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1 **Ultrasonic interference and critical attenuation in metal-plastic bilayer laminates**

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7 **ABSTRACT**

8 The resonance of ultrasonic waves in a metal-plastic bilayer laminate placed in an infinite medium
9 is theoretically investigated, and critical attenuation is shown to occur under specific conditions.

10 The bilayer laminate is subjected to normal wave incidence from the side of the plastic layer.

11 Based on a linear viscoelastic model, the amplitude of the reflection spectrum for the laminate is
12 formulated to obtain the exact results for wave resonance and critical attenuation by numerical

13 calculations. Furthermore, the approximate formulae for the conditions of the resonance and
14 critical attenuation are derived explicitly by Taylor expansions with respect to the loss factor of

15 the plastic. As a result, the exact solution shows that the resonance frequencies of the plastic layer
16 slightly increase with increasing loss factor. Their variation with the loss factor agrees well with

17 the second-order approximation result but is insignificant in the cases of common plastics with
18 relatively low loss factors. The exact results also show that the amplitude at each order resonance

19 depends on the loss factor and has a minimum at a critical loss factor. The obtained critical loss
20 factor decreases with increasing resonance order and is well reproduced by the zeroth-order

21 approximation. Experiments are performed on two types of metal-plastic bonded laminates,
22 demonstrating that the measured reflection spectra are in good agreement with the theoretical

23 predictions. In particular, the amplitude of the measured reflection spectrum decreases
24 significantly at a resonance frequency, which is theoretically predicted to satisfy the critical
25 attenuation condition.

26 *Keywords:* Layered structure; Interference; Critical attenuation; Reflection; Viscoelasticity

27 **1. Introduction**

28 Elastic wave propagation in layered structures is one of the classical issues in acoustics [1, 2],
29 yielding fundamental knowledge for various applications, e.g., the design of acoustic devices and
30 the ultrasonic spectroscopy of laminates. A basic case is that an elastic wave impinges on a single
31 planar layer placed in an infinite medium. For the normal incidence of a plane wave, layer
32 resonance occurs at frequencies depending on the ratio of the wave velocity to the thickness of
33 the layer [3, 4]. This phenomenon results in the local minima of the amplitude spectrum of the
34 reflected wave from the layer due to destructive interference. In structures with multiple layers,
35 the spectral characteristics of the elastic waves become more complicated.

36 While numerous studies have been carried out on the acoustic properties of layered structures
37 so far, the effect of viscoelastic layers on wave propagation [5–9] is not fully revealed compared
38 to purely elastic cases [10–16]. Viscoelastic materials transform the energy of sound waves into
39 heat and attenuate the propagating waves. However, in the cases of layered structures,
40 viscoelasticity sometimes facilitates wave resonances appearing in the reflection spectrum.
41 Lavrentyev and Rokhlin [17] investigated the wave interaction with a single planar viscoelastic
42 layer sandwiched by semi-infinite dissimilar materials. They showed theoretically that for plane
43 wave incidence on the layer from the side of the semi-infinite material with lower acoustic
44 impedance, the increase of the attenuation factor in the layer can make local minima of the
45 amplitude of the reflection spectrum deep and narrow. The amplitude of the reflection spectrum
46 at the layer resonance frequency becomes zero at a specific condition, called critical attenuation
47 [17]. The critical attenuation condition for a single viscoelastic layer was analytically derived
48 under several approximations.

49 To the authors' knowledge, however, the critical attenuation behavior has been reported only
50 for the case of the single layer between semi-infinite dissimilar materials. Recently, Mori *et al.*
51 [18] examined the wave propagation in a metal-plastic bilayer laminate placed in water
52 theoretically and experimentally, showing that the damping property of the plastic layer
53 contributes to the appearance of the layer resonance in the reflection spectrum. Nevertheless, the
54 quantitative effect of the viscoelasticity on the resonance frequencies of the plastic layer has not
55 been shown, and it remains unclear whether critical attenuation occurs in the bilayer laminates.

56 The present study aims to reveal the wave resonance and the critical attenuation behavior in
57 metal-plastic bilayer laminates. Theoretical analysis is performed to derive the conditions of the
58 layer resonance and the critical attenuation based on several approximations. The viscoelastic
59 effects on the responses of the laminates to normal wave incidence are examined by comparing
60 the exact and approximate results. Experimental results are shown to validate the theoretical
61 findings and to investigate how the wave resonance and the critical attenuation appear.

62 This paper is structured as follows. In Section 2, the modeling of a bilayer elastic-viscoelastic
63 laminate in an infinite medium is described. The loss factor is introduced when modeling the
64 viscoelastic material, and the interaction of a normally incident wave with the laminate is
65 formulated. Approximate formulae are obtained based on Taylor expansions with respect to the
66 loss factor. In Section 3, the theoretical results are shown for two different cases, i.e. purely elastic
67 and viscoelastic laminates. In Section 4, the results of the ultrasonic measurement are shown for
68 two types of metal-plastic laminates and are used to discuss the theoretical results.

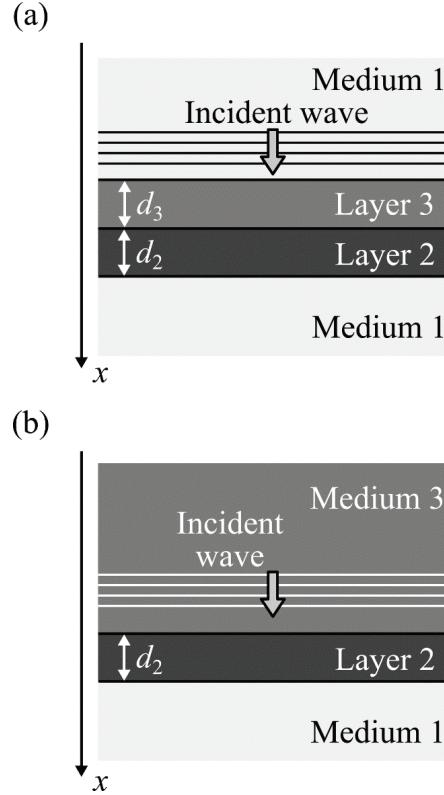
69 **2. Theory**

70 *2.1 Modeling and formulation*

71 As shown in Fig. 1(a), a bilayer laminate is placed in an infinite medium and is subjected to
72 normal wave incidence. The medium surrounding the laminate, called medium 1, is lossless. The
73 laminate consists of two planar homogeneous layers called layers 2 and 3 with thicknesses d_2 and
74 d_3 , respectively. The incident wave propagates along the x axis corresponding to the thickness
75 direction. When a longitudinal plane wave is incident on the laminate from the side of layer 3, the
76 wave propagation substantially becomes one-dimensional. In the frequency domain, the wave
77 propagation behavior obeys

$$-\rho\omega^2u = \frac{\partial\sigma}{\partial x}, \quad (1)$$

78 where ρ is the mass density of a medium, $\omega = 2\pi f$ is angular frequency, f is frequency, and $u(x, \omega)$
79 and $\sigma(x, \omega)$ are the frequency-domain representations of the displacement and normal stress
80 components in the x direction. Harmonic wave components propagating in the x direction are
81 represented by $\exp\{i(kx - \omega t)\}$, where k is wavenumber, t is time, and $i = \sqrt{-1}$.



83
84 Fig. 1 Theoretical models of (a) a bilayer laminate in an infinite medium and (b) a bottom layer
85 (layer 2) sandwiched by dissimilar media, subjected to normal wave incidence.
86

87 In this study, layers 3 and 2 are assumed to be plastic and metal, which are modeled as linear
88 viscoelastic and purely elastic bodies, respectively. If the temperature variation by the wave
89 attenuation in a linear viscoelastic material is negligible, its stress-strain relation can be modeled
90 by [19–21]

$$\sigma = (E - iD)\varepsilon = \rho c^2 (1 - i\zeta) \frac{\partial u}{\partial x}, \quad (2)$$

91 where $\varepsilon(x, \omega)$ is the frequency-domain representation of the normal strain, $E - iD$ is complex elastic
92 modulus ($D > 0$), $c = (E/\rho)^{1/2}$ is wave velocity, and $\zeta = D/E$ is loss factor. In purely elastic materials,
93 the loss factor can be set as $\zeta = 0$. In general, the loss factors of viscoelastic materials depend on
94 frequency, but ζ is assumed to be frequency-invariant in this study. The validity of this assumption
95 is mentioned in Section 4. The minus sign in the complex elastic modulus is based on the time-
96 dependent terms of $\exp(-i\omega t)$. By substituting Eq. (2) to Eq. (1) and considering a solution of a
97 plane wave $u = A \exp\{i(kx - \omega t)\}$, where A is a non-zero constant, the complex wavenumber can
98 be obtained as $k = \omega/v$, where

$$v = c \sqrt{1 - i\zeta} \quad (3)$$

99 is called complex wave velocity in this study. Let the mass density, wavenumber, wave velocity,
100 and complex wave velocity of layer j ($j = 2, 3$) be ρ_j , k_j , c_j , and v_j , respectively. The loss factors of
101 layers 3 and 2 are set as $\zeta_3 = \zeta$ and $\zeta_2 = 0$, respectively, which mean that $v_2 = c_2$ holds. The mass
102 density and wave velocity of medium 1 are expressed as ρ_1 and c_1 ($= v_1$), respectively.

