Kodama, A.
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A CHARACTERIZATION OF CERTAIN DOMAINS WITH GOOD BOUNDARY POINTS IN THE SENSE OF GREENE-KRANTZ, III

Dedicated to Professor Masaru Takeuchi on his 60th birthday

AKIO KODAMA*

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Introduction

This is a continuation of our previous papers [9, 10, 12]. For a domain D in $\boldsymbol{C}^n$, we denote by $\mathrm{Aut}(D)$ the group of all biholomorphic automorphisms of D and write ∂D (resp. $\bar{D}$) for the boundary (resp. closure) of D.

Let D be a bounded domain in $\boldsymbol{C}^n$ and $x \in \partial D$. Assume that x is an accumulation point of an $\mathrm{Aut}(D)$-orbit. *Can we then determine the global structure of D from the local shape of ∂D near x?* Of course, this is impossible without any further assumptions, as one may see in the examples such as the direct product of the open unit disk in $\boldsymbol{C}$ and an arbitrary bounded domain in $\boldsymbol{C}^{n-1}$. In the previous papers [2,8,9,10,12], this was exclusively studied in the case where ∂D near x coincides with the boundary of a generalized complex ellipsoid

$$E(n;n_1,\cdots,n_s;p_1,\cdots,p_s)$$
$$=\{(z_1,\cdots,z_s)\in \boldsymbol{C}^{n_1}\times\cdots\times \boldsymbol{C}^{n_s};\ \sum_{i=1}^{s}\|z_i\|^{2p_i}<1\}$$

in $\boldsymbol{C}^n=\boldsymbol{C}^{n_1}\times\cdots\times\boldsymbol{C}^{n_s}$, where $p_1,\cdots,p_s$ are positive real numbers and $n_1,\cdots,n_s$ are positive integers with $n=n_1+\cdots+n_s$.

The purpose of this paper is to establish the following extension of some results obtained in [2, 9, 10, 12]:

Theorem. *Let D be a bounded domain in $\boldsymbol{C}^n$ and $E=E(n;n_1,\cdots,n_s;p_1,\cdots,p_s)$ a generalized complex ellipsoid in $\boldsymbol{C}^n$. Let $x\in\partial D$. Assume that the following three conditions are satisfied:*

(1) *$p_1,\cdots,p_s$ are all positive integers;*

(2) *$x\in\partial E$ and there exists an open neighborhood Q of x in $\boldsymbol{C}^n$ such that*

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$D \cap Q = E \cap Q$; and

(3) *x is a good boundary point of D in the sense of Greene and Krantz* [6], *that is, there exist a point $b \in D$ and a sequence $\{\varphi_\nu\} \subset \mathrm{Aut}(D)$ such that $\varphi_\nu(b) \to x$ as $\nu \to \infty$.*

Then we have $D = E$ as sets. In particular, at least one of the p_i's must be equal to 1.

Note that the existences of a point $\tilde{b} \in E$ and a sequence $\{\tilde{\varphi}_\nu\} \subset \mathrm{Aut}(E)$ such that $\tilde{\varphi}_\nu(\tilde{b}) \to x$ as $\nu \to \infty$ are not assumed in the theorem. Hence, this does not follow directly from the results obtained in [7 or 12]; and also this gives an affirmative answer to Problem 1 in [11; p.62] in the case where ∂D near x is C^ω-smooth. In the special case $n_i = 1$ for all $i = 1, \cdots, s$, we know by [9, 10] that our theorem holds even for arbitrary $0 < p_1, \cdots, p_s \in \boldsymbol{R}$ (not necessarily integers). And, in its proof, Rudin's extension theorem [16] of holomorphic mappings defined near boundary points of the unit ball B^n in $\boldsymbol{C}^n$ played a crucial role. Notice that this theorem of Rudin can be applied no longer to the case $n_i \geq 1$ in general. However, employing a recent result due to Dini and Selvaggi Primicerio [3] instead of that due to Rudin and using the same scaling technique as in [12], we can prove the theorem above.

