{\frac{1}{\theta}}}\right] > 0. \tag{A-4'}$$

The last inequality of equation (A-4') is guaranteed by the assumption that $\overline{A}p_{t+1} > 1$.

Combining equations (A-2) and (A-3) gives us

$$\gamma^o_{t+1} = -p_{t+1}A^H x^o_{t+1} + \left[\frac{\beta\mu\left(A^H - A^L\right)p_{t+1}Q_{t+1}}{\overline{A}p_t\left(1-A^L p_{t+1}\right)}\right]^{\frac{1}{\theta}}. \tag{A-5}$$

Substituting equation (A-4') into (A-5), we obtain

$$\begin{aligned}
\gamma^o_{t+1} &= -A^H(A^H - A^L)^{\frac{1-\theta}{\theta}} \left(\frac{\beta p_{t+1}Q_{t+1}}{\overline{A}p_t}\right)^{\frac{1}{\theta}} \left[\left(\frac{\mu}{1-A^L p_{t+1}}\right)^{\frac{1}{\theta}} - \left(\frac{1-\mu}{A^H p_{t+1}-1}\right)^{\frac{1}{\theta}}\right] \\
&\quad + \left[\frac{\beta\mu\left(A^H - A^L\right)p_{t+1}Q_{t+1}}{\overline{A}p_t\left(1-A^L p_{t+1}\right)}\right]^{\frac{1}{\theta}} \\
&= \left(A^H - A^L\right)^{\frac{1-\theta}{\theta}} \left(\frac{\beta p_{t+1}Q_{t+1}}{\overline{A}p_t}\right)^{\frac{1}{\theta}} \left[\frac{(1-\mu)^{\frac{1}{\theta}}A^H\left(1-A^L p_{t+1}\right)^{\frac{1}{\theta}} - \mu^{\frac{1}{\theta}}A^L\left(A^H p_{t+1}-1\right)^{\frac{1}{\theta}}}{\left(A^H p_{t+1}-1\right)^{\frac{1}{\theta}}\left(1-A^L p_{t+1}\right)^{\frac{1}{\theta}}}\right].
\end{aligned} \tag{A-6}$$

The above equation can be rewritten as

$$\gamma^o_{t+1} = \left(A^H - A^L\right)^{\frac{1-\theta}{\theta}} \left(\frac{\beta p_{t+1}Q_{t+1}}{\overline{A}p_t}\right)^{\frac{1}{\theta}} \left[(1-\mu)^{\frac{1}{\theta}}A^H\left(A^H p_{t+1}-1\right)^{\frac{-1}{\theta}} - \mu^{\frac{1}{\theta}}A^L\left(1-A^L p_{t+1}\right)^{\frac{-1}{\theta}}\right]. \tag{10}$$

Now, we show that $\gamma^o_{t+1} > 0$ with equation (10) when $\frac{1}{\overline{A}} < p_{t+1} < \frac{1}{\overline{\overline{A}}}$ with $A^H > A^L$. We define $\overline{\overline{A}}$ as $\overline{\overline{A}} \equiv \frac{\mu A^H(A^L)^\theta + (1-\mu)A^L(A^H)^\theta}{\mu(A^L)^\theta + (1-\mu)(A^H)^\theta}$. Suppose that $p_{t+1} < \frac{1}{\overline{\overline{A}}}$. Then, we obtain

$$\begin{aligned}
&\mu A^H\left(A^L\right)^\theta p_{t+1} + (1-\mu)A^L(A^H)^\theta p_{t+1} < \mu(A^L)^\theta + (1-\mu)(A^H)^\theta \\
&\rightarrow (1-\mu)\left(A^H\right)^\theta\left(1-A^L P_{t+1}\right) > \mu\left(A^L\right)^\theta\left(A^H p_{t+1}-1\right) \\
&\rightarrow (1-\mu)^{\frac{1}{\theta}}A^H\left(A^H p_{t+1}-1\right)^{\frac{-1}{\theta}} - \mu^{\frac{1}{\theta}}A^L\left(1-A^L p_{t+1}\right)^{\frac{-1}{\theta}} > 0.
\end{aligned} \tag{A-7}$$

The last inequality of condition (A-7) shows that the right-hand side of equation (10) is positive and thus $\gamma^o_{t+1} > 0$, where $p_{t+1} < \frac{1}{\overline{\overline{A}}}$.

Finally, we derive s_t in equation (11). Substituting equations (A-4') and (A-6) into equation (6), we obtain

$$\left\{ \left(A^H - A^L\right)^{\frac{1-\theta}{\theta}} \left(\frac{\beta p_{t+1} Q_{t+1}}{\overline{A} p_t}\right)^{\frac{1}{\theta}} \left[\frac{(1-\mu)^{\frac{1}{\theta}} A^H \left(1 - A^L p_{t+1}\right)^{\frac{1}{\theta}} - \mu^{\frac{1}{\theta}} A^L \left(A^H p_{t+1} - 1\right)^{\frac{1}{\theta}}}{\left(A^H p_{t+1} - 1\right)^{\frac{1}{\theta}} \left(1 - A^L p_{t+1}\right)^{\frac{1}{\theta}}}\right] \right\}$$
$$+ \left\{ \left[p_{t+1}(A^H - A^L)\right]^{\frac{1-\theta}{\theta}} \left(\frac{\beta Q_{t+1}}{\overline{A} p_t}\right)^{\frac{1}{\theta}} \left[\frac{\mu^{\frac{1}{\theta}} \left(A^H p_{t+1} - 1\right)^{\frac{1}{\theta}} - (1-\mu)^{\frac{1}{\theta}} \left(1 - A^L p_{t+1}\right)^{\frac{1}{\theta}}}{\left(A^H p_{t+1} - 1\right)^{\frac{1}{\theta}} \left(1 - A^L p_{t+1}\right)^{\frac{1}{\theta}}}\right] \right\} = \frac{Q_{t+1}}{\lambda} s_t.$$

