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Evaluation of the precursor decay anomaly in single crystal lithium fluoride

Yukio Sano and Tomokazu Sano

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Evaluation of the precursor decay anomaly in single crystal lithium fluoride

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I. INTRODUCTION

The precursor decay anomaly in single crystal lithium fluoride (LiF) posed by Duvall and coworkers in 1972 is one of the most scientific questions in shock wave physics. The anomaly means extremely large difference in value between the measured dislocation density and the density at the impacted surface on the precursor decay curve calculated assuming impact between elastic bodies. An effort was made to resolve the anomaly. By simulating their experiments taking into consideration the generations of dislocations at the impact and rear surfaces of the pure crystal LiF sample and at the subgrain boundaries in the sample, Meir and Clifton suggested that the generations reduced the anomaly. Partom performed calculations of decay flow fields and precursor decay curves for 2024-T351Al and suggested that if a finite rate of dislocation generation was assumed, the anomaly was reduced.

Sano made an effort to resolve the anomaly in a single crystal LiF material (IIIb) (Ref. 1) using an approach that differed from those of Meir and Clifton and Partom. The dislocation density on the decay curve at the impacted surface estimated by Sano was extremely higher than the densities in recovered samples measured by Vorthman and Duvall. He considered that the anomaly was due to having based calculations on the extremely high and steep Asay’s decay curve that started upon shock loading. In his qualitative analysis of a smooth plane wave front in the vicinity of the impact surface, Sano revealed that the stress amplitude of the steady precursor in the wave front increased from the Hugoniot elastic limit to a maximum value and then decreased. The Sano’s decay curve obtained under the inference that it started from the maximum amplitude point was much lower than the Asay’s decay curve. Sano quantitatively analyzed the decay process in the material IIIb. This analysis revealed that as the decay was slow, the plastic strain rate at the leading edge of the follower was small. Thus, the studies of Sano demonstrated that the anomaly was reduced by using the Sano’s decay curve in the analysis.

In spite of the effort of the resolve of the anomaly by Sano, extremely high dislocation densities were recently reported by Gilman, et al.

II. THEORETICAL BACKGROUND

In this section, the equations for the particle velocity and the stress waves derived by Sano are described, together with the equations for the relaxation function and the dislo-
cation density derived by Sano.\textsuperscript{6} They are used in formulating the dislocation density on the decay curve in Sec. III.

A. Strain, particle velocity, and stress waves

Sano\textsuperscript{6} derived equations for the particle velocity and the stress waves, which correspond to linear strain waves, in the precursor and in the front part of the follower in a weak-discontinuity plane wave front during the decay process.\textsuperscript{12} He\textsuperscript{6} used a moving coordinate system expressed by

$$\xi = h - \tau, \quad \tau = \int_0^q c(q) dq,$$

where $h$ is the initial or Lagrangian position at $t=0$, at which the specimen is impacted, $q$ is the time that begins from the time $t$, when a kink has occurred in a smooth plane wave, that is, $q = t - t_s (\geq 0)$, and $c(q)$ is the velocity of the leading edge of the follower. The equation for the linear strain wave in the precursor $[0 < \xi \leq \xi_f]$ is

$$e(\xi, q) = e_i - \frac{E_i}{\xi_f} \xi,$$  

where $e(\xi, q) = e(\xi, \tau) = \varepsilon(h, q)$, $e_i(q) = e(0, q) = \varepsilon(h_i, q_i)$ is the strain at the leading edge of the follower on a precursor decay curve, whose location is expressed by $h = h_i(q)$ or $\xi = 0$, and $\xi_f(q)$ is the location of the leading edge of the precursor or the thickness of the precursor. The equation for the particle velocity wave corresponding to the strain wave is

$$v(\xi, q) = v_i - \left( \frac{cE_i}{\xi_f} + \dot{e}_i \right) \xi + \frac{\dot{e}_i \xi_f - e_i \xi_f}{2 \xi_f} \xi^2,$$  

where $v(\xi, q) = v(\xi, \tau) = u(h, q)$, the dots over the variables refer to differentiation with respect to $q$, $v_i(q)$ is the particle velocity at the leading edge of the follower expressed by $v_i = ce_i + (\dot{e}_i \xi_f + e_i \dot{\xi}_f) / 2$, and $\dot{\xi}_f = c_f - c$, where $c_f(q)$ is the velocity of the leading edge of the precursor. The equation for the stress wave corresponding to the strain wave is