As an immediate consequence of our theorem, we now obtain the following:

Corollary. *For arbitrary integers $p_1, \cdots, p_s \geq 2$, any bounded domain D in $\boldsymbol{C}^n$ with a point $x \in \partial D \cap \partial E$ $(n; n_1, \cdots, n_s; p_1, \cdots, p_s)$ near which ∂D coincides with ∂E $(n; n_1, \cdots, n_s; p_1, \cdots, p_s)$ cannot have any* $\mathrm{Aut}(D)$*-orbits accumulating at x.*

Clearly this gives an affirmative answer to the following conjecture of Greene and Krantz [6; p. 200]: *Let x be a boundary point of the domain $E = \{(z_1, z_2) \in \boldsymbol{C}^2; |z_1|^4 + |z_2|^4 < 1\}$. Then any weakly pseudoconvex bounded domain D in $\boldsymbol{C}^2$ with $x \in \partial D$ near which ∂D coincides with ∂E cannot have any* $\mathrm{Aut}(D)$*-orbits accumulating at x.*

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1. Preliminaries

For later purpose, in this section we shall recall a recent result on localization principle of holomorphic automorphisms of generalized complex ellipsoids due to Dini and Selvaggi Primicerio [3], which plays an essential role in our proof.

For convenience and without loss of generality, in the following we will always assume

(1.1) $$p_1=1<p_2,\cdots,p_s\in \boldsymbol{Z}. \qquad 0<n_2,\cdots,n_s\in \boldsymbol{Z}$$

and write a generalized complex ellipsoid E in the form

(1.2) $$E=E\ (n;n_1,n_2,\cdots,n_s;1,p_2,\cdots,p_s).$$

Here it is understood that 1 does not appear if $n_1=0$, and also this domain is the unit ball B^n in $\boldsymbol{C}^n$ if $s=1$.

For a generalized complex ellipsoid E as in (1.2), we denote by $\mathscr{W}(E)$ the set consisting of all weakly, but not strictly, pseudoconvex boundary points of E. Then it can be seen that

(1.3) $$\mathscr{W}(E)=\{(z_1,z_2,\cdots,z_s)\in\partial E;\ \|z_2\|\cdots\|z_s\|=0\}$$
$$\subset\bigcup_{i=2}^{s}\{(z_1,\cdots,z_i,\cdots,z_s)\in \boldsymbol{C}^{n_1}\times\cdots\times \boldsymbol{C}^{n_i}\times\cdots\times \boldsymbol{C}^{n_s};z_i=0\}.$$

We can now state the result due to Dini and Selvaggi Primicerio [3] in the following form:

Theorem D-S. *Let E_1, E_2 be generalized complex ellipsoids in $\boldsymbol{C}^n$ with $\boldsymbol{C}^\omega$-smooth boundaries and $\mathscr{W}(E_1)$, $\mathscr{W}(E_2)$ the sets of weakly pseudoconvex boundary points of E_1, E_2 respectively, as in* (1.3). *Let $x_1\in\partial E_1$, $x_2\in\partial E_2$ and U_1, U_2 open neighborhoods of x_1, x_2 in $\boldsymbol{C}^n$, respectively. Assume that:*

(1) *$\mathscr{W}(E_1)$ and $\mathscr{W}(E_2)$ are contained in the union of finitely many complex linear subspaces of $\boldsymbol{C}^n$ of codimension at least* 2;

(2) *$U_1\cap\partial E_1$ is a connected open subset of ∂E_1;*

(3) *$\Psi: U_1\cap E_1\to U_2\cap E_2$ is a biholomorphic mapping that can be extended to a continuous mapping $\tilde{\Psi}: U_1\cap\bar{E}_1\to\bar{E}_2$ with $\tilde{\Psi}(x_1)=x_2$ and $\tilde{\Psi}(U_1\cap\partial E_1)\subset\partial E_2$.*

Then Ψ extends to a biholomorphic mapping Φ from E_1 onto E_2.

