Restraint Stresses and Strains Due to Slit Weld in Rectangular Plate (Report II)†

– Effective Restraint Intensities of a Slit in a Thick Finite Plate –

Yukio UEDA*, Keiji FUKUDA** and You Chul KIM***

Abstract

In recent years, according to the scaling up of welding structures, the demand for welding of thicker plates increases. In case of the first pass weld of a thick plate, the throat thickness is smaller than the thickness of base plate. In this case, it is hard to presume that the thickness of base plate at the groove is entirely effective. For this reason, two dimensional restraint intensities estimated on the assumption that the plate thickness is entirely effective cannot be used as dynamical measures for cold cracking caused by the first pass weld in the case of a thick plate.

In this paper, two types of restraint intensities are analyzed by three dimensional elastic analysis with the aid of the finite element method. From the calculated results, the effect of the small throat thickness and the influence of the shape of groove on the restraint intensity are studied. Moreover, assuming that the first pass weld is laid deviated from the center of the plate thickness, the influence of the eccentricity is also investigated.

Based on these results, general formulae to evaluate effective restraint intensities are proposed. Henceforth, it is no more necessary to conduct a three dimensional elastic analysis which requires a huge amount of computing time and expense.

KEY WORDS: (Slit Weld) (Dynamical Measure for Cold Cracking) (Restraint Intensity) (Eccentricity) (Throat Thickness)

1. Introduction

In order to prevent cold cracking, it is desired from the dynamical viewpoint that welded stress and strain can be predicted prior to an actual welding. In this regard, a series of researches have been carried out on the first pass weld of a slit welded joint representing a two dimensional restraint state in which thermal expansion and shrinkage varies along the weld line[1-4]. (The same kind of problem in a one dimensional restraint state has been well studied.)

At present, restraint intensities are often used as a dynamical measure for cold cracking in a two dimensional restraint state. Restraint intensity R at a point can be defined as,

$$R = p/\delta$$

where,

p: load per unit weld length (kg/mm)
δ: dislocation in a groove (mm)

The value of the restraint intensity changes in accordance with the change of values applied to p and δ in the above equation. There are mainly three types of restraint intensities.

(1) Restraint intensity which can be calculated using measured residual stresses and dislocation produced by cutting the weld metal along the slit: R_a
(2) Restraint intensity which can be calculated on the assumption that residual stresses produced in the weld metal are uniform along the slit (restraint intensity under uniformly distributed loads): R_p
(3) Restraint intensity which can be calculated on the assumption that dislocation is uniform along the slit (restraint intensity under uniform dislocation): R_δ

On the other hand, restraint stresses and strains (elastic restraint strain + plastic restraint starin) produced in the weld metal are presumed to be dynamical measures which represent the severity of the dynamical restraint condition in a two dimensional restraint state. Using these dynamical measures, the dynamical background and the

† Received on April 30, 1983
* Professor
** Research Instructor (Formerly)
*** Research Associate

Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan

applicability of restraint intensities have been investigated[4),7)]. Consequently, it is clarified that restraint intensities can be applied as brief dynamical measures for cold cracking in the two dimensional restraint state within certain limits.

In recent years, according to the scaling up of welding structures such as industrial machinery and pressure vessels, the demand for welding of thicker plates increases. In case of the first pass weld of a thick plate, the throat thickness is smaller than the plate thickness of the base plate, so that the major part of the base plate thickness bears no stress. In this case, it is hard to presume that the plate thickness of the groove is entirely effective. For this reason, two dimensional restraint intensities estimated on the assumption that the plate thickness of a groove is entirely effective cannot be used as dynamical measures for cold cracking caused by the first pass weld of a thick plate. It is, therefore, necessary to investigate the effect of the throat thickness which is smaller than the plate thickness of the base plate[5)].

In this paper, among the formerly mentioned three types of restraint intensities, two types which can be determined only from the geometry under uniformly distributed loads and uniform dislocation are chosen and directly analyzed by a three dimensional elastic analysis with the aid of the finite element method. From the calculated results, the effect of the small throat thickness is investigated. In these analyses, I-groove is used, though there are various types of grooves for actual welding. Therefore, it is necessary to further investigate how different grooves influence restraint intensity. Moreover, assuming that the first pass weld is laid deviated from the center of the plate thickness, the influence of the eccentricity is investigated. Based on these results, more general equations to calculate effective restraint intensities (restraint intensities under consideration of the influence of the small throat thickness) are proposed. Henceforth, it is no more necessary to conduct a three dimensional elastic analysis which requires a huge amount of computing time and expense.

