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# A Predicting Method of Welding Residual Stress Using Source of Residual Stress (Report III)<sup>†</sup>

– Prediction of Residual Stresses in T- and I-joints Using Inherent Strains –

Yukio UEDA\* and Min Gang YUAN\*\*

## Abstract

*A method of predicting the residual stress in a butt-welded plate was previously proposed by the authors. In this method, an elastic analysis is performed using inherent strain as the source of residual stress. In this paper, the method is extended for the prediction of residual stresses in welded T- and I-joints.*

*The inherent strains in T- and I-joints were calculated inversely from the residual stresses obtained by thermal elasto-plastic analysis. It is found that the inherent strain distributions in the flange side and the web side are the same for both cases of T- and I-joints provided that the widths of the flange and the web are not considerably different. The inherent strain distributions in the vicinities of two welds of an I-joint are the same one another. Also they are the same as that in a T-joint with the same geometrical sizes.*

*Simple formulae were developed for predicting the distribution zone and the magnitude of inherent strain under a specified welding conditions. It is shown that the residual stresses in a T-joint and an I-joint can be predicted from the inherent strain that is given by the proposed formulae. For an I-joint, the inherent strain is imposed to the two welds in the same sequence as assembling procedure by welding.*

*The predicted residual stresses by the proposed method are compared with the results of thermal elasto-plastic analysis and the agreement is shown to be excellent. The validity of the method is also confirmed by experiments.*

**KEY WORDS:** ( Inherent strain ) ( Source of residual stress ) ( Welding residual stress ) ( T-joint )  
( I-joint ) ( Fillet weld ) ( Finite element method )

## 1. Introduction

The authors have proposed <sup>1), 2)</sup> a predicting method of welding residual stress in a butt-welded plate. In the method, the residual stress is predicted by an elastic analysis using the inherent strain as the source of the residual stress. Usually, numerical analysis is performed by means of the finite element method (FEM), in which the inherent strain is replaced to an equivalent load and imposed to the weld in stress-free state. It was shown that the inherent strain remains in the vicinity of weld and its distribution pattern is simple comparing with the residual stress distribution. The distribution zone and the magnitude of inherent strain under a specified welding condition may be obtained based on a couple of formulae which the authors derived theoretically. Then the welding residual stress can be predicted by an elastic analysis using the inherent strain.

The aim of this work is to develop the method for the prediction of residual stresses in fillet-welded T- and

I-shape joints of thin plates. The thickness of the plates is thin with respect to the heat input so that the temperature distribution through the thickness during welding can be regarded as uniform except near the weld. For predicting the residual stresses in such kinds of joints, some research work had been done. N. Rao et al. <sup>3)</sup> investigated the residual stress distributions in welded T- and I-joints under several welding conditions by experiments and led to some parametric formulae. K. Satoh et al. <sup>4)</sup> regarded a web and flange of a T-joint as a bead-on-edge and bead-on-center plate respectively, and expressed the residual stress distributions in the both plates by some parameters determined by experiments. However, these are not general methods since in these methods the relationship between residual stress distributions and welding conditions, plate dimensions and material properties is based on empirical formulae. Thus it is hard to apply these methods to the cases where the specifications are different from those used in the experiments.

In this paper, the authors presented a couple of

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formulae derived theoretically with some assumptions in order to predict the distribution zone and the magnitude of inherent strain in T- and I-joints. The factors of heat input, geometric sizes and material properties of the plates as well as the effect of longitudinal bending deflection during welding are taken account in these formulae. It was shown that the residual stress distribution can be predicted by an elastic analysis easily using the inherent strain determined by the proposed formulae. It was also shown that the formulae for predicting the inherent strain distributions in T- and I-joints have the similar form to those derived previously for butt joint. This indicates the possibility of using the experimental data of a butt joint that may be obtained from measurement or literature in predicting the residual stresses in a T-joint and an I-joint. The validity of the proposed method was demonstrated by numerical analysis and good accuracy of the method was recognized by experiments.

## 2. Analysis Model of Plane Deformation

### 2.1 Objective of Analysis

Figure 1 shows a fillet-welded T-joint and I-joint of steel plates where  $\bar{z}$  is the distance from the weld center to the neutral axis of transverse cross section. Usually, in the case of thin plates the longitudinal residual stress in the welding direction is a dominant component which influences the performances such as buckling strength of a welded structure rather than those in the directions perpendicular to the weld line. For this reason, this study is directed to the longitudinal residual stress distributions in T- and I-joints.

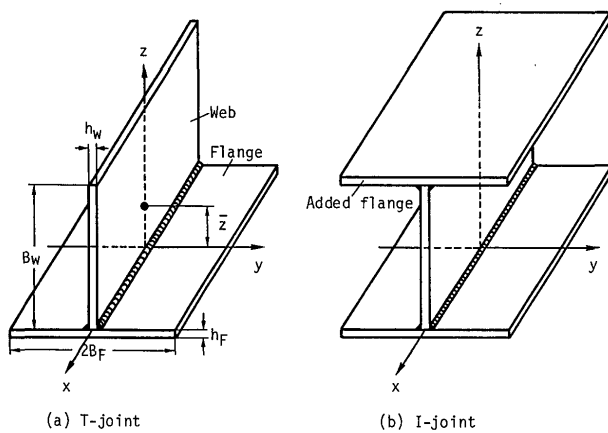


Fig.1 Fillet welded joints.

Numerical analysis is performed using FEM. The plates of T- and I-joints are made of mild steel. The

thicknesses of flange and web are assumed the same and set to 12 (mm). The temperature dependency of material properties used in the analysis are the same as those used in reference 5).

### 2.2 Welding Conditions

A heat source of welding is assumed to be applied to both corners of a T-joint simultaneously. Also an I-joint is assumed to be made from a fillet-welded T-joint and an additional flange. The heat input used in the succeeding welding of the I-joint is assumed to be the same as that used in the preceding welding of the T-joint.

In the numerical analysis, the quantity of heat input applied to each fillet per unit weld length is 1200 (J/mm) and the total quantity of heat input is then 2400 (J/mm). Usually, in the case of multi-passes welding the residual stresses are almost determined by the heat input of first pass 3) or by the largest heat input of multi-passes 6). Therefore, the residual stress distributions analyzed in this study can be considered applicable even to the case where only a single-pass heat source of 2400 (J/mm) is placed to one side of a T-joint or to the case where a two-passes heat source of 2400 (J/mm) is applied to both sides of the T-joint. Thus the quantity of heat input used in this numerical investigation may be regarded as one of first pass or the largest one of multi-passes.

