Initial Fatigue Crack Growth Behavior in a Notched Component (Report V)†

—Assessment of Initial Defect in Respect to Fatigue Life of a Notched Component—

Kohsuke HORIKAWA* and Sang-moung CHO**

Abstract

The present report is intended to apply Elasto-Plastic Fracture Mechanics(EPFM) for assessment of initial defect and for fatigue design in welding steel structures.

Considering fatigue design methods of welding steel structures, assessment methods of initial defect in notch field were classified into three cases.

In the Case 1 of fatigue design based on classification of joints, when quality level to initial defect was specified with class of joints concerned, safety factor to fatigue life varied with configuration of component.

In the Case 2A that fatigue design was based on classification of joints and frature mechanics was applied for assessment of initial defect, when allowable size of initial defect was assessed by ΔJ *and experiment, the size depended upon stress level and materials(SS41 & HT80).*

And in the Case 2B of fatigue based on EPFM to consider the previously determined size of initial defect, it could be confirmed that to raise section (or to lower sectional force) was beneficial when fatigue strengh in low cycle region was significant, and to lower tensile residual stress was beneficial when fatigue strength in high cycle region was significant.

KEY WORDS : (Welding Streel Structure)(Fatigue, Fatigue Design)(Defect)(Notch)(Fatigue Life) (Fracture Mechanics)(Elasto-Plastic Fracture Mechanics)(ΔJ)

1. Introduction

In welded steel structures subjected to cyclic load, it is needed to consider that initial defect grows, reaches the critical size and ultimately leads to final unstable fracture[1-3]. As the means to prevent the unstable fracture, Elasto-Plastic Fracture Mechanics(EPFM) is sometims applied[4-6]. It can be said that this approach is used to predict properly the critical crack size. In this case, allowable size of initial defect $\mathbf{a}_i$ can be, when $\Delta \mathbf{a}$ is referred to the crack length propagated by cyclic load during design life, determined as foollows.

$$\Delta \mathbf{a}_i < (\mathbf{a}_c - \Delta \mathbf{a}) \qquad (1)$$

Where, $\mathbf{a}_c$ is obtained by EPFM, but $\Delta \mathbf{a}$ is calculated by Linear Elastic Fracture Mechanics(LEFM) in some cases[4-6]. That is to say, to determine one parameter $\mathbf{a}_i$, two procedures of LEFM and EPFM are applied.

Therefore, in the previous reports of the present study[7-10], to calulate $\Delta \mathbf{a}$ also by EPFM, crack extension force ΔJ was evaluated in notch field. Futhermore it's applicability was examined, and. practically confirmed by a series of experiments of the present study[8-10].

The present report was intended to apply EPFM for assessment of initial defect and in fatigue design. The fundamental approach for it's application was proposed. And as shown in **Fig.1**, the assessment methods of initial defect were classified in consideration of fatigue design methods of welding steel structures as follows, and examined respectively.

Case 1 : Assessment of initial defect by specified level.

Case 2 : Assessment of initial defect by fracture mechanics.

Case 2A : Assessment of initial defect in the component designed by joint classification

Case 2B : Assessment of initial defect in the stage of fatigue design by fracture mechanics.

2. Specimens and Experimental Procedure

The data obtained by fatigue tests in our previous studies[8-10] were adjusted into fatigue life, and used in the present report. The materials considered in the present report were mild steel (SS41) and high tensile strength steel (HT80). Experimental results by companion specimens method ($R_\varepsilon = -1$) using smooth specimens in Re-

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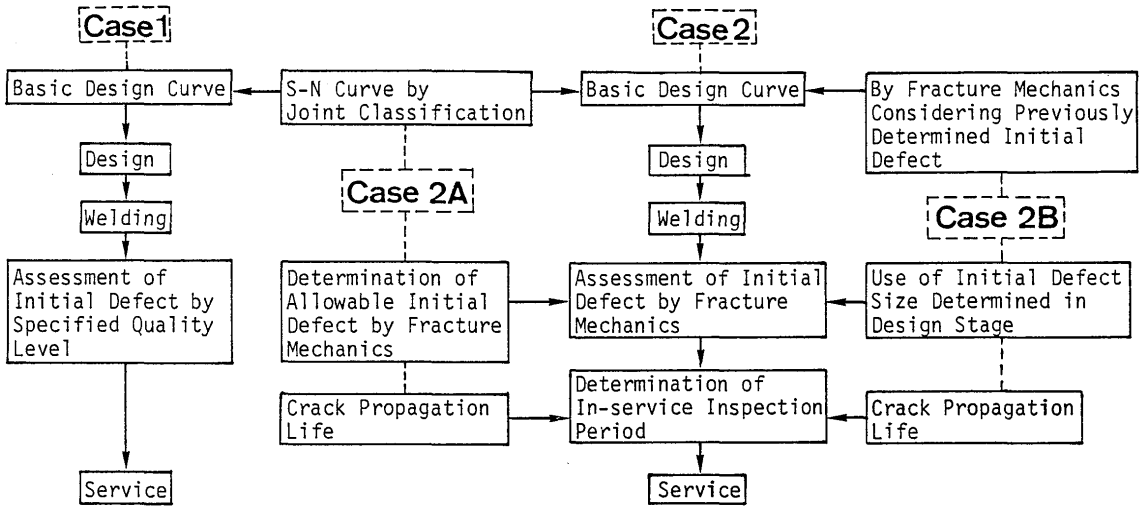