2. Two Dimensional Restraint Intensities Under Entirely Effective Plate Thickness of a Groove

An analytical method to accurately estimate restraint intensities R_p and R_δ, respectively under uniformly distributed loads and uniform dislocation, and highly accurate approximating equations have been already proposed[1)]. In the analysis, an analytical solution for an infinite plate and a finite element solution are superposed so that the boundary conditions are satisfied. They are reviewed here since their results are needed in Chapter 4.

2.1 Two dimensional restraint intensity under uniformly distributed loads

If the plate breadth B, the plate length L, and the slit length l are in the ratio of $B/l \geq 1.8$ and $L/l \geq 1.5$, this kind of two dimensional restraint intensity can be simply calculated with accuracy by the following equations.

1) $R_p = (1-\beta_p) R_p^\infty$ (1)

: restraint intensity of a finite plate

2) $\bar{R}_p = 2\dfrac{E}{\pi}(1-\beta_p)\dfrac{h}{l}$ (2)

: average restraint intensity of a finite plate

3) $R_p^\infty = \dfrac{E}{2}\dfrac{h}{l}\dfrac{1}{\sqrt{1-X^2}}$ (3)

: restraint intensity of an infinite plate (analytical solution)

where,

$\beta_p = 0.6/(L/l)^n + 0.75/(B/l)^{1.82}$ (4)

$n = 5.8/(B/l)^2 + 2.2$

β_p: correction factor to a finite plate

E: Young's modulus, h: plate thickness,

$X = 2x/l$

2.2 Two dimensional restraint intensity under uniform dislocation

With the same ratio of $B/l \geq 1.8$ and $L/l \geq 1.5$ as in 2.1, this kind of two dimensional restraint intensity can be simply and accurately calculated by the following equations.

1) $R_\delta = [1-(1-X^2)\beta_{\delta o}]R_\delta^\infty$ (5)

: restraint intensity in a finite plate

2) The average restraint intensity cannot be calculated because the integrated value of R_δ along the slit becomes infinite.

3) $R_\delta^\infty = \dfrac{E}{\pi}\dfrac{h}{l}\dfrac{1}{1-X^2}$ (6)

: restraint intensity in an infinite plate (analytical solution)

where,

$\beta_{\delta o} \doteqdot 2\beta_p$ (7)

: correction factor to finite plate

2.3 Relative relations between two types of restraint intensities, R_p and R_δ

In the region where $B/l \geq 1.8$ and $L/l \geq 1.5$, the correction factors to a finite plate, β_p and $\beta_{\delta o}$, have a relation shown by Eq. (7). In this regard, two restraint intensities, R_p and R_δ, can be related as follows.

$$\frac{R_p}{R_\delta} = \frac{\pi}{2} \frac{(1-\beta_p)\sqrt{1-X^2}}{1-2\beta_p(1-X^2)} \tag{8}$$

In this consequence, these restraint intensities can be converted each other using Eq. (8).

The relative relation (Eq. (8)) between these restraint intensities in a finite plate is in the proportion of $(\pi/2)[(1-\beta_p)/(1-2\beta_p)]$ at the center of the slit, though their distributions along the slit are complicated. If the correction factor β_p to a finite plate is set zero in Eq. (8), the relative relation between these restraint intensities in an infinite plate can be given as,

$$R_p^\infty / R_\delta^\infty = (\pi/2)\sqrt{1-X^2} \tag{9}$$

This equation indicates that the magnitude at the center of the slit is $(\pi/2)$ and the distribution along the slit is elliptical. On the other hand, if $B/l \geq 4$ and $L/l \geq 3.5$, the restraint intensity R_p approximates to that of an infinite plate. So is the restraint intensity R_δ with $B/l \geq 5$ and $L/l \geq 4.5$. Therefore, Eq. (9) can be applied to the region where $B/l \geq 5$ and $L/l \geq 4.5$.

3. Effective Restraint Intensities by Three Dimensional Elastic Analysis with the Aid of F.E.M.

Applying the finite element method (F.E.M.), effective restraint intensities are obtained by a three dimensional elastic analysis. The effectiveness of the throat thickness is investigated from the calculated results.