### 2.3 Boundary Conditions

Usually, the transverse cross section in the middle part of a T-joint or I-joint of plates can be regarded to behave "plane deformation" or "generalized plane strain" during welding if the length of plates is sufficiently long

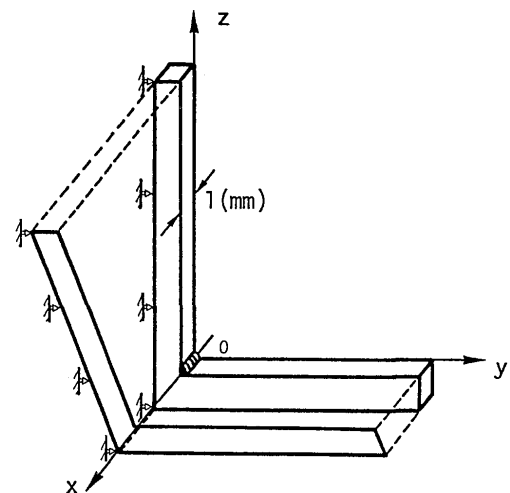


Fig.2 Plane deformation model for mechanical analysis.

comparing with the width of the plates. Moreover, in order to prevent a twisting deformation of welded plates, a proper welding sequence is designed so that the structure may deform symmetrically about the web. Thus, a two dimensional model of plane deformation with unit thickness is adopted in this study. The numerical analysis is carried out using a half model due to the symmetry as shown in Fig.2.

Concerning the restraint condition related to the angular distortion of a flange during fillet-welding, it is assumed to be free as shown in Fig.2 because the influence of angular distortion on the longitudinal residual stress in welding direction can be neglected according to the authors' experiences.

### 3. Characteristics of Inherent Strain Distribution

The characteristics of inherent strain distributions in fillet-welded T- and I-joints are investigated by numerical analysis. The calculations consist of heat conduction analysis to obtain transient temperature, thermal elasto-plastic mechanical analysis to obtain transient stresses using the calculated temperature as input data, and elastic analysis to obtain inherent strains using the calculated residual stresses as input data.

#### 3.1 Comparison of Inherent Strain Distributions between T-joint and Butt Joint

Figure 3 shows the calculated longitudinal inherent strain distributing in the flange and web of a fillet-welded T-joint by a solid line and a dashed line respectively. In the figure the average values through thickness of the plates are plotted. It is known that the magnitude and distribution of inherent strains in the flange and the web are almost the same one another provided that the widths of them are large enough relative to the heat input so that the heat flow may not be reflected at the edge of plates during welding.

In Fig.3 the inherent strain distribution calculated for a butt joint of flat plates is also shown by a broken line. The butt joint has the same average temperature rise  $T_{av}$  as that of the T-joint. This parameter  $T_{av}$  will be defined later which indicates a relative amount of heat input with respect to the geometrical sizes and material properties of the weldment. It is seen that the inherent strains at the weld parts of both joints are almost the same. The reason of this is that the yield stress of the both joints are the same. However, the width of inherent strain zones are different between the butt joint and the T-joint. The former is slightly larger than the latter. This is due to the

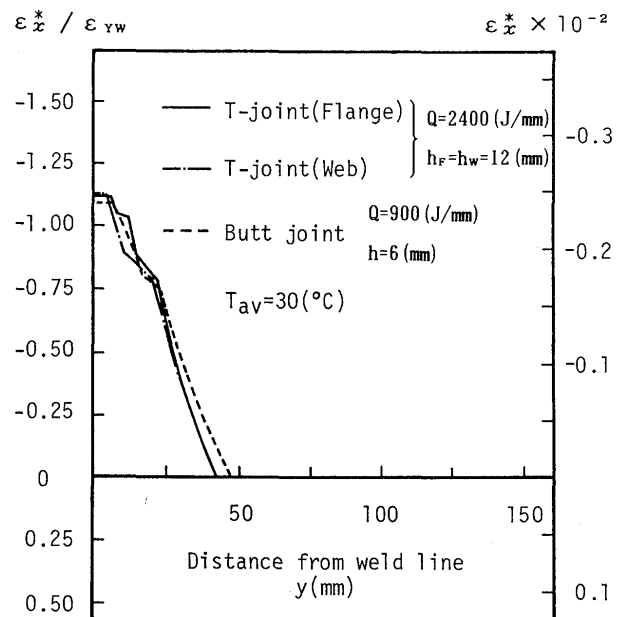


Fig.3 Comparison of inherent strains between T-joint and butt joint.

difference in heat inputs between the butt joint and the T-joint. In the case of thin plates the heat input  $Q$  applied to a welded T-joint can be divided into two parts approximately:  $Q_F$  conducted to the flange and  $Q_W$  to the web, both of which are proportional to the respective thicknesses, i.e.

$$Q_F = 2Qh_F / (2h_F + h_W) \quad (1)$$

$$Q_W = Qh_W / (2h_F + h_W) \quad (2)$$

Since the heat flows in two directions through the flange but flows in one direction through the web, the heat input per unit thickness in each plate is then  $Q / (2h_F + h_W)$  according to the above formulae. Supposing the same heat input  $Q$  is applied to a butt joint with thickness  $h$ , the heat input per unit thickness in each plate is  $Q / 2h$ . Hence the heat input per unit thickness for the T-joint is as  $2h / (2h_F + h_W)$  time as that for the butt joint. In other words, the heat input  $Q$  applied to a T-joint is equivalent to the heat input  $Q'$  applied to a butt joint. The equivalent heat input  $Q'$  is defined by the following formula.

$$Q' = 2Qh / (2h_F + h_W) \quad (3)$$

In Fig.3 the equivalent heat input of the T-joint is 800 (J/mm). However, the heat input of the butt joint is 900 (J/mm). This means that the heat input of the butt joint is larger than that of the T-joint. This made the distribution of inherent strain in the butt joint wider than that in the T-joint as shown in the figure. In other words, the inherent strain distributions should be the

same if the equivalent heat input of the T-joint is equal to the heat input of the butt joint.

### 3.2 Effect of Bending Moment on Inherent Strain Distribution in T-joint

According to the above deduction, the result of the previous investigations <sup>1), 2)</sup> into the influences of heat input, geometric sizes and material properties upon the inherent strain distribution in a butt-welded joint can be applied to the case of a fillet-welded joint. Meanwhile, additional two factors need to be considered in the cases of fillet-welded T- and I-joint. One is the effect of longitudinal bending deformation during welding because the weld center does not coincide with the neutral axis of transverse cross section. The other is the effect of welding sequence in the case of an I-joint which is made from an additional flange and a T-joint which contains residual stress caused by the preceding welding.

