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Electromagnetic Wave Localization in Photonic Fractals

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Abstract

3D fractal structures, which we have named photonic fractals, can strongly localize electromagnetic wave with a specific wavelength associated with the geometry and physical properties of the fractal structures. The photonic fractals can be fabricated with various materials including polymers, ceramics, metals, and their composites. The localization mechanism in a photonic fractal with self-similar structure is essentially different from the localization in a photonic crystal with a periodic structure. This new material is considered to be applicable in a wide area of communication, information, energy, sensing, medical care, and other fields.

KEY WORDS: (Photonic fractal), (Localization), (Microwave), (Menger sponge), (Stereolithography).

1. Introduction

Fractal is the term which describes rough and irregular structures with self-similarity such that the local configuration is similar to the whole configuration [1]. Fractal structures have no periodicity and no translational symmetry like crystal structures. Interaction of optical, electromagnetic, and acoustical waves with fractal or quasi-periodic structures has been of theoretical and practical interest in recent years. However, studies concerning the localization of electromagnetic waves in these structures have been restricted to one or two dimensions [2-6]. A 3D fractal structure is necessary to completely localize light or electromagnetic waves. Nevertheless, 3D fractal structures have not been fabricated probably due to the difficulties of construction.

Recently, we have succeeded in the fabrication of 3D fractal structures called Menger sponges, which are made of dielectric media such as epoxy and ceramics, and found a significant feature in that the incident microwave is strongly localized in the fractal with very large attenuations both in reflectance and transmittance at the same frequency [7,8]. We have named such fractals as photonic fractals because no other materials which can localize electromagnetic waves in the 3D fractal form is known. Photonic fractals may have large potential applications in communication, information, energy, sensing, medical care, and other fields. This paper reports on the design and fabrication of 3D fractals using CAD/CAM stereolithography, electromagnetic wave responses for such fractal structures made of polymer, ceramic, and metal materials, together with expected applications.

2. Fabrication of 3D Photonic Fractals by CAD/CAM Stereolithography

The 3D fractal structures were designed on a computer using a CAD program (Toyota Caelum Co. Ltd, thinkdesing ver.5.0). The designed structure is converted into a rapid prototyping format (STL file), sliced into a set of thin sections, and transferred to a stereographic machine (D-MEC Co.Ltd, Japan, SCS-300P). **Fig.1** illustrates the schematic form of a stereolithographic machine.

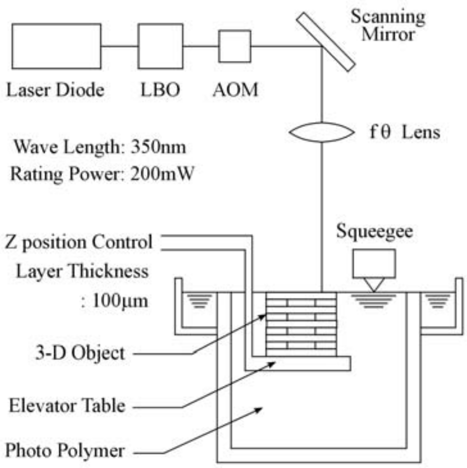

Fig.1 A schematic illustration of a stereolithographic machine.

This machine forms a three-dimensional object layer-by-layer by scanning a UV laser of 355 nm

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wavelength over a liquid photopolymer epoxy resin. The thickness of each layer is about 150 μm. The TiO_2-SiO_2 powders with a particle size of about 10 μm are dispersed into the liquid resin in order to increase the dielectric constant of the fractal structure.

We have fabricated 3D fractal structures called Menger sponges of a size comparable with the microwave range and investigated the propagation characteristics of electromagnetic waves in these structures. It is very difficult to fabricate such complex fractal structures by methods other than CAD/CAM systems. The Menger sponge is a 3D version of the Cantor bar fractal [9]. The Cantor bar fractal is formed by extracting the center segment of three equivalent segments divided from an initial bar and repeating this process to the remaining two side segments. The fractal dimension D of the Menger sponge is calculated by the following relationship: $N=S^D$, where N is the number of the self-similar units newly created when the size of the initial unit decreases to 1/S. In the present Menger sponge, N=20, and S=3, so that D=log20/log3 =2.7268.