103 For the incidence of a sinusoidal wave with angular frequency ω on the bilayer laminate, the
104 reflection spectrum of the stress component is theoretically expressed as [1]

$$R_b = \frac{Z_{321}^{\text{in}} - z_1}{Z_{321}^{\text{in}} + z_1}, \quad (4)$$

105 where $z_m = \rho_m v_m$ is the complex acoustic impedance of material m ($m = 1, 2, 3$),

$$\begin{aligned} Z_{321}^{\text{in}} &= \frac{Z_{21}^{\text{in}} - iz_3 \tan \phi_3}{z_3 - iz_{21}^{\text{in}} \tan \phi_3} z_3, \\ Z_{21}^{\text{in}} &= \frac{z_1 - iz_2 \tan \phi_2}{z_2 - iz_1 \tan \phi_2} z_2 \end{aligned} \quad (5)$$

106 are the effective acoustic impedances of two substructures, respectively, and $\phi_j = \omega d_j / v_j$ ($j = 2, 3$).
107 For example, the quantity Z_{321}^{in} is based on the subsurface structure in Fig. 1(a), i.e. layers 3 and
108 2, and lower semi-infinite medium 1. Substitution of Eq. (5) into Eq. (4) leads to [18]

$$R_b = \frac{r_{13} + r_{321} \exp(2i\phi_3)}{1 + r_{13}r_{321} \exp(2i\phi_3)}, \quad (6)$$

109 where

$$r_{mn} = \frac{z_n - z_m}{z_n + z_m} \quad (7)$$

110 is stress reflection coefficient at a boundary between materials m and n for normal wave incidence
111 from material m ($m, n = 1, 2, 3$), and

$$r_{321} = \frac{Z_{21}^{\text{in}} - z_3}{Z_{21}^{\text{in}} + z_3} = \frac{r_{32} + r_{21} \exp(2i\phi_2)}{1 + r_{32}r_{21} \exp(2i\phi_2)} \quad (8)$$

112 corresponds to stress reflection spectrum for a single layer shown in Fig. 1(b), i.e. layer 2
113 sandwiched by semi-infinite media 3 and 1. It is noted that the reflection spectrum r_{321} depends
114 on frequency, while the reflection coefficient r_{mn} is frequency-independent.

115 Due to the viscoelasticity of layer 3, the reflection coefficients r_{13} and r_{32} have non-zero
116 imaginary parts. For example, the reflection coefficient r_{13} can be expressed as $r_{13} = |r_{13}| \exp(i\theta_{13})$,
117 where θ_{13} is real. When the reflection spectrum r_{321} in Eq. (8) is given as $r_{321} = |r_{321}| \exp(i\theta_{321})$, Eq.
118 (6) can be rewritten as

$$R_b = \frac{|r_{13}| + |r_{321}| \exp(-2q) \exp[i(2p + \theta_{321} - \theta_{13})]}{1 + |r_{13}| |r_{321}| \exp(-2q) \exp[i(2p + \theta_{13} + \theta_{321})]} \exp(i\theta_{13}), \quad (9)$$

119 where $p = \text{Re}(\omega d_3 / v_3)$, $q = \text{Im}(\omega d_3 / v_3)$, and θ_{321} is real.

120 2.2 Resonance and critical attenuation in bilayer laminates

121 The amplitude of the reflection spectrum $|R_b|$ is a function of frequency, and wave resonances
 122 are closely associated with the local minima of $|R_b|$. As mentioned in Ref. [17], the numerator of
 123 R_b in Eq. (9) plays a dominant role when the amplitude of the reflection spectrum $|R_b|$ takes a local
 124 minimum. When the acoustic impedance of layer 2 is higher than those of media 1 and 3, the
 125 amplitude of the reflection spectrum for layer 2 shown in Fig. 1(b), i.e. $|r_{321}|$, takes a local
 126 minimum at the angular frequency $\omega = \omega_{2,n}$ ($n = 1, 2, \dots$), where [4]

$$\omega_{2,n} = \frac{n\pi}{\tau_2} \quad (10)$$

127 and $\tau_2 = d_2/c_2$. If the frequency dependence of $\exp[i(2p+\theta_{321}-\theta_{13})]$ is sufficiently weak in the
 128 vicinity of $\omega = \omega_{2,n}$, $|R_b|$ has a local minimum due to the resonance of layer 2.

129 On the other hand, the frequency dependence of $|r_{321}|$ is expected to be relatively insignificant
 130 in the off-resonance of layer 2. In this case, the reflection spectrum $|R_b|$ takes a local minimum
 131 when the two terms in the numerator of Eq. (9) are in opposite phases [17]. Namely, the resonance
 132 in layer 3 occurs at the angular frequency $\omega = \omega_{3,m}$ ($m = 1, 2, \dots$) that satisfies

$$\omega_{3,m}\tau_3 \cdot \operatorname{Re}\left(\frac{1}{\sqrt{1-i\zeta}}\right) = \left(m - \frac{1}{2}\right)\pi + \frac{\theta_{13} - \theta_{321}}{2}, \quad (11)$$

133 where $\tau_3 = d_3/c_3$. Furthermore, the reflection spectrum at the m th-order resonance angular
 134 frequency $\omega = \omega_{3,m}$ is expected to become $R_b = 0$ if the loss factor satisfies $\zeta = \zeta_m$, such that

$$\operatorname{Im}\left(\frac{1}{\sqrt{1-i\zeta_m}}\right) = \frac{1}{2\omega_{3,m}\tau_3} \ln \frac{|r_{321}|}{|r_{13}|}. \quad (12)$$

135 By replacing $|r_{321}|$ for $|r_{32}|$ and assuming $\zeta_m \ll 1$, Eq. (12) is reduced to the critical attenuation
 136 condition for a viscoelastic layer between dissimilar semi-infinite materials given in Ref. [17].
 137 Consequently, it is predicted that critical attenuation occurs not only in a single viscoelastic layer
 138 but also in the case of a bilayer laminate.

139 Equations (11) and (12) are nonlinear equations for the resonance angular frequency $\omega_{3,m}$ and
 140 the critical loss factor ζ_m , respectively, and cannot analytically be solved. In what follows,
 141 similarly to Ref. [17], Taylor expansion around the loss factor $\zeta = 0$ is employed to simplify the
 142 above conditions.

143 2.2.1 Conditions of resonance and critical attenuation: zeroth-order approximation

144 Under the zeroth-order approximation around the loss factor $\zeta = 0$, Eq. (11) gives the condition
 145 of the resonance in layer 3 as

$$\omega_{3,m}^{(0)}\tau_3 = \left(m - \frac{1}{2}\right)\pi + \frac{\theta_{13}^{(0)} - \theta_{321}^{(0)}}{2}, \quad (13)$$

146 where the superscript (0) represents the quantities at $\zeta = 0$. Namely, based on Eqs. (7) and (8),

$$\theta_{13}^{(0)} = \arg(r_{13}^{(0)}), \quad \theta_{321}^{(0)} = \arg(r_{321}^{(0)}), \quad (14)$$

147 where

$$r_{13}^{(0)} = \frac{z_3^{(0)} - z_1}{z_3^{(0)} + z_1}, \quad r_{321}^{(0)} = \frac{r_{32}^{(0)} + r_{21} \exp(2i\phi_{2m})}{1 + r_{32}^{(0)} r_{21} \exp(2i\phi_{2m})}, \quad (15)$$

$$r_{32}^{(0)} = \frac{z_2 - z_3^{(0)}}{z_2 + z_3^{(0)}} = \frac{1 - z_{32}}{1 + z_{32}}, \quad r_{21} = \frac{z_1 - z_2}{z_1 + z_2} = -\frac{1 - z_{12}}{1 + z_{12}}, \quad (16)$$

$$z_3^{(0)} = \rho_3 c_3, \quad z_{32} = z_3^{(0)} / z_2, \quad z_{12} = z_1 / z_2, \quad \phi_{2m} = \omega_{3,m}^{(0)} \tau_2. \quad (17)$$

148 Since $r_{13}^{(0)}$ is real, $\theta_{13}^{(0)} = 0$ if $z_3^{(0)} > z_1$, and otherwise $\theta_{13}^{(0)} = \pi$. Substitution of Eqs. (16) and 149 (17) into Eq. (15) leads to

$$r_{321}^{(0)} = \frac{(z_{12} - z_{32}) \cos \phi_{2m} - i(1 - z_{12} z_{32}) \sin \phi_{2m}}{(z_{12} + z_{32}) \cos \phi_{2m} - i(1 + z_{12} z_{32}) \sin \phi_{2m}}. \quad (18)$$

150 Thus,

$$|r_{321}^{(0)}| = \sqrt{\frac{(z_{12} - z_{32})^2 \cos^2 \phi_{2m} + (1 - z_{12} z_{32})^2 \sin^2 \phi_{2m}}{(z_{12} + z_{32})^2 \cos^2 \phi_{2m} + (1 + z_{12} z_{32})^2 \sin^2 \phi_{2m}}}, \quad (19)$$

$$\begin{aligned} \tan \theta_{321}^{(0)} &= -2z_{32}(1 - z_{12}^2)g(z_{12}, z_{32}), \\ g(z_{12}, z_{32}) &= \frac{\sin \phi_{2m} \cos \phi_{2m}}{(z_{12}^2 - z_{32}^2) \cos^2 \phi_{2m} + (1 - z_{32}^2 z_{12}^2) \sin^2 \phi_{2m}}. \end{aligned} \quad (20)$$

