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ON THE SEQUENCES INDUCED FROM AUSLANDER-REITEN SEQUENCES

Dedicated to Professor Hiroshi Nagao on his 60th birthday

KATSUHIRO UNO

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0. Introduction

Let kG be the group algebra of a finite group G over an algebraically closed field k of characteristic p , $p \neq 0$. Fix a normal subgroup N of G and a non-projective indecomposable kN -module V . Let $SV: 0 \rightarrow \Omega^2 V \rightarrow X \rightarrow V \rightarrow 0$ be the Auslander-Reiten sequence terminating at V . Here Ω denotes the Heller operator. In this paper, we study the induced sequence $0 \rightarrow (\Omega^2 V)^G \rightarrow X^G \rightarrow V^G \rightarrow 0$. We shall decompose it according to the decomposition of V^G and investigate the relation between the sequences appearing in the decomposition and the Auslander-Reiten sequences terminating at the indecomposable direct summands of V^G . For example, we shall give a condition which guarantees that some Auslander-Reiten sequences appear in the decomposition of the induced sequence. This result is related to the work of Knörr [6].

Notation is standard. All the kG -modules considered here are finite dimensional right modules. For kG -modules W and W' , we use $(W, W')^G$ to denote $\text{Hom}_{kG}(W, W')$. An element f of $(W, W')^G$ is said to be projective if there are a projective kG -module P and maps $\alpha \in (W, P)^G$ and $\beta \in (P, W')^G$ such that $f = \beta \circ \alpha$. We denote by $(W, W')^{1,G}$ the factor space of $(W, W')^G$ divided by the subspace consisting of projective homomorphisms. Note that $(W, W')^{1,G}$ is an $\text{End}_{kG}(W')$ - $\text{End}_{kG}(W)$ -bimodule. For any k -algebra R , we denote its radical by JR . Unless otherwise noted, \otimes means \otimes_{kN} .

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1. Decomposition of the induced sequence

Throughout this paper except Theorem 2.5, we deal with the situation in the Introduction. Let $E = \text{End}_{kG}(V^G)$ and $E_1 = \text{End}_{kN}(V)$. Then E_1 can naturally be considered as a subalgebra of E by the injection $\iota: E_1 \rightarrow E$ defined

by $\iota(f)=f \otimes \text{Id}_{kG}$ for all $f \in E_1$. We denote $(V^G, V^G)^{1,G}$ and $(V, V)^{1,N}$ by \underline{E} and \underline{E}_1 , respectively.

We begin with the following lemma, which is well-known and easy to see.

Lemma 1.1. $\Omega^n(V^G) \cong (\Omega^n V)^G$ for all $n=1, 2, \dots$.

Henceforth we write the above modules without parentheses.

Let P be the projective cover of V^G . For any $f \in E$, we can take $f_1 \in \text{End}_{kG}(P)$ and $f' \in \text{End}_{kG}(\Omega V^G)$ such that the following diagram is commutative.

$$\begin{array}{ccccccc} 0 & \longrightarrow & \Omega V^G & \longrightarrow & P & \longrightarrow & V^G \longrightarrow 0 & \text{(exact)} \\ & & f' \downarrow & & f_1 \downarrow & & f \downarrow & \\ 0 & \longrightarrow & \Omega V^G & \longrightarrow & P & \longrightarrow & V^G \longrightarrow 0 & \text{(exact)} \end{array}$$

In this case, f' corresponds to f under the isomorphism $\underline{E} \cong (\Omega V^G, \Omega V^G)^{1,G}$. (See the discussion following [1, 2.17.2].) Likewise we can find $f'' \in \text{End}_{kG}(\Omega^2 V^G)$ such that it corresponds to f' via $(\Omega^2 V^G, \Omega^2 V^G)^{1,G} \cong (\Omega V^G, \Omega V^G)^{1,G}$. Define left actions of \underline{E} on $(V^G, \Omega V^G)^{1,G}$ and on $\text{Ext}_{kG}(V^G, \Omega^2 V^G)$ via the above isomorphisms. Recall that we have the following. ([1, 2.17.5])

(1.2.a) $\underline{E}_1^* \cong \text{Ext}_{kN}(V, \Omega^2 V) \cong (V, \Omega V)^{1,N}$ as \underline{E}_1 - \underline{E}_1 -bimodules

(1.2.b) $\underline{E}^* \cong \text{Ext}_{kG}(V^G, \Omega^2 V^G) \cong (V^G, \Omega V^G)^{1,G}$ as \underline{E} - \underline{E} -bimodules

Here \underline{E}^* is the dual \underline{E} - \underline{E} -bimodule $\text{Hom}(\underline{E}, k)$.