As noted by themselves in [3], the assumption (1) cannot be dropped in general; and also, after shrinking U_1 if necessary, one may further assume that Ψ is defined on all of U_1.

We finish this section by the following:

DEFINITION. Let $E_1=E\ (n;n_1,n_2,\cdots,n_s;1,p_2,\cdots,p_s)$ and $E_2=E\ (n;m_1,m_2,\cdots,m_t;1,q_2,\cdots,q_t)$ be two generalized complex ellipsoids in $\boldsymbol{C}^n$. Then we say that E_1 *precedes* E_2 if $s\leq t$ and there exists a permutation σ of the set $\{2,\cdots,t\}$ such that $(p_i,n_i)=(q_{\sigma(i)},m_{\sigma(i)})$ for $i=2,\cdots,s$.

Note that every generalized complex ellipsoid precedes itself and that the unit ball B^n in $\boldsymbol{C}^n$ precedes any generalized complex ellipsoid in $\boldsymbol{C}^n$.

2. Proof of the Theorem

With the same assumption and notation as in (1.1) and (1.2), we write the given E and $x \in \partial D \cap \partial E$ in the form $E = E(n; n_1, n_2, \cdots, n_s; 1, p_2, \cdots, p_s)$ and $x = (x_1, x_2, \cdots, x_s) \in C^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}$.

In order to prove the theorem, we prepare the following:

Lemma. *The domain D is biholomorphically equivalent to a generalized complex ellipsoid $\tilde{E}$ that precedes E.*

Proof. The following proof will be presented in outline, since the details of the steps can be filled in by consulting the corresponding passages in the proof of [12; Theorem I].

If $s=1$, i.e., $E=B^n$, then x is a C^ω-smooth strictly pseudoconvex boundary point of D; and hence, D is biholomorphically equivalent to B^n by Rosay [15].

Assume that $s>1$. According to the form of x, we shall divide the proof into two cases as follows:

Case A. $x = (x_1, 0, \cdots, 0)$.

In this case, there exists a sequence $\{\tilde{\varphi}_\nu\} \subset \mathrm{Aut}(E)$ such that $\tilde{\varphi}_\nu(o) \to x$ as $\nu \to \infty$, where $o \in E$ denotes the origin of C^n. Hence, D is biholomorphically equivalent to E by Kodama, Krantz and Ma [12].

Case B. $x = (x_1, \cdots, x_i, \cdots, x_s)$ with some $x_i \neq 0$ $(2 \leq i \leq s)$.

First of all, passing to a subsequence if necessary, one may assume that $\varphi_\nu(b) \in D \cap Q = E \cap Q \subset E$ for all ν. So there exists a sequence $\{\psi_\nu\}$ in $\mathrm{Aut}(E)$ such that

(2.1) $\psi_\nu(\varphi_\nu(b)) = (0, z_2^\nu, \cdots, z_s^\nu)$ for $\nu = 1, 2, \cdots$;

(2.2) each ψ_ν can be written in the form

$$\psi_\nu(z) = ((A^\nu z_1 + b^\nu)/(c^\nu z_1 + d^\nu), \quad z_2/(c^\nu z_1 + d^\nu)^{1/p_2}, \cdots, z_s/(c^\nu z_1 + d^\nu)^{1/p_s})$$

for $z = (z_1, z_2, \cdots, z_s) \in E \subset C^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}$.

Moreover, if we define the holomorphic mappings $\psi_1^\nu : B^{n_1} \to C^{n_1}$ by

$$\psi_1^\nu(z_1) = (A^\nu z_1 + b^\nu)/(c^\nu z_1 + d^\nu) \quad \text{for} \quad z_1 \in B^{n_1}, \tag{2.3}$$

then $\psi_1^\nu \in \mathrm{Aut}(B^{n_1})$ for all $\nu = 1, 2, \cdots$. (For the structure of $\mathrm{Aut}(E)$, see [12].) Setting $y^\nu = \varphi_\nu(b) = (y_1^\nu, y_2^\nu, \cdots, y_s^\nu)$ for $\nu = 1, 2, \cdots$, we have now

(2.4) $\psi_1^\nu(y_1^\nu) = 0$ for all $\nu = 1, 2, \cdots$.