Fig. 1 Flows on assessment of initial defect

port III[9] were arranged into failure life curves. And experimental results by fatigue crack propagation tests (R=−1) using 3 kinds of center notched specimens in ReportIII, IV[9,10] were also arranged into fatigue life curves. In **Table 1**, the codes and details on 3 kinds of center notched specimens are shown.

Table 1 Codes of center notched specimens

Code	Remarks
C2.5	Circular hole($2a_0$=5.0, ρ=2.5, K_t=2.7), 2B=36
E0.25	Elliptical hole($2a_0$=5.0, ρ=0.25, K_t=6.0), 2B=36
C3.5	Circular hole($2a_0$=7.0, ρ=3.5, K_t=2.7)in tensile residual stress field, 2B=50

3. Assessment of Initial Defect by Specified Level

In this chapter, Case 1 in Fig.1 was investigated, namely initial defect was assessed by specified level, but not treated with fracture mechanics.

Welding steel structures are frequently designed by fatigue strength curves, which are determined according to joint classification[11-14]. **Figure 2** shows some faqigue strength curves[11], and this figure inidicates that fatigue strength curves vary with joint class. In Fig.2, Class B, Class C and Class F are different in specified quality level to initial defect as well as shape of joint. That is to say, in order to use a fatigue strength curve determined by joint class for fatigue design, specified quality level to initial defect in the joint must be satisfied.

These joints may be often used in structural discontinuity(Notch) as shown in **Fig.3.** In the present report, the joint in Fig.3 was assumed to be Class B in Fig.2, which was referred as plain steel in the following.

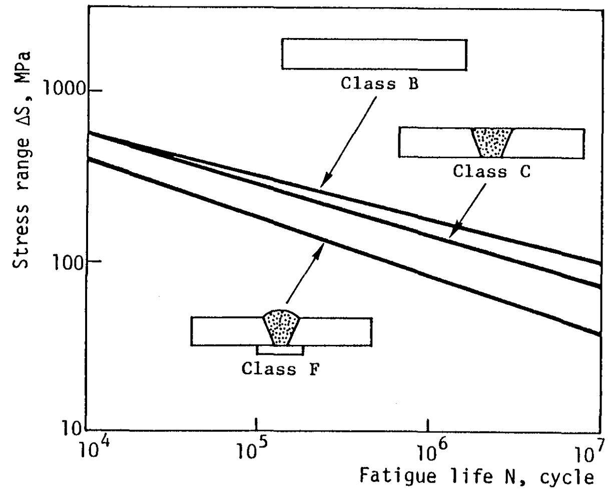


Fig. 2 Comparision of basic design curves to depend on joint class.

3.1 Fatigue strength of plain steel and specified quality level to initial defect

In this section, fatigue strength curve of plain steel was stated. It was regarded that the plain steel satisfy specified quality level defined as to be free from defect[11-13].

Figure 3 shows the structural discontinuity (Notch) subjected to net sectional stress range ΔS. In order to evaluate fatigue life for the notched component, mechanical quantity K_t ΔS may be used[11,12]. If the quantity K_t ΔS becomes higher than two times of cyclic yield strength, the quantity is not defined as stress itself. In this case, if Neuber's rule is applied and the joint in notched component of Fig.3 is supposed to be plain steel, K_t ΔS can be interpreted as mechanical meaning as follows[15].