The model used in this three dimensional analysis is shown by **Fig. 1**. Three dimensional analyses are carried

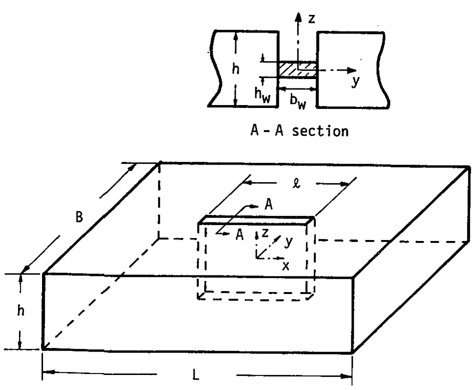


Fig. 1 Slit weld specimen of thick plate

out on the Y-groove weld cracking test specimen ($B = 150$, $L = 200$, $l = 80$ (mm)) keeping the throat thickness $h_w = 5$ mm constant and changing the plate thickness as $h = 30$, 50, and 100 (mm).

The first pass weld is presumed to be laid at the cneter of the plate thickness. In this consequence, the model to be analyzed becomes symmetric with respect to x, y and z axes, so that 1/8 of the whole specimen is analyzed.

3.1 Effective restraint intensity under uniformly distributed loads: $(R_p)_\eta$

Uniformly distributed stresses p_o (kg/mm^2) are loaded only for the throat thickness along the slit. The dislocation δ_s (z) produced in the throat thickness is obtained by a three dimensional elastic analysis, so that the effective restraint intensity $(R_p)_\eta$ can be calculated by the following equation.

$$(R_p)_\eta = p_o h_w / \frac{1}{h_w} \int_{-h_w/2}^{h_w/2} \delta_s(z)\, dz \tag{10}$$

In **Fig. 2**, the calculated results of the effective restraint intensity $(R_p)_\eta$ along the slit are shown by solid lines for respective cases when the plate thickness is 30 mm and 50 mm. The broken lines indicate the results of the two dimensional restraint intensity R_p calculated by Eq. (11) on the assumption that the plate thickness of the base plate is entirely effective.

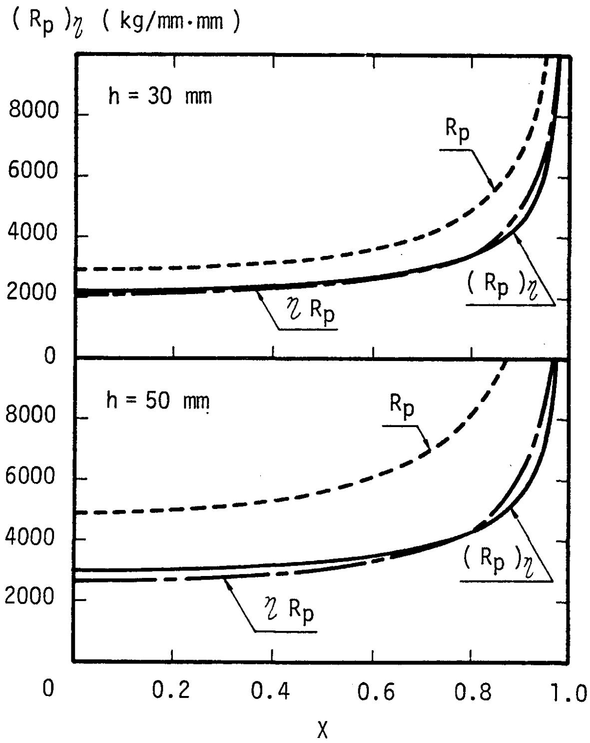


Fig. 2 Distribution of effective restraint intensities along slit (uniformly distributed loads)

3.2 Effective restraint intensity under uniform dislocation: $(R_\delta)_\eta$

A thick plate is loaded with uniform dislocation δ_0 for the throat thickness. According to the conventional definition, if the stress $\sigma_s(z)$ produced in a throat under such loading condition is obtained by a three dimensional elastic analysis, the effective restraint intensity $(R_\delta)_\eta$ is calculated by the following equation using the obtained stress $\sigma_s(z)$.

$$(R_\delta)_\eta = \frac{1}{h_w}\int_{-h_w/2}^{h_w/2} \sigma_s(z)\,dz \cdot h_w/\delta_0 \qquad (11)$$

However, an accurate analysis based on the theory of elasticity reveals that the stress σ_s produced in a throat under this loading condition is infinite ($\sigma_s(\pm h_w/2) = \infty$) on the top and bottom surfaces of the throat ($z = \pm h_w/2$)[6], so that the restraint intensity under uniform dislocation cannot be calculated. Contrary to this, if the finite element method is applied to a calculation, the reaction force produced in the throat can be estimated as the nodal reaction force F_i, so that the integrated value of the stress in the throat becomes finite. The nodal reaction force F_i can be obtained by a three dimensional elastic analysis. Therefore, the effective restraint intensity $(R_\delta)_\eta$ under uniform dislocation can be calculated by the following equation.