Figure 4 shows the calculated inherent strain distributions in T-joints. The magnitude of inherent strains,  $\hat{\epsilon}_x^*$ , and the width of inherent strain zones,  $b$ , of the T-joints with several kinds of sizes are shown in Table 1. It is seen that for the joint whose average temperature rise is small ( $T_{av} \leq 30^\circ\text{C}$ ), the inherent strain distribution varies little against a geometrical ratio of the joint,  $B_w/B_f$ . This implies that in this case the moment of inertia with respect to the longitudinal bending is large enough so that the effect of change in the ratio,  $B_w/B_f$ , may be ignored. However, for the joint whose average temperature rise is large, the inherent strain distribution varies also with the geometrical ratio. As the ratio  $B_w/B_f$  increases, the distance between the

weld center and the neutral axis of transverse cross section  $\bar{z}$  increases. In proportion to this, the bending moment resulted from ununiform thermal expansion during welding increases although the inertia moment of the cross section increases either to a certain extent. Thus larger bending deformation is permitted and the inherent strain is generated in a smaller area. On the other hand, the magnitude of the inherent strain at the weld part becomes larger with the increase of the geometrical ratio increases. This implies that to make the same residual stress at the weld, which usually reaches the yield stress, larger inherent strain is needed in order to contribute to the longitudinal bending. However, the change in the magnitude of inherent strain is relative small with respect to that in the distribution zone of the inherent strain as can be seen in Fig.4.

Table 1 Width and magnitude of inherent strains in various T-joints.

2B <sub>F</sub> (mm)	B <sub>w</sub> (mm)	T <sub>av</sub> (°C)	by F. E. M.		by formulae	
			b (mm)	- $\hat{\epsilon}_x^*$	b (mm)	- $\hat{\epsilon}_x^*$
600	800	30	46.6	0.00262	44.8	0.00265
400	1000	30	46.3	0.00266	43.6	0.00265
300	500	50	40.2	0.00279	40.8	0.00275
200	220	100	34.1	0.00298	34.5	0.00300
120	300	100	28.7	0.00309	29.2	0.00300
100	190	150	23.0	0.00326	22.3	0.00325
120	90	200	24.2	0.00328	24.6	0.00350
80	130	200	19.0	0.00350	16.0	0.00350
70	100	250	20.0	0.00348	17.3	0.00375

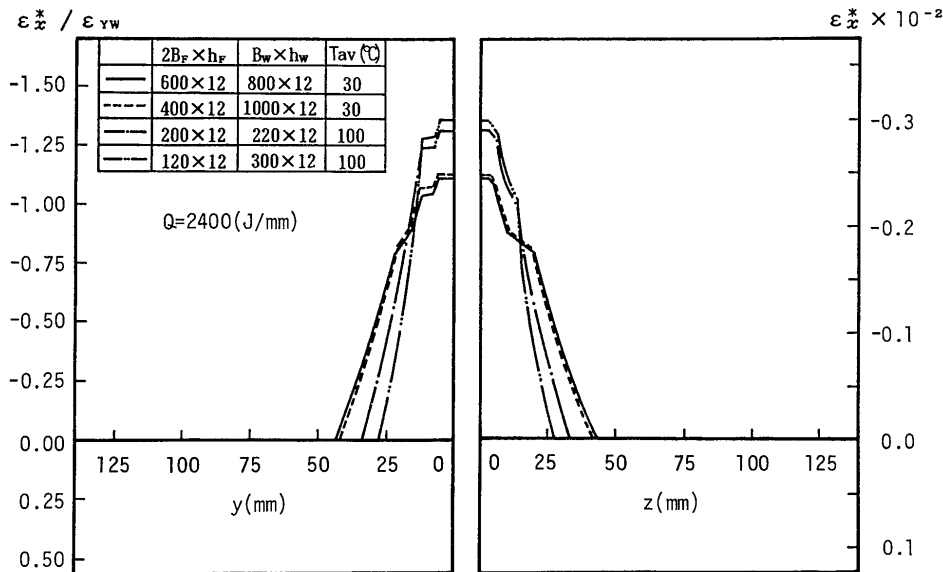


Fig.4 Inherent strain distributions in T-joints.

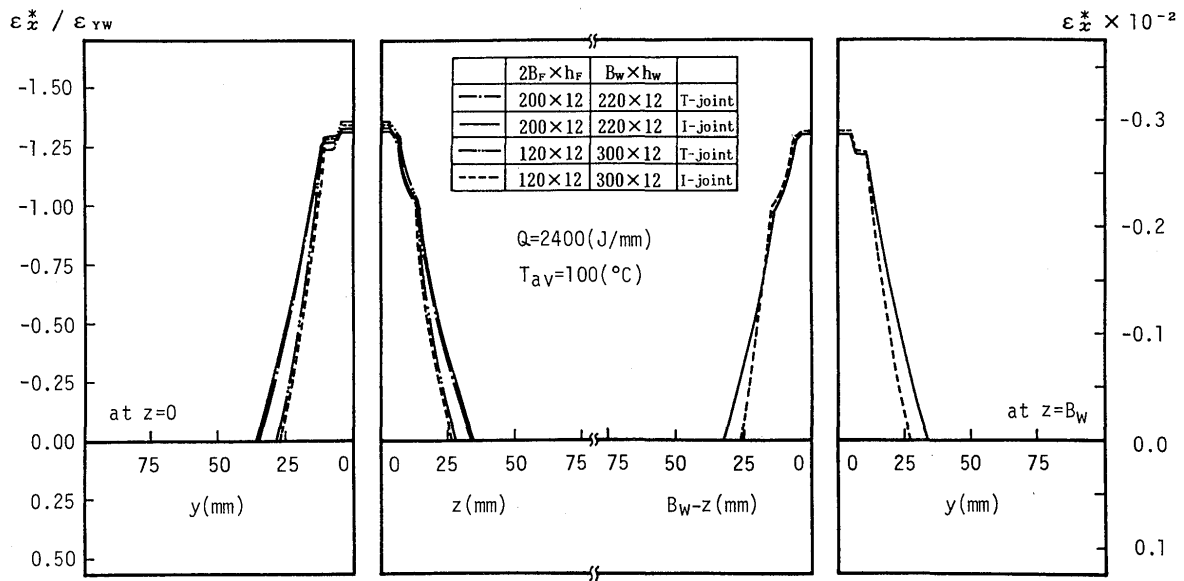


Fig.5 Inherent strain distributions in I-joints.

### 3.3 Effect of Welding Sequence on Inherent Strain Distribution in I-joint

Usually an I-joint is made by joining a flange with a T-joint which contains inherent strain and residual stress by fillet-welding as shown in Fig.1(b). Due to the welding, a new inherent strain distribution is introduced in an area nearby the weld while the inherent strain distribution remained in the T-joint may also change depending on the weld heat input and the joint geometry. In order to investigate the inherent strain distributions in the vicinities of two welds in an I-joint, numerical simulation is carried out using FEM. The procedure of FE-computation involves (1) thermal elasto-plastic analysis for a T-joint; (2) thermal elasto-plastic analysis for an I-joint which consists of the T-joint with initial stress obtained above and an additional flange in stress-free state; and (3) inverse analysis of inherent strain from the residual stress in the I-joint<sup>1)</sup>.

