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Endogenous Growth and Intellectual Property Rights*

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We examine how the intellectual property rights have an effect on the growth rate of the economy. We focus on a role of IPR protection that prevent the free riding. By using a simple endogenous growth model, we show that the existence of intellectual property rights essentially brings a high growth rate.

Keywords: Endogenous Growth Model, Intellectual Property Rights, Differential Game Analysis
I. Introduction

We examine the role of the intellectual property rights (IPR) protection in economic growth. Though it has been studied whether strong protections of intellectual property will stimulate or retard the innovation from both of empirical and theoretical approach, relatively fewer have mentioned the dynamic process of growth. A recent empirical study by Gould and Gruden (1996) clarifies that the intellectual property protection is positively related to economic growth. A theoretical model of Barro and Sala-i-Martin (1995) shows the positive relationship of intellectual property protection and growth. We also show that the IPR protection ameliorates the growth rate of the economy as they have proved, but in a different way.

In Barro and Sala-i-Martin's (1995) model, the technological improvement with the IPR protection is supposed to bring the monopoly profit to the inventor. In that model, the monopoly profit affects the motivation for investments and the economic growth. Insuring the monopoly profit is undoubtedly one of the outcomes of the IPR protection, but we pay attention to another outcome. Likewise Romer's (1986) or Benhabib and Rustichini's (1993) model, we assume that a new invention improves the representative's productivity. On this assumption, the representatives can enjoy higher consumption utility when a new technology is invented. They therefore have an incentive to invest even though there is no monopoly profit. In this case, the IPR protection seems to be unrelated to the growth. This model shows that eliminating the possibility of free riding, the IPR protection brings higher incentive to invest even in the case. When the IPR is not protected, representatives can access the fruit of the other representatives' R&D investments. Hence they may reduce the incentive to invest in their production technology. The IPR protection rules out such a free riding behavior.

We present a simple dynamic model. The structure of our model follows that of the AK model. The difference between our model and the AK model comes forth from the assumptions that the capital for production in our model is stock of technology, not the broadly defined capital (it includes physical and human capital) as the AK model supposes. The representatives are two groups and they produce and consume the goods. They also develop new technology. There exists the diffusion of knowledge which informs every group about the other group's invention.

If the IPR is protected, they cannot use the other group's stock of technology without the inventor's permission. We assume that when they use the other group's stock of technology, they exchange the license of their invention with the other group's license (cross licensing). When we investigate the case in which the IPR is not protected, since the diffusion of knowledge enables every group to access the other group's stock of technology, we regard the economy's whole stock of technology as a common state variable. Therefore, in such a case

we use a differential game model analysis. As a conclusion of this study, we show that the IPR protection brings a high rate of growth, but it needs the transactions of the technology stock that they possess.

Though the IPR protection is important for the development, it is also a source of monopoly distortion. Deardorff (1991) shows that while the protection increases the welfare of the inventing country, it may decrease that of the other country. Barro and Sala-i-Martin (1995) examine the role of government to prevent the distortion. Though our model does not include the distortion, we show that the IPR protection may bring a low growth rate in the case that the technology transactions are not executed.

II. The model

Suppose that two symmetric groups have the stock of technology denoted by $i = 1, 2$ and their production function is $y = Ad^i$. The coefficient "A" represents all the factors of production except the stock of technology. The cost to create a new type of technology is fixed at $\eta$ units of $y$ and the new technology improves the groups' productivity. We assume that the technologies which are invented by the different groups in a planning period are not identical and that the stock of technology invented by a group is also effective to improve the productivity of the other group. The objective function of the group is

$$\max_{U_i} = \int_0^\infty \frac{\sigma}{\sigma - 1} c_i^{\sigma - 1} e^{-\rho t} dt,$$

where $C_i^t$ is the consumption of each period, $\sigma (>0)$ is the elasticity of intertemporal substitution and $\rho$ is the discount rate. We assume that there is no negative consumption.

A. The economy with the IPR protection

We investigate two cases in both of which the IPR is protected in this section. In the first case (case A-1), the groups enclose their stock of technology and do not sell the licenses of the technologies and in the second case (case A-2), the groups trade their licenses with each other. In this study, we assume strong IPR protection that ensures perpetual monopoly rights for their inventions.