The next lemma is also easy to show.

Lemma 1.3. *Let H be a subgroup of G , V_1 and V_2 kH -modules, and let $f \in (V_1, V_2)^H$. Then f is projective if and only if $f \otimes_{kH} \text{Id}_{kG} \in (V_1^G, V_2^G)^G$ is projective.*

By the above lemma \underline{E}_1 can be regarded as a subalgebra of \underline{E} . Thus \underline{E}_1^* is a submodule of \underline{E}^* . Likewise we can and will regard the modules in (1.2.a) as submodules of the modules in (1.2.b).

Lemma 1.4. *Let $\gamma \in \text{Ext}_{kN}(V, \Omega^2 V)$ represent an extension $0 \rightarrow \Omega^2 V \rightarrow Y \rightarrow V \rightarrow 0$. Then considering γ as an element of $\text{Ext}_{kG}(V^G, \Omega^2 V^G)$, it represents the induced sequence.*

Proof. Take an element f of $(V, \Omega V)^N$ whose image in $(V, \Omega V)^{1,N}$ corresponds to γ under the isomorphism (1.2.a). Then we have the following pullback diagram.

$$\begin{array}{ccc} Y & \xrightarrow{\alpha} & V \\ \beta \downarrow & & \downarrow f \\ P' & \longrightarrow & \Omega V \end{array}$$

Here P' denotes the projective cover of ΩV . The above induces the following diagram, which is also pullback.

$$\begin{array}{ccc}
 Y^G & \xrightarrow{\alpha \otimes \text{Id}_{kG}} & V^G \\
 \beta \otimes \text{Id}_{kG} \downarrow & & \downarrow f \otimes \text{Id}_{kG} \\
 P'^G & \longrightarrow & \Omega V^G
 \end{array}$$

Note that P'^G is the projective cover of ΩV^G . Thus $f \otimes \text{Id}_{kG}$ gives the sequence $0 \rightarrow \Omega^2 V^G \rightarrow Y^G \rightarrow V^G \rightarrow 0$. This completes the proof.

As E_1 is local, the \underline{E}_1 - \underline{E}_1 -bimodules in (1.2.a) have irreducible socles which are of 1-dimensional over k . We denote the socles of those modules by L . Note that $J\underline{E}_1$ annihilates L from the both sides. A nonzero element γ of $\text{Ext}_{kN}(V, \Omega^2 V)$ represents the Auslander-Reiten sequence if and only if γ lies in L . (See the proof of [1, 2.17.7].)

Lemma 1.5. $xl = lx$ for all $l \in L$ and $x \in \underline{E}$.

Proof. We fix representatives G/N of cosets of N in G containing 1. Let T be the inertial subgroup of V in G . For any $t \in T/N$, there is a kN -isomorphism $\phi_t: V \rightarrow V \otimes t$. This gives a unit $u_t = \phi_t \otimes \text{Id}_{kT}$ of $E_T = \text{End}_{kT}(V^T)$. Let \underline{E}_T be $(V^T, V^T)^{t,T}$. Note that \underline{E}_T is naturally a subalgebra of \underline{E} . (See Lemma 1.3.) We first claim that;

$$(1.5.a) \quad xl = lx \quad \text{for all } l \in L \text{ and } x \in \underline{E}_T.$$

Recall that $E_1/JE_1 \cong k$. For all $m \in E_1/JE_1$ and $t \in T/N$, we have $u_t^{-1}mu_t = m$ in E_1/JE_1 . Since L is dual to E_1/JE_1 , we have $\bar{u}_t l = l\bar{u}_t$ for all $l \in L$ and $t \in T/N$, where \bar{u}_t is the image of u_t in \underline{E}_T . We also have $ul = lu$ for all $l \in L$ and $u \in \underline{E}_1$. Thus (1.5.a) holds since \underline{E}_T is generated by \underline{E}_1 and $\{u_t\}_{t \in T/N}$.