Figures 5 (a), (b) and (c) show the inherent strain distributions in two I-joints of different sizes by a solid line and broken line respectively. The strains distributing near the weld center  $y=0, z=0$  are produced by the preceding welding for fabricating the T-joints, and those near the another weld center  $y=B_W, z=0$  are produced by the succeeding welding for fabricating the I-joints. It is seen that the inherent strain distributions near the both welds of the I-joints are almost the same. In Fig.5(a) and the left side of Fig.5(b) the inherent strain distributions analyzed for T-joints with the same sizes as the I-joints are also shown by dashed lines. Comparing the results of T-joints and I-joints respectively, it is seen that the inherent strain

distributions remaining in the T-joints are independent of the succeeding welding of the I-joints.

### 3.4 Magnitude and Distribution of Inherent Strain

According to the above results, it is found that the inherent strain distributions in T- and I-joints may be approximated in a trapezoidal form. The peak of this trapezium appears at the weld portion including the heat affected zone (HAZ). This peak can be used as a parameter for measuring the magnitude of inherent strain. The trapezium spreads over an area by a certain width. This width is again a parameter for measuring the distribution of the inherent strain. It is evident that the whole inherent strain distribution can be obtained if the above two key parameters are determined for the specified welded joint. By recent research work, the authors succeeded in expressing the magnitude of inherent strain and the width of inherent strain distribution by a couple of formulae, in which the welding conditions, joint geometry and material properties are taken into account based on some assumptions.

#### 3.4.1 Formulae for butt joint

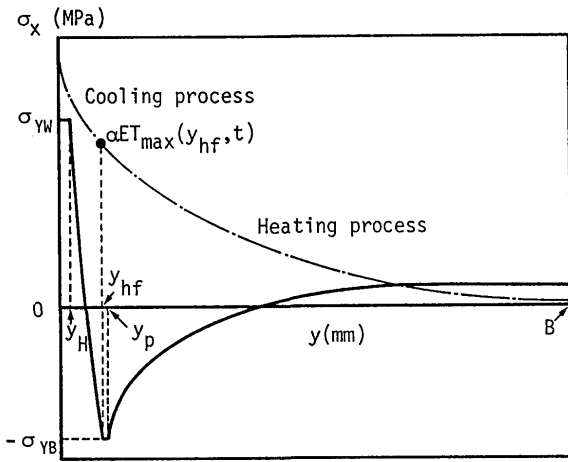
To explain this algorithm, a butt joint of thin plate is considered. The joint is subjected to a instantaneous heat source at the weld center line ( $y=0$ ) and performs uniformly as plane deformation (i. e.  $d\epsilon/dy=0$ ) in the middle part of the plate during the welding<sup>2)</sup>. The temperature distribution along the middle cross section ( $x=0$ ) is assumed as shown by a dashed line in Fig.6 which is expressed by

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$$T(y,t) = \frac{Q}{\sqrt{4\pi ktc\rho h}} \exp\left(-\frac{y^2}{4kt}\right) + T_0 \quad (4)$$

where,

- Q heat input (J/mm)
- c specific heat (J/g°C)
- $\rho$  density (g/mm<sup>3</sup>)
- k thermal diffusion rate (mm<sup>2</sup>/sec)
- h thickness of plate (mm)
- t present time (sec)
- T<sub>0</sub> room temperature (°C)



**Fig.6** Assumed transient stress distribution in butt joint.

And the transient stress distribution in welding direction caused at the same moment is assumed as shown by a solid line in Fig.6 which is expressed by

$$\sigma_x = \begin{cases} \sigma_{YW} & 0 \leq y \leq y_H \\ \alpha E [T_{\max}(y, y^2/2k) - T(y, t)] - \sigma_{YB} & y_H < y \leq y_{hf} \\ -\sigma_{YB} & y_{hf} \leq y \leq y_p \\ \alpha E [T(y_p, t) - T(y, t)] - \sigma_{YB} & y_H \leq y \leq y_{hf} \end{cases} \quad (5)$$

where,

- $\sigma_{YW}$  yield stress of weld metal and HAZ (MPa)
- $\sigma_{YB}$  yield stress of base metal (MPa)
- $\alpha$  linear thermal expansion coefficient (1/°C)
- E Young's modulus (GPa)
- y<sub>H</sub> width of HAZ (mm)
- y<sub>hf</sub>(=√2kt) position of maximum temperature at present time (mm)
- y<sub>p</sub> width of transient plastic zone (mm)
- B width of plate (mm)

The width of transient plastic zone y<sub>p</sub> can be determined from equilibrium condition of force, i. e.

$$\int_0^B \sigma_x h dy = 0 \quad (6)$$

To simplify the calculation, temperature dependency of the material properties are neglected. From the vanishing condition of the derivative of y<sub>p</sub> with respect to t, and the maximum value of y<sub>p</sub> is obtained. This becomes also the width of inherent strain distribution, b, and is expressed by

$$b = \xi b_0 \quad (7)$$

$$\xi = 1 / \left\{ 1 + \frac{\alpha E T_{av}}{2.066 \sigma_{YB}} \left[ \ln \frac{\sigma_{YB}}{\xi \alpha E (T_m - T_0)} - 4.1327 \times \phi \left( \frac{\sigma_{YB}}{\xi \alpha E (T_m - T_0)} \right) - \frac{\sigma_{YW} + \sigma_{YB}}{\alpha E (T_m - T_0)} + 3.0644 \right] \right\} \quad (8)$$

$$b_0 = 0.242 \alpha E Q / (c \rho h \sigma_{YB}) \quad (9)$$

where,

- T<sub>m</sub> temperature over which the metal becomes HAZ and behaves viscoplastic (°C)
- T<sub>av</sub>(=Q/c ρA) average temperature rise (°C)
- A area of transverse cross section (mm<sup>2</sup>)

$$\phi(u) = \frac{1}{\sqrt{2\pi}} \int_0^u \exp(-x^2/2) dx \quad \text{Gaussian function}$$

b<sub>0</sub> is a limit value in case where the plate width tends to infinite (i. e. T<sub>av</sub> = 0). ξ is a factor referred to as normalized width of inherent strain distribution. This ξ may be solved by specifying an initial value less than or equal to one at the right side of Eq.(8) and iterating the calculation until the value at the left is equal to that involved in the right side.