First, we investigate the case in which the IPR is protected and groups never trade their licenses with each other. In this case, the model is similar to the AK model. The stock of technology accumulation constraint of each group is

$$d_i^t = \frac{1}{\eta} (Ad_i^t - c_i^t),$$

where $d_i^t$ is the technology stock of each group and $c_i^t$ is the consumption of technology.

The result of maximization shows

$$c_i^t = \frac{(1 - \sigma) A}{\eta + \rho \sigma} d_i^t$$

and

$$d_i^t = d_i^t \exp\left(\frac{A}{\eta} - \rho \sigma t\right),$$

where $A$ is the rate at which the technology stock flows out to common technology in every period and the rate of the efflux is externally decided. In that model, the rate of efflux affects the ratio of protected technology stock and common technology stock but the growth rate of consumption is identical to the model that assumes the perpetual protection.
We assume that the growth rate of the stock of technology is positive. For the consumption to be positive, it is necessary to assume \( (1-\sigma)A/\eta + \rho \sigma \geq 0 \). The indirect utility function is

\[
U_i^{A-1} = \frac{\sigma}{\sigma - 1} \left( (1-\sigma) \frac{A}{\eta} + \rho \sigma \right)^{-\frac{1}{\sigma}} (\eta d_i^i \sigma)^{-\frac{1}{\sigma}}
\]

\( i = 1,2 \).

Now we investigate the case in which the groups license their patented technologies each other. Suppose that the stock of technology that is exchanged has an equal productivity for both of the groups. Thus, as long as the stock of technology is smaller than or equal to the other group’s stock, the upper bound of stock which the group can receive from the other group by the cross licensing is a function of the group’s own stock\(^5\). We study the case that the groups trade the licenses at their upper bound\(^6\). Since the technology is a type of knowledge and it has a nonrival character, providing the license does not reduce the provider’s stock of technology. The constraint for the each group is

\[
d_i^i = \frac{1}{\eta} \left[ A(d_i^i + f_i^i(d_i^e)) - c_i^i \right] \quad i = 1,2.
\]

Where \( f_i^i(d_i^e) \) is the stock of technology that the group receives by cross licensing. Because of the assumption that the groups are symmetric and that the stock of technology they exchanged provides equal rate of productivity improvement to the donor and the recipient, let \( f_i^i(d_i^e) = d_i^i \). We rewrite the above equation as:

\[
d_i^i = \frac{1}{\eta} (2A d_i^i - c_i^i) \quad i = 1,2.
\]

The maximization of the utility function under the above constraint yields

\[
c_i^i = \left( (1-\sigma) \frac{2A}{\eta} + \rho \sigma \right) d_i^i \quad i = 1,2
\]

and

\[
d_i^i = d_i^e \exp \left( \frac{2A}{\eta} - \rho \sigma \right) t \quad i = 1,2.
\]

For the positive consumption, we assume that \( (1-\sigma)2A/\eta + \rho \sigma \geq 0 \). The indirect utility function of case A-2 is

\[
U_i^{A-2} = \frac{\sigma}{\sigma - 1} \left( (1-\sigma) \frac{2A}{\eta} + \rho \sigma \right)^{-\frac{1}{\sigma}} (\eta d_i^e \sigma)^{-\frac{1}{\sigma}}
\]

\( i = 1,2 \).

**B. The economy with no IPR protection**

In this section we investigate the case that the IPR is not protected and that the diffusion of knowledge enables the groups to use the other group’s knowledge without any costs or lags (case B). Because of no IPR protection and the diffusion of knowledge, they can use the entire stock of technology that exists in the economy. The invention of each group is described as:

\[
d_i^i = \frac{1}{\eta} (AD_i - c_i^i) \quad i = 1,2,
\]

where, \( D_i = d_i^i + d_i^e \). In this case, the state variable for the each group is \( D \) and we regard it as a common state variable. Thus, a con-

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5) How about the case where the other one’s stock of the technology is smaller than the one’s stock? Whether the developed group neglect or use the other one’s stock of technology, as long as they transact their technology with each other, the following group will catch up to the leading group. The equality of the groups will hence be attained in the long run.