Now note that $V^G_N = \bigoplus_{g \in G/N} V \otimes g$ as kN -modules. So by the Frobenius reciprocity, we have the following isomorphisms.

$$E \simeq (V, V^G_N)^N \simeq \bigoplus_{g \in G/N} (V, V \otimes g)^N.$$

Letting E_g be the inverse image of $(V, V \otimes g)^N$ in E , we obtain $E = \bigoplus_{g \in G/N} E_g$. (Note: our previous E_1 coincides with the new one.) Then it is easy to check that $E_g E_{g'} \subseteq E_{gg'}$ for all $g, g' \in G/N$. Since $E = E_T \oplus (\bigoplus_{g \in G/N \setminus T/N} E_g)$ as k -spaces, to complete the proof, it suffices to show that

$$(1.5.b) \quad \bar{x}l = l\bar{x} = 0 \quad \text{for all } l \in L \text{ and } x \in E_g \text{ with } g \notin T/N,$$

where \bar{x} is the image of x in \underline{E} .

Fix $g \in G/N \setminus T/N$ and $x \in E_g$. Then for any $g' \in G/N$ and any $y \in E_{g'}$, it follows that

$$(1.5.c) \quad \begin{aligned} &xy \text{ and } yx \text{ lie in } JE_1, \text{ if } Ng' = Ng^{-1}, \text{ and} \\ &xy \text{ and } yx \text{ lie in } \bigoplus_{g \neq 1} E_g, \text{ otherwise.} \end{aligned}$$

Now consider $l \in L$ as an element of \underline{E}^* , i.e., as a k -linear map from \underline{E} into k . Since $l \in \underline{E}_1^*$, for all $z \in \bigoplus_{g \neq 1} E_g$, l takes \bar{z} to zero. Further, l vanishes on $J\underline{E}_1$. Hence by (1.5.c) we can conclude that, for all $y \in E$, l maps both $\bar{x}\bar{y}$ and $\bar{y}\bar{x}$ to zero. By the definition of the action of \underline{E} on \underline{E}^* , this means that the elements $l\bar{x}$ and $\bar{x}l$ of \underline{E}^* both send \bar{y} to zero for all $y \in E$. Therefore, we can conclude that (1.5.b) holds. This completes the proof.

Now we decompose the sequence $0 \rightarrow \Omega^2 V^G \rightarrow X^G \rightarrow V^G \rightarrow 0$. Let e_1, \dots, e_n be orthogonal primitive idempotents of E with $\text{Id}_V^G = e_1 + \dots + e_n$. We can find orthogonal primitive idempotents e'_1, \dots, e'_n of $\text{End}_{kG}(\Omega^2 V^G)$ such that each \bar{e}'_i corresponds to \bar{e}_i via $\underline{E} \cong (\Omega^2 V^G, \Omega^2 V^G)^{1,G}$. Remark that the left actions of \bar{e}_i and \bar{e}'_i on the modules in (1.2.b) are equal to each other.

Theorem 1.6. *For each $i, 1 \leq i \leq n$, there exists a non-split exact sequence $S_i: 0 \rightarrow e'_i \Omega^2 V^G \rightarrow Y_i \rightarrow e_i V^G \rightarrow 0$ such that their direct sum $0 \rightarrow \Omega^2 V^G \rightarrow \bigoplus_i Y_i \rightarrow V^G \rightarrow 0$ is equivalent to the induced sequence $(SV)^G: 0 \rightarrow \Omega^2 V^G \rightarrow X^G \rightarrow V^G \rightarrow 0$. Moreover, this gives the unique (up to equivalence) decomposition of $(SV)^G$ with respect to e_1, \dots, e_n .*