$$\sigma(\xi, q) = \sigma_i + \rho_0 \left( -\frac{c^2 \dot{e}_i}{\xi_f} + A \right) \xi + B \xi^2 + D \xi^3,$$  

where $\sigma(\xi, q) = \sigma(\xi, \tau) = \sigma(h, q)$, $\sigma_i(q)$ is the stress at the leading edge of the follower expressed by $\sigma_i = \rho_0 (c^2 \dot{e}_i - A \xi_f - B \xi^2 - D \xi^3)$. As for coefficients $A(q)$, $B(q)$, and $D(q)$, see Ref. 6.

The equation for the linear strain wave in the follower $[\xi \leq 0]$ is

$$e(\xi, q) = e_i - \gamma \xi,$$  

where $\gamma(q)$ is the angle of incidence of the strain wave. The equations for the particle velocity and the stress waves corresponding to the strain wave are

$$v(\xi, q) = v_i - (c \gamma + \dot{e}_i) \xi + \frac{1}{2} \gamma \xi^2,$$  

$$\sigma(\xi, q) = \sigma_i + \rho_0 (E \dot{\xi} + F \dot{\xi}^2 + G \dot{\xi}^3).$$  

As for coefficients $E(q)$, $F(q)$, and $G(q)$, see Ref. 6.

B. Dislocation density

Sano\textsuperscript{6} derived an equation for stress relaxation function $F(h, q)$ by incorporating the equations of conservation of mass and momentum into the constitutive relation of Duvall,\textsuperscript{13}

$$\alpha \frac{D\sigma}{Dq} + (1 - \alpha) \left( \frac{\partial \sigma}{\partial q} \right)_h = -2 \mu \left( \frac{\partial \tau}{\partial q} \right)_h = -F,$$  

where $D/Dq$ represents differentiation along a path in time $[h = h(q)]$ in the $(h, q)$ coordinate system and $\sigma(h, q)$ is the plastic component of the natural strain in the direction of wave propagation $\varepsilon(h, q)$. $\mu$ is the shear modulus, and $\alpha(h, q)$ is the velocity ratio expressed by

$$\alpha = \frac{c_L^2}{c_w c_{uc}},$$  

where $c_w = dh_{\sigma}/dq$ (Ref. 6) and $c_{uc}(h, q)$ is the phase velocity at a constant particle velocity derived by Fowles,\textsuperscript{14} which is expressed by

$$c_{uc} = \frac{\left( \frac{\partial \varepsilon}{\partial q} \right)_h}{\left( \frac{\partial \varepsilon}{\partial h} \right)_q},$$  

and where $c_{uc}(h, q)$ is the Lagrangian wave speed in a uniaxial strain state expressed by $c_{uc} = (\rho_0/\rho) c_s$, where $\rho_0$ is the initial material density, $\rho_0 h$ is the material density, and $c_s$ is the Eulerian wave speed.

Asay et al.\textsuperscript{1} and Gupta et al.\textsuperscript{2} related the relaxation function $F$ to the dislocation density $N_m$.

$$N_m = \eta F,$$  

where $\eta = 1/(2 b v_d)$, where $b$ is the Burgers vector and $v_d$ is the average dislocation velocity expressed by $v_d = v_s \exp(-D/\tau_s)$, where $v_s$ is the shear wave velocity, $D$ is the drag stress, and $\tau_s$ is the resolved shear stress that is related to stress $\sigma$ by $\tau_s = (221/760) \sigma$ in LiF. For LiF III, we have $b = 2.85 \times 10^{-10} \text{ m}$, $\mu = 11.05 \text{ GPa}$, and therefore $\eta = 10/(63 v_d) \text{ Pa}^{-1} \text{ m}^{-2}$, and $v_s = 3280 \text{ m/s}$. The value of $D$ is discussed in Sec. IV B.