6) Appendix A shows the reason of this assumption.
straint for each group is
\[ D_i = d_i^i + d_i^j \]
\[ = \frac{1}{\eta} [ (AD_t - c_i^i) + (AD_t - c_i^j) ] \]
\[ = \frac{1}{\eta} (2AD_t - c_i^i - c_i^j) \]
\[ i = 1, 2 \quad i \neq j. \]

This is the case that the groups may have the temptation of free riding. Because of the lack of the IPR protection, every group has the chance to use the stock of technology that the other one invented. Both of them may hence suspect the other one's will to invest in the innovation. To investigate this sort of model, we use a differential game model analysis. The difficulty of the differential games is that the unknown strategy of the other player is involved in the one's optimization problem. Following the Tornell and Velasco's (1992) model, we focus on the equilibrium in the Markovian strategy and assume that the best response depends linearly on the current value of the state variable, \( c_i^t = \alpha D_t \). The optimization of the problem gives following equations (see Appendix B).
\[ c_i^t = \frac{(1-\sigma)2A/\eta + \rho \sigma}{2 - \sigma} D_t \quad i = 1, 2 \]
\[ D_t = D_0 \exp 2(\frac{A}{\eta} - \rho) \cdot \frac{\sigma}{2 - \sigma} t \quad i = 1, 2 \]

We assume that\( ((1-\sigma)2A/\eta + \rho \sigma)/(2 - \sigma) > 0. \)
The indirect utility function of this case is
\[ U_i^g = \frac{\sigma}{\sigma - 1} \left( \frac{(1-\sigma)2A/\eta + \rho \sigma}{2 - \sigma} \right)^{\frac{1}{\eta}} \left( \eta D_0^g \right)^{\frac{\sigma - 1}{\sigma}} \]
\[ i = 1, 2. \]

Here we will compare the above 3 cases. The order of the growth rates of the cases is
\[ G_{A-2} \geq G_b \geq G_{A-1}, \]

Where \( G_{(*,*)} \) is the growth rate of the group's and consumption stock of knowledge of case ( * ). When the initial conditions of these cases are identical, the group's welfare levels of the cases follow the above order (see Appendix C). If they enclose their stock of technology and do not exchange their licenses, the codification of the IPR brings low growth rate and the rate is inferior to the rate of the no IPR protection economy. Therefore, the authorities of the economy groups have to pay attention to the condition of the technology transaction between the groups.

III. The case of asymmetric initial endowments

As mentioned above, the codification of IPR and the technology transaction bring higher growth rate and increased welfare when the groups are symmetric. In this section, we show that a group which has very small initial endowments in comparison to the other group's endowments may not prefer the codification of the IPR. We investigate the case that the initial endowments of technology stock of group 2 are smaller than that of group 1. The result of the investigation shows what kind of IPR institution the inferior group prefers when a barrier which has isolated the inferior group from the developed group for some (e.g., political or geographical) reasons disappears. Supposing that \( d_i^1 > d_i^2 \) and \( d_i^2/d_i^1 = m \), we can rewrite...
the indirect utility functions of case A-2 and case B as:
\[ U_{2}^{A-2} = \frac{\sigma}{\sigma-1} \left[ (1-\sigma) \frac{2A}{\eta} + \rho_0 \right] \frac{1}{\sigma} \left[ \eta md_2 \right] \frac{\sigma-1}{\sigma} \]
and
\[ U_{1}^{B} = \frac{\sigma}{\sigma-1} \left[ \frac{(1-\sigma)A}{\eta} + \rho_0 \right] \frac{1}{2-\sigma} \left[ \eta (1+m)d_1 \right] \frac{\sigma-1}{\sigma}. \]

When \( m=1 \), the conditions are identical to that of the section 2 and \( U_{1}^{B} \) is larger than \( U_{1}^{A} \). When \( m<1 \) and the following inequality is satisfied, the order of the utility functions will change.
\[ m < \frac{(2-\sigma)^{\frac{1}{\sigma-1}}}{1-(2-\sigma)^{\frac{1}{\sigma-1}}} \]

Namely, when initial endowments of group 2 (\( d_0^2 \)) is sufficiently small in comparison to that of group 1 (\( d_0^1 \)), the IPR protection is not profitable for the group 2. This result shows that the group that possesses a small amount of stock of technology does not want the codification of the IPR protection.

IV. Conclusion

By using a simple dynamic model, we investigate the role of the IPR protection in economic growth. We show that the IPR protection improves the growth rate when the technology transaction is sufficiently executed. Therefore, for the high-rate of economic growth, authorities of the country have to uphold the IPR protection and get rid of the factors that disturb the technology transaction.