151 The function $g(z_{12}, z_{32})$ is expressed as Taylor expansion around $(z_{12}, z_{32}) = (0, 0)$, i.e.

$$g(z_{12}, z_{32}) = \frac{1}{\tan \phi_{2m}} + \frac{z_{12}^2}{2} \frac{\partial^2 g(0,0)}{\partial z_{12}^2} + \frac{z_{32}^2}{2} \frac{\partial^2 g(0,0)}{\partial z_{32}^2} + \dots. \quad (21)$$

152 If the acoustic impedance of layer 2 is sufficiently higher than those of layer 3 and medium 1, i.e.

153 $z_{12}, z_{32} \ll 1$, Eq. (20) can be simplified as

$$\tan \theta_{321}^{(0)} = -2z_{32}(1 - z_{12}^2) \left(\frac{1}{\tan \phi_{2m}} + \dots \right) \cong -\frac{2z_{32}}{\tan \phi_{2m}} \equiv -A_m. \quad (22)$$

154 This leads to

$$\theta_{321}^{(0)} = -\tan^{-1} A_m. \quad (23)$$

155 As a result, the zeroth-order resonance condition of Eq. (13) is rewritten as

$$\omega_{3,m}^{(0)} \tau_3 = \left(m - \frac{1}{2} \right) \pi + \frac{\theta_{13}^{(0)}}{2} + \frac{1}{2} \tan^{-1} A_m, \quad (24)$$

156 which is an explicit form of a nonlinear equation with the resonance angular frequency $\omega_{3,m}^{(0)}$. If 157 the third term in the right-hand side is negligible, Eq. (24) is reduced to

$$\omega_{3,m}^{(0)} \tau_3 = \left(m - \frac{1}{2} \right) \pi + \frac{\theta_{13}^{(0)}}{2}, \quad (25)$$

158 which corresponds to the resonance condition of a single layer [18]. Namely, $\tan^{-1}A_m$ in Eq. (24)
 159 represents the coupling effect between layers 3 and 2.

160 The zeroth-order approximation of the resonance angular frequency is applied to the critical
 161 attenuation condition of Eq. (12). If the critical loss factor for the m th-order resonance is assumed
 162 to be sufficiently low, it can be expressed as

$$\zeta_m^{(0)} = \frac{1}{\omega_{3,m}^{(0)} \tau_3} \ln \frac{|r_{321}^{(0)}|}{|r_{13}^{(0)}|}. \quad (26)$$

163 If the coupling effect of layers 3 and 2 is weak, i.e. $|\tan \phi_{2m}| \gg 1$, Eq. (19) is simplified as

$$|r_{321}^{(0)}| \cong \left| \frac{1 - z_{12}z_{32}}{1 + z_{12}z_{32}} \right|, \quad (27)$$

164 leading to

$$\zeta_m^{(0)} = \frac{1}{\omega_{3,m}^{(0)} \tau_3} \ln \left| \frac{(1 - z_{12}z_{32})(z_{12} + z_{32})}{(1 + z_{12}z_{32})(z_{12} - z_{32})} \right|. \quad (28)$$

165 This formula gives the zeroth-order approximation of the critical loss factor if the corresponding
 166 resonance frequency of layer 3 is already calculated.

167 2.2.2 Second-order approximation of resonance condition

168 The zeroth-order approximation of the resonance frequency in layer 3, given by Eq. (24), does
 169 not provide the dependence on the loss factor ζ . In this section, the terms up to the second order
 170 are taken into account. The m th-order resonance condition of Eq. (11) becomes

$$\omega_{3,m}^{(2)} \tau_3 = \left(1 + \frac{\zeta^2}{2} \right) \left[\left(m - \frac{1}{2} \right) \pi + \frac{\theta_{13}^{(2)} - \theta_{321}^{(2)}}{2} \right], \quad (29)$$

171 where the superscript (2) represents the second-order approximation around $\zeta = 0$. The reflection
 172 coefficient r_{13} can be expanded as

$$\begin{aligned} r_{13}^{(2)} &= r_{13}^{(0)} + \zeta \frac{dr_{13}}{d\zeta} \Big|_{\zeta=0} + \frac{\zeta^2}{2} \frac{d^2 r_{13}}{d\zeta^2} \Big|_{\zeta=0} \\ &= r_{13}^{(0)} \left[1 + \frac{\zeta^2}{16} \left(\frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} \right) \left(3 - t_{13}^{(0)} \right) - i \frac{\zeta}{4} \left(\frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} \right) \right], \end{aligned} \quad (30)$$

173 where $t_{13}^{(0)} = 2z_1/(z_3^{(0)} + z_1)$, and its phase is obtained as

$$\theta_{13}^{(2)} = \theta_{13}^{(0)} + \arg \left[1 + \frac{\zeta^2}{16} \left(\frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} \right) \left(3 - t_{13}^{(0)} \right) - i \frac{\zeta}{4} \left(\frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} \right) \right]. \quad (31)$$

174 Similar expansion is performed on the reflection spectrum r_{321} . Since

$$\begin{aligned}
r_{321}^{(2)} &= r_{321}^{(0)} + \zeta \frac{dr_{321}}{d\zeta} \Big|_{\zeta=0} + \frac{\zeta^2}{2} \frac{d^2 r_{321}}{d\zeta^2} \Big|_{\zeta=0} \\
&= r_{321}^{(0)} \left[1 - \frac{\zeta^2}{16} \left(\frac{1}{r_{321}^{(0)}} - r_{321}^{(0)} \right) (3 - t_{321}^{(0)}) + i \frac{\zeta}{4} \left(\frac{1}{r_{321}^{(0)}} - r_{321}^{(0)} \right) \right], \tag{32}
\end{aligned}$$

175 where $t_{321}^{(0)} = 2z_{21}^{\text{in}}/(z_3^{(0)} + z_{21}^{\text{in}})$, its phase is expressed by

$$\theta_{321}^{(2)} = \theta_{321}^{(0)} + \arg \left[1 - \frac{\zeta^2}{16} \left(\frac{1}{r_{321}^{(0)}} - r_{321}^{(0)} \right) (3 - t_{321}^{(0)}) + i \frac{\zeta}{4} \left(\frac{1}{r_{321}^{(0)}} - r_{321}^{(0)} \right) \right]. \tag{33}$$

176 Substitution of Eqs. (31) and (33) into Eq. (29) leads to

$$\omega_{3,m}^{(2)} \tau_3 = \left(1 + \frac{\zeta^2}{2} \right) \left(\omega_{3,m}^{(0)} \tau_3 + \frac{\Phi_{13}^{(2)} - \Phi_{321}^{(2)}}{2} \right). \tag{34}$$

177 where

$$\begin{aligned}
\Phi_{13}^{(2)} &= \arg \left[1 + \frac{\zeta^2}{16} \left(\frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} \right) (3 - t_{13}^{(0)}) - i \frac{\zeta}{4} \left(\frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} \right) \right], \\
\Phi_{321}^{(2)} &= \arg \left[1 - \frac{\zeta^2}{16} \left(\frac{1}{r_{321}^{(0)}} - r_{321}^{(0)} \right) (3 - t_{321}^{(0)}) + i \frac{\zeta}{4} \left(\frac{1}{r_{321}^{(0)}} - r_{321}^{(0)} \right) \right]. \tag{35}
\end{aligned}$$

178 When more than third-order terms with respect to the loss factor are neglected, Eq. (34) becomes

$$\omega_{3,m}^{(2)} \tau_3 = \omega_{3,m}^{(0)} \tau_3 + \zeta \chi_1 + \frac{\zeta^2}{2} \omega_{3,m}^{(0)} \tau_3, \tag{36}$$

179 where

$$\chi_1 = -\frac{1}{8} \left\{ \frac{1}{r_{13}^{(0)}} - r_{13}^{(0)} + \left(\frac{1}{|r_{321}^{(0)}|} - |r_{321}^{(0)}| \right) \cos \theta_{321}^{(0)} \right\}. \tag{37}$$

180 If $|\theta_{321}^{(0)}| \ll 1$ holds, it is further simplified as

$$\chi_1 \cong -\frac{1}{8} \left(r_{13}^{(0)} + |r_{321}^{(0)}| \right) \left(\frac{1}{r_{13}^{(0)} |r_{321}^{(0)}|} - 1 \right). \tag{38}$$

181 If $z_3^{(0)} > z_1$, the zeroth-order reflection coefficient $r_{13}^{(0)}$ satisfies $0 < r_{13}^{(0)} < 1$. Since $0 <$

182 $|r_{321}^{(0)}| < 1$, the coefficient χ_1 becomes $\chi_1 < 0$. On the other hand, the second-order term in the

183 right-hand side of Eq. (36) is always positive.