C. Validity of use of the constitutive relation of Duvall

Since the assumption that stresses are maintained by elastic strains alone is used in deriving the constitutive relation of Duvall,\textsuperscript{13} strain rate and acceleration must not be included in $\sigma$ in Eq. (7). Armstrong et al.\textsuperscript{15} interpreted the dislocation generation rate to be of controlling importance in constitutive equation modeling of shock-induced plasticity. This means that the effects of strain rate and acceleration are included in shock-induced stress $\sigma$. However, the effects on the stress on the decay curve, that is, on the stress at the rear of the near elastic precursor are small. Therefore, Eq. (7) holds on the decay curve to a good approximation.

III. FORMULATION OF DISLOCATION DENSITIES ON THE DECAY CURVE

In this section, an equation for the relaxation function at the leading edge of the follower is first derived from Eq. (7) using Eqs. (5) and (6). Next, an inequality for the function at
the rear of the precursor is derived, and then expressions for the lower and upper bounds of the inequality are obtained using Eqs. (1)–(3).

A. At the leading edge of the follower

The following equation for the relaxation function $F_i = F(h_i)$ at the leading edge of the follower on the decay curve is obtained from Eq. (7):

$$\alpha_i \partial \gamma + (1 - \alpha_i) \left( \frac{\partial \alpha}{\partial q} \right)_{h=h_i} = -F_i,$$

where $F_i = 2\mu(\partial^2 \gamma / \partial q)^2_{h=h_i}$, and $\alpha_i = \alpha(h_i) = c_L \gamma_{\text{eq}}$. $\gamma_{\text{eq}}$ is obtained from Eq. (8), where $c_L = c_L(h_i)$, $c_{uc} = c_u(h_i)$, and $\gamma_{\text{eq}} = dh_i / dq = c$. In Eq. (7), $D/Dq$ represented the differentiation along the path $[h=h_i(q)]$, but in Eq. (11), it is along the decay curve, so that $D/Dq$ in Eq. (7) becomes $\dot{\gamma}(q)$ in Eq. (11). Since the velocity of the leading edge of the follower $c$ is the Lagrangian wave speed, that is, since $c_L = c$, the relation for $\alpha_i$, which is obtained from Eq. (8), reduces to

$$\alpha_i = c_{uc}.$$

An equation for $\alpha_i$ is first derived. Equation $c_{uc} = c_{uc}[1 - (\partial \gamma / \partial q)_{\tau_{eq}=0}]$, which is obtained from Eq. (9), where $(\partial \gamma / \partial q)_{\tau_{eq}=0} = \dot{\gamma} / c$ and $(\partial \gamma / \partial q)_{\tau_{eq}=0} = -\dot{\gamma} - \dot{\gamma}_i$, were obtained from Eq. (5). Substitution of the equation for $c_{uc}$ above into Eq. (12) yields

$$\alpha_i = c_{uc} (\gamma + \dot{\gamma} / c) + \dot{\gamma}_i.$$

Next, an equation for $\alpha_i$ is derived using $\dot{\gamma}(q)_{\tau_{eq}=0} = \dot{\gamma}_i$, and $\dot{\gamma}(q)_{\tau_{eq}=0} = -\dot{\gamma}_i$, which are obtained from Eq. (11). Finally, substitution of Eq. (13) and the equation for $(\partial \gamma / \partial q)_{h=h_i}$ above into Eq. (11) yields an equation for $F_i$

$$F_i = (\rho_0 c \dot{\gamma}_i + \dot{\gamma}_i).$$

The same equation as the above is also derived from the strain wave that is expressed by a power series up to the nth ($\geq 2$) order with respect to $\xi$.

The equation for the dislocation density at the leading edge of the follower $N_{\text{ml}} = N_m(h_i)$ is

$$N_{\text{ml}} = \eta_i F_i,$$

where $\eta_i = 10(63v_{di})$. Here $v_{di} = v_s \exp(-D/\tau_{si})$, where $\tau_{si} = (221/760)\sigma_i$.