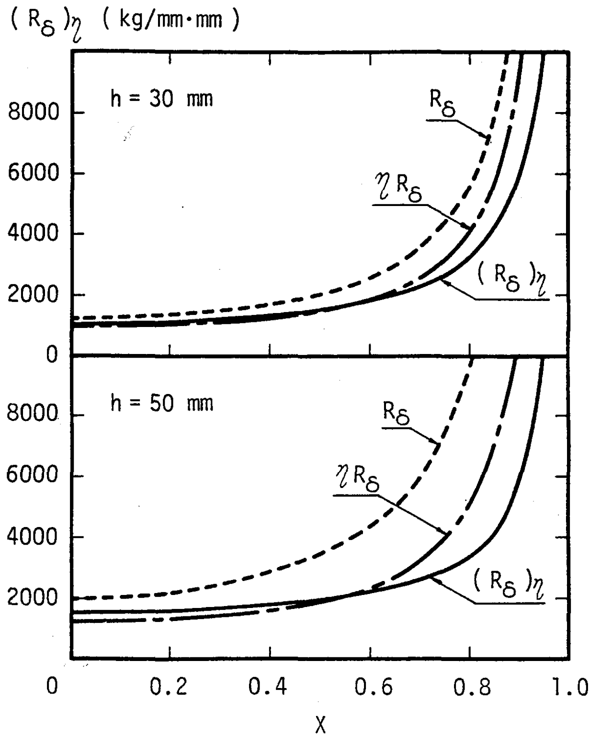


Fig. 3 Distribution of effective restraint intensities along slit (uniform dislocation)

$$(R_\delta)_\eta = \frac{1}{k}\sum_{k=1}^{k} F_k/\delta_0 \qquad (12)$$

The effective restraint intensity $(R_\delta)_\eta$ distributed along the slit is shown by solid lines in **Fig. 3** for respective cases when the plate thickness h is 30 mm and 50 mm. The broken lines show the two dimensional restraint intensity R_δ calculated by Eq. (5). Both restraint intensities become infinite at the slit end.

As is stated above, when the throat thickness is smaller than the plate thickness of the base plate, the plate thickness of the groove is not entirely effective and restraint intensities decrease a great deal. In this consequence, as far as the first pass weld of a thick plate is concerned, the effect of the smaller throat thickness needs be corrected. The method of correction is stated in the following chapter.

4. Effectiveness of Base Plate and Proposal of a Correction Factor for Smaller Throat Thickness

When the throat thickness is smaller than the plate thickness of the base plate, restraint intensities decrease a great deal. In this chapter, it is shown that the analytically derived correction factor η for the effect of smaller throat thickness of a one dimensional restraint joint (**Fig. 6**) which has been already proposed in Ref. 1) is also applicable to the two dimensional restraint state. The applicability of the factor is thus to be proved.

The above mentioned correction factor η is combined with respective two dimensional restraint intensities, R_p (Eq. (1)) and R_δ (Eq. (5)), so that the effective restraint intensities $(R_p)_\eta$ and $(R_\delta)_\eta$ are obtained.

$$(R_p)_\eta = \eta R_p \qquad (13)$$

: effective restraint intensity of a finite plate under uniformly distributed loads

$$(R_\delta)_\eta = \eta R_\delta \qquad (14)$$

: effective restraint intensity of a finite plate under uniform dislocation

where,

$$\eta = \frac{1}{1+\dfrac{h}{B}\displaystyle\sum_{m=1}^{\infty}\frac{8}{m\pi}\left(\frac{\sin m\pi h_w/h}{m\pi h_w/h}\right)^2 \frac{\sinh^2 m\pi B/h}{\sinh 2m\pi B/h + 2m\pi B/h}}$$

: correction factor for the effect of smaller throat thickness (15)

The effective restraint intensities calculated by Eqs. (13) and (14) are shown by the chain line respectively in Figs. 2 and 3. They indicate that Eq. (13) gives an accurate effective restraint intensity under uniformly distributed loads (Fig. 2), while Eq. (14) provides one under uniform dislocation somehow overestimated near the slit end (Fig. 3). Nevertheless, Eq. (14) is more practical than a three dimensional analysis which requires a huge amount of computing time and expense.