Though the IPR protection brings higher growth rate, groups that have inferior initial endowment of stock of technology may not prefer it. Such groups may oppose the codification of the IPR. Actually, Helpman (1993) and Gould and Gruden (1996) point out that many of the developing countries have traditionally offered shorter periods of protection for patents than have the developed countries, and that in the recent round of trade negotiations, developing countries criticize the fact that tight IPR protections strengthen the monopoly power of foreign developed countries.

The IPR protection is increasingly realized as a fundamental condition of the high-level innovation and growth. The reason is not only that it ensures the monopoly profit, but that it rules out the possibility of free riding.

Though our study is very partial and ignoring many aspects of the patent rights, it may help to examine the issues of such the free riding.

Appendix

A. The decision of the license trade

When they trade a part of their stock, we can rewrite the constraint as
\[ \delta_i = \frac{1}{\eta} \left[ A \left( d_i^i + f_i^i (g'd_i^i) - c_i^i \right) \right], \]
where \( f_i^i(g'd_i^i) = g'd_i^i \) and \( g^i \leq 1 \). The coefficient \( g' \) presents the ratio of the traded stock to the whole stock and groups can control it. In this case, we can rewrite the indirect utility function of case A-2 as
\[ U_{1}^{A-2} = \frac{\sigma}{\sigma-1} \left[ (1-\sigma) \frac{1+g^i}{\eta} (1+g^i)A + \rho_0 \right] \frac{1}{\sigma} \left[ \eta d_1^{i-1} \right] \frac{\sigma-1}{\sigma}. \]

Because the utility function is assumed to be positive, one can see that \( dU_{1}^{A-2} / dg_i > 0 \). Hence when they choose the level of \( g^i \), \( g^i = 1 \) is a feasible choice.
B. No IPR protection economy

The problem in this section is

\[
\begin{aligned}
\text{Max.} & \quad \int_0^\infty \frac{\sigma}{\sigma - 1} c_t^{\sigma - 1} \Delta t \\
\text{s.t.} & \quad \dot{D}_t = \frac{1}{\eta} (2AD_t - c_t^i - c_t^j)
\end{aligned}
\]

As mentioned in chapter 2, we assume that best response depends linearly on the current value of the state variable. Thus, we assume that \(c_t^j = aD_t\) and the groups are symmetric.

The Hamiltonian of group i is

\[H = \frac{\sigma}{\sigma - 1} c_t^{\sigma - 1} + \phi_i \left(2AD_t - c_t^i - aD_t\right).\]

The first order conditions are

\[b-1 \quad c_t^{i(1)} = \frac{1}{\eta} \phi_i,\]

\[b-2 \quad \phi_i = \gamma \phi_i - \phi_i \frac{1}{\eta} (2A - \alpha)\]

and T.V.C.

From \(c_t^i = \alpha D_t\) and b-1, we obtain

\[b-3 \quad \frac{\dot{D}_t}{\Delta t} = \frac{c_t^i}{c_t^j} = -\frac{\phi_i}{\phi_j}.\]

Combining this with the factor constraints, b-2 and b-3, we obtain

\[\alpha = \frac{2A(1 - \sigma) + \rho \sigma}{2 - \sigma} \eta\]

Therefore,

\[c_t^i = \frac{2A(1 - \sigma) + \rho \sigma}{2 - \sigma} \eta D_t\]

For the utility index to be bounded, we assume that \(2A(1 - \sigma) + \rho \sigma \eta) / (2 - \sigma) > 0\).

C. The order of the indirect utility function

At first, we compare the indirect utility function of case B and case A-2. The residual of them is

\[U_t^{A-2} - U_t^B = \frac{\sigma}{\sigma - 1} \left[\frac{2A}{\eta} + \rho \sigma\right]^{\sigma - 1} [\eta D_t]^{\sigma - 1} \left[1 - 2\left(\frac{2 - \sigma}{2}\right)^{\sigma - 1}\right].\]

From the assumption of no negative consumption and \(d_t^i > 0\), the first and second brackets are positive. Thus, the sign of this residual depends on the first term \(\left(\sigma / (\sigma - 1)\right)\) and third brackets.