B. At the rear of the precursor

The equation for the function $F_i^+ = F(h_i^+)$ at the rear of the precursor, whose location is expressed by $h = h_i(q)^*$ or $\xi = 0^+$, is

$$\alpha_i^* \dot{\gamma} + (1 - \alpha_i^*) \left( \frac{\partial \alpha}{\partial q} \right)_{h=h_i^+} = -F_i^+,$$

where $F_i^+ = 2\mu(\partial^2 \gamma / \partial q)_{h=h_i^+}$, and $\alpha_i^* = \alpha(h_i^*)$ satisfies inequalities

$$\alpha_i^* \leq \alpha_i^+ \leq \alpha_{ui}^*.$$

where $\alpha_i^*$ is the lower bound of $\alpha_i^*$ and $\alpha_{ui}^*$ is the upper bound. The lower bound is given by $\alpha_{ui} = \alpha_{ui}^*$, $\alpha_{ui} \leq \alpha_{ui}^*$, or $\alpha_{ui} = \alpha_{ui}^*$, where $\alpha_{ui} = c_{uc} / c_{uc}(h_i^*)$ and $\alpha_{ui} = c_{uc} / c_{uc}(h_i^*)$, and $\alpha_{ui}^* = c_{uc} / c_{uc}(h_i^*)$, where $c_{uc} = c_u(h_i^*)$. $\alpha_{ui} = c_{uc}(h_i^*)$ are the phase velocities at constant strain, constant particle velocity, and constant stress at the rear of the precursor, respectively, which are expressed as

$$c_{uc} = c \left( 1 - \frac{(d\gamma / \partial \tau)_{\tau_{eq}=0}}{(d\gamma / \partial \tau)_{\tau_{eq}=0}^*} \right), c_{uc} = c \left( 1 - \frac{(d\gamma / \partial \tau)_{\tau_{eq}=0}}{(d\gamma / \partial \tau)_{\tau_{eq}=0}^*} \right),$$

The following relations are derived from Eqs. (1)–(3), respectively,

$$\gamma_{eq} = \gamma \gamma_{eq}, \gamma_{eq} = \gamma \gamma_{eq}, \gamma_{eq} = \gamma \gamma_{eq}, \gamma_{eq} = \gamma \gamma_{eq}.$$
On the other hand, the equation for the upper bound $F_{iu}$ is

$$F_{iu} = \alpha_{iu} \left( \frac{\partial \bar{\sigma}}{\partial q} \bigg|_{h=h_i^*} - \bar{\sigma}_f \right) - \left( \frac{\partial \bar{\sigma}}{\partial q} \bigg|_{h=h_i^*} \right).$$  \hspace{1cm} (23)

By substituting Eq. (19) and the equation for $(\partial \bar{\sigma}/\partial q)_{h=h_i^*}$, which is obtained from Eq. (3), into Eq. (23), an equation is obtained,

$$F_{iu} = -(\rho_0 c \dot{v}_i + \dot{\bar{\sigma}}).$$  \hspace{1cm} (24)

The same equation as the equation above is also derived from the strain wave in the precursor that is expressed by a power series up to the $n$th $(\geq 2)$ order with respect to $\xi$. Equation (24) is identical to Eq. (14), namely,

$$F_{iu} = F_i.$$  \hspace{1cm} (25)

The identity of $F_{iu}$ to $F_i$ is justified by the fact that the form and the slope of the precursor are not included in the equation for $F_{iu}$ as well as the fact that those of the follower are not in the equation for $F_i$. If $\bar{\sigma}_f \equiv \rho_0 c \dot{v}_i$, then $F_{iu} \equiv -2\bar{\sigma}_f$, so that $0 < F_{iu}^+ \leq -2\bar{\sigma}_f$ for a thin precursor.

The following inequalities are obtained for the dislocation density at the rear of the precursor $N_{mi}^{-}\equiv N_{mi}(h_i^*)$:

$$N_{mil} \leq N_{mi}^+ \leq N_{mi},$$ \hspace{1cm} (26)

where $N_{mil} = \eta_i F_{il}$ and $N_{mi} = \eta_i F_{iu}$. From Eq. (25),

$$N_{mi} = N_{mi}.$$ \hspace{1cm} (27)