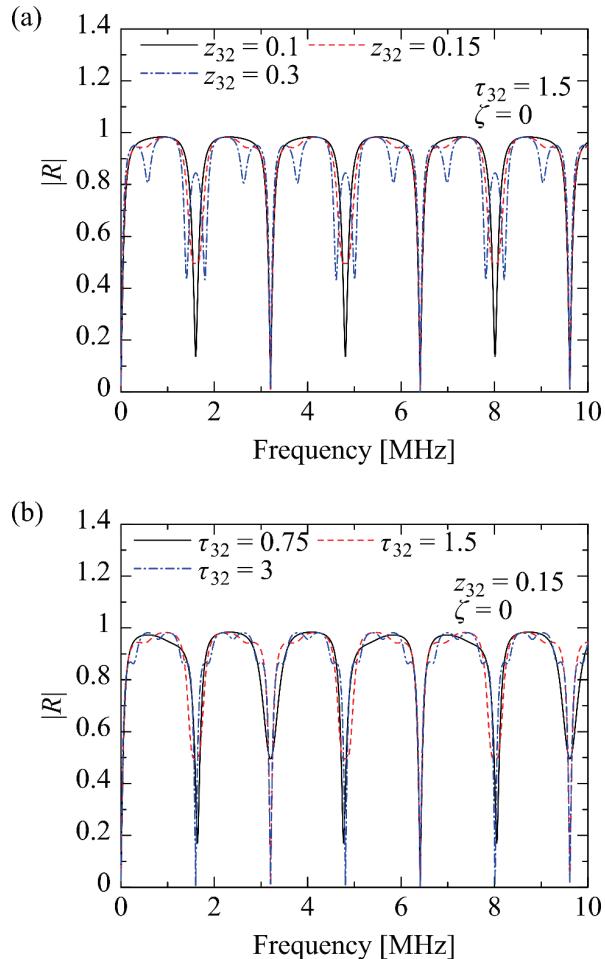
184 3. Theoretical results

185 3.1 Case of purely elastic laminates

186 Before considering viscoelastic cases, the responses of purely elastic bilayer laminates are
187 investigated in this section. Layer 2 is modeled by an elastic layer of thickness $d_2 = 2$ mm, mass
188 density $\rho_2 = 2.65 \times 10^3$ kg/m³, and wave velocity $c_2 = 6.41$ km/s, which are based on the properties

189 of aluminum alloy 5052. The surrounding medium is assumed to be water with $\rho_1 = 1.00 \times 10^3$
190 kg/m^3 and $c_1 = 1.48 \text{ km/s}$. In this case, the acoustic impedance ratio z_{12} is $z_{12} = 0.0871$. At $\zeta = 0$,
191 the reflection spectrum for the bilayer laminate is calculated for different acoustic impedance
192 ratios z_{32} and normalized quantities $\tau_{32} = \tau_3/\tau_2$. In this study, τ_{32} is called the normalized thickness
193 ratio. If layer 3 is polystyrene of thickness $d_3 = 1.2 \text{ mm}$, mass density $\rho_3 = 1.02 \times 10^3 \text{ kg/m}^3$, and
194 wave velocity $c_3 = 2.16 \text{ km/s}$, the above quantities are obtained as $z_{32} = 0.130$ and $\tau_{32} = 1.78$.

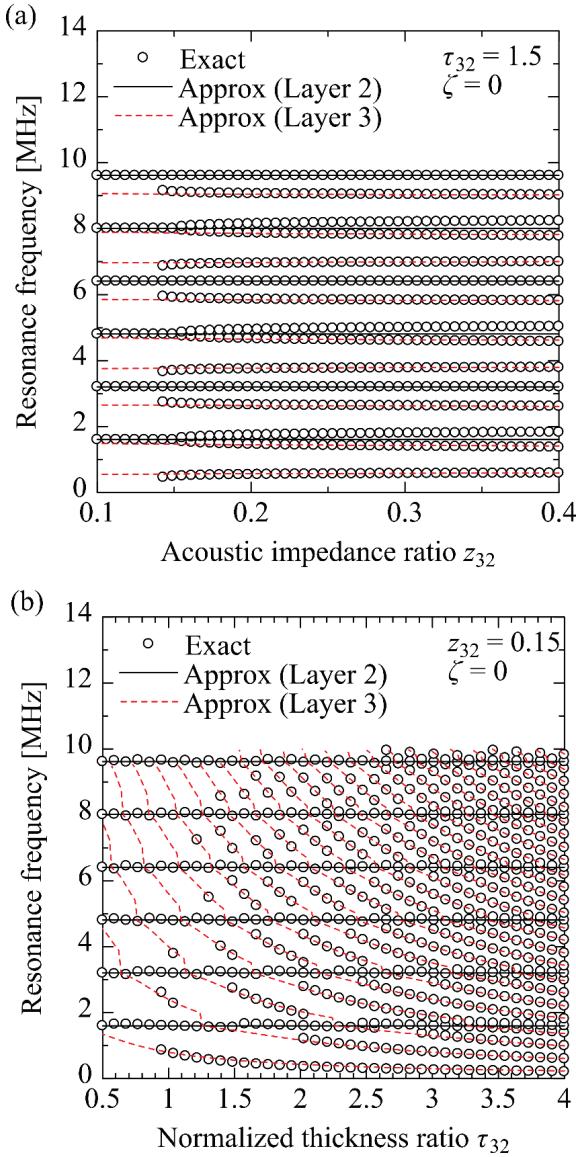
195 Fig. 2(a) shows the amplitudes of the reflection spectrum as functions of frequency, obtained
196 by Eq. (6) at a fixed normalized thickness ratio $\tau_{32} = 1.5$. When the acoustic impedance ratio is
197 low, i.e. $z_{32} = 0.1$, local minima lower than 0.2 appear periodically at 1.602 MHz, 3.204 MHz,
198 4.808 MHz, etc. These drops result from the resonance in layer 2, and the resonance frequencies
199 are in good agreement with Eq. (10). However, at the higher acoustic impedance ratio, i.e. $z_{32} =$
200 0.3, additional local minima emerge. Namely, if the acoustic impedance of layer 3 is closer to that
201 of layer 2, the effect of layer 3 on the reflection spectrum becomes more significant. This feature
202 is further examined later.



203
204 Fig. 2 Amplitudes of the reflection spectrum for purely elastic laminates calculated for (a)
205 different acoustic impedance ratios z_{32} at $\tau_{32} = 1.5$ and (b) different normalized thickness
206 ratios τ_{32} at $z_{32} = 0.15$.
207

208 Fig. 2(b) shows the amplitudes of the reflection spectrum calculated by Eq. (6) for different
209 normalized thickness ratios τ_{32} at a fixed acoustic impedance ratio $z_{32} = 0.15$. In this figure, local
210 minima resulting from the resonance in layer 2, which have also been confirmed in Fig. 2(a), are
211 clear if the normalized thickness ratio τ_{32} changes. The normalized thickness ratio τ_{32} seems to
212 affect the reflection spectra, particularly in the vicinity of the local maxima.

213 The frequencies at which local minima appear were extracted below 10 MHz from the
214 amplitudes of the reflection spectrum calculated by Eq. (6) in various conditions. Fig. 3(a) and
215 (b) show the local minimum frequencies as functions of the acoustic impedance ratio z_{32} and the
216 normalized thickness ratio τ_{32} , respectively. In Fig. 3(a), the normalized thickness ratio is fixed at
217 $\tau_{32} = 1.5$, and in Fig. 3(b), the acoustic impedance ratio is set as $z_{32} = 0.15$. It is found that the
218 variation of the resonance frequencies is almost flat in Fig. 3(a), but some of the resonance
219 frequencies decrease significantly with increasing normalized thickness ratio τ_{32} in Fig. 3(b).



220
221 Fig. 3 Relations of the local minimum frequencies to (a) the acoustic impedance ratio z_{32} at z_{32}
222 $= 1.5$ and (b) the normalized thickness ratio τ_{32} at $z_{32} = 0.15$ extracted from the amplitudes
223 of the reflection spectrum calculated by Eq. (6) for purely elastic laminates, together with
224 the zeroth-order approximate results for the resonance frequencies of layers 2 and 3.
225

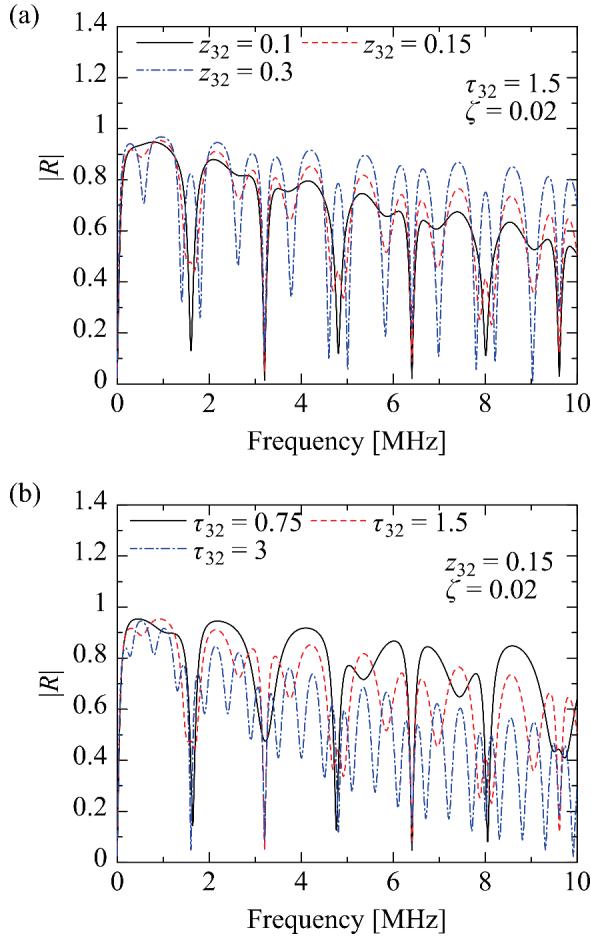
226 For comparison, the zeroth-order approximation results for the resonance frequencies of
227 layers 2 and 3 were obtained by Eqs. (10) and (24), respectively. The nonlinear equation of Eq.
228 (24) was numerically solved with the bisection method. The obtained results are shown together
229 in Fig. 3(a) and (b). On the whole, the exact values of the resonance frequencies are in good
230 agreement with the approximate results. In particular, the approximate results in Fig. 3(b) fairly
231 reproduce that the resonance frequencies of layer 2 are almost unchanged if the normalized
232 thickness ratio τ_{32} varies, while those of layer 3 decrease with decreasing τ_{32} .