The above equation can be rewritten as

$$s_t \frac{Q_{t+1}}{\lambda} = \left[p_{t+1}(A^H - A^L)\right]^{\frac{1-\theta}{\theta}} \left(\frac{\beta Q_{t+1}}{\overline{A} p_t \left(A^H p_{t+1} - 1\right)\left(1 - A^L p_{t+1}\right)}\right)^{\frac{1}{\theta}}$$
$$\times \left[\left(A^H p_{t+1} - 1\right)(1-\mu)^{\frac{1}{\theta}} \left(1 - A^L p_{t+1}\right)^{\frac{1}{\theta}} + \left(1 - p_{t+1} A^L\right) \mu^{\frac{1}{\theta}} \left(A^H p_{t+1} - 1\right)^{\frac{1}{\theta}}\right]$$
$$\rightarrow s_t = \lambda \left[(A^H - A^L) p_{t+1} Q_{t+1}\right]^{\frac{1-\theta}{\theta}} \left(\frac{\beta}{\overline{A} p_t}\right)^{\frac{1}{\theta}} \left[(1-\mu)^{\frac{1}{\theta}} \left(A^H p_{t+1} - 1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}} \left(1 - p_{t+1} A^L\right)^{\frac{\theta-1}{\theta}}\right]. \tag{11}$$

Appendix B. Derivation of equation (16) and the properties of $p(Q_t)$

First, we derive equation (16). From equation (1) and the definition of k_t, we obtain

$$\frac{Y_t}{X_t} = \chi \left(\frac{K_t}{L_t}\right)^{\alpha} \left(\frac{X_t}{L_t}\right)^{-\alpha} = \chi k_t^{\alpha} \left(\frac{X_t}{L_t}\right)^{-\alpha}. \tag{B-1}$$

From equations (1) and (2) and the definition of k_t, we obtain

$$q_t = \alpha\chi \left(\frac{K_t}{L_t}\right)^{-(1-\alpha)} \left(\frac{X_t}{L_t}\right)^{1-\alpha} = \alpha\chi k_t^{-(1-\alpha)} \left(\frac{X_t}{L_t}\right)^{1-\alpha}.$$

The above equation can be rewritten as

$$\frac{X_t}{L_t} = \left(\frac{q_t}{\alpha\chi}\right)^{\frac{1}{1-\alpha}} k_t. \tag{B-2}$$

Plugging equations (B-1) and (B-2) into equation (3), we obtain

$$p_t = (1-\alpha)\chi k_t^{\alpha} \left(\frac{q_t}{\alpha\chi}\right)^{\frac{-\alpha}{1-\alpha}} k_t^{-\alpha} = \chi^{\frac{1}{1-\alpha}} (1-\alpha) (\alpha)^{\frac{\alpha}{1-\alpha}} (Q_t - 1)^{-\frac{\alpha}{1-\alpha}}. \tag{16}$$

The second equality of the above equation is given by the definition of Q_t.

Next, we examine the relationship between the intermediate goods price p_t and gross interest rate Q_t. Let $p(Q_t)$ be the right-hand side of equation (16) such that $p(Q_t) \equiv \chi^{\frac{1}{1-\alpha}} (1-\alpha)(\alpha)^{\frac{\alpha}{1-\alpha}} (Q_t - 1)^{-\frac{\alpha}{1-\alpha}}$, where $Q_t > 1$. Then, differentiating $p(Q_t)$ with respect to Q_t yields $p'(Q_t) = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p(Q_t)}{Q_t-1}\right) < 0$. Using this and differentiating $p'(Q_t)$ with respect to Q_t yields $p''(Q_t) = \left(\frac{\alpha}{(1-\alpha)^2}\right)\left(\frac{p(Q)}{(Q-1)^2}\right) > 0$. Thus, the intermediate goods price function $p(Q_t)$ is a decreasing and convex function of the gross interest rate Q_t.

Appendix C. Derivations of equations (17) and (18)

We first derive equation (17) based on the intermediate goods and capital market-clearing conditions (12) and (15). Dividing both sides of equation (12) by L_t gives us the intermediate goods market-clearing condition per young individual as follows:

$$\overline{A} x_t^y + \overline{A} \frac{\lambda}{1+n} x_t^o = \frac{X_t}{L_t}, \tag{C-1}$$

where $\frac{L_{t-1}}{L_t} = \frac{1}{1+n}$.

Substituting the resource constraints $x_t^y = \overline{w} - s_t$ and $x_t^o = \frac{Q_t}{\lambda} s_{t-1} - \gamma_t^o$ into (C-1) yields

$$\overline{A}\left(\overline{w} - s_t\right) + \overline{A} \frac{\lambda}{1+n}\left(\frac{Q_t}{\lambda} s_{t-1} - \gamma_t^o\right) = \frac{X_t}{L_t}. \tag{C-2}$$

Using equation (B-2) and the definition of Q_t, equation (C-2) can be rewritten as

$$\overline{A}\left(\overline{w} - s_t\right) + \overline{A} \frac{\lambda}{1+n}\left(\frac{Q_t}{\lambda} s_{t-1} - \gamma_t^o\right) = \left(\frac{Q_t - 1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}} k_t. \tag{C-3}$$

The right-hand side of equation (C-3) represents demand for intermediate goods per young individual.

Next, we rewrite equation (C-3) with the variables in periods t and $t+1$, not including the variables in period $t-1$. We first divide both sides of equation (C-3) by s_{t-1}:

$$\overline{A}\left(\frac{\overline{w}}{s_{t-1}} - \frac{s_t}{s_{t-1}}\right) + \overline{A} \frac{\lambda}{1+n}\left(\frac{Q_t}{\lambda} - \frac{\gamma_t^o}{s_{t-1}}\right) = \left(\frac{Q_t - 1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}} \frac{k_t}{s_{t-1}}. \tag{C-4}$$

From individuals' optimizing conditions (10), (11), and (16), we obtain

$$\frac{\lambda\gamma_t^o}{s_{t-1}} = p(Q_t)Q_t\left[\frac{(1-\mu)^{\frac{1}{\theta}}A^H\left(A^Hp(Q_t)-1\right)^{\frac{-1}{\theta}} - \mu^{\frac{1}{\theta}}A^L\left(1-A^Lp(Q_t)\right)^{\frac{-1}{\theta}}}{(1-\mu)^{\frac{1}{\theta}}\left(A^Hp(Q_t)-1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}}\left(1-A^Lp(Q_t)\right)^{\frac{\theta-1}{\theta}}}\right]. \tag{C-5}$$