As the plate is cooled down to the room temperature the inherent strain remaining in the vicinity of weld may distribute in a trapezoidal form shown in Fig.7(a) approximately. Usually, the residual stress distribution caused by this inherent strain may be shown in Fig.7(b) because of plane deformation. From the relation between inherent strain and residual stress (i. e.  $\sigma_x = E(\epsilon_x - \epsilon_x^*)$ ,  $\epsilon_x^* < \epsilon_x < 0$ ) and the equilibrium condition of force (i. e. Eq.(6)), the magnitude of inherent strain,  $\hat{\epsilon}_x^*$ , may be expressed by

$$\hat{\epsilon}_x^* = \zeta \hat{\epsilon}_{x0}^* \quad (10)$$

$$\zeta = -1 / \left\{ 1 - \frac{\alpha E T_{av}}{4.1327 \sigma_{YB}} \left[ \zeta + \frac{\sigma_{YB}}{\xi \alpha E (T_m - T_0)} \right] \right\} \quad (11)$$

$$\hat{\epsilon}_{x0}^* = \sigma_{YW} / E \quad (12)$$

The magnitude of inherent strain converges to a limit value  $\hat{\epsilon}_{x0}^*$  in case where the plate width tends to infinite

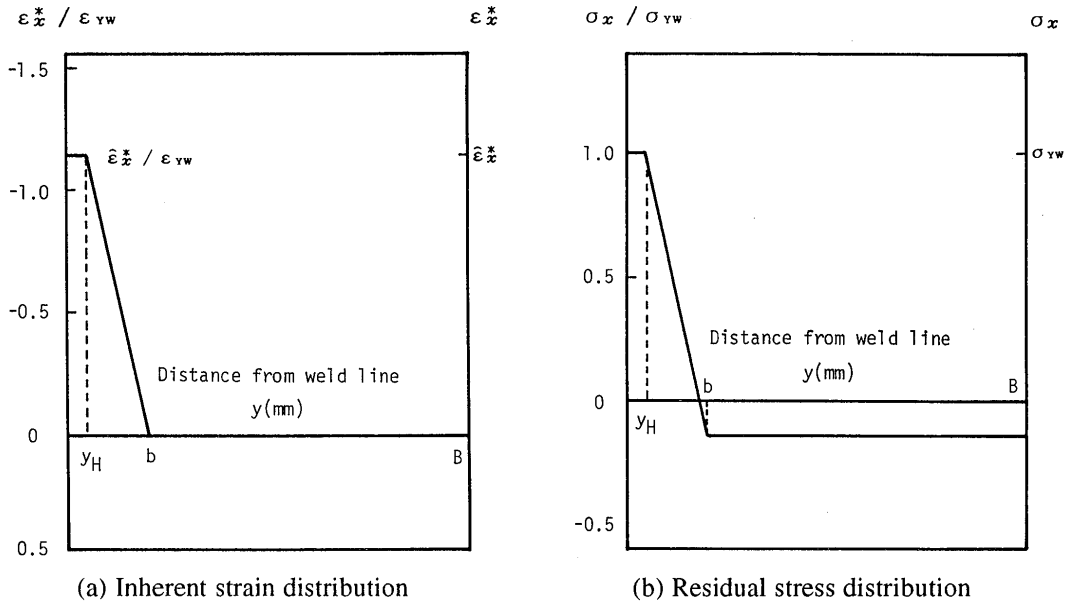


Fig.7 Relation between inherent strain and residual stress.

(i.e.  $T_{av}=0$ ).  $\zeta$  is a factor referred to as normalized magnitude of inherent strain.

Equations (8) and (11) contain parameters of welding heat input, joint geometry and material properties. According to the authors' investigation into the influences of these parameters, where  $T_{av} = 0\sim 250$  ( $^{\circ}C$ ),  $T_m = 500\sim 900$  ( $^{\circ}C$ ),  $\sigma_{yB} = 230\sim 430$  (MPa) and  $\sigma_{yW} / \sigma_{yB} = 1.45$  are considered, it was found that  $\xi$  and  $\zeta$  are dominantly dependent on the parameter  $\alpha ET_{av} / \sigma_{yB}$  and can be approximately expressed by the following simplified formulae <sup>7)</sup>.

$$\xi = 1 - 0.27\alpha ET_{av} / \sigma_{yB} \quad (8')$$

$$\zeta = -1 - 0.27\alpha ET_{av} / \sigma_{yB} \quad (11')$$

### 3.4.2 Formulae for T-joint

It was indicated that if the average temperature rise  $T_{av}$  of a T-joint is small, that is to say, if the widths of both web and flange are relatively large with respect to the heat input applied, the effect of change in the ratio,  $B_w / B_f$ , upon the inherent strain distribution may be neglected (Fig.4). In this case, the inherent strain distribution in the T-joint may be the same as that caused in such a butt joint whose equivalent heat input is equal to the heat input  $Q$  of the T-joint (Fig.3). These facts imply that Eqs.(9) and (12) are valid even for a T-joint in the case where  $T_{av}$  is small so that  $\xi$  and  $\zeta$  approach unit values. However, for the case where  $T_{av}$  is relatively large, the inherent strain distribution also depends upon the geometrical ratio  $B_w / B_f$  of the T-joint, particularly the width of inherent strain zone rather than the magnitude of the inherent strain may be greatly

influenced as mentioned in 3.2. For this reason, Eq.(8)' needs to be modified in order to take the effect of longitudinal bending deformation into account.

Let's consider the generation of transient strain in the flange of a T-joint which is being welded at the central line. The strain in longitudinal direction created during welding may be divided into two parts. One is due to uniform deformation along the transverse cross section as what happened in a butt joint, and another is due to bending deformation because the weld center disagree with the neutral axis of the cross section. Inherent strain created near the weld is influenced by the deformations. The effect of uniform deformation is the same as that in the case of butt joint and thus may be measured with average temperature rise  $T_{av}$ . As shown in Eqs.(8)' and (11)', the larger the average temperature rise (i.e. the smaller the plate width in respect to the heat input), the smaller the inherent strain zone but the larger the inherent strain magnitude. The influence of bending deformation is considered in the following. The bending moment is expressed by

$$M = \iint \alpha ET(y,t)(\bar{z}-z) dy dz \approx \alpha ET_{av} A \bar{z} \quad (13)$$

where,

$\bar{z}$  distance between weld center and neutral axis of transverse cross section (mm)

According to the beam theory, the longitudinal strain of flange due to the bending moment may nearly be expressed by



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$$\Delta \epsilon = M \bar{z} / E I \approx \alpha \beta T_{av} \quad (14)$$

$$\beta = \bar{z}^2 A / I \quad (15)$$

where,

I inertia moment of transverse cross section (mm<sup>4</sup>)

$\beta$  is a dimensionless factor showing the flexibility against to longitudinal bending deformation due to fillet welding. From Eq.(14),  $\Delta \epsilon$  may be regarded to be caused by an extra average temperature rise,  $\Delta T_{av}$  ( $=\beta T_{av}$ ), instead of by the bending deformation imaginarily. Substituting the following average temperature rise

$$T_{av}' = T_{av} + \Delta T_{av} \approx (1+\beta) T_{av} \quad (16)$$

for  $T_{av}$  in Eq.(8)', and  $Q'$  defined by Eq.(3) for  $Q$  in Eq.(9), the width of inherent strain zone in the T-joint may be obtained with the effect of bending deformation being taken into account. It is expressed by

$$b = \xi b_0 \quad (17)$$

$$\xi = 1 - 0.27 \alpha E T_{av} (1+\beta) / \sigma_{YB} \quad (18)$$

$$b_0 = 0.484 \alpha E Q / [c \rho (2h_F + h_W) \sigma_{YB}] \quad (19)$$

While the magnitude of the inherent strain in the T-joint may be obtained by Eqs.(10), (11)' and (12) for the reason mentioned above. According to the investigation in 3.2 and 3.3, the derived formulae are valid for both flange and web of a T-joint as well as an I-joint.