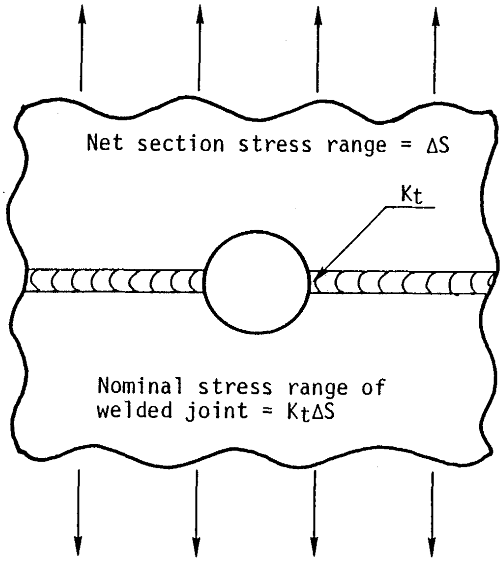


Fig. 3 Explanation of nominal stress range when welded joint is situated in region of structural discontinuity

$$K_t\Delta S=\sqrt{K_\sigma \Delta S K_\varepsilon \Delta S}=\sqrt{E\Delta\sigma\varepsilon} \qquad (2)$$

where, K_σ, K_ε : stress and strain concentration factor

$\Delta\sigma$, $\Delta\varepsilon$: stress and strain range in smooth plate or notch tip

The application of Eq.(2) allows one to be able to use $\sqrt{E\Delta\sigma\Delta\varepsilon}-N_f$ relation of plain streel for evaluation of fatigue life in notched component. Fatigue strength reduction factor K_f (or notch factor) is sometimes substituted for the factor K_t in Eq.(2)[15-18]. But, in the present report, the factor K_t itself was used to evaluate fatigue life of notched component as shown in Fig. 3.

Figure 4 shows the relation of $\sqrt{E\Delta\sigma\Delta\varepsilon}-N_f$, that was obtained from axial strain control fatigue test ($R=-1$) for plain steel of SS41 and HT80. The stress range $\Delta\sigma$ was taken as the value at the half of failure life. The experimental results on SS41 (○ marks) could be approximated to a straight line. This straight line is nearly the same that B-curve of UKDOE is shifted in parallel toward the side of high strength by 〈3 X Standard deviation〉 , and extended up to lower cycles than 10^4 cycles. In some fatigue designs for welding steel structures, the difference of static strength is not reflected in fatigue strength[11-13]. Therefore, the fatigue stregth curve of HT80 is also assumed to be it of SS41(solid line in Fig.4).

While the B-curve is the fatigue strength curve for plain steel which is free from defect. Stating this in reverse, in order to employ the solid line in Fig.4 for evaluation of fatigue life, specified quality level defindd as to be free from defect is required.

3.2 Fatigue strength of notched component and examination for quality level to initial defect

In this section, when plain steel was used as a kind of joint in notched component, fatigue strength of the notched component was evaluated.

Figure 5 indicates the fatigue life evaluated for the notched components of E0.25 and C2.5 Basic fatigue strength curve (solid line in Fig.5) used for the life prediction of the component was the failure life curve of plain steel shown in Fig.4. Experimental results of ▲△marks were the number of cycles from notch tip to the crack length of 0.1mm. In Fig.5. if the solid line of basic fatigue strength curve of the notched component is employed for the life prediction, it can be regarded that fatigue life up to the crack length of 0.1mm is evaluated. However, in the plain steel of $K_t=1.0$, as known in Fig.4, failure life is evaluated. In this way, when fatigue life is evaluated by

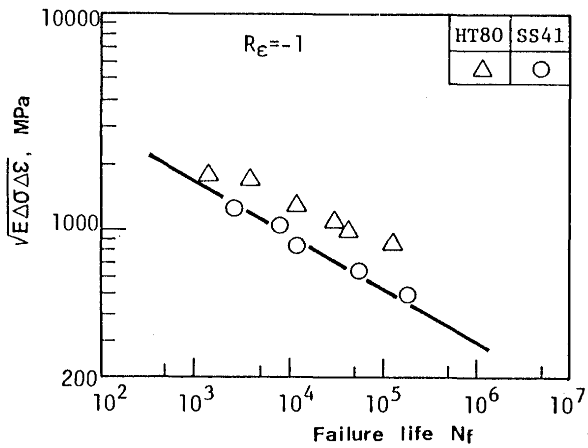


Fig. 4 Relatipn of $\sqrt{E\Delta\sigma\Delta\varepsilon}$ and failure life of smooth specimens by strain control fatigue tests