Let $\Gamma(Q_t)$ be the right-hand side of equation (C-5):

$$\Gamma(Q_t) \equiv p(Q_t)Q_t\left[\frac{(1-\mu)^{\frac{1}{\theta}}A^H\left(A^Hp(Q_t)-1\right)^{\frac{-1}{\theta}} - \mu^{\frac{1}{\theta}}A^L\left(1-A^Lp(Q_t)\right)^{\frac{-1}{\theta}}}{(1-\mu)^{\frac{1}{\theta}}\left(A^Hp(Q_t)-1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}}\left(1-A^Lp(Q_t)\right)^{\frac{\theta-1}{\theta}}}\right].$$

Substituting $\frac{\lambda\gamma_t^o}{s_{t-1}} = \Gamma(Q_t)$ into (C-4) yields

$$\overline{A}\left(\frac{\overline{w}}{s_{t-1}} - \frac{s_t}{s_{t-1}}\right) + \overline{A}\frac{Q_t}{1+n} - \overline{A}\frac{\Gamma(Q_t)}{1+n} = \left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\frac{k_t}{s_{t-1}}. \tag{C-4'}$$

From the capital market equilibrium condition (15), $k_{t+1}(1+n) = s_t$. Combining it with equation (C-4'), we obtain

$$\overline{A}\left(\frac{\overline{w}}{(1+n)k_t} - \frac{k_{t+1}}{k_t}\right) + \overline{A}\frac{Q_t}{1+n} - \overline{A}\frac{\Gamma(Q_t)}{1+n} = \left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}(1+n)^{-1}. \tag{C-6}$$

Multiplying both sides of equation (C-6) by $\frac{k_t}{\overline{A}}$ yields

$$\frac{\overline{w}}{(1+n)} - k_{t+1} + \frac{Q_tk_t}{1+n} - \frac{\Gamma(Q_t)k_t}{1+n} = \left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\frac{k_t}{\overline{A}(1+n)}. \tag{C-7}$$

Equation (C-7) can be rewritten as

$$k_{t+1} = \frac{\overline{w}}{(1+n)} + \frac{Q_tk_t}{1+n} - \frac{\Gamma(Q_t)k_t}{1+n} - \left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\frac{k_t}{\overline{A}(1+n)}.$$

From this, we obtain equation (17):

$$k_{t+1} = \frac{\overline{w}}{1+n} + \frac{k_t}{1+n}\left[(Q_t - \Gamma(Q_t)) - \frac{1}{\overline{A}}\left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right]. \tag{17}$$

Next, we derive equation (18) from equation (17) and saving function (11) with equation (16). From the capital market equilibrium condition (15) and (17), we obtain

$$s_t = \overline{w} + k_t\left[(Q_t - \Gamma(Q_t)) - \frac{1}{\overline{A}}\left(\frac{Q_t-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right]. \tag{C-8}$$

Combining equations (11) and (16) yields

$$s_t = \lambda \left[(A^H - A^L)p\left(Q_{t+1}\right)Q_{t+1}\right]^{\frac{1-\theta}{\theta}} \left(\frac{\beta}{\overline{A}p(Q_t)}\right)^{\frac{1}{\theta}} \left[(1-\mu)^{\frac{1}{\theta}} \left(A^H p\left(Q_{t+1}\right) - 1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}} \left(1 - A^L p\left(Q_{t+1}\right)\right)^{\frac{\theta-1}{\theta}}\right] \tag{C-9}$$

Because the right-hand side of equation (C-8) equals that of (C-9), we obtain

$$\begin{aligned} &\overline{w} + k_t \left[(Q_t - \Gamma(Q_t)) - \frac{1}{\overline{A}}\left(\frac{Q_t - 1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right] \\ &= \lambda \left[(A^H - A^L)p\left(Q_{t+1}\right)Q_{t+1}\right]^{\frac{1-\theta}{\theta}} \left(\frac{\beta}{\overline{A}p(Q_t)}\right)^{\frac{1}{\theta}} \left[(1-\mu)^{\frac{1}{\theta}} \left(A^H p\left(Q_{t+1}\right) - 1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}} \left(1 - A^L p\left(Q_{t+1}\right)\right)^{\frac{\theta-1}{\theta}}\right]. \end{aligned}$$

From this, we obtain

$$\begin{aligned} &\lambda \left[(A^H - A^L)p\left(Q_{t+1}\right)Q_{t+1}\right]^{\frac{1-\theta}{\theta}} \left[(1-\mu)^{\frac{1}{\theta}} \left(A^H p\left(Q_{t+1}\right) - 1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}} \left(1 - A^L p\left(Q_{t+1}\right)\right)^{\frac{\theta-1}{\theta}}\right] \\ &= \left(\frac{\beta}{\overline{A}p(Q_t)}\right)^{\frac{-1}{\theta}} \left\{\overline{w} + k_t \left[(Q_t - \Gamma(Q_t)) - \frac{1}{\overline{A}}\left(\frac{Q_t - 1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right]\right\}. \end{aligned} \tag{18}$$

Appendix D. Derivations of equations (22) and (23)

Since we derive equations (22) and (23) from the intermediate goods and capital market-clearing conditions, we evaluate the two conditions at the steady state. Using the definition of $\tilde{s}(Q)$, for the capital market, evaluating equation (15) in the steady state, we obtain

$$k = \frac{\lambda \tilde{s}(Q)}{1+n}. \tag{23}$$

We next derive equation (22). As Appendix C shows, equation (17) can be rewritten as equation (C-3), which comes from the intermediate goods market-clearing condition. Evaluating equation (C-3) at the steady state, we obtain

$$\overline{A}\left(\overline{w} - s\right) + \overline{A}\frac{\lambda}{1+n}\left(\frac{Q}{\lambda}s - \gamma^o\right) = \left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}} k. \tag{D-1}$$

Combining equations (D-1) and (23) with the definitions of $\tilde{s}(Q)$ and $\gamma^o(Q)$, we obtain

$$\overline{A}\left(\overline{w} - \lambda\tilde{s}(Q)\right) + \overline{A}\frac{\lambda}{1+n}\left(Q\tilde{s}(Q) - \gamma^o(Q)\right) = \frac{\lambda\tilde{s}(Q)}{1+n}\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}. \tag{22}$$

As Appendix C shows, equation (C-8) is derived from equations (15) and (17) and equation (18) can be rewritten as equation (C-8). Thus, when equations (22) and (23) hold at the steady state, equations (17) and (18) hold.