Proof. It follows from Lemma 1.5 that $\bar{e}_i l = l \bar{e}_i$ for all $l \in L$ and $i, 1 \leq i \leq n$. Hence we have

$$l = \left(\sum_i \bar{e}_i \right) l \left(\sum_j \bar{e}_j \right) = l \sum_{i,j} \bar{e}_i \bar{e}_j = \sum_i \bar{e}_i l \bar{e}_i$$

for all $l \in L$. For each i , the element $\bar{e}_i l \bar{e}_i$ gives an extension $S_i: 0 \rightarrow e'_i \Omega^2 V^G \rightarrow Y_i \rightarrow e_i V^G \rightarrow 0$ and their sum $\sum_i \bar{e}_i l \bar{e}_i$ corresponds to the direct sum of those sequences. Hence it follows by Lemma 1.4 that the direct sum $0 \rightarrow \Omega^2 V^G \rightarrow \bigoplus_i Y_i \rightarrow V^G \rightarrow 0$ is equivalent to $(SV)^G$ if l represents SV .

Now suppose that some S_i splits, i.e., $l \bar{e}_i = 0$. Then we have $l \underline{E} \bar{e}_i = 0$ by Lemma 1.5. This implies that the following sequence is exact.

$$0 \rightarrow (e_i V^G, \Omega^2 V^G)^G \rightarrow (e_i V^G, X^G)^G \rightarrow (e_i V^G, V^G)^G \rightarrow 0$$

By the Frobenius reciprocity law, there holds

$$0 \rightarrow (e_i V^G, \Omega^2 V)^N \rightarrow (e_i V^G, X)^N \rightarrow (e_i V^G, V)^N \rightarrow 0 \quad (\text{exact}).$$

Since V is isomorphic to a direct summand of $(e_i V^G)_N$, the above contradicts our assumption that SV is an Auslander-Reiten sequence. Therefore each S_i does not split.

To see that this gives the unique decomposition, note that if we have

$l = \sum_i \bar{e}_i x_i \bar{e}_i$ for some $x_i \in \text{Ext}_{kG}(V^G, \Omega^2 V^G)$, then $\bar{e}_i x_i \bar{e}_i = \bar{e}_i l \bar{e}_i$ for all $i, 1 \leq i \leq n$.

Now the proof is complete.

2. The sequences appearing in the decomposition of $(SV)^G$

In this section, we shall discuss how S_i in Theorem 1.6 is far from $S(e_i V^G)$, the Auslander-Reiten sequence terminating at $e_i V^G$.

For any subgroup H of G and any kG -module W , let $\text{Tr}_H^G: (W, W)^H \rightarrow (W, W)^G$ denote the trace map. We begin with the following general result.

Lemma 2.1. *For an indecomposable kG -module W , suppose that $J(\text{End}_{kG}(W)) = \sum_{H <_G \text{vtx}(W)} \text{Im Tr}_H^G$. Then a short exact sequence $S: 0 \rightarrow \Omega^2 W \rightarrow Z \rightarrow W \rightarrow 0$ is an Auslander-Reiten sequence if and only if the following two conditions hold.*

- (i) S does not split.
- (ii) S splits on the restriction to H for all $H <_G \text{vtx}(W)$.

Proof. It is well known that the above two hold if S is an Auslander-Reiten sequence ([1, 2.17.10]). To see the converse, we first prove that any map f in $J(\text{End}_{kG}(W))$ factors through σ . By the assumption, we may assume that $f = \text{Tr}_H^G(h)$ for some $H <_G \text{vtx}(W)$ and $h \in \text{End}_{kH}(W)$. We can take $h' \in (W_H^G, W)^G$ corresponding to h by the Frobenius reciprocity law. Also, let ξ be the element of $(W, W_H^G)^G$ corresponding to $\text{Id}_W \in \text{End}_{kH}(W)$. Then it is routine to check that $f = \text{Tr}_H^G(h) = \text{Tr}_H^G(h \circ \text{Id}_W) = h' \circ \xi$. Since W_H^G is H -projective, the condition (ii) yields that there exists $\phi \in (W_H^G, Z)^G$ such that $\sigma \circ \phi = h'$. Thus we obtain $f = \sigma \circ \phi \circ \xi$. Therefore f factors through σ . Now by (i), the only elements of $\text{End}_{kG}(W)$ that factor through σ are precisely those that lie in $J(\text{End}_{kG}(W))$.