The Menger sponge structures were designed from a cube with a side length a. The cube is divided into 27 identical cube pieces, and the seven pieces at the body- and face-centers are extracted making air cube rods. By repeating the same extraction process for the remaining twenty pieces we create the Menger sponge. Model structures of a Menger sponge with four different stages of the repeating process (0, 1, 2, 3) are illustrated in **Fig.2**. The side length a of the real samples is 27mm. The lengths of the longest, middle, and shortest sides of the square air rods are 9mm, 3mm, and 1mm, respectively.

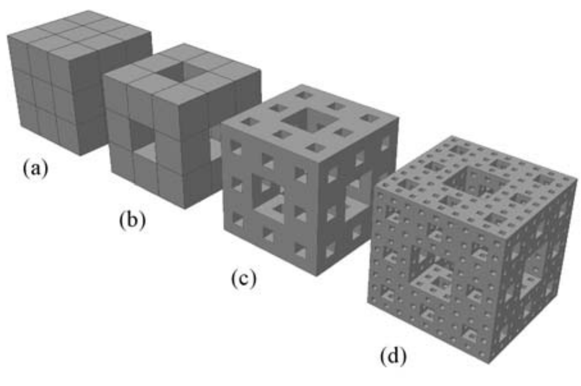

Fig. 2 Model images of the Menger sponges (a) stage 0 (b) stage1 (c) stage 2 (d) stage3.

3. Localization of Microwaves in 3D Dielectric Fractals

Normally incident reflection and transmission spectra of the Menger sponge for stages1, 2, and 3 were measured in a metal cavity by using a network analyzer and two mono-pole antennae. The result for the stage 3 Menger sponge made of epoxy with the dielectric constant of 2.8 is shown in **Fig.3**. The reflection and transmission spectra are normalized to that of free space. In the transmission spectrum, a sharp transmission attenuation as large as -31dB and a relatively broad attenuation of reflection as large as -7dB was observed at about 12.8 GHz. In addition, for the stage 2 Menger sponge, a sharp attenuation as large as -15dB and a corresponding broad attenuation of reflection as large as -5dB were both observed at around 13 GHz. In contrast, such anomalous attenuations were not observed for the Menger sponge of stage 0 and 1 in the measured frequency range.

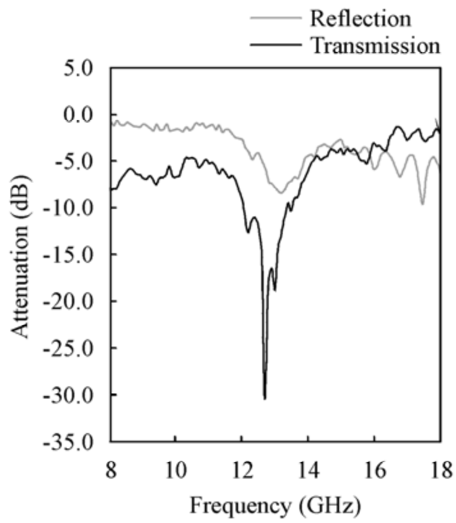

Fig. 3 Reflection and transmission spectra of the stage 3 dielectric Menger sponge.

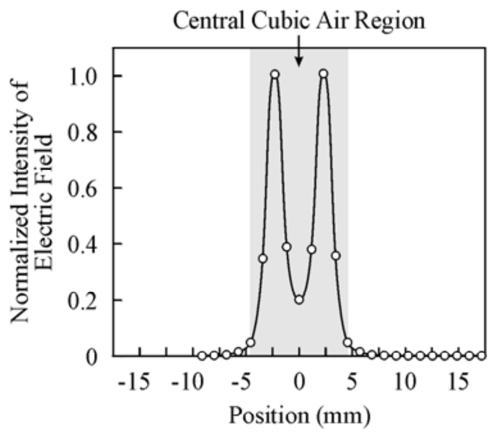

Fig. 4 Spatial intensity profile of the electric field at 12.8 GHz along the central air rod axis of the stage 3 epoxy Menger sponge. The electromagnetic wave at 12.8 GHz is strongly localized within the central cubic region. The electric field was measured to the right, outside of the Menger sponge.

Fig.4 shows the spatial distribution of the electric field at 12.8 GHz in the stage 3 Menger sponge, which was measured at positions along the central air-rod by a probe method using two mono-pole antennae.

A mono-pole antenna for emission was placed at the same position in the case of the transmission measurement and another antenna was inserted into the largest air rod at the center. The electrical field intensity shows a typical double maximum curve symmetric for the center point. Apparently this mode is strongly localized almost within the central largest cubic air-cage region and the electric field decreases abruptly outside the central air cube. The localized mode has a cubic-shell shape whose edges and corners are rounded. **Fig.5** shows the normally incident reflection and transmission spectra of the stage 3 Menger sponge made of 10vol.% TiO_2-SiO_2 dispersed epoxy. Both sharp attenuations for reflection and transmission to -40dB were observed at about 8 GHz.