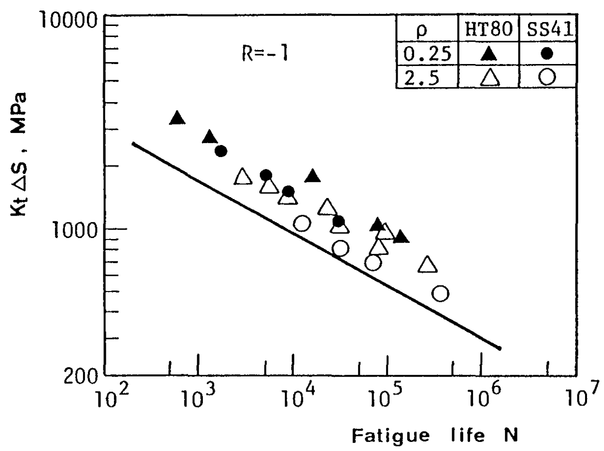


Fig. 5 Relation of K_t ΔS and fatigue life N in notched specimens

basic fatigue strength curve and local stress, the criterion of the crack length is not defined as to be constant, varies with the configuration of component. In other words, it is implied that even though a basic fatigue strength curve for an identical joint with a specified quality level is used for the evaluation of fatigue life, safety factor to life varies with the configuration of component in which the joint is included.

Therefore, if to keep up the same safety factor to life is desirable, quality level to initial defect in a joint must not be specified with an identical level, but must be determined in consideration of the configuration of component and the difficulty of welding process.

4. Assessment of Initial Defect by Fracture Mechanics

In this chapter, Case 2 in Fig.1 to assess initial defect by fracture mechanics, was stated.

In order to evaluate the effect of initial defect on fatigue life, fatigue crack propagation life must be evaluated properly by appling fracture mechanics. And rationality and validity on defect assessment may depend upon the evaluating accuracy of fatigue crack propagation life. Therefore, first of all, fatigue crack propagation life was evaluated bv ΔK, ΔJ and experiment. Next, Case 2 was classified into Case 2A and Case 2B as shown in Fig.1. And assessment of initial defect was carried out for each case. Moreover, for Case 2B the approach for fatigue design considering initial defect was examined.

In this Case 2, because it is regarded that initial defect grows during design life of component, in-service inspection to consider growth property of the defect is required.

4.1 Evaluation of fatigue crack propagation life in notched component

In this section, the influence of ΔK and ΔJ on the evaluation accuracy of crack propagation life in notch field was investigated by comparing with experimental results. The used crack propagation rules were as follows.

$$da/dn = C(\Delta K^{m} - \Delta K_{th}^{m}) \quad , \ \mathrm{m/cycle} \qquad (3a)$$

$$da/dn = A(\Delta J^{B} - \Delta J_{th}^{B}) \quad , \ \mathrm{m/cycle} \qquad (3b)$$

Crack opening ratio U was considered in calculation of ΔJ only. When stress ratio was $R = -1$, material constants used in Eq.(3) for SS41 and HT80 were indicated on the case with and without tensile residual stress, as shown in **Table 2.**

Figure 6 (a) (b) show the crack propagation life N_p from initial crack length $\mathbf{a}_1$ to critical crack length $\mathbf{a}_c$ in C2.5 of two materials. The evaluation of N_p was done by numerical integral using ΔK and ΔJ, and by experiments. The quantity $K_t\ \Delta S$ on vertical axis in Fig.6 was the value calculated in disregard of defect in notch field. And the bold solid line to mean a strength curve needed for the notched component, was temporarily given by the basic fatigue strength curve in the component shown in Fig.4 and Fig.5. (a) was the results for SS41, ○● marks were experimental results. In (a), the broken line was obtained by ΔK, and the fine solid line was calculated by ΔJ. In the case that to be $\mathbf{a}_i = 0.1$mm was presumed, there was great difference between crack propagation life calculatedby ΔK and ΔJ. But in the case of $\mathbf{a}_i = 2.0$mm, crack propagation life did not become very different with the evaluation method. On the whole, the results evaluated by ΔJ rather than ΔK coincided well with experiments. (b) was the results for HT80. Also in this case, the fine solid line by ΔJ showed good correspondence with experimental results. From Fig.6, it can be mentioned that the higher loading stress is, and the shorter the initial crack length is, the more the difference between the life evaluated by ΔK and ΔJ arise.

In **Fig.7**, the effect of tensile residual stress on crack propagation life N_p was examined for C3.5 of SS41. The life N_p was the number of cycles from $\mathbf{a}_i = 0.2$mm to $\mathbf{a}_c = 3.5$mm. Experimental result (○ marks) were obtained from the case that the peak tensile residual stress at notch tip was $\sigma_{res} = 255$MPa. The fine solid lines mean the trend of N_p evaluated by ΔJ, and it is recognized to coincide very well with experimental results. For comparision, the life N_p for C3.5 of HT80 was also evaluated on two cases of $\sigma_{res} = 294$MPa and $\sigma_{res} = 0$, and showed in Fig.7. The effect of residual stress on N_p is not very considerable in the low cycle (high stress range) region. But it can be known that the effect is very remarkable in the high cycle (low stress range) region. As the results, the slope of life curve when tensile residual stress is present become greater than that when residual stress is not present.