C. Results extracted from Eq. (14)

Three important results are extracted from Eq. (14). This equation only includes the slopes of the decay curves for particle velocity and stress as variables. Thus, the relaxation function is independent of the form and the angle of incidence of the follower. Therefore, the first result extracted is that Eq. (14) holds irrespective of the kind of the follower [contraction (compression) wave C, degenerate contraction waves I and II, subcritical rarefaction wave $R'$, and rarefaction wave $R_b$]. This is justified by the demonstration of Sano\textsuperscript{8} that the constitutive relation of Duvall\textsuperscript{13} holds in any of the five elementary waves. The jumps in particle velocity and stress across the precursor in LiF III, both satisfy the Rankine–Hugoniot (RH) jump conditions, $\dot{v}_i = \varepsilon c \dot{v}_i$ and $\bar{\sigma}_f = \rho_0 c^2 \dot{\varepsilon}_i$, to a good approximation,\textsuperscript{8} indicating that if the $\varepsilon_i$-$t$ curve is steep, then both the $v_i$-$t$ and $\sigma_i$-$t$ curves are also steep and hence the value of $F_i$ is large at any time during the decay process. In short, Eq. (14) implies that as the decay is steep, plastic strain rate is large. Therefore, it is found easily from Eq. (14) that the values of $F_i$ are larger along the Asay’s decay curve than along the Sano’s decay curve. This is the second extracted, although it was already revealed in the analysis of Sano.\textsuperscript{8} If the decay curve is accurately determined, relaxation functions would be precisely evaluated from Eq. (14) because neither the form nor the angle of incidence of the follower are included in the equation for the function as mentioned above in this paragraph. This is the third (final) extracted.
tion velocities are predicted from \( v_d = v_s \exp(-D/\tau_s) \),\(^{2,16,17}\) small values of the drag stress \( D \) are required. In fact, there is a small critical value of \( D_{cr} = 0.036 \) GPa. The change in time of the density \( N_{mi} \) for any value of \( D \) in a range of \( D \geq D_{cr} \) increases with time after a time between \( t = 0.3 \) and \( t = 1.0 \) \( \mu \)s, which approaches to 1.0 \( \mu \)s as the value of \( D \) decreases to the value of \( D_{cr} \). The reason for the occurrence of such unreasonable changes is that the value of the function \( F_i \) becomes small with time (see Fig. 1), whereas the value of the coefficient \( \eta \) becomes large. In other words, for any value of \( D \) in a range of \( D \leq D_{cr} \), we predict a reasonable \( N_{mi} \) distribution that decreases with time up to \( t = 1.0 \) \( \mu \)s, indicating that values of \( D \) in the range of \( D \leq D_{cr} \) should be used. In the next section, calculations are performed using the value of \( D = 0.036 \) GPa.

Velocity \( v_d = 2670 \) m/s is calculated from \( v_d = v_s \exp(-D/\tau_s) \) using the values of \( D = 0.036 \) GPa and \( \tau_s = (221/760)\sigma_i \) obtained from a middle value of \( \sigma_i = 0.6 \) GPa between the values of stresses \( \sigma_i \) at \( t = 0.3 \) and \( t = 1.0 \) \( \mu \)s. The velocity of 2670 m/s comparable with \( v_s = 3280 \) m/s obtained illustrates the indication of Granato\(^{16}\) that dislocation drag effects are not effective under shock loading conditions.

### C. Dislocation density

Changes in time of the lower and upper bounds of the dislocation density \( N_{mil} \) and \( N_{miu} (= N_{mi}) \) at the rear of the precursor on the Sano’s decay curve are shown in Fig. 2. The density \( N_{mi} \) decreases rapidly with time from a maximum value of about \( 2.0 \times 10^{11} \) m\(^{-2} \) at \( t = 0.015 \) \( \mu \)s to a value of about \( 0.65 \times 10^{11} \) m\(^{-2} \) at \( t = 0.3 \) \( \mu \)s. This rapid decrease in \( N_{mi} \) is evident from changes in \( v_d \) and \( \sigma_i \) included in Eq. (24), whose absolute values decrease rapidly up to 0.3 \( \mu \)s.

