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**CORRECTION TO  
 A NOTE ON GALOIS COVERING**

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Prof. L.N. Gupta has pointed out the author that there is a gap in the proof of Theorem 2.2 [3]. In this theorem we need to put certain assumptions. To give it, we shall extend to the case of preschemes certain notions defined by Chase, Harrison and Rosenberg in [1].

Let  $\sigma, \tau: X \rightarrow Z$  be two morphisms of preschemes where  $X$  is affine. Then  $\sigma$  and  $\tau$  are called to be strongly distinct if, for any sum  $X \cong X_1 \amalg X_2$  such that  $X_i$  are affine schemes ( $i=1, 2$ ),  $\sigma\varphi$  and  $\tau\varphi$  are distinct where  $\varphi$  is the canonical morphism:  $X_1 \rightarrow X$ .

Let  $X$  be a Galois covering of a prescheme  $Y$  with a Galois group  $\mathfrak{G}$ ,  $\Phi: X \rightarrow Y$  the structure morphism and  $Z$  an intermediate prescheme between  $X$  and  $Y$  with the structure morphism  $\Phi_1: X \rightarrow Z$ ,  $\Phi_2: Z \rightarrow Y$ . We shall say that  $Z$  is  $\mathfrak{G}$ -strong if there is an affine open covering  $\{V_\gamma\}_{\gamma \in I}$  of  $Y$  such that for any pair  $\sigma, \tau \in \mathfrak{G}$ ,  $\Phi_1\sigma$  and  $\Phi_1\tau$  are equal or their restrictions to  $\Phi^{-1}(V_\gamma)$  are strongly distinct for all  $\gamma \in I$ .

One can show that  $\mathfrak{G}$ -strongness is independent on an affine open covering.

Let  $\varphi: X \rightarrow Y$  be a surjective morphism of preschemes which is finite and locally free. Let  $Z_1, Z_2$  be two intermediate preschemes between  $X$  and  $Y$  such that the structure morphisms  $\psi_i: Z_i \rightarrow Y$  are affine for  $i=1, 2$ .  $Z_1$  and  $Z_2$  are said to be isomorphic as intermediate preschemes if there is a  $Y$ -isomorphism  $\psi: Z_1 \rightarrow Z_2$  such that the diagram

$$\begin{array}{ccc} & X & \\ \swarrow & & \searrow \\ Z_1 & \xrightarrow{\psi} & Z_2 \end{array}$$

is commutative where the unadorned morphisms are structural. We shall call an intermediate covering between  $X$  and  $Y$  an isomorphism class of intermediate preschemes between  $X$  and  $Y$ .

A correct form of Theorem 2.2 in [3] can be obtained by strengthening

hypotheses in the following manner.

**Theorem 2.2.** *Let  $Y$  be a prescheme and  $X$  a Galois covering of  $Y$  with a Galois group  $\mathfrak{G}$ . Let  $Z$  be an intermediate covering between  $X$  and  $Y$ . If  $Z$  is a quasi-unramified covering of  $Y$  which is  $\mathfrak{G}$ -strong, then there exists a unique subgroup  $\mathfrak{H}$  of  $\mathfrak{G}$  such that  $Z$  is the quotient prescheme  $X/\mathfrak{H}$  of  $X$  by  $\mathfrak{H}$ .*

Proof. It follows from modifying the proof of Theorem 2.2. in [3], noting the following facts;

1) Since a union of two disjoint affine open sets in a prescheme is also affine, we can choose an affine open covering  $\{V_\gamma\}_{\gamma \in I}$  of  $Y$  satisfying that, for any pair  $(\alpha, \beta) \in I \times I$ , there is a sequence  $V_\alpha = V_{\gamma_0}, V_{\gamma_1}, \dots, V_{\gamma_\lambda} = V_\beta$  with  $V_{\gamma_i} \cap V_{\gamma_{i+1}} \neq \emptyset$  for  $\gamma_i \in I$ .

2) Let  $\varphi: Z \rightarrow Y$  be the structure morphism. For an affine open set  $V$  in  $Y$ , the ring of  $\varphi^{-1}(V)$  is  $\mathfrak{G}$ -strong in sense of Chase, Harrison and Rosenberg [1].

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