Appendix E. Proof of Lemma 1

PROOF OF LEMMA 1. Let $\Gamma_o(Q)$ and $\Gamma_y(Q)$ define the following:

$$\Gamma_o(Q) \equiv (1-\mu)^{\frac{1}{\theta}} A^H \left(A^H p(Q) - 1\right)^{\frac{-1}{\theta}} - \mu^{\frac{1}{\theta}} A^L \left(1 - A^L p(Q)\right)^{\frac{-1}{\theta}};$$

$$\Gamma_y(Q) \equiv (1-\mu)^{\frac{1}{\theta}} \left(A^H p(Q) - 1\right)^{\frac{\theta-1}{\theta}} + \mu^{\frac{1}{\theta}} \left(1 - A^L p(Q)\right)^{\frac{\theta-1}{\theta}}.$$

Then, equations (20) and (21) can be rewritten as

$$\gamma^o(Q) = \left(A^H - A^L\right)^{\frac{1-\theta}{\theta}} \left(\frac{\beta Q}{\overline{A}}\right)^{\frac{1}{\theta}} \Gamma_o(Q); \tag{E-1}$$

$$\lambda \tilde{s}(Q) = \lambda \left[(A^H - A^L)Q\right]^{\frac{1-\theta}{\theta}} (p(Q))^{-1} \left(\frac{\beta}{\overline{A}}\right)^{\frac{1}{\theta}} \Gamma_y(Q). \tag{E-2}$$

Differentiating $\tilde{s}(Q)$ with respect to Q, we obtain

$$\frac{d\tilde{s}(Q)}{dQ} = \left[\frac{1-\theta}{\theta Q} + \frac{\frac{1-\theta}{\theta}\left(-p'(Q)\right)\Gamma_o(Q)}{\Gamma_y(Q)} + \frac{-p'(Q)}{p(Q)}\right]\tilde{s}(Q). \tag{E-3}$$

Here, we use $\Gamma_y'(Q) = \frac{1-\theta}{\theta}\left(-p'(Q)\right)\Gamma_o(Q)$.

First, we consider an economy in which $0 < \theta \leq 1$. Because $p'(Q) < 0$ from Appendix B, if $0 < \theta \leq 1$, we obtain

$$\frac{d\lambda\tilde{s}(Q)}{dQ} > 0. \tag{E-4}$$

This shows Lemma 1 (b).

Next, we show Lemma 1 (a). Consider an economy in which $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ with Assumptions 1 and 3. Then, using $p'(Q) = -\left(\frac{\alpha}{1-\alpha}\right)\frac{p(Q)}{Q-1} < 0$, equation (E-3) can be rewritten as

$$\frac{d\tilde{s}(Q)}{dQ} = \left\{ \underbrace{-\frac{\theta-1}{\theta}(-p'(Q))\Gamma_o(Q)Q}_{<0} - \frac{\Gamma_y(Q)}{Q-1}\left[\left(\frac{\theta-1}{\theta} - \frac{\alpha}{1-\alpha}\right)Q - \frac{\theta-1}{\theta}\right]\right\}\tilde{s}(Q). \tag{E-5}$$

$\left(\frac{\theta-1}{\theta}-\frac{\alpha}{1-\alpha}\right)>0$ because $\theta>\frac{1-\alpha}{1-2\alpha}$ and Assumption 1. From Assumption 3, $\left(\frac{\theta-1}{\theta}-\frac{\alpha}{1-\alpha}\right)Q-\frac{\theta-1}{\theta}>0$ because we consider the economy in which $\overline{\overline{Q}}<Q$, which means that $\gamma^o>0$. Hence, we obtain

$$\frac{d\lambda\tilde{s}(Q)}{dQ}<0, \tag{E-6}$$

when $\theta>\frac{1-\alpha}{1-2\alpha}>1$. This shows Lemma 1 (a).

Appendix F. Proof of Lemma 2

Proof of Lemma 2. Differentiating equation (20) with respect to Q, we obtain

$$\frac{d\gamma^o}{dQ}=\frac{\gamma^o(Q)}{\theta Q}$$
$$+\frac{-p'(Q)}{\theta}\left(A^H-A^L\right)^{\frac{1-\theta}{\theta}}\left(\frac{\beta Q}{\overline{A}}\right)^{\frac{1}{\theta}}\left[(1-\mu)^{\frac{1}{\theta}}\left(A^H\right)^2\left(A^Hp(Q)-1\right)^{\frac{-1-\theta}{\theta}}+\mu^{\frac{1}{\theta}}\left(A^L\right)^2\left(1-A^Lp(Q)\right)^{\frac{-1-\theta}{\theta}}\right]>0. \tag{F-1}$$

The last inequality follows from $p'(Q)<0$ in equation (19) (see Appendix B).

Appendix G. Lemma 4

We define ϵ_{sQ} and $\epsilon_{Q\lambda}$ as the elasticity of savings when young with respect to the gross interest rate and that of the gross interest rate to longevity, that is, $\epsilon_{sQ}\equiv\frac{\frac{-d\tilde{s}}{dQ}}{\frac{\tilde{s}}{Q}}$ and $\epsilon_{Q\lambda}\equiv\frac{\frac{dQ}{d\lambda}}{\frac{Q}{\lambda}}$. Then, equation (24) gives the following lemma.

Lemma 4.
1. *Suppose that $\epsilon_{sQ}\epsilon_{Q\lambda}>1$. Then, increasing longevity decreases capital stock per young individual: $\frac{dk}{d\lambda}<0$.*
2. *Suppose that $\epsilon_{sQ}\epsilon_{Q\lambda}<1$. Then, increasing longevity increases capital stock per young individual: $\frac{dk}{d\lambda}>0$.*

Proof of Lemma 4. We can rewrite equation (24) as follows:

$$\frac{dk}{d\lambda}=\frac{1}{1+n}\left[\tilde{s}(Q)-\lambda\left(\frac{-d\tilde{s}(Q)}{dQ}\right)\frac{dQ}{d\lambda}\right]=\frac{k}{\lambda}\left(1-\epsilon_{sQ}\epsilon_{Q\lambda}\right). \tag{G-1}$$

From this equation, we obtain (i) and (ii).