233 It is noted that the approximate results for the resonance frequencies of layer 2, given by Eq.
234 (10), do not depend on the acoustic impedance ratio z_{32} . In Fig. 3(a), however, if the approximate
235 result for a resonance frequency of layer 2 is close to that for layer 3, the exact result for the
236 resonance frequency of layer 2 tends to increase as the acoustic impedance ratio z_{32} increases. For
237 example, the resonance frequency of 1.603 MHz at $z_{32} = 0.1$ becomes 1.831 MHz at $z_{32} = 0.4$.
238 This deviation is probably attributed to the fact that the coupling effects between layers 2 and 3
239 are neglected in the approximate formula of Eq. (10). The modification of Eq. (10) could lead to
240 better agreement between the exact and approximate results for the resonance frequencies of layer
241 2. However, this is not further explored because the present study has an interest in the resonance
242 phenomenon occurring in layer 3.

243 In Fig. 3(a), the resonance frequencies of layer 3 are well reproduced by the approximate
244 results if the acoustic impedance ratio z_{32} increases. This can be attributed to the coupling term in
245 Eq. (24). On the other hand, in $z_{32} < 0.14$, the exact results for the resonance frequencies of layer
246 3 could not be obtained because the local minima do not appear in the amplitude of the reflection
247 spectrum. This phenomenon corresponds to the result in Ref. [18] that the amplitude spectrum of
248 the reflected wave from a purely elastic bilayer laminate is not affected by the resonance of the
249 layer with lower acoustic impedance. The resonance effect is expected to emerge if the loss factor
250 of layer 3 is non-zero [18]. In the next section, the effect of the loss factor ζ of layer 3 on the
251 reflection spectrum is examined.

252 3.2 Case of viscoelastic laminates

253 The amplitudes of the reflection spectrum for viscoelastic-elastic bilayer laminates were
254 calculated by Eq. (6). Based on the measured result for polystyrene, the loss factor was set to be
255 $\zeta = 0.02$. Fig. 4(a) and (b) show the obtained results for different acoustic impedance ratios z_{32}
256 and normalized thickness ratios τ_{32} , respectively. The parameters except for the loss factor ζ in
257 Fig. 4(a) and (b) are the same as those in Fig. 2(a) and (b), respectively. In Fig. 2(a), only local
258 minima corresponding to the resonance of layer 2 have appeared at $z_{32} = 0.1$. However, in Fig.
259 4(a), the reflection spectrum has additional local minima at $z_{32} = 0.1$ when $\zeta = 0.02$. This result
260 implies that the effect of the non-zero loss factor ζ deepens the local minima of the reflection
261 spectrum due to the resonance of layer 3. The additional local minimum frequencies appear to be
262 unchanged at the high acoustic impedance ratio $z_{32} = 0.3$. In Fig. 4(b), the normalized thickness
263 ratio τ_{32} affects the intervals of local minima in the reflection spectrum. This trend is different
264 from the case of $\zeta = 0$ in which only clear local minima corresponding to the resonance of layer
265 2 appear even if the normalized thickness ratio τ_{32} changes, as shown in Fig. 2(b). In what follows,
266 the characteristics of the resonance and critical attenuation are separately investigated.

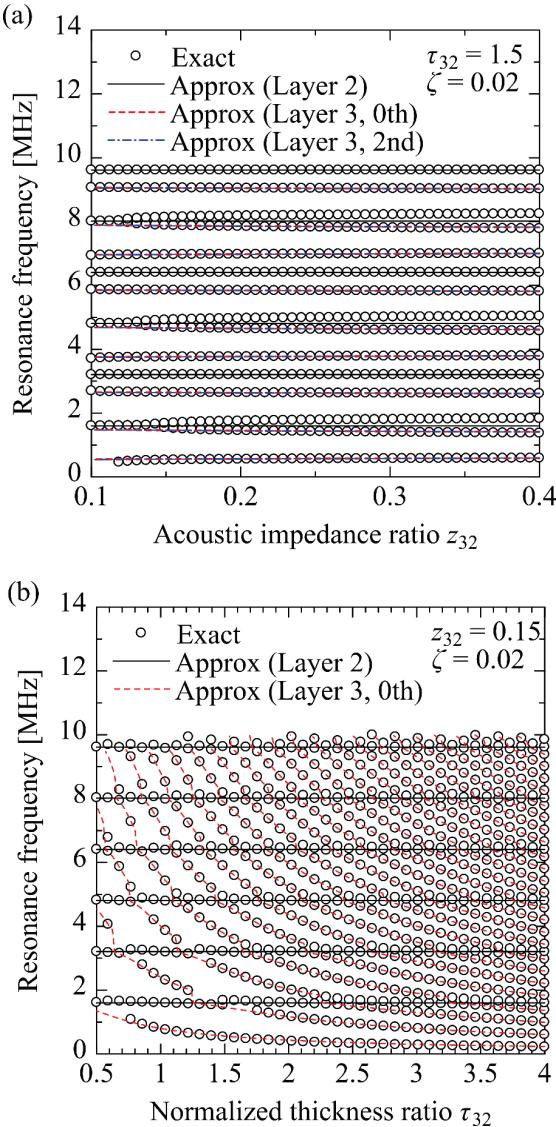


267

268 Fig. 4 Amplitudes of the reflection spectrum for viscoelastic-elastic bilayer laminates of $\zeta =$
 269 0.02, calculated for (a) different acoustic impedance ratios z_{32} at $\tau_{32} = 1.5$ and (b) different
 270 normalized thickness ratios τ_{32} at $z_{32} = 0.15$.

271 *3.2.1 Resonance behavior*

272 The resonance frequencies lower than 10 MHz were extracted from the reflection spectrum
 273 calculated by Eq. (6) for viscoelastic laminates of the loss factor $\zeta = 0.02$. The obtained results
 274 are shown as the functions of the acoustic impedance ratio z_{32} and the normalized thickness ratio
 275 τ_{32} in Fig. 5(a) and (b), respectively. In Fig. 5(a), the normalized thickness ratio is fixed at $\tau_{32} =$
 276 1.5, and in Fig. 5(b), the acoustic impedance ratio is set as $z_{32} = 0.15$. For comparison, the second-
 277 order approximation of the resonance frequencies of layer 3 is calculated by Eqs. (36) and (37)
 278 and is shown in Fig. 5(a), together with the zeroth-order resonance frequencies obtained by Eqs.
 279 (10) and (24). It is found in Fig. 5(a) that the differences between the second- and zeroth-order
 280 approximate results are trivial. This fact is attributed to the sufficiently low loss factor $\zeta = 0.02$.
 281 Accordingly, only the zeroth-order approximate results are shown in Fig. 5(b) for comparison.



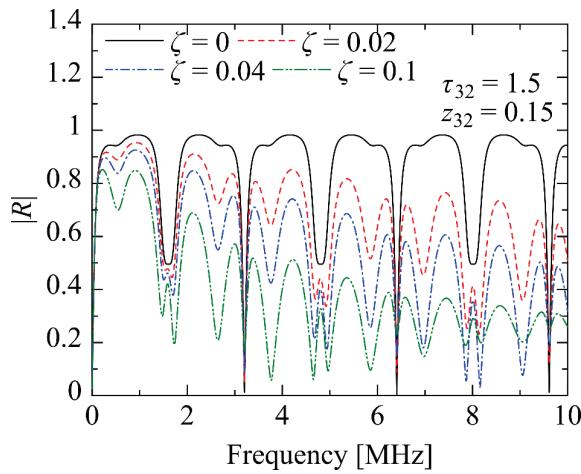
282

283 Fig. 5 Relations of the local minimum frequencies to (a) the acoustic impedance ratio z_{32} at z_{32}
 284 = 1.5 and (b) the normalized thickness ratio τ_{32} at $z_{32} = 0.15$, extracted from the amplitudes
 285 of the reflection spectrum calculated by Eq. (6) for viscoelastic laminates of $\zeta = 0.02$,
 286 together with the approximate results for the resonance frequencies of layers 2 and 3.

287

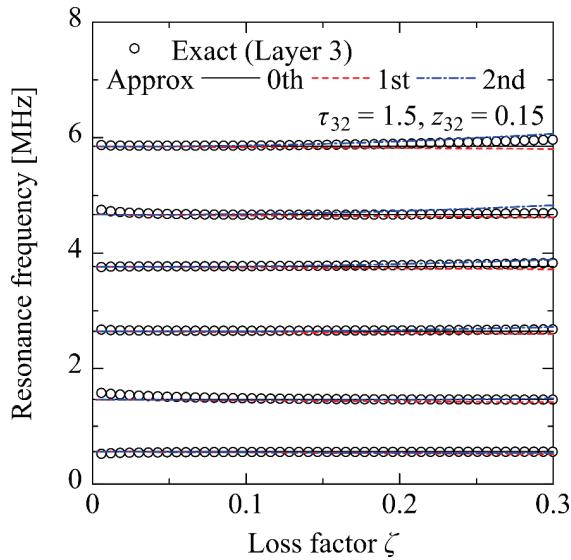
288 The exact resonance frequencies in Fig. 5(a) and (b) are fairly reproduced by the zeroth-order
 289 approximation for layers 2 and 3. This tendency is analogous to the cases of purely elastic
 290 laminates in Fig. 3(a) and (b). However, the exact results in Fig. 5(a) and (b) have more numbers
 291 of resonance frequencies of layer 3 than those in Fig. 3(a) and (b). For example, in Fig. 3(a), the
 292 exact values of the resonance frequencies of layer 3 are missing in $z_{32} < 0.14$ when $\zeta = 0$, but they
 293 appear when $\zeta = 0.02$ in Fig. 5(a). This difference results from the viscoelastic effect of layer 3
 294 [18].