In consequece, in the case that tensile residual stress is present as well as the case that residual stress is not, crack

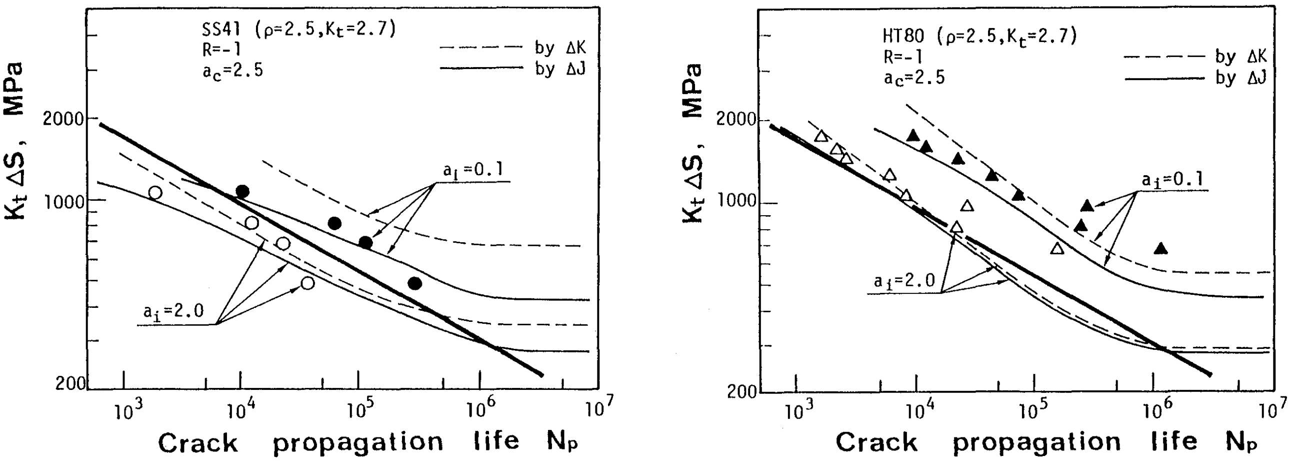
Table 2 Material constants for crack propagation($R = -1$)

(a) $\sigma_{res} = 0$

	C	m	ΔK_{th} (MPa√m)	A	B	ΔJ_{th} (J/m2)
SS41	1.2×10^{-13}	3.7	14.0	7.5×10^{-13}	1.6	85
HT80	1.0×10^{-12}	2.9	10.8	4.6×10^{-12}	1.3	72

(b) $\sigma_{res} > 0$

	A	B	ΔJ_{th} (J/m^2)
SS41, σ_{res}=255 MPa	7.5×10^{-13}	1.6	36
HT80, σ_{res}=294 MPa	4.6×10^{-12}	1.3	28



(a) SS41 (b) HT80

Fig. 6 Variation of crack propagation life from a_i to a_c in notch field (C2.5)

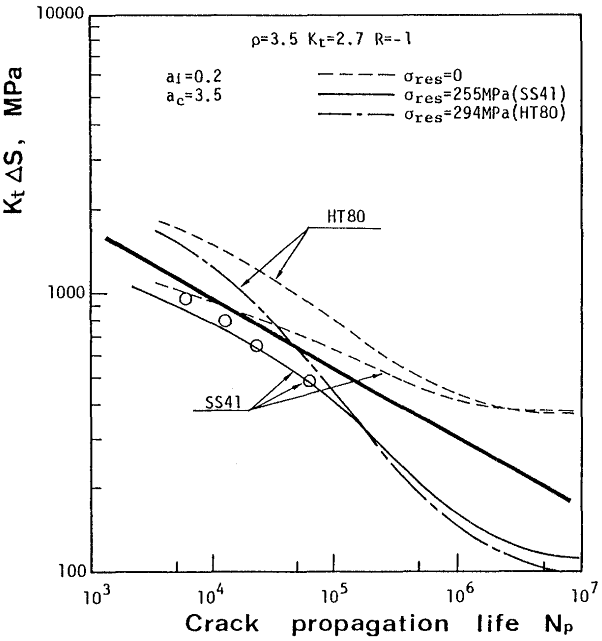


Fig. 7 Effect of residual stress on crack propagation life in notch field (C3.5)

propagation life in notched component could be evaluated with accuracy by appling ΔJ in the present study.