Let γ be the element of $\text{Ext}_{kG}(W, \Omega^2 W)$ corresponding to S . Then the above shows that $J(\text{End}_{kG}(W))$ annihilates γ from the right. Hence γ generates a semisimple module. Because $\text{Ext}_{kG}(W, \Omega^2 W)$ has a simple socle, γ must be a generator of the socle. This completes the proof. (See also the proof of [1, 2.17.7].)

The above lemma implies the following.

Theorem 2.2. *Suppose that $J(\text{End}_{kG}(e_i V^G)) = \sum_{H <_G \text{vtx}(V)} \text{Im Tr}_H^G$. Then the sequence S_i is an Auslander-Reiten sequence.*

Proof. Note that $\text{vtx}(V) = \text{vtx}(e_i V^G)$. If $H <_G \text{vtx}(V)$, then since $\text{vtx}(V) \leq N$, it easily follows from [1, 2.17.10] that $0 \rightarrow (\Omega^2 V^G)_H \rightarrow X_H^G \rightarrow V_H^G \rightarrow 0$ splits. (Note that $0 \rightarrow \Omega^2 V \otimes g \rightarrow X \otimes g \rightarrow V \otimes g \rightarrow 0$ is an Auslander-Reiten sequence for all $g \in G$.) Thus by Theorem 1.6 each S_i is H -split. Moreover,

S_i itself does not split. Therefore the result follows from Lemma 2.1.

For any exact sequence $S: 0 \rightarrow \Omega^2 W \rightarrow Z \xrightarrow{\sigma} W \rightarrow 0$, let $V^G \cdot S$ denote the cokernel of σ_* : $(V^G, Z)^G \rightarrow (V^G, W)^G$. So $V^G \cdot S$ is naturally a right E -module.

Let I be the two-sided ideal of E generated by JE_1 . In the case where V is G -invariant, $\bar{E} = E/I$ is isomorphic to a twisted group algebra of G/N over k . Now we have;

Proposition 2.3. *Suppose that V is G -invariant. Then;*

- (i) *For each $i, 1 \leq i \leq n, V^G \cdot S_i$ is a projective indecomposable right \bar{E} -module.*
- (ii) *A sequence $S: 0 \rightarrow e'_i \Omega^2 V^G \rightarrow Z \rightarrow e_i V^G \rightarrow 0$ is an Auslander-Reiten sequence if and only if $V^G \cdot S$ is a simple E -module. Hence in this case $V^G \cdot S$ is a simple \bar{E} -module.*

Proof. (i) We first claim that $V^G \cdot (SV)^G$ is isomorphic to \bar{E} . By the Frobenius reciprocity law, we have $V^G \cdot (SV)^G \cong (V^{G_N}) \cdot SV$ as E_1 - E -bimodules. Since V is G -invariant, $(V^{G_N}) \cdot SV \cong (V \cdot SV)^{|G:N|}$ as E_1 - E_1 -bimodules. Thus JE_1 annihilates $V^G \cdot (SV)^G$ from the right, and hence $V^G \cdot (SV)^G$ is an \bar{E} -module. Since it is a factor module of E having the dimension $|G:N|$ over k , it must coincide with \bar{E} . Now Theorem 1.6 yields that $V^G \cdot S_i$ is a direct summand of $V^G \cdot (SV)^G$. Therefore $V^G \cdot S_i$ is projective. Since $I \subseteq JE$, the image of e_i in \bar{E} is a nonzero idempotent of \bar{E} for all $i, 1 \leq i \leq n$. Hence $V^G \cdot S_i$ is an indecomposable \bar{E} -module.

(ii) Note that $V^G \cdot S_i$ is a factor module of a projective indecomposable E -module $e_i E = (V^G, e_i V^G)^G$. On the other hand, S is an Auslander-Reiten sequence if and only if $e_i JE e_i$ is contained in the kernel of the epimorphism $e_i E \rightarrow V^G \cdot S$ and $V^G \cdot S \neq 0$. These hold if and only if $V^G \cdot S$ is simple. Now the proof is complete.