295 To clarify the quantitative effect of the loss factor ζ , the amplitudes of the reflection spectrum
 296 for the bilayer laminates were calculated for different loss factors by Eq. (6). Fig. 6 shows the
 297 obtained results at fixed acoustic impedance ratio $z_{32} = 0.15$ and normalized thickness ratio $\tau_{32} =$
 298 1.5. At the resonance frequencies of layer 3 located below 6 MHz, e.g. 2.64 MHz and 3.76 MHz,
 299 the amplitude of the reflection spectrum decreases with increasing loss factor in $\zeta < 0.1$. Namely,
 300 the loss factor contributes to the generation of the local minima. However, this tendency is not
 301 confirmed at higher frequencies. For example, when $\zeta = 0.04$, the amplitude of the reflection
 302 spectrum at the resonance frequency of 9.06 MHz is smaller than that for $\zeta = 0.1$.



303
 304 Fig. 6 Amplitudes of the reflection spectrum for viscoelastic laminates calculated for different
 305 loss factors ζ at fixed acoustic impedance ratio $z_{32} = 0.15$ and normalized thickness ratio
 306 $\tau_{32} = 1.5$.
 307

308 The resonance frequencies were extracted from the reflection spectra calculated for different
 309 loss factors ζ at $z_{32} = 0.15$ and $\tau_{32} = 1.5$. Fig. 7 shows the results only for the resonance frequencies
 310 of layer 3 as functions of the loss factor in $\zeta < 0.3$. The frequency range was set to be down to 6
 311 MHz to show the dependence on the loss factor more clearly. It is found in this figure that the
 312 effect of the loss factor ζ on the resonance frequencies is not significant, but some of the resonance
 313 frequencies slightly increase with increasing loss factor. For example, the resonance frequency
 314 5.86 MHz at $\zeta = 0.006$ becomes 5.96 MHz at $\zeta = 0.3$. The resonance frequencies in the vicinity
 315 of 1.46 MHz and 4.67 MHz slightly decrease in $\zeta < 0.05$, but this behavior appears probably
 316 because the resonance frequencies of layer 2 exist nearby, as shown in Fig. 5(a). The coupling
 317 effect of layers 2 and 3 would be significant in these cases.



318

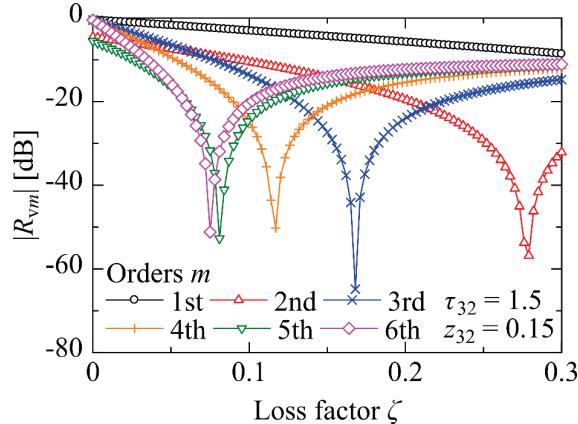
319 Fig. 7 Relations of the resonance frequencies of layer 3 to the loss factor ζ at fixed acoustic
 320 impedance ratio $z_{32} = 0.15$ and normalized thickness ratio $\tau_{32} = 1.5$, extracted from the
 321 amplitudes of the reflection spectra calculated by Eq. (6), together with the approximate
 322 results.

323

324 The approximate results for the resonance frequencies of layer 3 with three different orders
 325 are shown together in Fig. 7. The zeroth-order resonance frequencies are invariant with the loss
 326 factor ζ . The resonance frequencies under the first-order approximation, obtained by eliminating
 327 the second-order term in Eq. (36), linearly decrease with increasing loss factor because the
 328 coefficient χ_1 is negative in this case. Consequently, the second-order term is necessary to
 329 reproduce the increasing behavior of the resonance frequencies with increasing loss factor in $\zeta <$
 330 0.3. However, the loss factor of common plastics is lower than $\zeta = 0.1$, implying that the higher-
 331 order terms do not play an apparent role. In other words, the zeroth-order approximation seems
 332 to be sufficient in predicting the resonance frequencies of a common plastic layer.

333 3.2.2 Critical attenuation behavior

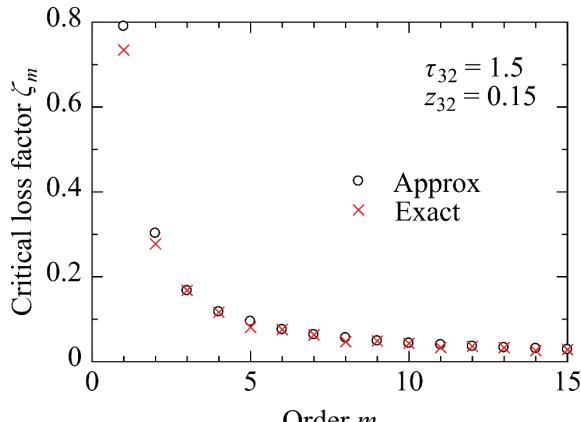
334 The amplitudes of the reflection spectrum at the m th-order resonance of layer 3, denoted as
 335 $|R_{vm}|$ ($m = 1, 2, \dots$), are further examined in this section. Based on the results in the previous
 336 section, $|R_{vm}|$ was calculated as the amplitude of the reflection spectrum at the zeroth-order
 337 resonance angular frequency of layer 3, $\omega = \omega_{3,m}^{(0)}$, which is given by Eq. (24). Following Fig. 7,
 338 the amplitudes $|R_{vm}|$ for $m = 1-6$ are shown in Fig. 8 as the functions of the loss factor in $\zeta < 0.3$.
 339 The acoustic impedance ratio z_{32} and the normalized thickness ratio τ_{32} in Fig. 8 are the same as
 340 those in Fig. 7. It is shown in Fig. 8 that the amplitudes $|R_{vm}|$ at the second- to sixth-order
 341 resonances ($m = 2-6$) have deep minima at different loss factors. This phenomenon corresponds
 342 to critical attenuation in bilayer laminates.



343
 344 Fig. 8 Relations of the amplitudes of the reflection spectrum at different-order resonance
 345 frequencies of layer 3 to the loss factor ζ at fixed acoustic impedance ratio $z_{32} = 0.15$ and
 346 normalized thickness ratio $\tau_{32} = 1.5$, calculated by Eq. (6).

347

348 The variation of the amplitude $|R_{vm}|$ with the loss factor ζ was calculated for each order m , and
 349 the critical loss factor $\zeta = \zeta_m$ which minimizes $|R_{vm}|$ was sought in a range of $0.01 < \zeta < 1$. The
 350 obtained results are plotted with the resonance orders $m = 1-15$ in Fig. 9. The critical loss factor
 351 ζ_m decreases as the resonance order m increases. Larger loss factors would be necessary to realize
 352 the critical attenuation at low resonance frequencies.



353
 354 Fig. 9 Critical loss factors for different resonance orders extracted from the amplitudes of the
 355 reflection spectrum at fixed acoustic impedance ratio $z_{32} = 0.15$ and normalized thickness
 356 ratio $\tau_{32} = 1.5$, together with the zeroth-order approximate results based on Eq. (28).

357

358 Furthermore, the zeroth-order approximate results of the critical loss factor $\zeta_m^{(0)}$, calculated
 359 by Eq. (28), are shown together in Fig. 9. As a result, the zeroth-order results agree well with the
 360 exact critical loss factors. Namely, the decrease of the critical loss factor ζ_m with increasing
 361 resonance order m is well reproduced by the inverse proportion to the resonance angular frequency,
 362 as expressed in Eq. (28). In Fig. 9, the deviation between the exact and approximate results is

363 relatively distinct at low resonance orders probably because the loss factor is not sufficiently small
364 to satisfy the zeroth-order approximation. The agreement might become better if higher-order
365 terms are considered in the formula of the critical loss factor. However, in a practical sense, this
366 extension could not be that essential because the loss factors of common plastics are not so high
367 as $\zeta = 0.1$. The modification of the formula is not further explored in this study.

368 **4. Experimental validation**

369 *4.1 Specimens and measurement setup*

370 Ultrasonic measurements were performed on metal-plastic bonded specimens to examine and
371 discuss the theoretical findings. The details of the specimens used in the experiments are
372 summarized in Table 1. PS1.2/AL2.0 is a bonded specimen of polystyrene (PS) and aluminum
373 alloy (AL) plates with thicknesses of 1.2 mm and 2.0 mm, respectively [18]. A cyanoacrylate
374 adhesive (Henkel, Loctite 406) was used to join the two plates. Microscopic observation of side
375 faces showed that PS1.2/AL2.0 had an adhesive layer with a thickness of 5.1 μm . PC1.5/AL0.8
376 corresponds to a bonded specimen of polycarbonate (PC) and AL plates with thicknesses of 1.55
377 mm and 0.8 mm, respectively, which was produced in a similar manner to PS1.2/AL2.0. The PS
378 and PC sheets have square shapes with side lengths of 30 mm.

380 **Table 1** Specimens used in the experiments.