Figure 2 also shows a change in time of the difference \( \Delta N_{mi} = N_{miu} - N_{mil} \). Since \( F_{il} = F_{il} \) (see Sec. IV A), we have \( N_{mil} = N_{mil} \) and hence \( \Delta N_{mi} = N_{mi} - N_{mi}^* \). The large values of \( \Delta N_{mi} \), which are shown in Fig. 2, reveal that the density decreases largely near the leading edge of the follower. This large increase results from the generations of dislocations near the impact surface and at the subgrain boundaries as well as in the bulk.\(^{4,5}\)

Figure 3 shows changes in time of three different lower bounds, \( N_{mil}^* \), \( N_{mil} \), and \( N_{mil} \), calculated up to \( t = 0.3 \) \( \mu \)s, where \( N_{mil} = \eta F_{il} \), \( N_{mil} = \eta F_{il} \), and \( N_{mil} = \eta F_{il} \), where \( F_{il} \), \( F_{il} \), and \( F_{il} \) are given by Eq. (21) where \( \alpha_{il} \), \( \alpha_{il} \), \( \alpha_{il} \), and \( \alpha_{il} \) are defined, respectively. The negative values of \( N_{mil} \) have no physical meaning. As shown in Fig. 3, inequalities \( N_{mil} > N_{mil} > N_{mil} \) hold, indicating that \( N_{mil} \) is a small critical value of \( D \) in a range of \( D \leq D_{cr} \). The density \( N_{mil} \) decreases almost linearly with time from a maximum value of about \( 2.1 \times 10^{10} \) m\(^{-2} \) at the beginning to a value of about \( 1.3 \times 10^{10} \) m\(^{-2} \) at \( t = 0.3 \) \( \mu \)s. The values of \( N_{mil}^* \) (see Sec. IV A) are considerably larger than that of the initial density \( 2 \times 10^{10} \) m\(^{-2} \) in the bulk. The dislocation generations near the impact surface and at the subgrain boundaries as well as in the bulk are also responsible for the larger values of \( N_{mil}^* \).

### V. PRECURSOR DECAY ANOMALY

#### A. Dislocation generation

The densities \( N_{mi} \) on the Asay’s decay curve that begins at \( t = 0 \) are evaluated from Eq. (15). The strain \( \varepsilon \) induced at the impacted surface upon shock loading has a value of \( \varepsilon = 0.024 \) that is obtained from the RH jump condition \( \varepsilon_{max} = c_0 \varepsilon_s \) using \( c_0 \approx 7000 \) m/s and \( \varepsilon_{max} = 166 \) m/s, where \( c_0 \) is the velocity of an shock-induced wave at \( t = 0 \) and \( \varepsilon_{max} \) is the peak particle velocity at the impacted surface.\(^{18}\) A change in time of \( N_{mi} \) for \( \varepsilon = 0.024 \) is shown by a dashed line in Fig. 4. However, Sano\(^7\) revealed \( \varepsilon_{max} = 0.80c_0\varepsilon_s \) at the impacted surface. In this case, the value of \( \varepsilon \) is \( \varepsilon = 0.030 \). A change in time of \( N_{mi} \) for \( \varepsilon = 0.030 \) is shown by a solid line. The difference in value between both the changes is not large. The value of the density \( N_{mi} \) decreases from about \( 2.0 \times 10^{11} \) m\(^{-2} \) at \( t = 0 \) to about \( 0.3 \times 10^{11} \) m\(^{-2} \) at \( t = 0.3 \) \( \mu \)s. The value of 2.0
$10^{12}$ m$^{-2}$ would provide the maximum value of the density that can be evaluated on the decay curve. Based on the value of $2.0 \times 10^{12}$ m$^{-2}$, it is decided that LiF III, has no mechanism that generates dislocations as many as those reported by Gilman, Shehadeh et al., and Bringa et al.

B. Consideration of the anomaly

Vorthman and Duvall estimated the density in the bulk of about $10^{10}$ m$^{-2}$ in the postshock analysis of a LiF sample impacted at a projectile velocity of 186 m/s. The value of $10^{10}$ m$^{-2}$ is not significantly larger than the preshock value $15 \times 10^{9}$ m$^{-2}$. In short, the high densities at a projectile velocity of 340 m/s predicted by Duvall and co-workers were not observed in their recovery experiments.

The value of $10^{10}$ m$^{-2}$ measured by Vorthman and Duvall is considerably smaller than a maximum value of $N_{mi}$ of 2.0 $\times 10^{11}$ on the Sano’s decay curve and that of 2.0 $\times 10^{12}$ m$^{-2}$ for Duvall's decay curve. The main reason for this may lie in the difference between impact velocities of 186 m/s in their experiment and 340 m/s in this analysis. In short, it is inferred from the mechanism in LiF III, by which many dislocations are not generated that the density is of the order of $10^{10}$ m$^{-2}$ at the velocity of 186 m/s.