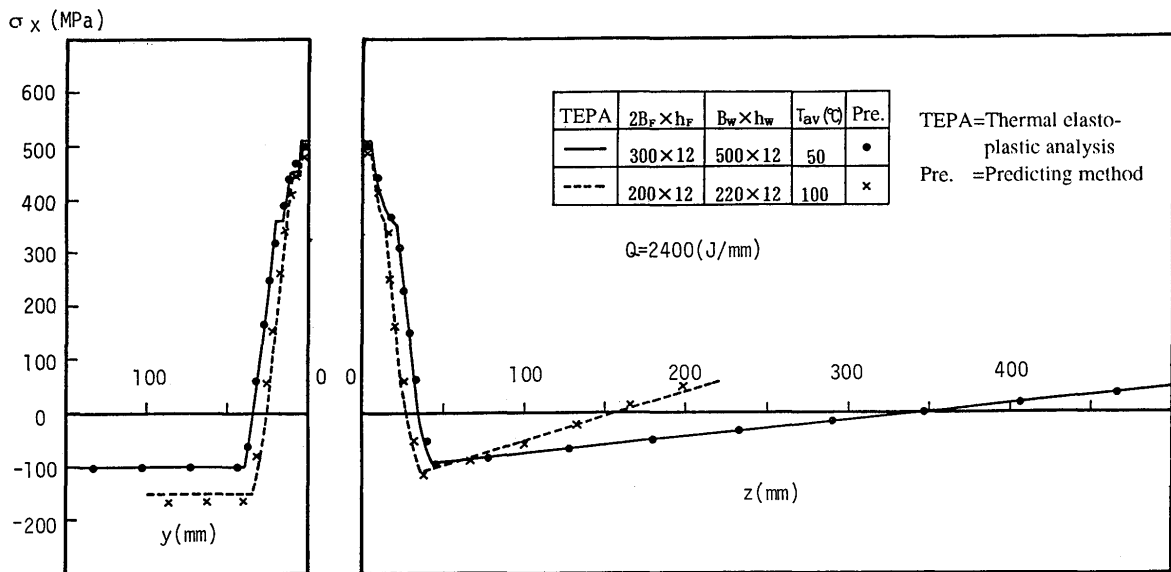
In Table I the width of inherent strain distribution,  $b$ , and magnitude of the inherent strain,  $\hat{\epsilon}_x^*$ , calculated

by the proposed formulae for several kinds of T-joints are shown assuming  $Q=2400$ (J/mm),  $c=0.63$  (J/g°C),  $\rho=7.82$ (mg/mm),  $\alpha=12$ ( $\mu$ /°C),  $E=210$ (GPa),  $\sigma_{YB}=330$  (MPa) and  $\sigma_{YW}=520$ (MPa). Comparing the calculated values with the results of FE-analysis shown in the same table, good agreement is recognized. This implies that the obtained formulae are applicable for a wide range of plate sizes and the geometrical ratio  $B_W/B_F$ .

#### 4. Predicting Method of Residual Stresses Using Inherent Strains

##### 4.1 Prediction of Residual Stresses in T-joints

The aim of this paper is to propose a convenient method of predicting residual stresses in T- and I-joints by elastic analysis, in which inherent strain is used as an external load instead of weld heat input. The inherent strain distribution under a specified joint may be obtained by the formulae derived in 3.4 or by modifying a reference inherent strain distribution in an arbitrary butt joint or a T-joint which may be known from literature or through experiment if more accurate prediction is desired. In the latter case, the reference inherent strain is modified in such a way that the width of inherent strain distribution of the specified joint,  $b$ , is proportional to that of the reference one,  $b'$ , by a ratio of  $(\xi b_0) / (\xi b_0)'$ , and the magnitude of the inherent strain of the specified joint,  $\hat{\epsilon}_x^*$ , is proportional to that of the reference one,  $\hat{\epsilon}_x^{*'}$ , by a ratio of  $(\zeta \hat{\epsilon}_x^*) / (\zeta \hat{\epsilon}_x^{*'})$ , in order to match the specified heat input, materials properties and geometric sizes.



**Fig.8** Comparison of residual stresses in T-joints obtained by proposed method and by thermal elasto-plastic analysis for cases of  $T_{av}=50, 100$ (°C).

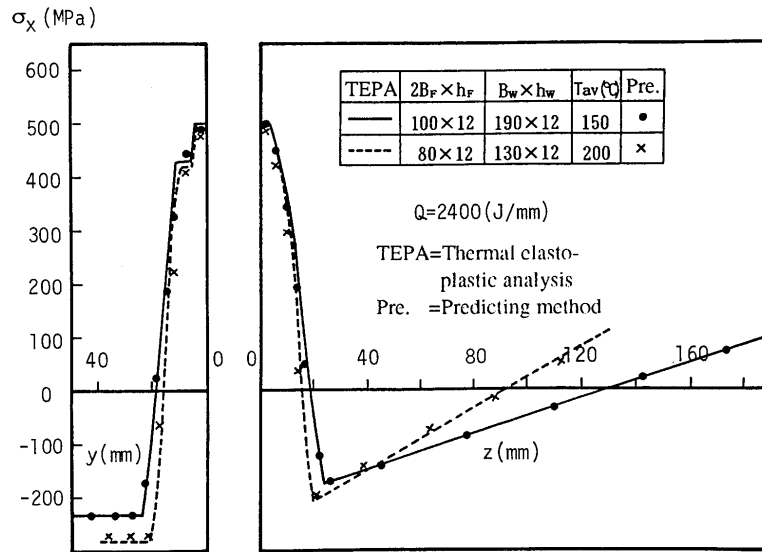


Fig.9 Comparison of residual stresses in T-joints obtained by proposed methos and by thermal elasto-plastic analysis for cases of T<sub>av</sub>=150, 200(°C).

Figures 8 and 9 shows the residual stress distributions in T-joints predicted by the proposed method using the inherent strain distribution of T<sub>av</sub>=30(°C) shown in Fig.4 as a reference one by marks. In the same figure, the residual stress distributions obtained by thermal elasto-plastic analysis are also shown by lines. It is seen that the results by the two methods coincide well one another.