The average effective restraint intensity $(\bar{R}_p)_\eta$ of a finite plate subjected to uniformly distributed loads can be calculated as follows.

$$(\bar{R}_p)_\eta = 2\eta(1-\beta_p)\frac{E}{\pi}\frac{h}{l} \tag{16}$$

: average effective restraint intensity of a finite plate under uniformly distributed loads

In **Fig. 4**, the average effective restraint intensity calculated by Eq. (16) is shown by a solid line and the result

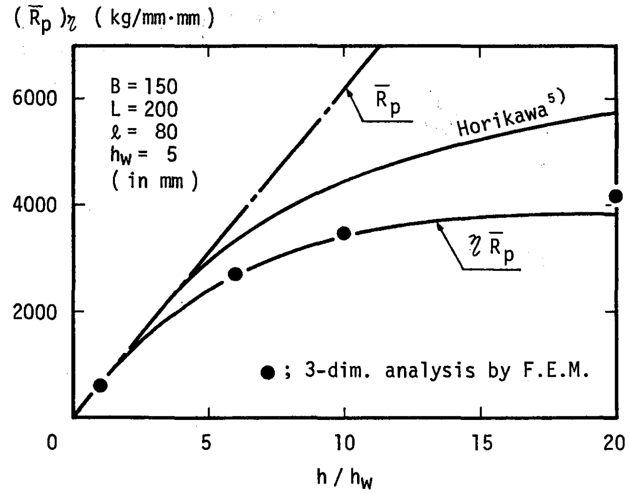


Fig. 4 Effect of plate thickness on $(\bar{R}_p)_\eta$

of a three dimensional elastic analysis by 「●」. The chain line shows the result calculated by Eq. (16) with $\eta = 1$, which corresponds to the two dimensional average restraint intensity $\bar{R}_p$ (Eq. (2)) calculated on the assumption that the plate thickness of the groove is entirely effective.

In accordance with the increase of the ratio between the thickness of the base plate h and the throat thickness h_w, h/h_w, the effective restraint intensity increases monotonously so as to converge to a certain value (Fig. 4). As far as h/h_w is less than 3.5 ($h/h_w < 3.5$), the plate thickness of the groove can be considered to be entirely effective.

The solid line in **Fig. 5** shows the correction factor for the effect of smaller throat thickness, η, calculated by Eq. (15), which corresponds to the increase of h/h_w. While 「●」 in Fig. 5 shows the result of the three dimensional elastic analysis which is the average of η along the slit length. The correction factor η calculated by Eq. (15) represents well the effect of throat thickness also for the two dimensional restraint state.

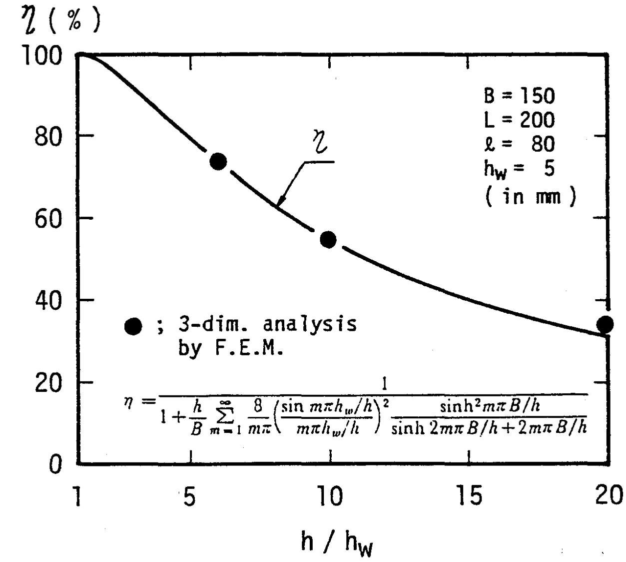


Fig. 5 Correction factor: η for throat thickness

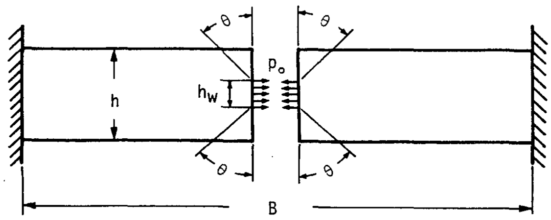
5. Effect of Grooves and Influence of Eccentricity from the Center of Plate Thickness

I-groove is used to the three dimensional elastic analysis. However, there are many kinds of grooves which are used in practical cases. In this chapter, therefore, it is investigated how the effective restraint intensities are influenced by the shape of groove and the eccentric distance of the first pass weld from the center of the plate thickness.