4.2 Assessment of initial defect in the component designed by joint classification

In this section, Case 2A in Fig.1 was stated.

If quality level to initial defect in a joint is required by specified level to be constant in any where, safety factor to fatigue life varies with the configuration of component as mentioned in Section 3.2. By the way, as known from $a_i = 0.1$mm in Fig.6 (a), even though the same defect size is existent in the same component, the crack propagation life N_p satisfies a strength curve needed in the component (bold solid line in the figure) in high cycle region, but does not in low cycle region. Comparing (a) with (b) in Fig..6, in (b) of HT80 the life N_p satisfies the basic strength curve in the case of $a_i = 0.1$mm, but in (a) of SS41 the life N_p does not in the same case. Accordingly, even though quality level to initial defect in the same component is satisfied with specified level to be constant, safety factor to fatigue life varies with material and loading stress level of the component. In reverse, if safety factor to fatigue life is kept up with constant level, allowable quality level to initial defect vary with the configuration, material and stress level of the component.

Therefore, in order to make crack propagation life N_p identical with a strength curve, namely in order to make safety factor to the life to be 1.0, allowable size of initial defect a_i was evaluated, shown in **Fig.8.** ○□ marks in Fig.8 are the experimental results. The broken lines and solid lines are the trends obtained by Δ K and Δ J respectively. In the case of using Δ J for SS41 in which elasto-plastic effect appears remarkably, initial defect is hardly allowed when the quantity K_t Δ S is higher than 1000MPa. As the whole trend of Fig.8, the allowable size of initial defect is evaluated to be smaller when Δ J is applied rather than ΔK. And in high stress region, larger initial defect in HT80 than SS41 is allowed. But in low stress region, it can be known that the state is reversed.

As a result, when this design method is used, it may be more reasonable that basic fatigue strength curve is determined in consideration of material property in the form of coefficient as well as the shape of joint.

The above matters may be the approach to be able to consider when defect with lower quality level than specified level is detected in the component designed by application of joint classification and fatigue strength curve. Therefore, this approach can be considered in the stage quality assurance after welding.

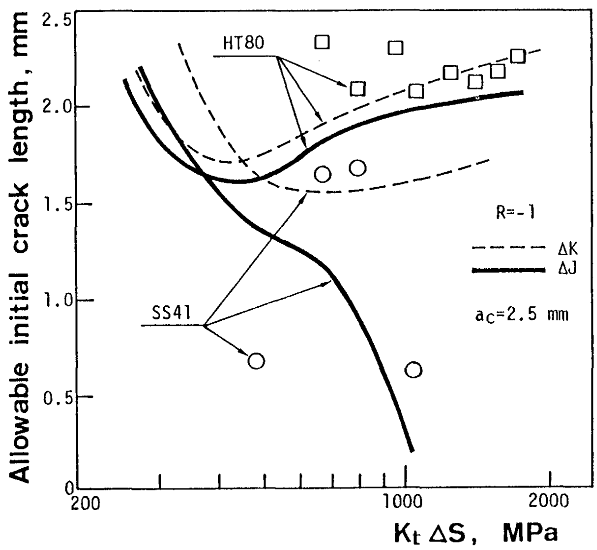


Fig. 8 Allowable initial crack length assessed by ΔK, ΔJ and experiment

4.3 Assessment of initial defect in the stage of fatigue design by fracture mechanics

In this section, Case 2B in Fig.1 was investigated.

According to welding procedure and the preision of nondestructive test, initial defect size $\mathbf{a}_i$ to be potential in welded joint may be provided. By supposing the defect size $\mathbf{a}_i$ in design stage, design stress and residual stress can be determined so that the crack propagation life N_p from $\mathbf{a}_i$ to critical crack size $\mathbf{a}_c$ is equal to design life of the component. In the folllowing, it was presumed that $\mathbf{a}_i$ and $\mathbf{a}_c$ was 0.5mm and 3.5mm respectively. And the effect of component size considered in design stage and of residual stress related to welding procedure on the life N_p was investigated.

Figure 9 shows the effect of component size on the life N_p when sectional force to act in the component is fixed. The original size of the component was assumed to be the size of C3.5. And thickness t only was changed as the typical measure of the component. Accordingly, load range on vertical axis in Fig.9 was the range of sectional force to act in net section of C3.5 with original size (43mm net section breadth × 5mm thickness). In this case, residual stress was assumed to be not present, and the life N_p was evaluated by ΔJ. As the fatigue strength curve needed in the component, the basic fatigue strength curve in Fig.4 and Fig.5 was temporarity used as shown by bold solid line in Fig.9.