#### 4.2 Prediction of Residual Stresses in I-joints

For predicting residual stress distribution in an I-joint, it is necessary to apply the inherent strain in the same sequence as welding as illustrated in Fig.10 since the residual stress distribution caused by the preceding welding may influence the subsequent residual stress distribution caused by the succeeding welding. In step (a) the inherent strain is applied to a T-joint which is in stress-free state and a stress distribution  $\sigma_x'$  is obtained. In step (b) the inherent strain is applied to an I-joint which is in stress-free state either and a stress distribution  $\sigma_x''$  is obtained. Finally in step (c) the obtained stress distributions are superposed into  $\sigma_x$ . This is the residual stress distribution in the welded I-joint.

Figures 11 and 12 show the residual stress distributions in I-joints predicted by the proposed method using the same reference inherent strain distribution as that used above by marks. In the same figure the residual stress distributions obtained by thermal elasto-plastic analysis are also shown by lines.

It is seen that the results by the two methods agree well each other.

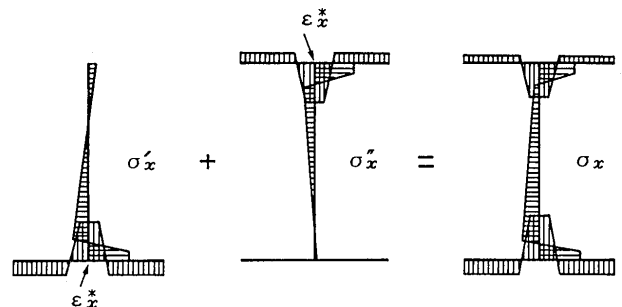


Fig.10 Process of predicting residual stress in I-joint using inherent strain.

### 5. Comparison of Predicted Results and Experimental Data

#### 5.1 Description of Experimental Details

Experimental investigation was carried out for validating the predicting method of welding residual stresses proposed in this paper. Two specimens of fillet-welded T-joints were used in the experimental measurements. GMAW was performed for joining the specimens with current 400(A) and voltage 40(V). Double heat sources were placed at the right and left sides of web (Fig.1) and moved symmetrically by the same speed 13.33(mm/sec). Both fillet weld legs were 6(mm). Mild steel was used for the base plate and the corresponding weld rod. A heat treatment was performed before welding for releasing the initial stresses remained in the plates.

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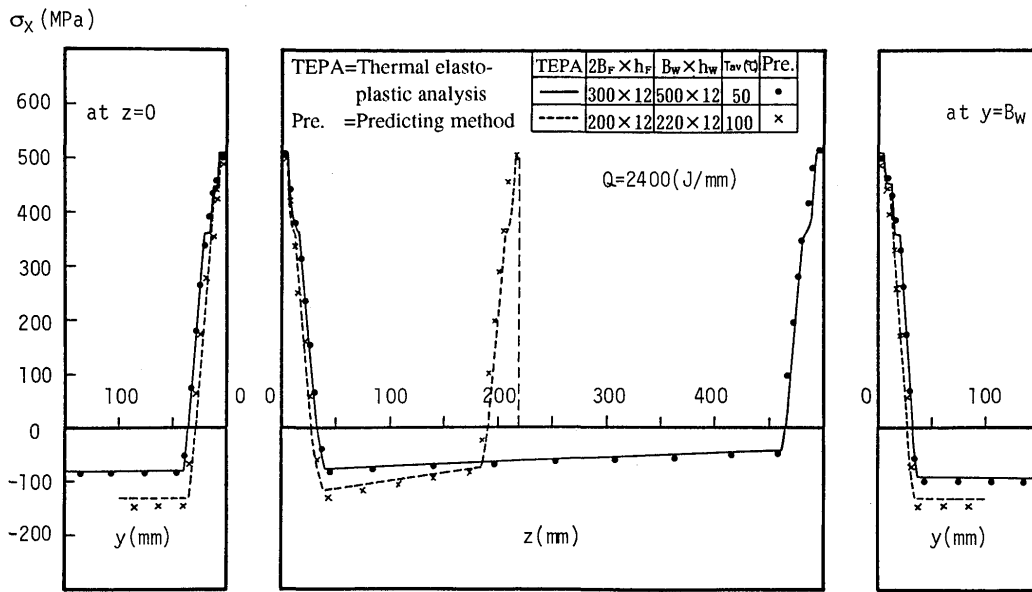


Fig.11 Comparison of residual stresses in I-joints obtained by proposed methos and by thermal elasto-plastic analysis for cases of  $T_{av}=50, 100(^\circ C)$ .

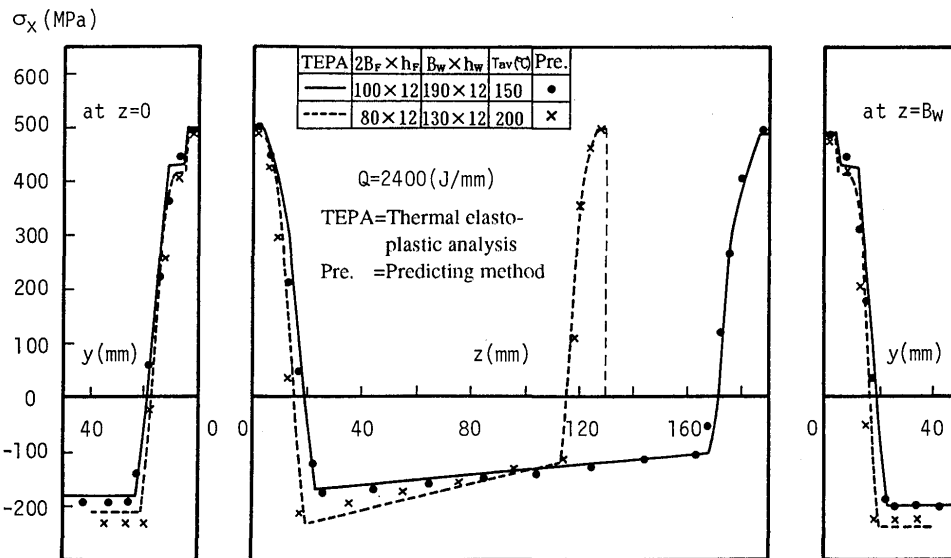


Fig.12 Comparison of residual stresses in I-joints obtained by proposed methos and by thermal elasto-plastic analysis for cases of  $T_{av}=150, 200(^\circ C)$ .

The length and thickness of the plates were 600(mm) and 12(mm) respectively for both the flanges and webs of the T-joints. Two kinds of sectional ratios were adopted, as described at the table in Fig.13, keeping the sectional area constant in order to observe the effect of bending deformation upon the inherent strains and the residual stresses.

#### 5.2 Results

The residual strains in the central transverse cross section of each specimen ( $x=0$ ) were measured with

strain gauges. The details about cutting sequence and other conditions are referred to reference <sup>8)</sup>. The inherent strains calculated from the measured elastic strains <sup>1)</sup> are shown in Fig.13. Comparing these with those shown in Fig.4 the characteristics of inherent strain distributions in T-joints studied in section 3 by thermal elasto-plastic analysis are confirmed. The inherent strain distributions in the flange and web of the T-joints are almost the same which vary with the ratio  $B_w/B_f$  as a result of bending deformation for the same average temperature rise  $T_{av}$ .