On the other hand, since $\|x_1\|^2+\sum_{i=2}^{s}\|x_i\|^{2p_i}=1$ and $x_i\neq 0$ for some $2\leq i\leq s$, we see that

(2.5) the point $x_1=\lim_{v\to\infty}y_1^v$ is contained in B^{n_1} ,

which implies that $\{y_1^v\}$ lies in a compact subset of B^{n_1}. This combined with (2.4) guarantees that $\{\psi_1^v\}$ has a convergent subsequence in $\mathrm{Aut}(B^{n_1})$ [13; p.82]. Here we assert that, after taking a subsequence if necessary, $\{\psi_v\}$ converges to some $\psi\in\mathrm{Aut}(E)$. In fact, this can be seen as follows. With the same notation as in section 1 of [12], we can express $\mathrm{Aut}(B^{n_1})=U(n_1,1)/S^1$, where $U(n_1,1)$ is a special kind of linear Lie group and S^1 is closed normal subgroup of $U(n_1,1)$. Hence $\mathrm{Aut}(B^{n_1})$ is the base space of the principal fiber bundle $\pi: U(n_1,1)\to U(n_1,1)/S^1$. Let us assume that $\lim_{v\to\infty}\psi_1^v=\psi_1\in\mathrm{Aut}(B^{n_1})$. Then there exists a C^ω-smooth local cross section γ of $\pi: U(n_1,1)\to\mathrm{Aut}(B^{n_1})$ defined on an open neighborhood O of ψ_1. Without loss of generality, we may assume that

$$\{\psi_1^v\}\subset O \quad\text{and in (2.3)}\quad \gamma(\psi_1^v)=\begin{pmatrix}A^v & b^v\\ c^v & d^v\end{pmatrix}\quad\text{for } v=1,2,\cdots.$$

Then we have

$$\lim_{v\to\infty}\begin{pmatrix}A^v & b^v\\ c^v & d^v\end{pmatrix}=\gamma(\psi_1)\in U(n_1,1).$$

This combined with (2.2) assures that $\{\psi_v\}$ converges to some $\psi\in\mathrm{Aut}(E)$, as desired. Now, notice here that each ψ_v as well as ψ are difined on $B^{n_1}\times C^{n_2}\times\cdots\times C^{n_s}$; and, in fact,

(2.6) $\{\psi,\psi_v; v=1,2,\cdots\}\subset\mathrm{Aut}(B^{n_1}\times C^{n_2}\times\cdots\times C^{n_s})$;

(2.7) $\psi_v(z)\to\psi(z)$ (resp. $\psi_v^{-1}(z)\to\psi^{-1}(z)$) uniformly on compact subsets of $B^{n_1}\times C^{n_2}\times\cdots\times C^{n_s}$.

Hence we have $z^o:=\lim_{v\to\infty}\psi_v(y^v)=\psi(x)\in\partial E$, because the set $\{x,y^v; v=1,2,\cdots\}$ is now compact in $B^{n_1}\times C^{n_2}\times\cdots\times C^{n_s}$ by (2.5). Therefore, Case I in the proof of [12; Theorem I] does not occure in our Case B. Once it is shown that there exists a small open neighborhood U of z^o such that $\psi_v^{-1}(E\cap U)\subset E\cap Q=D\cap Q$ for all sufficiently large v, the rest of our proof can be done with exactly the same arguments as in the proof (Case II, pp.181–190) of [12; Theorem I] only by setting $\Gamma=\mathrm{id}_{C^n}$ throughout. Therefore, it is enough to prove the existence of such a neighborhood U of z^o. To this end, taking (2.5) into account, we choose an open neighborhood V of x with compact closure in $(B^{n_1}\times C^{n_2}\times\cdots\times C^{n_s})\cap Q$. Then, by (2.6) and (2.7) we see that $\psi(V)=\lim_{v\to\infty}\psi_v(V)$ is an open neighborhood of z^o and

$\psi_\nu^{-1}(\psi(V)) \subset (B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}) \cap Q$ for all sufficitently large ν. Hence, every open neighborhood U of z^o with $U \subset \psi(V)$ satisfies the requirement above. This completes the proof of the lemma in the case $n_1 > 0$.