In the scope of fatigue life shown in Fig.9, when the thickness is greater than about 7.2mm in SS41 and about 5.0mm in HT80, the life N_p in the component exceeds the needed strength curve. HT80 become superior to SS41 in the low cycle region to depend on cyclic yield strength of material. But, in the high cycle region to depend on ΔJ_{th},

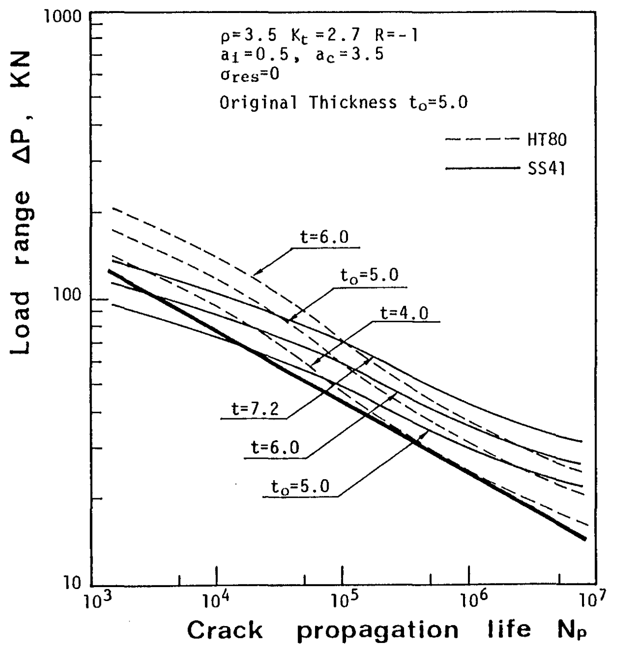


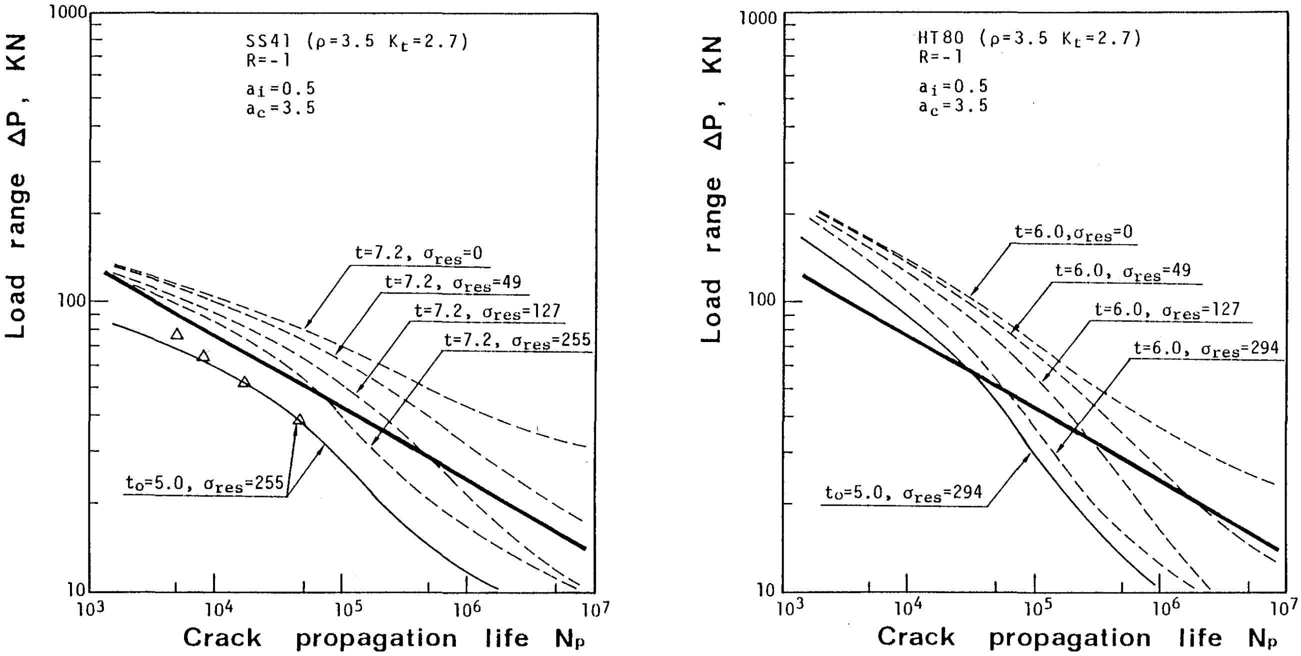
Fig. 9 Assessment for effect of plate thickness on crack propagation life in notch field (C3.5)

SS41 is superior to HT80 because ΔJ_{th} of SS41 is generally higher than it of HT80.