Lemma 4 (i) indicates the conditions when increasing longevity decreases capital stock per young individual.

Appendix H. Condition of $\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} > 0$

From the definition of $\Psi^D(Q;\lambda)$ in equation (22), differentiating $\Psi^D(Q;\lambda)$ with respect to Q, we obtain

$$\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} = \frac{\Psi^D(Q;\lambda)}{\tilde{s}(Q)}\frac{d\tilde{s}(Q)}{dQ} + \frac{\Psi^D(Q;\lambda)}{(Q-1)(1-\alpha)} = \frac{\Psi^D(Q;\lambda)}{Q}\left[\frac{Q}{(Q-1)(1-\alpha)} - \epsilon_{sQ}\right]. \tag{H-1}$$

From Lemma 1 (b), when $0 < \theta \leq 1$, $\epsilon_{sQ} < 0$. Thus, when $0 < \theta \leq 1$, $\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} > 0$.

We next consider an economy when $\theta > \frac{1-\alpha}{1-2\alpha}$ under Assumptions 1 and 3. From Lemma 1 (a) under Assumptions 1 and 3, $\epsilon_{sQ} > 0$. We next show $\epsilon_{sQ} < 1$. From the definition of ϵ_{sQ} and equations (E-1), (E-2), and (E-3), we obtain

$$\begin{aligned}
\epsilon_{sQ} - 1 &= \frac{\frac{-d\tilde{s}(Q)}{dQ}}{\frac{\tilde{s}(Q)}{Q}} - 1 \\
&= -\frac{1}{\theta} + \frac{(-p'(Q))\,Q}{\theta p(Q)}\left[\frac{(\theta-1)\Gamma_o(Q)p(Q)}{\Gamma_y(Q)} - \theta\right] \\
&= -\frac{1}{\theta} - \frac{(-p'(Q))\,Q}{\theta p(Q)}\left[\frac{-(\theta-1)\gamma^o(Q) + \theta Q\tilde{s}(Q)}{Q\tilde{s}(Q)}\right] \\
&= -\frac{1}{\theta} - \frac{(-p'(Q))\,Q}{\theta p(Q)}\left[\frac{\gamma^o(Q) + \theta\left(Q\tilde{s}(Q) - \gamma^o(Q)\right)}{Q\tilde{s}(Q)}\right] < 0.
\end{aligned} \tag{H-2}$$

The last inequality of (H-2) holds because $p'(Q) < 0$ (see Appendix B) and $x^o = Q\tilde{s}(Q) - \gamma^o(Q) > 0$ from equations (6), (21), and (A-4') (see Appendix A). Thus, $\epsilon_{sQ} < 1$ for $\overline{\overline{Q}} < Q < \overline{Q}$ from equation (H-2). This is summarized in the following lemma.

Lemma 5. *Let ϵ_{sQ} define that $\epsilon_{sQ} \equiv \frac{\frac{-d\tilde{s}(Q)}{dQ}}{\frac{\tilde{s}(Q)}{Q}}$. Then, from Assumptions 1 and 3 and Lemma 1, savings when young are less elastic with respect to the gross interest rate; that is, $\epsilon_{sQ} < 1$ for $\overline{\overline{Q}} < Q < \overline{Q}$.*

From equation (H-1), $\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} > 0$ is equivalent to $\frac{Q}{(1-\alpha)(Q-1)} > \epsilon_{sQ}$. Then, from equation (H-1) and Lemma 5, $\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} > 0$ holds because $\frac{Q}{(1-\alpha)(Q-1)} > 1 > \epsilon_{sQ}$ for all $Q > 1$.

Appendix I. Properties of the supply of intermediate goods $\boldsymbol{\Psi^S(Q;\lambda)}$

From $\Psi^S(Q;\lambda)$ in equation (22), differentiating $\Psi^S(Q;\lambda)$ with respect to Q, we obtain

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} = \frac{\overline{A}\lambda}{1+n}\left\{\underbrace{\tilde{s}(Q) + [Q-(1+n)]\frac{d\tilde{s}(Q)}{dQ}}_{(*)} - \frac{d\gamma^o(Q)}{dQ}\right\}. \tag{I-1}$$

We first examine the sign of the term $(*)$. When $\theta > \frac{1-\alpha}{1-2\alpha}$, the term $(*)$ takes a positive value for all $Q \in (\overline{\overline{Q}}, \overline{Q})$. This term can be rewritten as follows: $\frac{\tilde{s}(Q)}{Q}\left[Q(1-\epsilon_{sQ}) + (1+n)\epsilon_{sQ}\right]$, where $\epsilon_{sQ} \equiv \frac{\frac{-d\tilde{s}(Q)}{dQ}}{\frac{\tilde{s}(Q)}{Q}}$. Because $0 < \epsilon_{sQ} < 1$ from Lemma 1 (a) (see (E-6)) and Lemma 5 (see Appendix H), the term $(*)$ takes a positive value. When $0 < \theta \leq 1$ and $Q-(1+n) > 0$, the term $(*)$ is positive because $\frac{d\tilde{s}(Q)}{dQ} > 0$ from Lemma 1 (b) (see (E-4)). On the contrary, when $0 < \theta \leq 1$ and $Q-(1+n) \leq 0$, the term $(*)$ can be negative. When the term $(*)$ is negative, from (I-1) and $\frac{d\gamma^o(Q)}{dQ} > 0$ from Lemma 2, $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} < 0$.

When the term $(*)$ takes a positive value, equation (I-1) implies that

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} \gtreqless 0 \leftrightarrow \tilde{s}(Q) + [Q-(1+n)]\frac{d\tilde{s}(Q)}{dQ} \gtreqless \frac{d\gamma^o(Q)}{dQ}. \tag{I-2}$$

In this case, the right-hand side of (I-2) is also positive since $\frac{d\gamma^o(Q)}{dQ} > 0$ from Lemma 2. From (I-2), $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q}$ is positive (negative) when $\frac{d\gamma^o(Q)}{dQ}$ is small (large). From equation (F-1) in Appendix F, $\frac{d\gamma^o(Q)}{dQ}$ decreases as θ increases. Thus, $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q}$ becomes larger (smaller) than 0 as θ increases (decreases).