Specimen	Description
PS1.2/AL2.0	Bonded specimen of 1.20 mm thick polystyrene plate and 2.00 mm thick aluminum alloy A5052 plate [18]
PC1.5/AL0.8	Bonded specimen of 1.55 mm thick polycarbonate plate and 0.80 mm thick aluminum A1050 plate
PS1.2	Polystyrene plate with a thickness of 1.20 mm
PC1.5	Polycarbonate plate with a thickness of 1.55 mm

381
382 The same experimental setup as Ref. [18] was used in the present study. The measurement
383 was performed at room temperature, i.e. approximately 28 °C. A specimen was placed in a water
384 tank and subjected to normal wave incidence. An Olympus immersion transducer V311-SU with
385 an element diameter of 0.5 inches and a nominal frequency of 10 MHz was used to emit the
386 incident wave and detect the reflected waves from the specimen. The distance between the
387 transducer and the specimen was approximately 20 mm. A spike pulse voltage was supplied to
388 the transducer by a JSR Ultrasonics pulser/receiver DPR300. The reflection waveform was passed
389 in a lowpass filter of 22.5 MHz and was recorded by a Tektronix oscilloscope MDO3014 after

390 averaging over 64 synchronized signals. The measured data were transferred to a PC via
391 LabVIEW.

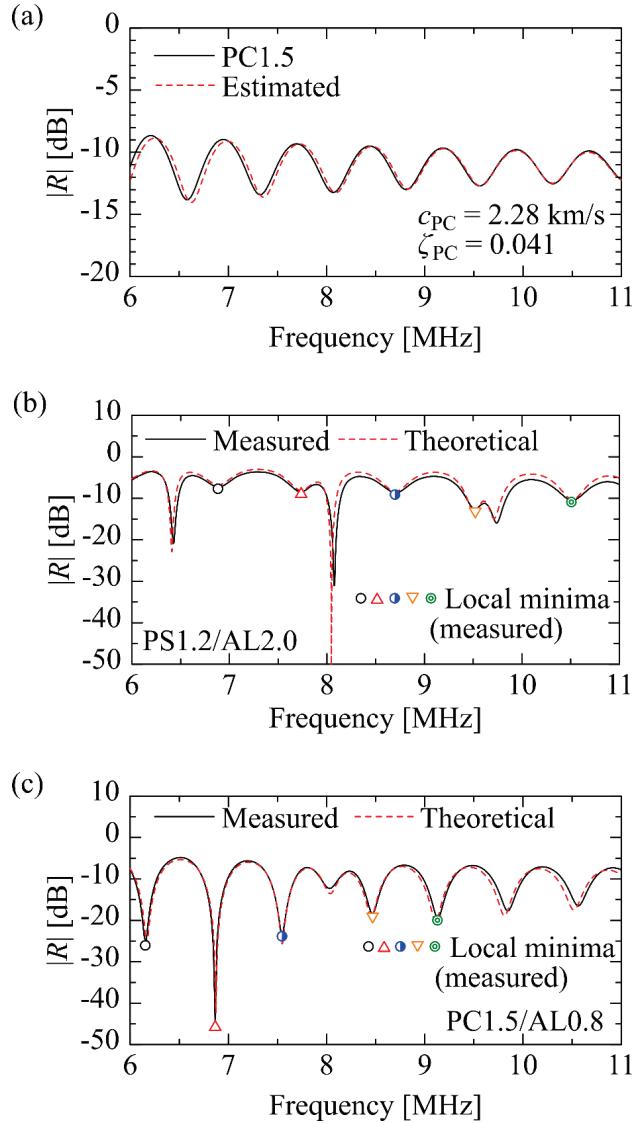
392 The measured waveform was analyzed by fast Fourier transform (FFT) to obtain its spectrum
393 $P_L(f)$, where f is frequency. The reflection spectrum

$$394 R(f) = \frac{P_L(f)}{I(f)}, \quad (39)$$

395 was calculated, where $I(f)$ is the spectrum of the incident wave. The incident spectrum $I(f)$ was
396 obtained based on the surface reflection waveform from a known material. In this study, a 10 mm
397 thick stainless steel 303 block was used to measure the spectrum of the surface reflection
398 waveform $P_s(f)$. The spectrum of the incident wave $I(f)$ was calculated by $I(f) = P_s(f)/r_{ws}$, where
399 $r_{ws} = 0.938$ was the reflection coefficient between water and stainless steel obtained by Eq. (7)
and the acoustic impedances of the two materials.

400 4.2 Experimental results and discussions

401 The measurements were first performed on single plastic and metal plates to obtain their
402 material properties. Fig. 10(a) shows the amplitude of the reflection spectrum $|R|$ measured for a
403 1.55 mm thick PC plate, called PC1.5, in a frequency range of 6–11 MHz. It is noted that the
404 measured reflection spectrum is shown in dB scales. This result was compared to the theoretical
405 curve obtained by Eq. (8). The wave velocity c_{PC} and loss factor ζ_{PC} that reproduce the measured
406 reflection spectrum were sought in a range of $2 \text{ km/s} < c_{PC} < 3 \text{ km/s}$ with an increment of 0.01
407 km/s and $0 < \zeta_{PC} < 0.08$ with an increment of 0.001. As a result, $c_{PC} = 2.28 \text{ km/s}$ and $\zeta_{PC} = 0.042$
408 were obtained as optimal values for PC. The theoretical reflection spectrum calculated with these
409 quantities is shown together in Fig. 10(a). It is found in this figure that the measured data are well
410 reproduced by the theoretical reflection spectrum calculated with the estimated properties. This
411 agreement implies that it is consistent to assume the frequency invariance regarding the loss factor
412 of PC. Similar measurements were performed on a polystyrene plate of thickness 1.20 mm, called
413 PS1.2, and two aluminum plates. The properties of each material were identified based on the
414 measured spectra, as summarized in Table 2. The mass densities of the specimens were obtained
415 by measuring the weights and dimensions. The material properties given in Table 2 are used when
416 comparing the experimental data for the metal-plastic bonded specimens to the theoretical
417 predictions.



418

419 Fig. 10 Amplitudes of the reflection spectra measured for (a) PC1.5, (b) PS1.2/AL2.0, and (c)
 420 PC1.5/AL0.8, together with the corresponding theoretical curves. Some of the measured
 421 local minima at the resonance of the plastic layers are shown by symbols in (b) and (c).
 422

423 Table 2 Material properties identified by ultrasonic measurements.

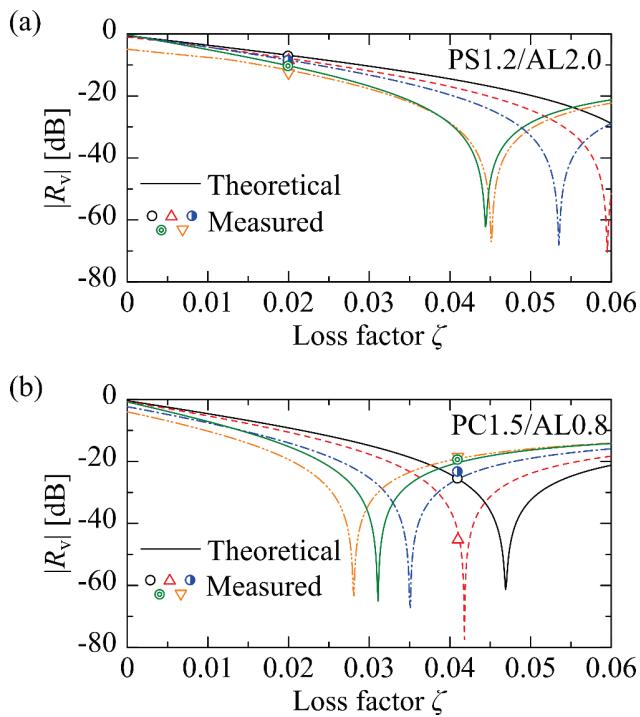
	c [km/s]	ζ	ρ [kg/m ³]
Polycarbonate (PC)	2.28	0.041	1.15×10^3
Polystyrene (PS)	2.16	0.020	1.02×10^3
Aluminum alloy 5052	6.41	0	2.65×10^3
Aluminum 1050	6.46	0	2.65×10^3

424

425 Fig. 10(b) and (c) show the amplitudes of the reflection spectra measured for the two bonded
426 specimens, PS1.2/AL2.0 and PC1.5/AL0.8, respectively. The measured data for PS1.2/AL2.0 in
427 Fig. 10(b) are based on Ref. [18]. It is found that local minima appear in the reflection spectra for
428 both PS1.2/AL2.0 and PC1.5/AL0.8. The theoretical reflection spectrum was calculated for each
429 specimen by Eq. (6) based on the material properties in Table 2. The obtained results when the
430 thicknesses of the PS and PC layers are set to be 1.20 mm and 1.57 mm, respectively, are shown
431 in Fig. 10(b) and (c), respectively. The experimental results are fairly reproduced by the
432 theoretical curves. This agreement implies that the theoretical formulation regarding the reflection
433 spectrum is sufficiently valid to reproduce the responses of the PS/AL and PC/AL laminates to
434 normal wave incidence. Namely, the adhesive layers of the two specimens are sufficiently thin
435 that they can be modeled as bilayer laminates.

436 In Section 3.2, the amplitudes of the reflection spectrum at the resonance of layer 3 (surface
437 layer), i.e. $|R_{vm}|$, were theoretically examined for different loss factors ζ . However, it was difficult
438 to change the loss factor continuously in the present experiment. Thus, the amplitudes $|R_v|$ for
439 different resonance orders were investigated for the two specimens. In each case, five of the
440 measured amplitudes at the resonance frequencies of the plastic layer were extracted, as
441 represented by symbols in Fig. 10(b) and (c). The local minima at around 6.4 MHz and 8 MHz in
442 Fig. 10(b) and 8 MHz in Fig. 10(c) correspond to the resonances of the AL layers, which are not
443 considered here.