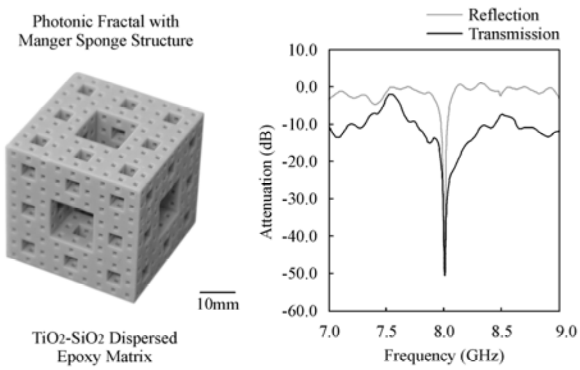

Fig. 5 Photo of a stage 3 Menger sponge fractal made of TiO_2-SiO_2 particles dispersed in epoxy, and the reflection and transmission spectra.

These localizations of electromagnetic waves in the 3D fractal structures can not be explained by Bragg deflection and band gap formation in photonic crystals [10, 11]. Probably such localization occurs due to the interference of multiple reflections in the fractal structure. Though the localization mechanism of electromagnetic waves in the dielectric Menger sponge structure is not clarified yet, we succeeded in deriving the following empirical equation to predict the frequencies of the localized modes in the Menger sponge [12]. This is the extended equation to high order localized modes from the first equation, $\lambda_{conf} = 2^{\ell}/3\, a\sqrt{\varepsilon_{eff}}$, derived in our previous study [7].

$$\lambda_{conf} = 2^{\ell}/3\, a\sqrt{\varepsilon_{eff}}/S^{2\ell-1} \qquad (1)$$

where λ_{conf} is the wavelength of the localized mode in air. ℓ is the order number of the localized modes. The effective dielectric constant ε_{eff} is the spatially averaged dielectric constant of the Menger sponge structure.

The filling factor f of the remaining parts of the n-dimensional fractal like the Menger sponge of stage m is expressed by $f=(N/S^n)^m$. Thus, the filling factor of the dielectric medium in the stage 3 Menger sponge is $(20/27)^3$. The effective dielectric constant ε_{eff} is given by the equation $\varepsilon_{eff} = f\varepsilon_A + (1-f)\varepsilon_B$, where ε_A and ε_B are the dielectric constants of the remaining and extracted parts, respectively. For a Menger sponge of epoxy, $\varepsilon_A = \varepsilon_{epoxy}=2.8$, and $\varepsilon_B = \varepsilon_{air}=1$. Estimated from the above equation, the effective dielectric constant of the stage 3 epoxy Menger sponge is 1.73. When the parameters of a=27mm, ε_{eff} =1.73, ℓ =1 are substituted into equation (1), we can obtain λ_{conf}=23.6mm. The frequency of this localized mode corresponds to 12.7 GHz, which well agrees with the measured frequency.

In the case of the TiO_2-SiO_2 dispersed epoxy Menger sponge, ε_A=8.8. Therefore, the effective dielectric constant is 4.17. The wavelength for ℓ =1 is calculated to be 36.7 mm and the corresponding frequency is 8.1 GHz. This calculated frequency coincides with the measured frequency as well. The frequencies of the second order modes for the stage 3 Menger sponge of the epoxy and the TiO_2-SiO_2 dispersed epoxy are calculated to be 57.0 GHz and 36.8 GHz, respectively. Unfortunately, these frequencies are out of the measurable range in this experiment. We infer that the major energy of higher order localized modes are concentrated in the central air cube, however it is necessary to investigate their intensity profiles in the Menger sponge structure to confirm this.

4. Integration of Photonic Fractals

It is necessary to integrate photonic fractals for certain applications. **Fig.6** shows the reflection and transmission spectra for a larger sample of the stage 4 Menger sponge structure composed of 10 vol.% TiO_2-SiO_2 dispersed epoxy, which were measured by using two horn antennae. Both reflection and transmission show sharp attenuations at 14 GHz.