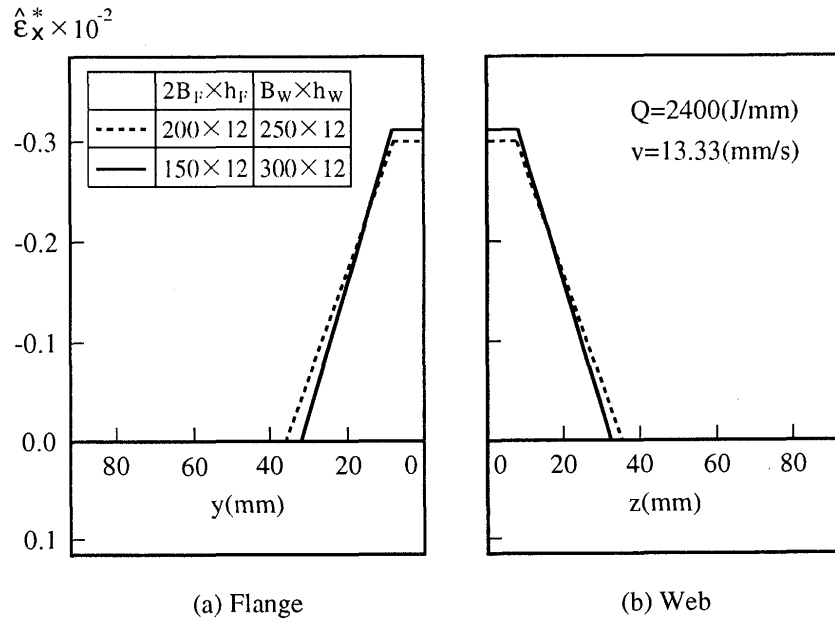


Fig.13 Inherent strains determined from measured elastic strains in T-joints.

In Table 2 the width of inherent strain zone and the magnitude of the inherent strain of the T-joints obtained by experimental measurements and by the derived formulae using the same material properties as those assumed in 3.4.2 are shown. It is seen that the calculated results coincide well with the experimental data. This reveals that the formulae in this paper are valid which was derived taking account not only the average temperature rise but also the longitudinal bending deformation.

In Fig.14 the measured residual stress distributions at the central transverse cross sections of the T-joints are shown by marks. The residual stress distributions predicted by the proposed method using the idealized inherent strain distributions (Fig.7(a)), whose widths and magnitudes are determined by the formulae (Table 2), are shown in the same figure by lines. Good agreement between the results obtained through experimental and proposed methods are recognized.

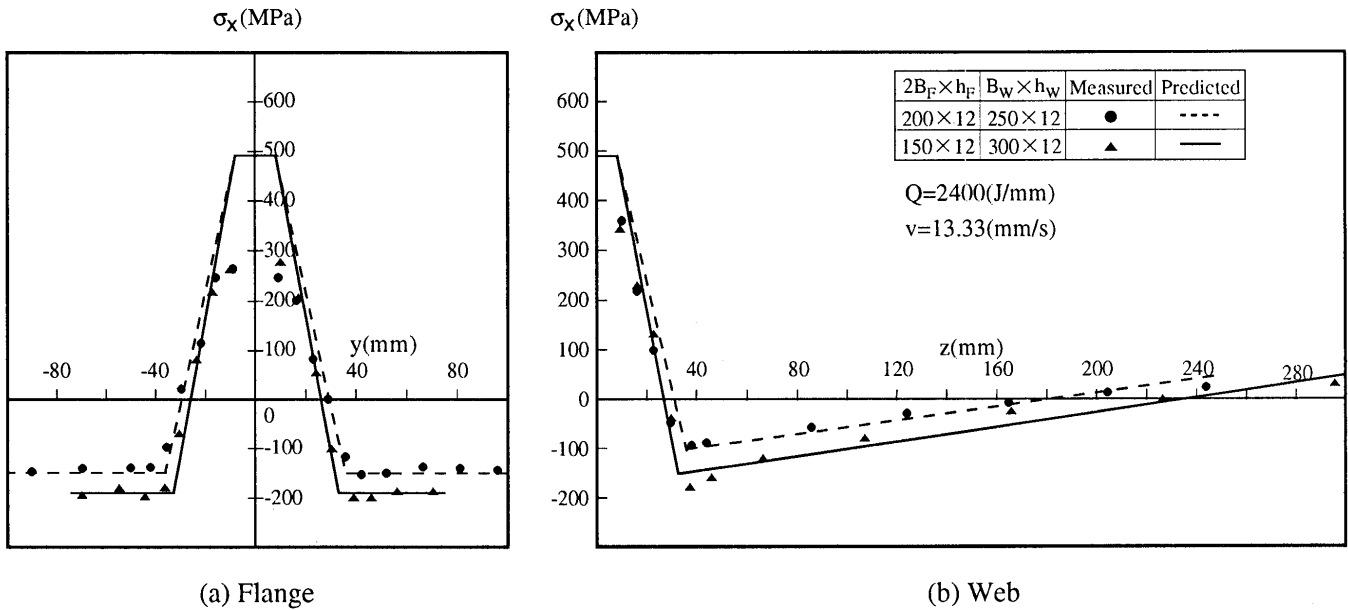
6. Conclusion

- (1) The inherent strain distributions in the flange and the web of a T-joint are the same which are dependent upon not only the average temperature rise as in the case of a butt joint but also with the sectional ratio  $B_W/B_F$  related to the longitudinal bending deformation.
- (2) The inherent strain distributions produced near the two welds of an I-joint are both the same as that produced near the weld of a T-joint.
- (3) Formulae of the width of inherent strain zone and magnitude of the inherent strain for T- and I-joints are derived. The influences of the weld heat input, material properties and geometrical sizes including bending deformation are taken into account in these formulae.
- (4) The residual stress distributions in various T- and I-joints can be predicted by the proposed method in which inherent strains are applied to the objects as

Table 2 Comparison of measured and calculated width and magnitude of inherent strain in T-joints.

2B <sub>F</sub> (mm)	B <sub>W</sub> (mm)	T <sub>av</sub> (°C)	β	by measurement				by formulae	
				Web		Flange		b (mm)	-ε <sub>x</sub> *
				b (mm)	-ε <sub>x</sub> *	b (mm)	-ε <sub>x</sub> *		
200	250	90	0.63	35	0.0030	36	0.0030	35.3	0.00295
150	300	90	0.92	33	0.0031	32	0.0031	32.6	0.00295

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**Fig.14** Comparison of measured and predicted residual stresses in T-joints.

the source of the welding residual stress in the same sequence as the real welding.

- (5) The accuracy of the derived formulae of inherent strain distributions for T- and I-joints and the proposed method for predicting the residual stress distributions are confirmed through experiments.

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