Appendix J. Proof of Lemma 3

Proof of Lemma 3. From the definition of $\Psi^S(Q;\lambda)$ in equation (22), differentiating $\Psi^S(Q;\lambda)$ with respect to λ, we obtain

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} = \frac{\overline{A}}{1+n}\left\{[Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q)\right\}. \tag{J-1}$$

From equation (J-1), we obtain

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} < 0 \leftrightarrow [Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q) < 0. \tag{J-2}$$

When $Q-(1+n) \leq 0$, inequality (J-2) holds for $\overline{\overline{Q}} < Q < \overline{Q}$. We next consider an economy when $Q-(1+n) > 0$. We first show that $\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} < 0$ holds when $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ and $Q-(1+n) > 0$ from Assumptions 1–4. Using equations (E-1) and (E-2) and the definitions $\Gamma_o(Q)$ and $\Gamma_y(Q)$ in Appendix E, we obtain $Qp(Q)\frac{\Gamma_o(Q)}{\Gamma_y(Q)} = \frac{\gamma^o(Q)}{\tilde{s}(Q)}$. Using this equation, condition (J-2) can be rewritten as follows:

$$Qp(Q)\frac{\Gamma_o(Q)}{\Gamma_y(Q)} > Q-(1+n). \tag{J-3}$$

From equation (E-3) and using $p'(Q) = -\left(\frac{\alpha}{1-\alpha}\right)\frac{p(Q)}{Q-1} < 0$, $\frac{d\tilde{s}(Q)}{dQ} < 0$ leads to

$$-\frac{\theta-1}{\theta Q} - \frac{\frac{\theta-1}{\theta}\left(\frac{\alpha}{1-\alpha}\frac{p(Q)}{Q-1}\right)\Gamma_o(Q)}{\Gamma_y(Q)} + \frac{\alpha}{1-\alpha}\frac{1}{Q-1} < 0.$$

This condition can be rewritten as

$$Qp(Q)\frac{\Gamma_o(Q)}{\Gamma_y(Q)} > \left(\frac{\theta}{\theta-1} - \frac{1-\alpha}{\alpha}\right)Q + \frac{1-\alpha}{\alpha}. \tag{J-4}$$

From Lemma 1 (a), (J-4) holds when $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ with Assumptions 1 and 3.

From Assumption 4 and the definition of $\hat{\theta}$,

$$\frac{\overline{Q}-1-\alpha n}{\overline{Q}-1-\alpha n-\alpha\overline{\overline{Q}}} \geq \theta.$$

Here, Assumptions 1 and 2 guarantee $\hat{\theta} > \frac{1-\alpha}{1-2\alpha} > 1$ (see footnote 21). This leads to the following:

$$\begin{aligned} \overline{Q}-1-\alpha n &\geq \theta\left[\overline{Q}-1-\alpha n-\alpha\overline{Q}\right] \\ \rightarrow \frac{(\theta-1)(1+\alpha n)}{(\theta-1)-\alpha\theta} &\geq \overline{Q}. \end{aligned} \tag{J-5}$$

Since we consider an economy in which $Q < \overline{Q}$, from (J-5), we obtain

$$\frac{(\theta-1)(1+\alpha n)}{(\theta-1)-\alpha\theta} > Q. \tag{J-6}$$

Subtracting the right-hand side of (J-3) from the right-hand side of (J-4), we obtain

$$Q-(1+n)-\left(\frac{\theta}{\theta-1} - \frac{1-\alpha}{\alpha}\right)Q - \frac{1-\alpha}{\alpha} = [(\theta-1)-\alpha\theta]Q-(\theta-1)(1+\alpha n) < 0. \tag{J-7}$$

Inequality (J-7) holds from (J-6). Since conditions (J-4) and (J-7) hold, conditions (J-3) and thus (J-2) hold when $\theta > \frac{1-\alpha}{1-2\alpha}$ with Assumptions 1–4. Thus, we obtain $\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} < 0$ for $\overline{\overline{Q}} < Q < \overline{Q}$ when $\theta > \frac{1-\alpha}{1-2\alpha}$ with Assumptions 1–4. This shows Lemma 3 (a).

We next show that $\frac{\partial \Psi^S(Q;\lambda)}{\partial \lambda} < 0$ holds when $0 < \theta \leq 1$ and $Q - (1+n) > 0$ with Assumption 6. Differentiating the left-hand side of (J-2) with respect to Q, we obtain

$$\frac{d\left\{[Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q)\right\}}{dQ} = \tilde{s}(Q) + [Q-(1+n)]\,\frac{d\tilde{s}(Q)}{dQ} - \frac{d\gamma^o(Q)}{dQ} < 0. \tag{J-8}$$

The inequality of (J-8) holds because the signs of $\frac{d\{[Q-(1+n)]\tilde{s}(Q)-\gamma^o(Q)\}}{dQ}$ correspond to the signs of $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q}$ from (I-2) and (J-8). As shown in Appendix I, $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} < 0$ as θ becomes small. Thus, $[Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q)$ is a decreasing function of Q when $0 < \theta \leq 1$ from equation (J-8). When $0 < \theta \leq 1$ and $Q-(1+n) > 0$, $\lim_{Q \to \overline{\overline{Q}}} [Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q) = \left[\overline{\overline{Q}} - (1+n)\right] \tilde{s}(\overline{\overline{Q}}) > 0$ because $\gamma^o(\overline{\overline{Q}}) = 0$ and $\lim_{Q \to \overline{Q}} [Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q) = \left[\overline{Q} - (1+n)\right] \tilde{s}(\overline{Q}) - \gamma^o(\overline{Q}) < 0$ with Assumption 6. From these boundary conditions and (J-8), there exists $\hat{Q}$ for $\overline{\overline{Q}} < \hat{Q} < \overline{Q}$ such that

$$[\hat{Q} - (1+n)]\tilde{s}(\hat{Q}) - \gamma^o(\hat{Q}) = 0, \tag{J-9}$$

and for $\hat{Q} < Q < \overline{Q}$, inequality (J-2) holds. This shows Lemma 3 (b).

Appendix K. Proof of Proposition 1

Proof of Proposition 1. We show that $\frac{dk}{d\lambda} < 0$ when $\theta > \frac{1-\alpha}{1-2\alpha}$ and $\overline{\overline{Q}} < Q < \overline{Q}$ with Assumptions 1–5 and 7.

