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On Wendt's Theorem of Knots, II

By Shin'ichi Kinoshita

1. Recently R. H. Fox introduced in his paper [1] an operation τ called a single twist. Using this operation τ , we introduce now a numerical knot¹⁾ invariant $\bar{s}(k)$ defined as the minimal number of τ^n which change the given knot k to the trivial one, where the natural number n is not fixed. By definition of $\bar{s}(k)$ and $s(k)^2$

$$\bar{s}(k) \leq \bar{s}(k) \leq s(k)$$
.

Then the purpose of this note is to prove

$$(*) e_g \leq (g-1)\bar{s}(k),$$

where e_g is the minimal number of essential generators of the 1-dimensional homology group of the g-fold cyclic covering space of S, branched along k. From the above inequality (*) it follows that

$$e_{\rm g} \leq (g-1)\bar{s}(k) \;, \qquad e_{\rm g} \leq (g-1)s(k) \;, \label{eq:eg}$$

where the former is proved in [2] and the latter is due to H. Wendt [3].

2. Now we prove our inequality (*). Let k be a knot. Suppose that k is deformed into k' by τ^n . Then we are only to prove that

$$e_g(k') \leq e_g(k) + (g-1)$$
.

Let F(S-k) be the fundamental group of S-k. By [1] we may assume that

$$F(S-k) = (a, b, A, B, x_1, x_2, \cdots);$$

$$a = A, b = B, r_1 = 1, r_2 = 1, \cdots),$$

$$F(S-k') = (a, b, A, B, x_1, x_2, \cdots);$$

$$a = A, b = A^nB, r_1 = 1, r_2 = 1, \cdots).$$

The 1-dimensional homology groups of S-k and S-k' are infinite cyclic. We denote by t a generator of either group. Then abelianization of F(S-k) or F(S-k') maps A into t^q and B into 1, where q is an integer. By usual methods the presentations of F(S-k) and F(S-k') can be transformed to the following one:

¹⁾ A knot is a polygonal simple closed curve in the 3-sphere S.

²⁾ $\bar{s}(k)$ and s(k) are defined in [2].

$$F(S-k) = (t, \ \bar{a}, \ \bar{b}, \ \bar{A}, \ \bar{B}, \ \bar{x}_1, \ \bar{x}_2, \cdots :$$

$$\bar{a} = \bar{A}, \ \bar{b} = \bar{B}, \ \bar{r}_1 = 1, \ \bar{r}_2 = 1, \cdots, \ tf_1^{-1} = 1),$$

$$F(S-k') = (t, \ \bar{a}, \ \bar{b}, \ \bar{A}, \ \bar{B}, \ \bar{x}_1, \ \bar{x}_2, \cdots :$$

$$\bar{a} = \bar{A}, \ t^{nq}\bar{b} = (t^q\bar{A})^n\bar{B}, \ \bar{r}_1 = 1, \ \bar{r}_2 = 1, \cdots, \ tf_2^{-1} = 1).$$

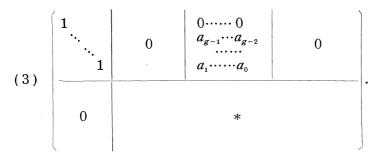
Furthermore we may suppose that $f_1 = f_2$. Then the 1-dimensional homology group of the g-fold cyclic covering space of S, branched along k, is given by the matrix:

and that of S, branched along k', is given by the following one:

$$(1') \begin{array}{|c|c|c|c|c|c|c|}\hline \bar{B} & \bar{b} & \bar{A} & \bar{a} & \bar{x}_i \\\hline 1 & -1 & & a_0 \cdots \cdots a_{g-1} & & \\ \ddots & & \ddots & & a_{g-1} \cdots a_{g-2} & & \\ 1 & -1 & a_1 \cdots \cdots a_0 & & & \\\hline 0 & & * & & & \\\hline\end{array}$$

Putting $\sum_{i=0}^{q-1} a_i = \alpha$, we can transform (1) and (1') to the following one, respectively:

(2') is equivalent to



From (2) and (3) it is easy to see that

$$e_{\sigma}(k') \leq e_{\sigma}(k) + (g-1)$$
.

Thus our proof is complete.

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References

- [1] R. H. Fox: Congruence classes of knots, Osaka Math. J. 10, 37-41 (1958).
- [2] S. Kinoshita: On Wendt's theorem of knots, Osaka Math. J. 9, 61-66 (1957).
- [3] H. Wendt: Die gordische Auflösung von Knoten, Math. Z. 42, 680-696 (1937).