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CORRECTION TO
A NOTE ON GALOIS COVERING

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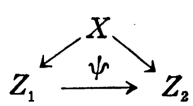
Prof. L.N. Gupta has pointed out the author that there is a gap in the proof of Theorem 2.2 [3]. In this theorem we need to put certain assumptions. To give it, we shall extend to the case of preschemes certain notions defined by Chase, Harrison and Rosenberg in [1].

Let σ, τ: $X \to Z$ be two morphisms of preschemes where X is affine. Then σ and τ are called to be strongly distinct if, for any sum $X \cong X_1 \amalg X_2$ such that X_i are affine schemes ($i=1, 2$), $\sigma\varphi$ and $\tau\varphi$ are distinct where φ is the canonical morphism: $X_1 \to X$.

Let X be a Galois covering of a prescheme Y with a Galois group $\mathfrak{G}$, Φ: $X \to Y$ the structure morphism and Z an intermediate prescheme between X and Y with the structure morphism Φ_1: $X \to Z$, Φ_2: $Z \to X$. We shall say that Z is $\mathfrak{G}$-strong if there is an affiine open covering $\{V_\gamma\}_{\gamma \in I}$ of Y such that for any pair $\sigma, \tau \in \mathfrak{G}$, $\Phi_1\sigma$ and $\Phi_1\tau$ are equal or their restrictions to $\Phi^{-1}(V_\gamma)$ are strongly distinct for all $\gamma \in I$.

One can show that $\mathfrak{G}$-strongness is independent on an affine open covering.

Let φ: $X \to Y$ be a surjective morphism of preschemes which is finite and locally free. Let Z_1, Z_2 be two intermediate preschemes between X and Y such that the structure morphisms ψ_i: $Z_i \to Y$ are affine for $i=1, 2$. Z_1 and Z_2 are said to be isomorphic as intermediate preschemes if there is a Y-isomorphism ψ: $Z_1 \to Z_2$ such that the diagram



is commutative where the unadorned morphisms are structural. We shall call an intermediate covering between X and Y an isomrphism class of intermediate preschemes between X and Y.

A correct form of Theorem 2.2 in [3] can be obtained by strengthening hypotheses in the following manner.

Theorem 2.2. *Let Y be a prescheme and X a Galois covering of Y with a Galois group $\mathfrak{G}$. Let Z be an intermediate covering between* X *and Y. If Z is a quasi-unramified covering of Y which is $\mathfrak{G}$-strong, then there exists a unique subgroup $\mathfrak{H}$ of $\mathfrak{G}$ such that Z is the quotient prescheme $X/\mathfrak{G}$ of X by $\mathfrak{H}$.*

Proof. It follows form modifying the proof of Theorem 2.2. in [3], noting the following facts;

1) Since a union of two disjoint affine open sets in a prescheme is also affine, we can choose an affine open covering $\{V_\gamma\}_{\gamma\in I}$ of Y satisfying that, for any pair $(\alpha, \beta)\in I\times I$, there is a sequence $V_\alpha=V_{\gamma_0}, V_{\gamma_1}, \cdots, V_{\gamma_\lambda}=V_\beta$ with $V_{\gamma_i}\cap V_{\gamma_{i+1}}\neq\phi$ for $\gamma_i\in I$.

2) Let φ: $Z\rightarrow Y$ be the structure morphism. For an affine open set V in Y, the ring of $\varphi^{-1}(V)$ is $\mathfrak{G}$-strong in sense of Chase, Harrison and Rosenberg [1].

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