Using equation (23), because the left-hand side of equation (22) is $\Psi^S(Q;\lambda)$ and the right-hand side of equation (22) is $\Psi^D(Q;\lambda)$, we can rewrite each side of equation (22) as follows:

$$\Psi^S(Q;\lambda) = \overline{A}\overline{w} - \overline{A}(1+n)k + \overline{A}Qk - \frac{\overline{A}\lambda}{1+n}\gamma^o(Q), \tag{K-1}$$

and

$$\Psi^D(Q;\lambda) = k\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}. \tag{K-2}$$

Given $\overline{A}$, $\overline{w}$, and n, totally differentiating equation (K-1) yields

$$\frac{d\Psi^S(Q;\lambda)}{d\lambda} = \overline{A}\left[Q-(1+n)\right]\frac{dk}{d\lambda} + \frac{\overline{A}\lambda}{1+n}\left[\frac{(1+n)k}{\lambda} - \frac{d\gamma^o(Q)}{dQ}\right]\frac{dQ}{d\lambda} - \frac{\overline{A}}{1+n}\gamma^o(Q).$$

From equation (23), the above equation can be rewritten as

$$\frac{d\Psi^S(Q;\lambda)}{d\lambda} = \overline{A}\left[Q-(1+n)\right]\frac{dk}{d\lambda} + \frac{\overline{A}\lambda}{1+n}\left[\tilde{s}(Q) - \frac{d\gamma^o(Q)}{dQ}\right]\frac{dQ}{d\lambda} - \frac{\overline{A}}{1+n}\gamma^o(Q). \tag{K-3}$$

Given α and χ, from equation (23), totally differentiating equation (K-2) yields

$$\frac{d\Psi^D(Q;\lambda)}{d\lambda} = \left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\frac{dk}{d\lambda} + \frac{\frac{\lambda\tilde{s}(Q)}{1+n}}{(1-\alpha)(Q-1)}\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\frac{dQ}{d\lambda}. \tag{K-4}$$

In the steady state, $\frac{d\Psi^S(Q;\lambda)}{d\lambda} = \frac{d\Psi^D(Q;\lambda)}{d\lambda}$ holds. Then, from equations (K-3) and (K-4), we obtain

$$\underbrace{\left\{\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}} - \overline{A}\left[Q-(1+n)\right]\right\}}_{(*)}\frac{dk}{d\lambda}$$
$$= \frac{\lambda}{1+n}\underbrace{\left\{\overline{A}\left[\tilde{s}(Q) - \frac{d\gamma^o(Q)}{dQ}\right] - \frac{\tilde{s}(Q)}{(1-\alpha)(Q-1)}\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right\}}_{(**)}\frac{dQ}{d\lambda} - \frac{\overline{A}}{1+n}\gamma^o(Q). \tag{K-5}$$

We now show that the terms $(*)$ and $(**)$ are both positive from Assumptions 1–5 and 7. The term $(*)$ is positive when $Q-(1+n) \leq 0$. The term $(*)$ becomes positive even if $Q-(1+n) > 0$ when

$$\frac{\overline{A}n}{Q-1} > \Phi(Q) \equiv \overline{A} - (Q-1)^{\frac{\alpha}{1-\alpha}}(\alpha\chi)^{\frac{-1}{1-\alpha}}. \tag{K-6}$$

Condition (K-6) holds when $\overline{\overline{Q}} < Q < \overline{Q}$ with Assumptions 1 and 5. $\Phi'(Q) < 0$ and $\Phi''(Q) = -\left(\frac{1-2\alpha}{1-\alpha}\right)\left(\frac{\Phi'(Q)}{Q-1}\right) > 0$ when $1 < Q$ from Assumption 1. From the definitions of $\overline{Q}$ and $\overline{\overline{Q}}$ and Assumptions 1 and 5, $\Phi(\overline{Q}) = \overline{A}\left(1 - \left(\frac{1-\alpha}{\alpha}\right)\right) < 0$ and $\Phi(\overline{\overline{Q}}) = \overline{A} - \left(\frac{1-\alpha}{\alpha}\right)\overline{\overline{A}} < 0$. Thus, $\Phi(Q) < 0$ when $\overline{\overline{Q}} < Q < \overline{Q}$. Since the left-hand side of (K-6) takes a positive value, condition (K-6) holds when $\overline{\overline{Q}} < Q < \overline{Q}$ with Assumptions 1 and 5.

We next show the condition that the term $(**)$ is negative from Assumptions 1, 5, and 7. If the following inequality holds, the term $(**)$ becomes negative:

$$\overline{A}\left[\tilde{s}(Q) - \frac{d\gamma^o(Q)}{dQ}\right]\frac{(1-\alpha)(Q-1)}{\tilde{s}(Q)} < \left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}. \tag{K-7}$$

Since the term $(*)$ is positive from Assumptions 1 and 5, the right-hand side of equation (K-7) is larger than $\overline{A}[Q-(1+n)]$. When the left-hand side of equation (K-7) is smaller than $\overline{A}[Q-(1+n)]$, the following condition holds:

$$[-\alpha(Q-1)+n]\,\tilde{s}(Q) \leq (1-\alpha)(Q-1)\frac{d\gamma^o(Q)}{dQ}. \tag{K-8}$$

The left-hand side of (K-8) is negative since we consider an economy in which $\overline{\overline{Q}} < Q$ and $\frac{n}{\alpha} + 1 \leq \overline{\overline{Q}}$ hold from Assumption 7. The right-hand side of equation (K-8) is positive from Lemma 2. Thus, (K-8) holds in an economy in which $\overline{\overline{Q}} < Q$ with Assumption 7. Combining conditions (K-6) and (K-7) with (K-8), from Assumptions 1, 5, and 7, the following inequalities hold:

$$\overline{A}\left[\tilde{s}(Q) - \frac{d\gamma^o(Q)}{dQ}\right]\frac{(1-\alpha)(Q-1)}{\tilde{s}(Q)} \leq \overline{A}[Q-(1+n)] < \left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}. \tag{K-9}$$

Condition (K-9) shows that the term $(*)$ in equation (K-5) is positive and the term $(**)$ in equation (K-5) is negative from Assumptions 1, 5, and 7.