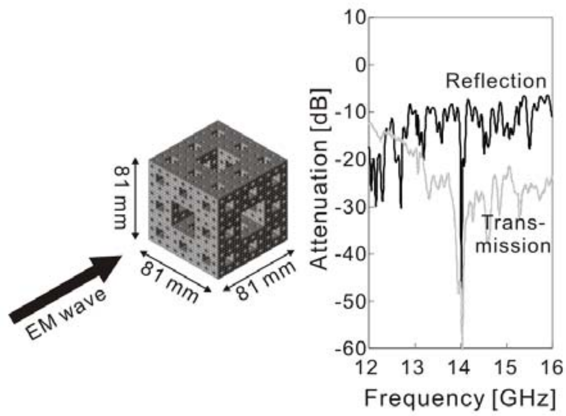

Fig. 6 Schematic illustration of a stage 4 Menger sponge fractal made of TiO_2-SiO_2 particles dispersed in epoxy, and the corresponding reflection and transmission spectra.

This frequency coincides with the calculated one for the second order mode (ℓ =2). Probably the first order mode at 3.0 GHz was not detected because such a lower frequency is out of the measurable range in this study. **Fig. 7** shows an integrated structure which shares 1/3 part of a stage 4 Menger sponge and its electromagnetic wave responses. In this case, a sharp confinement appears at the same frequency of 14 GHz as the single stage 4 Menger sponge. However, when stage 4 Menger sponges were simply integrated without structure sharing, no attenuation in reflection and transmission appeared. Probably, the simply integrated structure has no self-similarity and changes to the periodic structure and this results in the loss of the localization function of electromagnetic waves. These results suggest that it is important for integration of photonic fractals to combine them without producing periodicity.

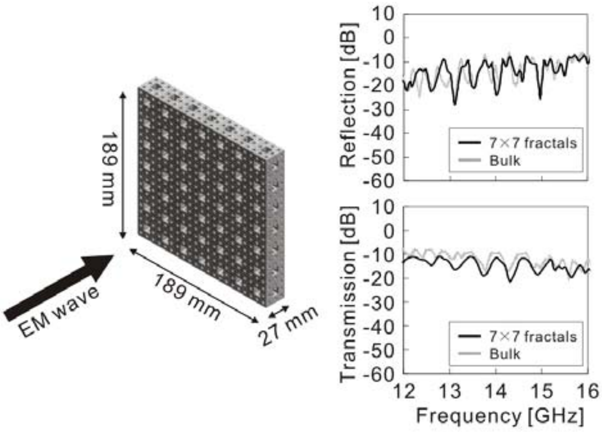

Fig. 7 Schematic illustration of the 3x3 wall structure which shares 1/3 of a stage 4 Menger sponge fractal made of TiO_2-SiO_2 particles dispersed in epoxy, and the reflection and transmission spectra.

5. Localization of Microwaves in 3D Metal Fractals

We fabricated Menger sponge structures with metals and found similar localization of microwave to the same structures of dielectric materials. **Fig.8** shows a stage 3 Menger sponge of a Bi-Pb-Sn alloy with the melting point of 70℃.

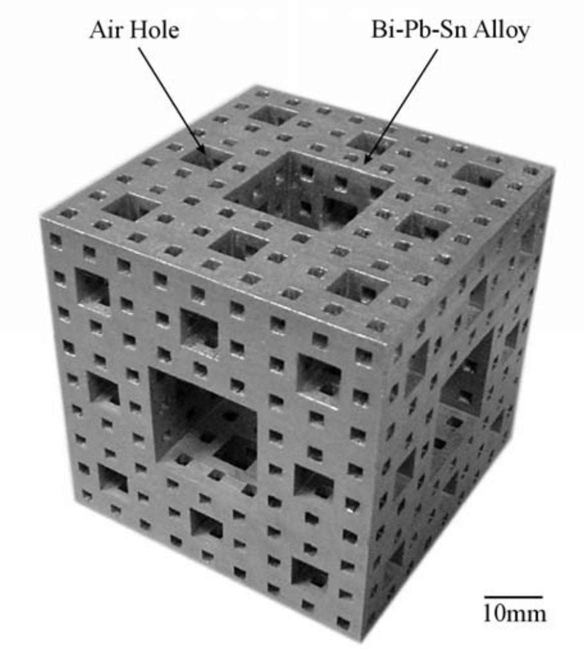

Fig.8 Photo of a stage 3metal Menger sponge structure.