444 The obtained amplitudes $|R_v|$ for PS1.2/AL2.0 and PC1.5/AL0.8 are plotted in Fig. 11(a) and
445 (b), respectively. The loss factors of the plastic layers in PS1.2/AL2.0 and PC1.5/AL0.8 are
446 assumed to be $\zeta = 0.020$ and $\zeta = 0.041$, respectively, based on Table 2. The types of symbols in
447 Fig. 11(a) and (b) are identical to those in Fig. 10(b) and (c), respectively. The theoretical relations
448 of $|R_v|$ at the corresponding resonance orders to the loss factor ζ are shown together in Fig. 11(a)
449 and (b). In the theoretical calculation, the zeroth-order approximate resonance frequencies were
450 used to obtain the amplitudes. The colors of the lines and symbols in the figures correspond to the
451 orders of the resonance. Fig. 11(a) and (b) show that the measured amplitudes $|R_v|$ are located
452 almost on the theoretical curves. For PS1.2/AL2.0 in Fig. 11(a), the loss factor is much lower than
453 the critical attenuation conditions in $\zeta > 0.04$. On the other hand, the experimental results for
454 PC1.5/AL0.8 in Fig. 11(b) demonstrate that the amplitude represented by a red triangle almost
455 satisfies the theoretical condition of the critical attenuation ($\zeta_m = 0.0418$). The amplitude at this
456 resonance order would be sensitive to the loss factor in the vicinity of $\zeta = 0.0418$. The critical loss
457 factors of the other orders in Fig. 11(b) are predicted to be in $0.025 < \zeta < 0.05$.



458
459 Fig. 11 Theoretical relations of the amplitudes $|R_v|$ at five different resonance orders for (a)
460 PS1.2/AL2.0 and (b) PC1.5/AL0.8 to the loss factor ζ , and their comparison to the
461 measured results. The experimental results correspond to the symbols shown in Fig. 10(a)
462 and (b), and the colors of the lines and symbols represent the orders of the resonance. (For
463 interpretation of the references to color in this figure legend, the reader is referred to the
464 web version of this article.)

465 **5. Conclusions**

466 In this study, the resonance of ultrasonic waves in metal-plastic bilayer laminates placed in an
467 infinite medium has been theoretically investigated, and critical attenuation has been shown to
468 occur in the laminates under specific conditions. Based on the viscoelastic model using the loss
469 factor, the approximate conditions of the wave resonance and critical attenuation have been
470 derived theoretically and compared to the exact results. For the normal wave incidence from the
471 side of the plastic layer, it has been shown that the viscoelastic nature facilitates the wave
472 interference, and the resonance frequencies of the plastic layer slightly increase with increasing
473 loss factor. Nevertheless, the effect of the viscoelasticity on the resonance frequencies has not
474 been significant if the loss factor is within the range of common plastics. On the other hand, it has
475 been shown that the amplitude of the reflection spectrum at the resonance frequencies of the
476 plastic layer depends on the loss factor, taking a minimum at a certain loss factor. The critical loss
477 factors calculated by the reflection spectrum have been successfully predicted by the approximate
478 results. Furthermore, ultrasonic measurements have been performed on two different metal-
479 plastic bonded specimens to validate the theoretical results. The measured reflection spectra have

480 agreed well with the theoretical results based on the material properties obtained by other
481 ultrasonic measurements. The drop of the amplitude at the resonance due to the critical attenuation
482 has been successfully observed and well reproduced by the theoretical prediction.

483 **Declaration of competing interest**

484 The authors declare that they have no known competing financial interests or personal
485 relationships that could have appeared to influence the work reported in this paper.

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489 **References**

- 490 [1] L.M. Brekhovskikh, Waves in Layered Media, Academic Press, New York, 1976.
- 491 [2] R.D. Borcherdt, Viscoelastic Waves in Layered Media, Cambridge University Press,
492 Cambridge, 2009.
- 493 [3] C.C.H. Guyott, P. Cawley, Evaluation of the cohesive properties of adhesive joints using
494 ultrasonic spectroscopy, *NDT Int.* 21 (1988) 233–240. [https://doi.org/10.1016/0308-9126\(88\)90336-7](https://doi.org/10.1016/0308-9126(88)90336-7).
- 496 [4] T. Pialucha, P. Cawley, The detection of thin embedded layers using normal incidence
497 ultrasound, *Ultrasonics* 32 (1994) 431–440. [https://doi.org/10.1016/0041-624X\(94\)90062-0](https://doi.org/10.1016/0041-624X(94)90062-0).
- 498 [5] N.F. Haines, J.C. Bell, P.J. McIntyre, The application of broadband ultrasonic spectroscopy
499 to the study of layered media, *J. Acoust. Soc. Am.* 64 (1978) 1645–1651.
<https://doi.org/10.1121/1.382131>.
- 501 [6] R. Fiorito, W. Madigosky, H. Überall, Theory of ultrasonic resonances in a viscoelastic layer,
502 *J. Acoust. Soc. Am.* 77 (1985) 489–498. <https://doi.org/10.1121/1.391868>.
- 503 [7] F. Simonetti, P. Cawley, Ultrasonic interferometry for the measurement of shear velocity and
504 attenuation in viscoelastic solids, *J. Acoust. Soc. Am.* 115 (2004) 157–164.
<https://doi.org/10.1121/1.1631944>.
- 506 [8] S. Dixon, B. Lanyon, G. Rowlands, Coating thickness and elastic modulus measurement
507 using ultrasonic bulk wave resonance, *Appl. Phys. Lett.* 88 (2006) 141907.
<https://doi.org/10.1063/1.2192144>.
- 509 [9] S. Dixon, B. Lanyon, G. Rowlands, Ultrasonic resonance in thin two-layer dynamic systems,
510 *J. Phys. D. Appl. Phys.* 39 (2006) 506–514. <https://doi.org/10.1088/0022-3727/39/3/014>.
- 511 [10] A.I. Lavrentyev, S.I. Rokhlin, Ultrasonic spectroscopy of imperfect contact interfaces
512 between a layer and two solids, *J. Acoust. Soc. Am.* 103 (1998) 657–664.
<https://doi.org/10.1121/1.423235>.

514 [11] C. Han, C.T. Sun, Attenuation of stress wave propagation in periodically layered elastic
515 media, *J. Sound Vib.* 243 (2001) 747–761. <https://doi.org/10.1006/jsvi.2000.3420>.

516 [12] J. Kaplunov, A. Krynnik, Resonance vibrations of an elastic interfacial layer, *J. Sound Vib.*
517 294 (2006) 663–677. <https://doi.org/10.1016/j.jsv.2005.11.030>.

518 [13] N. Mori, N. Matsuda, T. Kusaka, Effect of interfacial adhesion on the ultrasonic interaction
519 with adhesive joints: A theoretical study using spring-type interfaces, *J. Acoust. Soc. Am.* 145
520 (2019) 3541–3550. <https://doi.org/10.1121/1.5111856>.

521 [14] R. Hodé, S. Raetz, N. Chigarev, J. Blondeau, N. Cuvillier, V. Gusev, M. Ducoussو, V. Tournat,
522 Laser ultrasonics in a multilayer structure: Plane wave synthesis and inverse problem for
523 nondestructive evaluation of adhesive bondings, *J. Acoust. Soc. Am.* 150 (2021) 2076–2087.
524 <https://doi.org/10.1121/10.0005975>.

525 [15] E.V. Glushkov, N.V. Glushkova, Multiple zero-group velocity resonances in elastic layered
526 structures, *J. Sound Vib.* 500 (2021) 116023. <https://doi.org/10.1016/j.jsv.2021.116023>.

527 [16] N. Mori, D. Wakabayashi, T. Hayashi, Tangential bond stiffness evaluation of adhesive lap
528 joints by spectral interference of the low-frequency A0 Lamb wave, *Int. J. Adhes. Adhes.* 113
529 (2022) 103071. <https://doi.org/10.1016/j.ijadhadh.2021.103071>.

530 [17] A.I. Lavrentyev, S.I. Rokhlin, Anomalous attenuation effect on reflectivity of an ultrasonic
531 wave from a thin layer between dissimilar materials, *J. Acoust. Soc. Am.* 101 (1997) 3405–
532 3414. <https://doi.org/10.1121/1.418351>.

533 [18] N. Mori, Y. Iwata, T. Hayashi, N. Matsuda, Viscoelastic wave propagation and resonance in
534 a metal-plastic bonded laminate, *Mech. Adv. Mater. Struct.* (2022) in press.
535 <https://doi.org/10.1080/15376494.2022.2084191>.

536 [19] B.A. Auld, *Acoustic Fields and Waves in Solids*, John Wiley and Sons, New York, 1973.

537 [20] C. Bacon, B. Hosten, P.-A. Bernard, Acoustic wave generation in viscoelastic rods by time-
538 gated microwaves, *J. Acoust. Soc. Am.* 106 (1999) 195–201.
539 <https://doi.org/10.1121/1.427073>.

540 [21] M. Castaings, C. Bacon, B. Hosten, M.V. Predoi, Finite element predictions for the dynamic
541 response of thermo-viscoelastic material structures, *J. Acoust. Soc. Am.* 115 (2004) 1125–
542 1133. <https://doi.org/10.1121/1.1639332>.