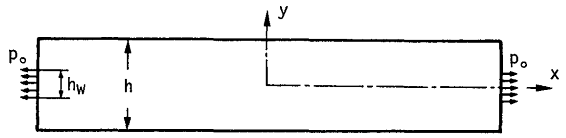
5.1 Effect of the shape of groove

The influence of the shape of groove on the effective restraint intensities is investigated based on the stress distribution produced in a one dimensional restraint joint from which the formerly mentioned correction factor for the effect of smaller throat thickness, η, is derived (Fig. 6).

When uniformly distributed stresses p_o are imposed only for the throat thickness (Fig. 6 (b)), stresses are produced in an arbitrary position (x, y) of the plate. Using the Fourie series, the stresses are calculated. The results are shown in the appendix of the reference 1). As in **Fig. 7**, the principle stresses can be calculated by using these stresses. The solid lines in Fig. 7 represent the



(a) R.R.C. test specimen



(b) Model for analysis

Fig. 6 Model for analysis of correction factor: η

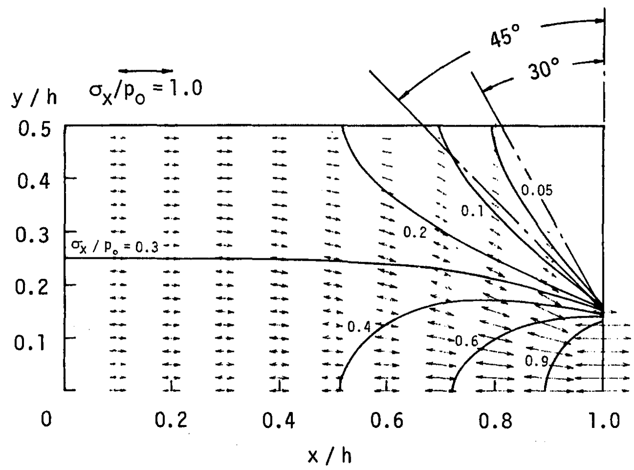


Fig. 7 Isostress lines and principle stresses

isoquant lines of the stress ratio, σ_x/p_o, in the loading direction (x-axis). The principle stresses extend in the 45° direction from the top and bottom surfaces of the throat. Therefore, if the practical groove angle θ is smaller than 45 degree in one side ($\theta<45°$), stresses produced in the region with 45° is so little that the effective restraint intensities are no more decreased than the case of I-groove. As the result of two dimensional elastic analyses actually conducted by the finite element method on this kind of joints with various groove angles, it is known that if the groove angle θ is within 45 degree, the effective restraint intensities are decreased to be lower than those with the I-groove only by 3% at the most.

On the contrary to this, unlike the above mentioned two dimensional stress problem, the throat in a slit of a thick plate is a three dimensional stress problem in which stress distributions become complicated. However, difference in the phenomenon is considered to be little. Therefore, if the actual groove angle is within 45° in one side ($\theta<45°$), the effective restraint intensities are no further decreased.

5.2 Influence of eccentricity from the center of plate thickness

Assuming that the first pass weld is laid deviated from the center of the plate thickness, a three dimensional elastic analysis is conducted in order to investigate the influence of the eccentricity on the effective restraint intensities.

In the analysis, a Y-groove weld cracking test specimen is used keeping the plate thickness $h = 50$ mm and the throat thickness $h_w = 5$ mm constant and changing the distance from the center of plate thickness of the base plate to the center of throat thickness, h', to $h' = 0$, 7.5, 15.0, and 22.5 (mm). The analysis is actually conducted on 1/4 of the whole specimen because it is bi-axially symmetric with respect to x and y. Uniformly distributed stresses p_o are loaded only over the throat thickness h_w. The calculated results are shown by **Fig. 8**.