Finally, consider the case $n_1 = 0$. Then, setting $\Gamma = \mathrm{id}_{C^n}$ and also $\psi_\nu = \mathrm{id}_{C^n}$ for all ν, and proceeding along exactly the same line as in the proof (Case II, pp.181–190) of [12; Theorem I], we can check that D is biholomorphically equivalent to some generalized complex ellipsoid $\tilde{E}$ that precedes E; thereby completing the proof of the lemma.

Q.E.D.

Proof of the Theorem. After relabeling the indices, one may assume that

(2.8) $n_2 = \cdots = n_k = 1 < n_{k+1}, \cdots, n_s$ for some integer k $(1 \leq k \leq s)$.

Here it is understood that all $n_2, \cdots, n_s \geq 2$ if $k=1$, and also $n_2 = \cdots = n_s = 1$ if $k = s$.

By virtue of the Lemma, D is now biholomorphically equivalent to a generalized complex ellipsoid $\tilde{E}$ in C^n that precedes E. Therefore, remembering the definition of precedence and renaming the indices if necessary, we may assume that D is biholomorphically equivalent to the generalized complex ellipsoid E^* in C^n defined by

$$E^* = \{z = (z_1, \cdots, z_s) \in C^{n_1} \times \cdots \times C^{n_s} = C^n ; \rho(z) < 1\},$$

where

$$\rho(z) = \|z_1\|^2 + \sum_{a=2}^{j} |z_a|^2 + \sum_{a=j+1}^{k} |z_a|^{2p_a} + \sum_{b=k+1}^{l} \|z_b\|^2 + \sum_{b=l+1}^{s} \|z_b\|^{2p_b}$$

for some integers j, l $(1 \leq j \leq k \leq l \leq s)$, with the natural understanding that some of summands may vanish (for example, $\sum_{a=2}^{j} |z_a|^2 = 0$ if $j=1$). Let us fix a biholomorphic mapping $F: D \to E^*$ and take a point

$$z^* = (z_1^*, \cdots, z_s^*) \in Q \cap \partial D \quad \text{with} \quad \|z_1^*\| \cdots \|z_s^*\| \neq 0.$$

There is a sequence $\{z^i\}$ in D such that

$z^i \to z^*$ and $F(z^i) \to w^*$ for some point $w^* \in \partial E^*$.

Since z^* is a C^ω-smooth strictly pseudoconvex boundary point of D and since w^* satisfies Condition (P) in the sense of Forstnerič and Rosay [5], the inverse mapping $F^{-1}: E^* \to D$ of F has a continuous extension $G: W \cap \bar{E}^* \to \bar{D}$ by [5], where W is a sufficiently small open neighborhood of w^* in C^n. Clearly $G(w^*) = z^*$. So there exist open neighborhoods U^*, W^* of z^*, w^* in C^n, respectively, such that

$$U^* \subset Q \cap \{(z_1,\cdots,z_s)\in \boldsymbol{C}^n;\ \|z_1\|\cdots\|z_s\| \neq 0\};$$
$$W^* \subset W \text{ and } G(W^* \cap \bar{E}^*) \subset U^*.$$

Take a point

$$w^{**}=(w_1^{**},\cdots,w_s^{**})\in W^* \cap \partial E^* \text{ with } \|w_1^{**}\|\cdots\|w_s^{**}\| \neq 0$$

and set $z^{**}=G(w^{**})\in U^* \cap \partial D$. Then z^{**} and w^{**} are $\boldsymbol{C}^\omega$-smooth strictly pseudoconvex boundary points of D and E^*, respectively. Applying again the extension theorem of Forstnerič and Rosay [5] to the biholomorphic mappings $F: D \to E^*$ and $F^{-1}: E^* \to D$, one can find open neighborhoods U^{**}, W^{**} of z^{**}, w^{**} respectively in $\boldsymbol{C}^n$ such that

(2.9) $U^{**} \subset U^*$, $W^{**} \subset W^*$ and $U^{**} \cap \partial D$ is a connected subset of ∂D;

(2.10) F extends to a homeomorphism $H: U^{**} \cap \bar{D} \to W^{**} \cap \bar{E}^*$ with $H^{-1}=G$ on $W^{**} \cap \bar{E}^*$.