In Fig.9, it was considered to decrease the stress acting in the component. Besides, it may also be available to decrease sectional force itself by attaching cover plates or ribs.

Figure 10 (a) (b) show the effect of residual stress related to welding procedure on the life N_p. This investigation must be carried out in the stage of design. The life N_p was evaluated by using ΔJ which had been calculated in Report IV in consideration of residual stress. △marks in (a) of SS41 are the experimental results for C3.5 with 5.0mm original thickness. The bold soliid line in Fig.10 was also used temporarity as the fatigue strength curve needed in the component. When residual stress was kept under 49MPa, in the case of SS41 with t=7.2mm the life N_p satistied the fatigue strength curve needed in the component. In Fig.10 (b), crack propagation life N_p for HT80 was evaluated in the same way. When residual stress of 49MPa is present in C3.5 HT80, thickness is required to be greater than 6.0mm. However, in the region below 10^5 cycles, the residual stress of 127MPa can be permitted to be present although the same thickness of 6.0mm is kept.

From the above Fig.9 and Fig.10, in the case that initial defect size $\mathbf{a}_i$ is previously determined, it could be confirmed that to raise section (or to lower sectional force) was reasonable when fatigue strength in low cycle region was important, and to lower tensile residual stress was reasonadle when fatigue strength in high cycle region was important.



(a) SS41 (b) HT80

Fig. 10 Assessment for effect of residual stress on crack propagation life in notch field (C3.5)

Because tensile residual stress is not completely released by post-heat treatment, this has to be considered in the stage when sectional size is determined. Accordingly, it may be stated that the determination of post-heat treatment condition and sectional size must be judged by external force, characteristics of structure, environment, etc..

In consequence, when this Case 2B is applied, greater defects than the size determined in design stage have only to be detected in inspection stage after welding process.

In the following, in-service inspection was brifly discussed.

When the Case 1 is applied, even if there is the initial defect to satisfy the allowable size in the welded joint, the relation between the growth property of the defect and the life of component is not grasped. Therefore, it vs difficult to determine the period and the location of in-service inspection in consideration of the relation.

While, when the Case 2 is applied, the growth property of initial defect can be predicted at the beginning of service. If the predicted growth property of the defect coincides perfectly with the pratical case, in-service inspection is not needed. However, because of various errors to result from the prediction of external force, the evaluation of environmental effect, structural analysis, etc., the defect may not grow as predicted. And so, the in-service inspection may have significance to confirm whethere the defect grows as predicted or not. Moreover, the cause for the difference between prediction and practicce can be investigated by the in-service inspection. By using the results of the investigation, the growth property of the defect after the inspection can be predicted anew.

5. Conclusions

Taking account of fatigue design of welding structures, the assessment methods of initial defect were classified into three cases, and the relative merits of each case were examined. In order to carry out fatigue design considering the growth property of initial defect in notch field, elasto-plastic fracture mechanics was applied, and the results by the application were compered with the experiment results.

The obtained results are as follows.

(1) When fatigue design was performed by using local stress based on joint classification, safety factor to fatigue life varied with configuration of the component. Therefore, it is desirable to determine quality level to initial defect in a joint in consideration of configuation of the component and difficulty of welding process.(Fig.4 &Fig.5)

(2) Initial crack propagation life was evaluated by ΔK and ΔJ in notch field, and differenc between the two parameters became large when initial crack length was short and when high stress was loaded (in low cycle region). The case by ΔJ corresponded well to the experimental results.

And it was confirmed that also initial crack propagation life in tensile residual stress field could be evluated with accuracy by ΔJ. Therefore, it is conservative to evaluate the prapagation life of small defect in notch field by appling ΔJ. (Fig.6 & Fig.7)

(3) In the case of fatigue design to be based on joint classification and to take local stress, allowable size of initial defect was assessed by ΔJ and expreriment in notch field. The difference of the allowable size between SS41 and HT80 was remarkable in high stress (low cycle) region. Therefore, when this design method is applied, it is reasonble to determine a basic fatigue strength curve in consideration of material property in the form of coefficient as well as the shape of joint. (Fig.8)

(4) In the case of fatigue design by frature mechanics, by appling elastoplastic fracture mechanics, it could be confirmed that to raise section (or to lower sectional force) was beneficial when fatigue strength in low cycle region was significant,and to lower tensile residual stress was beneficial when fatigue strength in high cycle region was significant.(Fig.9 & Fig.10)

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