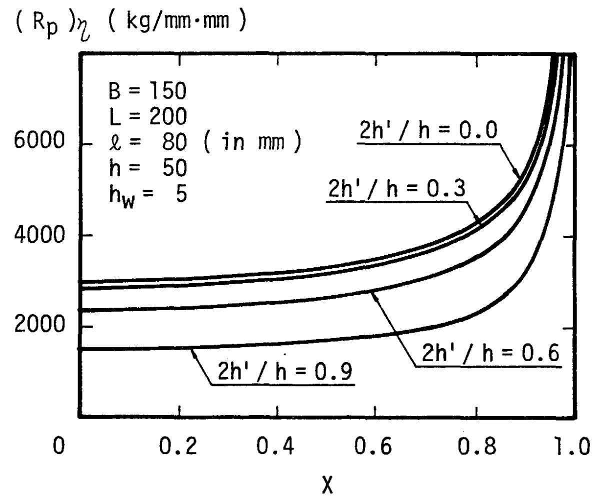


Fig. 8 Effects of eccentricity on effective restraint intensity

According as the eccentric distance h' increases, the out-of-plane bending deformation enlarges and the effective restraint intensities decrease than the case without any eccentricity. However, the distributions of the effective restraint intensity along the slit show no great difference.

The influence of the eccentric distance from the center of thickness of the base plate to that of the throat thickness, h', on the effective restraint intensities, that is the correction factor η' for eccentricity $e(= 2h'/h)$ from the center of plate thickness, is estimated from the above mentioned calculated results and shown by ┌●┘ in **Fig. 9**.

As far as eccentricity $e(= 2h'/h)$ is less than 0.4 ($e<0.4$),

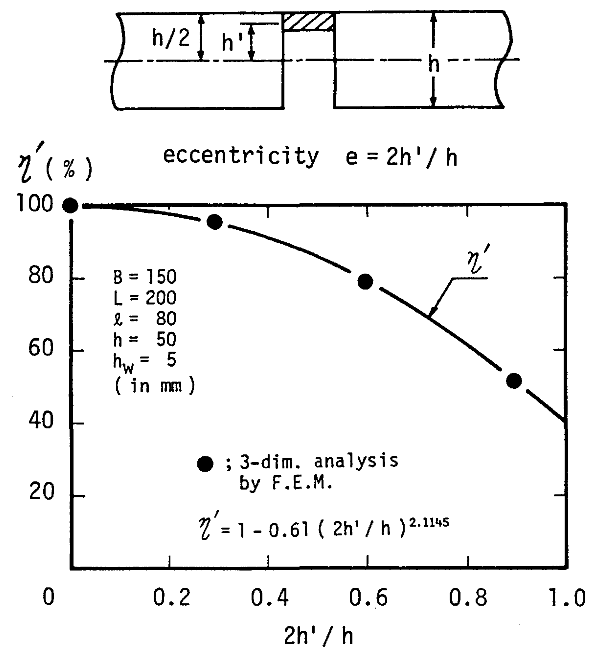


Fig. 9 Correction factor: η' for effect of eccentricity

the effect restraint intensities are almost the same as those produced when the first pass weld is laid in the center of plate thickness. While in case of $e = 0.9$ (the highest), the effective restraint intensities at the center of the base plate thickness are decreased approximately by 50%.

In order to facilitate the quantitative evaluation of the correction factor η' for eccentricity, the following equation is formulated.

$$\eta' = 1 - 0.61(2h'/h)^{2.1145} \tag{17}$$

6. Proposal of Approximating Equations for Generalized Effective Restraint Intensities

Though restraint intensities are used as a dynamical measure for cold cracking caused by the first pass weld in a slit welded joint of a thick plate, the two dimensional restraint intensity calculated on the assumption that the plate thickness of the groove is entirely effective cannot be applied as it is. For this reason, a three dimensional elastic analysis are carried out to clarify the following influences.

1) The influence of the throat thickness which is smaller than the base plate
2) The influence of the eccentricity from the center of plate thickness of the base plate

At the same time, the validity and the applicability of correction factors, η and η', for respective influences are shown. Moreover,

3) the influence of the shape of groove is investigated.