If \(\sigma > 1\), the first term is positive. From the assumption of positive consumption of case B, we get \(1 < \sigma < 2\). To investigate the sign of the third brackets, we define following function.

\[s(\sigma) = 1 - 2\left(\frac{2 - \sigma}{2}\right)^{\frac{\sigma - 1}{\sigma}}\]

Since the function satisfies

\[s_{(\sigma)} = 1\]

and

\[\frac{\partial s(\sigma)}{\partial \sigma} = -\frac{2 - \sigma}{\sigma} < 0,\]

the function is smaller than 1 and the sign of the third brackets is positive. Because the first term and the third brackets are positive, the residual of the utilities \((c-1)\) is positive.

If \(\sigma < 1\), the residual is positive because the above argument shows that the first term and the third brackets are both negative.

If \(\sigma = 1\), the utility function is given by \(lnC_t\). Then the residual of their utility functions is

\[U_t^{A-2} - U_t^B = \left(\frac{\ln(d_t \rho)}{\rho} + \frac{2A}{\eta} - 2\rho\right) - \left(\frac{\ln(D_t \rho)}{\rho} + \frac{2A}{\eta} - 3\rho\right) = \left(ln \frac{1}{\rho} + 1\right) / \rho > 0\]

and we see that it is positive.
Thus the indirect utility of case A-2 is larger than that of case B from this equation.

Second, we investigate the residual of the indirect utility of the case B and case A-1.

\[ U_{i}^{B} - U_{i}^{A-1} = \frac{\sigma}{\sigma - 1} \left[ \frac{[1 - \sigma \frac{2A}{\eta} + \rho \alpha]}{2 - \sigma} \right] \]

\[ - \left[ (1 - \sigma) \frac{A}{\eta} + \rho \alpha \right] \left[ m\left( \sigma \right) \right] \]

The first term and the first braces decide the sign of the residual.

If \( \sigma > 1 \), the residual is positive if the first term of the braces is larger than the second term of it.

\[ c-3 \left[ (1 - \sigma) \frac{2A}{\eta} + \rho \alpha \right] > \left[ (1 - \sigma) \frac{A}{\eta} + \rho \alpha \right] \]

As mentioned above, we get \( 1 < \sigma < 2 \) from the assumption that \( C_{j}^{i} \) is positive. We rewrite the equation as

\[ c-4 \]

\[ [(1 - \sigma) \frac{2A}{\eta} + \rho \alpha]^{2 - \sigma} < \left[ (1 - \sigma) \frac{A}{\eta} + \rho \alpha \right] \]

To investigate c-4, we define a following function.

\[ m(\sigma) = \frac{[(1 - \sigma) \frac{2A}{\eta} + \rho \alpha]^{2 - \sigma}}{[(1 - \sigma) \frac{A}{\eta} + \rho \alpha]^{2 - \sigma}} - \left[ (1 - \sigma) \frac{A}{\eta} + \rho \alpha \right] \]

As the function satisfies

\[ m_{\sigma}(\sigma) = 0 \]

\[ \frac{\partial m(\sigma)}{\partial \sigma} = -\frac{A}{\eta} \left( \frac{1}{2} - \frac{(1 - \sigma)(2 - \sigma)}{2} \right) \]

\[ + \rho \left( \frac{1}{2} - \frac{(1 - \sigma)(2 - \sigma)}{2} \right) \]

the sign of \( m(\sigma)(1 < \sigma < 2) \) is negative. Hence the inequality c-3 is satisfied and we can show that indirect utility of case B is larger than that of case A-1.

If \( 1 < \sigma \), similar explanations show that c-1 is positive and the indirect utility of case B is larger than that of case A-1.

If \( \sigma = 1 \), we see that it is positive by rewriting the residual as follows;

\[ U_{i}^{B} - U_{i}^{A-1} = \frac{\ln(D_{e} \rho)}{\rho} + \frac{2A/\eta - 3\rho}{\rho^{2}} \]

\[ - \frac{\ln(D_{e} \rho)}{\rho} + \frac{A/\eta - 2\rho}{\rho^{2}} \]

\[ = \{ \ln 2 + \left( \frac{A}{\eta \rho} - 1 \right) \}/\rho > 0 \]

Then, the utility of case B is larger than that of case A-1.

Reference:
Gould, D. and Gruben, W., (1996), "The role of intellectual property rights in economic