We next show the condition that $\frac{dQ}{d\lambda} > 0$ when $\theta > \frac{1-\alpha}{1-2\alpha}$ with Assumptions 1–5 and 7. In the steady state, $\frac{d\Psi^S(Q;\lambda)}{d\lambda} = \frac{d\Psi^D(Q;\lambda)}{d\lambda}$. Thus, from equations (25) and (J-1) (see Lemma 3 (a) and Appendix J), this condition can be rewritten as

$$\left(\frac{\partial\Psi^S(Q;\lambda)}{\partial Q} - \frac{\partial\Psi^D(Q;\lambda)}{\partial Q}\right)\frac{dQ}{d\lambda} = -\underbrace{\frac{\overline{A}}{1+n}\left\{[Q-(1+n)]\,\tilde{s}(Q) - \gamma^o(Q)\right\}}_{>0} + \frac{\Psi^D(Q;\lambda)}{\lambda}. \tag{K-10}$$

The right-hand side of equation (K-10) is positive because condition (J-2) holds when $\theta > \frac{1-\alpha}{1-2\alpha}$ with Assumptions 1–4. Thus, (K-10) implies that $\frac{dQ}{d\lambda} > 0$ when $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ if and only if $\frac{\partial\Psi^S(Q;\lambda)}{\partial Q} - \frac{\partial\Psi^D(Q;\lambda)}{\partial Q} > 0$. Combining equations

(H-1) and (I-1) with the definition of $\Psi^S(Q;\lambda)$ in equation (22), we obtain

$$\begin{aligned}&\frac{\partial \Psi^S(Q;\lambda)}{\partial Q}-\frac{\partial \Psi^D(Q;\lambda)}{\partial Q}\\ &=\frac{\overline{A}\lambda}{1+n}\left\{-(1+n)\frac{d\tilde{s}(Q)}{dQ}+\tilde{s}(Q)+Q\frac{d\tilde{s}(Q)}{dQ}-\frac{d\gamma^o(Q)}{dQ}\right\}\\ &-\frac{\lambda}{1+n}\frac{d\tilde{s}(Q)}{dQ}\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}-\frac{\lambda\tilde{s}(Q)}{(1+n)(1-\alpha)(Q-1)}\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}. \quad \text{(K-11)}\end{aligned}$$

(K-11) implies that $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q}-\frac{\partial \Psi^D(Q;\lambda)}{\partial Q}>0$ if and only if

$$\underbrace{\left\{\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}-\overline{A}\left[Q-(1+n)\right]\right\}}_{(*)\text{in (K-5)}}\left(\frac{-d\tilde{s}(Q)}{dQ}\right)+\underbrace{\left\{\overline{A}\left[\tilde{s}(Q)-\frac{d\gamma^o(Q)}{dQ}\right]-\frac{\tilde{s}(Q)}{(1-\alpha)(Q-1)}\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}\right\}}_{(**)\text{in (K-5)}}>0. \quad \text{(K-12)}$$

Substituting equations (K-5) and (24) into equation (K-12), we can rewrite equation (K-12) as

$$\underbrace{\left\{\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}-\overline{A}\left[Q-(1+n)\right]\right\}}_{(*)\text{in (K-5)}}\left\{\left(\frac{-d\tilde{s}(Q)}{dQ}\right)+\frac{1+n}{\lambda}\frac{\frac{1}{1+n}\left[\tilde{s}(Q)-\lambda\left(\frac{-d\tilde{s}(Q)}{dQ}\right)\frac{dQ}{d\lambda}\right]}{\frac{dQ}{d\lambda}}\right\}+\frac{\overline{A}}{\lambda}\frac{\gamma^o(Q)}{\frac{dQ}{d\lambda}}>0.$$

The above condition can be rewritten as

$$(\lambda)^{-1}\left(\frac{dQ}{d\lambda}\right)^{-1}\left\{\underbrace{\left\{\left(\frac{Q-1}{\alpha\chi}\right)^{\frac{1}{1-\alpha}}-\overline{A}\left[Q-(1+n)\right]\right\}}_{(*)\text{in (K-5)}}\tilde{s}(Q)+\overline{A}\gamma^o(Q)\right\}>0. \quad \text{(K-13)}$$

(K-13) implies that $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q}-\frac{\partial \Psi^D(Q;\lambda)}{\partial Q}>0$ and $\frac{dQ}{d\lambda}>0$ since the term $(*)$ is positive from Assumptions 1 and 5. This shows that the term $(*)$ is positive, the term $(**)$ is negative, and $\frac{dQ}{d\lambda}>0$ when $\theta>\frac{1-\alpha}{1-2\alpha}>1$ and $\overline{\overline{Q}}<Q<\overline{Q}$ with Assumptions 1–5 and 7. Thus, because $\frac{dQ}{d\lambda}>0$, $\frac{dk}{d\lambda}<0$ holds from equation (K-5).

Appendix L. $\frac{dQ}{d\lambda}<0$ when $0<\theta\leq 1$ with Assumptions 1–7

When $0<\theta\leq 1$ and $\hat{Q}<Q<\overline{Q}$, from Assumptions 1–7, equations and inequalities (K-1)–(K-10) hold. Similarly to the case of $\theta>\frac{1-\alpha}{1-2\alpha}>1$, the term

(∗) in equation (K-5) is positive and the term (∗∗) in equation (K-5) is negative when $0 < \theta \leq 1$. However, when $0 < \theta \leq 1$, $\frac{d\tilde{s}(Q)}{dQ} > 0$ from Lemma 1 (b) in contrast to the case in which $\theta > \frac{1-\alpha}{1-2\alpha} > 1$ (see (K-11) to (K-13) in Appendix K). Hence, the following holds:

$$\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} - \frac{\partial \Psi^D(Q;\lambda)}{\partial Q} < 0, \tag{L-1}$$

since $\frac{\partial \Psi^S(Q;\lambda)}{\partial Q} < 0$ and $\frac{\partial \Psi^D(Q;\lambda)}{\partial Q} > 0$ (see Appendices H and I). Combining condition (L-1) with equation (K-10), we obtain $\frac{dQ}{d\lambda} < 0$ when $0 < \theta \leq 1$ because the first term on the right-hand side of (K-10) is positive with Assumption 6 (see Lemma 3 (b)).

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