As a result, approximating equations are formulated in order to simply estimate generalized effective restraint intensities in slit welded joints of finite thick plates. Consequently, it is no more necessary to conduct a three dimensional elastic analysis which requires a huge amount of computing time and expense. The formulae are,

(1) Generalized effective restraint intensity of a finite plate under uniformly distributed loads

$$(R_p)_\eta = \eta\,\eta' R_p \tag{18}$$

(2) Generalized effective restraint intensity of a finite plate under uniform dislocation

$$(R_\delta)_\eta = \eta\,\eta' R_\delta \tag{19}$$

(3) Generalized average effective restraint intensity under uniformly distributed loads

$$(\overline{R}_p)_\eta = 2\eta\,\eta'(1-\beta_p)\frac{E}{\pi}\frac{h}{l} \tag{20}$$

7. Conclusion

In this report, a series of theoretical analyses are conducted on the restraint intensities which are effective within limited conditions as a dynamical measure for cold cracking in the first pass weld of a thick plate. The main results are as follows.

(1) The result of a three dimensional elastic analysis indicates that if the throat thickness is smaller than the thickness of a base plate, restraint intensities are not in a simple proportion to the thickness of the base plate but increases monotonously as to converge to a certain value. If the ratio of the thickness of the base plate h to the throat thickness h_w is $h/h_w < 3.5$, the effect of difference in thickness can be neglected. However, in case of $h/h_w > 3.5$, restraint intensities calculated on the assumption that the plate thickness of the base plate is entirely effective need be corrected with consideration of the effect of the smaller throat thickness.

(2) The correction factor η analytically derived for a welded joint in the one dimensional restraint state can be also applied to the two dimensional restraint state with high accuracy.

(3) If the practical groove angle is less than 45° in one side, the effect of the shape of groove on the effective restraint intensities can be disregarded.

(4) If the first pass weld is laid eccentrically from the

center of the plate thickness, the out-of-plane bending deformation enlarges according to the increase of eccentricity $e(=2h'/h)$ so as to decrease the effective restraint intensities estimated at the center of thickness. However, if eccentricity if $e<0.4$, the influence of the eccentricity can be neglected because the effective restraint intensities are approximately equal to those estimated without consideration of eccentricity. The correction factor η' for eccentricity is formulated.

(5) In order to calculate generalized effective restraint intensities in slit welded joints of finite rectangular thick plates, simple calculating equations are proposed. Consequently, effective restraint intensities can be simply calculated without conducting a three dimensional analysis which requires a huge amount computing time and expense. Above all, effective restraint intensities under uniformly distributed loads can be estimated with extremely high accuracy.

Acknowledgement

The authors wish to thank Mr. T. Taenaka, Department of Naval Architecture of Osaka University, for his efforts in performing this research as a part of his graduate study.

References

1) Y. Ueda, k. Fukuda and Y.C. Kim; Restraint Intensities of a Slit in a Finite Plate, Trans. JWRI (Welding Research Institute of Osaka University), Vol. 7, No. 1 (1978) 11-16, and J. JWS (Japan Welding Society), 48-10 (1979) 851-858 (in Japanese).

2) Y. Ueda, K. Fukuda and J.K. Low; Mechanism of Production of Residual Stress Due to Slit Weld, IIW (International Institute of Welding), Doc. X-734-74, and J. JWS, 44-7 (1975) 580-587 (in Japanese).

3) Y. Ueda, K. Fukuda and Y.C. Kim; Analytical Calculation Method of Restraint Stresses and Strains Due to Slit Weld in Rectangular Plates, Trans. JWRI, Vol. 11, No. 1 (1982) 105-113, and J. JWS, 50-9 (1981) 930-937 (in Japanese).

4) Y. Ueda, K. Fukuda, Y.C. Kim and R. Koki; Characteristics of Restraint Stress-Strain of the Slit Weld in a Finite Rectangular Plate and the Significance of Restraint Intensities as a Dynamical Measure, Trans. JWRI, Vol. 11, No. 2 (1982) 105-113, and J. JWS, 51-8 (1982) 636-643 (in Japanese).

5) K. Horikawa, S. Matsui, S. Murase, H. Nakajima and K. Satoh; Application of the Y-Groove Restraint Cracking Test for Heavy Plates, IIW Doc. IX-965-76.

6) e.x. Timoshenko and Goodier; Theory of Elasticity, Third Edition, 104-109.

7) Y. Yeda, K. Fukuda and Y.C. Kim; Dynamical Characteristics of Y-Groove Cracking Test Specimen of Arbitrary Thickness, J. JWS, 51-8 (1982) 644-650 (in Japanese).