Now we give an application of the above results, which is related to the work of Knörr [6].

Corollary 2.4. *Suppose that N is a p -group. Let $H = NC_G(N)$ and let B_i be the block of kG containing $e_i V^G$. Then, if S_i is an Auslander-Reiten sequence, the blocks of H covered by B_i have N as their defect groups.*

Proof. By [5, Satz 2.2] and [2, § 6, Exercise 14], we may assume that each e_i lies in $E_T = \text{End}_{kT}(V^T)$, where T is the inertial subgroup of V in G . Let S'_i be the sequence $0 \rightarrow \Omega^2 e_i V^T \rightarrow Y'_i \rightarrow e_i V^T \rightarrow 0$ appearing in the decomposition of $(SV)^T$. (By Theorem 1.6, S'_i is determined uniquely up to equivalence.) We claim that S'_i is also an Auslander-Reiten sequence. Let $\bar{e}_i, l\bar{e}_i \in \text{Ext}_{kT}(e_i V^T, \Omega^2 e_i V^T)$ represent S'_i . By the proof of Theorem 1.6, $\bar{e}_i, l\bar{e}_i$ also represents S_i . Now $\text{End}_{kT}(e_i V^T)$ is naturally considered as a subalgebra of $\text{End}_{kG}(e_i V^G)$, and hence $J(\text{End}_{kT}(e_i V^T)) \subseteq J(\text{End}_{kG}(e_i V^G))$. Since $J(\text{End}_{kG}(e_i V^G))$

annihilates \bar{e}_i/\bar{e}_i by the assumption, so does $J(\text{End}_{kT}(e_i V^T))$. This implies that S'_i is an Auslander-Reiten sequence. Thus by Proposition 2.3, $V^T \cdot S'_i$ is a simple projective E_T/I' -module, where I' is the ideal of E_T generated by JE_1 . Therefore the result follows by [6, Cor. 2.2].

Our final result concerns relative projectivity of Auslander-Reiten sequences. Recall that each Auslander-Reiten sequence gives a (finitely presented) simple object of the category $\text{MMod}(kG)$ of contravariant k -linear functors from the category of kG -modules into the category of k -spaces. (See [4, §1], for example.) In [4], Green defined relative projectivity of finitely presented objects of $\text{MMod}(kG)$ and showed that each of those indecomposable objects S has vertex $\text{vt}x(S)$, which is a p -subgroup of G determined uniquely up to G -conjugate. (See [4, §4] for detail.) He also proved that for any non-projective indecomposable kG -module W , there holds $\text{vt}x(SW) \geq_G \text{vt}x(W)$, [4, Theorem 5.12]. Here we identify the sequence SW with the corresponding simple object. The following was suggested by the referee.

Theorem 2.5. *Let W be a non-projective indecomposable kG -module. Suppose that $J(\text{End}_{kG}(W)) = \sum_{H <_G \text{vt}x(W)} \text{Im Tr}_H^G$. Then $\text{vt}x(W) =_G \text{vt}x(SW)$. In particular, if W is simple, then $\text{vt}x(W) =_G \text{vt}x(SW)$.*

Proof. Let P be a vertex of W , $M = N_G(P)$, and W' the Green correspondent of W with respect to (G, P, M) . Since $J(\text{End}_{kM}(W')) = \sum_{H <_M \text{vt}x(W')} \text{Im Tr}_H^M$ by [3, Chap. III, Lemma 5.10 (i)], Theorem 2.2 yields that $S(W')$ appears in the decomposition of $(SV_0)^M$, where V_0 is the P -source of W . This shows that $S(W')$ is P -projective. On the other hand, it follows from [4, Theorem 7.8] that $\text{vt}x(SW) \leq_G \text{vt}x(S(W'))$. Hence we have $\text{vt}x(SW) \leq_G P = \text{vt}x(W)$.

Therefore, the proof is completed by [4, Theorem 5.12].

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