VI. CONCLUSIONS

The calculations of the dislocation density on the decay curve in LiF III, indicated that many dislocations were not generated in the material through the predicted maximum value of the density of at most $2.0 \times 10^{12}$ m$^{-2}$ at a projectile velocity of 340 m/s. On the other hand, the value of the density measured by Vorthman and Duvall was about $10^{10}$ m$^{-2}$ at a projectile velocity of 186 m/s. The mechanism in the material that does not generate many dislocations suggests that the measured value is not unreasonable. It is inferred from this suggestion that the difference in values between both the densities of $2.0 \times 10^{12}$ and $10^{10}$ m$^{-2}$ is caused by the difference in the projectile velocity between 340 and 186 m/s and therefore that the precursor decay anomaly does not exist.

APPENDIX: DECAY CURVE FOR STRAIN

The decay curve for strain $\epsilon_i(q)$ that was formulated by Sano is described. A quadratic equation and a linear equation are connected at $q=q_1$ under the condition that the slopes of the quadratic and linear curves are equal there,

$$
\epsilon_i(q) = aq^2 + bq + c \quad (0 \leq q < q_1),
$$

$$
\epsilon_i(q) = dq + e \quad (q \geq q_1),
$$

where

$$
a = \frac{\epsilon_i - \epsilon_1}{q_1^2}, \quad b = d - 2aq_1, \quad c = \epsilon_1,
$$

$$
d = \frac{\epsilon_1 - \epsilon_2}{q_1 - q_2}, \quad e = \epsilon_1 - dq_1,
$$

where $0 < q_1 < q_2$, $\epsilon_i = \epsilon_i(s)$ is the strain at $q=0$ on the Sano’s decay curve, and $\epsilon_1 = \epsilon_1(s)$ and $\epsilon_2 = \epsilon_2(s)$ are the strains at $q=q_1$ and $q=q_2$ that are also on the Asay’s decay curve.

The values of $\epsilon_1$, $\epsilon_2$, and $\epsilon_i$ that were determined by Sano are described. Relation $\epsilon_i = [u_i]_{\text{max}}/\epsilon_m$ is derived from the RH jump condition $\epsilon_i = c_m \epsilon_m$, where $\epsilon_i = \langle \tilde{u}_i \rangle, [u_i]_{\text{max}} = \frac{\tilde{u}_i}{\epsilon_i}$, $c_m = (c+1)/2$, and $\epsilon_i$ is the peak particle velocity at the impact surface. First, value $[u_i]_{\text{max}}$ is obtained from $[u_i] = (\tilde{u}_R \tilde{u}_\text{max})/[u_i]_R$ using the value $[u_i]_R = \tilde{u}_R / \epsilon_i = 0.31$, which was determined in Ref. 7, and the value $\epsilon_i = 10^{-2}$ is obtained from $\epsilon_i = [u_i]_{\text{max}}/\epsilon_m$ using the value $[u_i]_{\text{max}} = 0.39$ and the value $\epsilon_i = 166$ m/s, which was measured using an interferometer.

On the other hand, on the Asay’s decay curve, the value of $[u_i]$ is equal to the measured value $[\tilde{u}_i]$ at the same time, where $[\tilde{u}_i] = i/t_{\text{max}}$, where $t_{\text{max}}$ is the peak current at the impact surface. The values of $q_1 = 3 \times 10^{-7} - t_s$ and $q_2 = 10^{-6} - t_s$ s are taken. Value $\epsilon_1 = 0.68 \times 10^{-2}$ is obtained from $\epsilon_1 = [u_i]_{\text{max}}/\epsilon_m$ using $[\tilde{u}_i] = [u_i]_{\text{max}} = 0.27$ in LiF, and value $\epsilon_2 = 0.43 \times 10^{-2}$ is obtained using $[\tilde{u}_i] = [u_i]_{\text{max}} = 0.16$.

The values of coefficients $d$ and $e$ are first determined using the values of $\epsilon_1$ and $\epsilon_2$, and those of coefficients $a$, $b$, and $c$ are then determined.