The edge length is 81mm. It was fabricated by vacuum casting the molten alloy into the epoxy mold made by stereolithography. The metal fractal structures were well formed without distortions, cracks, and pores. Very sharp attenuations of transmission and reflection appeared at the same frequency of about 15 GHz as seen in Fig.9. The strong intensity profile of the electric field at 15GHz was observed in the central air cube similar to that seen in Fig.4. Stage 2 and 4 metal Menger sponge samples with the same sizes as the stage 3 showed more shallow and deep dips, respectively, at the same 15GHz, though the stage 1 sample showed no localization.

P. Sheng and his co-workers reported on anomalous behaviors in reflectance and transmittance of microwaves for 2D Cu metal fractals [6]. While, Shalaef's group found extremely strong localizations of light for random fractal films of nano-sized Ag metal particles [5]. In the case of dielectric fractals, the localized wavelength and frequency depended on the effective dielectric constant which changes with the stage number even though the size and material of the Menger sponge structure are the same. While the metal Menger sponges showed no dependence of the localized frequency on the stage numbers. The dielectric constant of metal is negative and electromagnetic waves can not transmit through metal and totally reflect at the surface. **Fig.9** shows the existence of a cut-off frequency at about 6GHz.

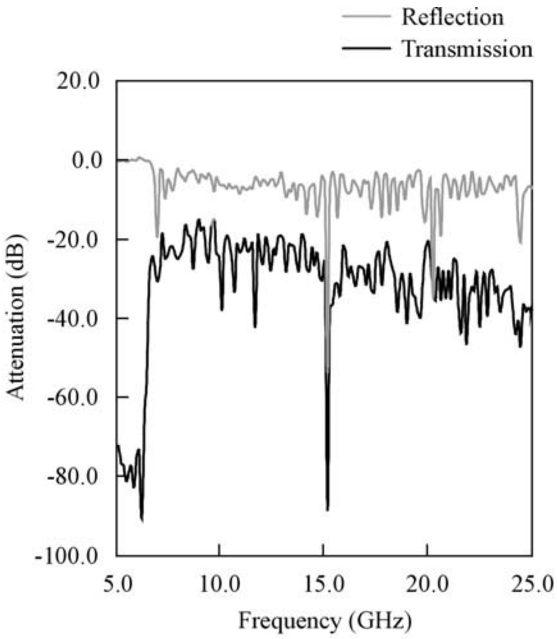

Fig.9 Reflection and transmission spectra of the stage 3 metal Menger sponge.

It corresponds to the wavelength of 50mm. The edge length of the largest center air rod is 27mm near the half wavelength. While, the wavelength of the localized mode is about 20mm. The localized modes with the 2/3 and 1/2 wavelength may exist in the largest (27mm) and second (9mm) air cubes, respectively. We can not explain yet why electromagnetic waves localize in the Menger sponge fractals, but it seems true that the 3D fractal structure plays an essential role because both dielectric and metal fractal structures can produce the localization of electromagnetic waves.

Potential Applications of Photonic Fractals

The localization function of electromagnetic waves in 3D fractal structures is applicable to various fields of information, communication, energy, medical care, etc. Fractal structures such as Menger sponges can provide ideal absorbers without reflectance and transmission for electromagnetic waves. Of course, when the fractal structure is made at optical wave scales, the light can be localized and confined and used for various photonic applications. It may be possible to localize and accumulate the electromagnetic energy in the 3D fractal structure, if the structure is composed of an efficiently low loss material and the number of stages m is sufficient to reduce the medium of the fractal. Such energy accumulation in the 3D fractal structure may provide various new heat treatment devices for industrial, home and medical uses depending on the power. If high efficiency in energy accumulation is realized, photonic energy storage, capacitor, and collectors could be made.

6. Summary

The localization function of electromagnetic waves in 3D dielectric fractal structures and its potential applications are demonstrated. The obtained results can be summarized as follows.

1. The dielectric and metal Menger sponge structures can strongly localize the incident electromagnetic wave with a specific wavelength.
2. The wavelength and frequency of the localized modes in dielectric Menger sponges can be predicted using an empirical equation associated with the fractal geometry and spatially averaged dielectric constant, thus the frequency of the localized modes can be designed. For metal Menger sponges, the localized frequency may be determined mainly by the geometrical factors.
3. It is important to combine fractal structures without producing a periodic structure for integrating photonic fractals in order to keep the localization function of electromagnetic waves. The periodic arrangement of fractals cannot localize electromagnetic waves.
4. Photonic fractals have a large potential for applications in information, communication, energy, medical and other various fields.

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