Now, define the mappings Π_1, $\Pi_2: \boldsymbol{C}^n \to \boldsymbol{C}^n$ by setting

$$\Pi_1(z)=(z_1,(z_2)^{p_2},\cdots,(z_k)^{p_k},z_{k+1},\cdots,z_s),$$
$$\Pi_2(z)=(z_1,\cdots,z_j,(z_{j+1})^{p_{j+1}},\cdots,(z_k)^{p_k},z_{k+1},\cdots,z_s)$$

for $z=(z_1,\cdots,z_s)\in \boldsymbol{C}^{n_1}\times\cdots\times \boldsymbol{C}^{n_s}=\boldsymbol{C}^n$; and consider the generalized complex ellipsoids E_1, E_2 in $\boldsymbol{C}^n$ defined by

$$E_1=E(n;n_1+\cdots+n_k,n_{k+1},\cdots,n_s;1,p_{k+1},\cdots,p_s),$$
$$E_2=E(n;n_1+\cdots+n_l,n_{l+1},\cdots,n_s;1,p_{l+1},\cdots,p_s).$$

Since $n_2=\cdots=n_k=1$ by (2.8) and since $2\le p_2,\cdots,p_k\in \boldsymbol{Z}$ by (1.1), both Π_1 and Π_2 are proper holomorphic mappings from $\boldsymbol{C}^n$ onto $\boldsymbol{C}^n$ such that

(2.11) $\Pi_1(E)=E_1$ and $\Pi_2(E^*)=E_2$;

(2.12) Π_1 and Π_2 are injective near z^{**} and w^{**}, respectively.

After shrinking U^{**} and W^{**} if necessary, we can therefore assume that the restrictions $\Pi_1|U^{**}: U^{**}\to\Pi_1(U^{**})$ and $\Pi_2|W^{**}: W^{**}\to\Pi_2(W^{**})$ are biholomorphic mappings. Consider here the homeomorphism

$$\Psi := \Pi_2\circ H\circ(\Pi_1|U^{**}\cap\bar{D})^{-1}: \Pi_1(U^{**})\cap\bar{E}_1 \to \Pi_2(W^{**})\cap\bar{E}_2.$$

Then, it is obvious that the hypotheses (2) and (3) of Theorem D-S hold with $x_1=\Pi_1(z^{**})$, $x_2=\Pi_2(w^{**})$, $U_1=\Pi_1(U^{**})$ and $U_2=\Pi_2(W^{**})$. Moreover, in view of (1.3), the set $\mathscr{W}(E_1)$ (resp. $\mathscr{W}(E_2)$) is contained in the union of finitely many complex linear subspaces of $\boldsymbol{C}^n$ of codimension at least 2 if and only if all

$n_{k+1},\cdots,n_s\geq 2$ (resp. $n_{l+1},\cdots,n_s\geq 2$), which is now guaranteed by (2.8). (Note that $p_{k+1},\cdots,p_s\geq 2$ and $l\geq k\geq 1$.) Therefore, Theorem D-S can be applied to obtain a biholomorphic mapping $\Phi: E_1\to E_2$ such that $\Psi(z)=\Phi(z)$ for all $z\in\Pi_1(U^{**})\cap E_1$, or equivalently

$$\Phi^{-1}\circ\Pi_2\circ F(z)=\Pi_1(z) \quad \text{for all} \quad z\in U^{**}\cap D;$$

consequently $\Phi^{-1}\circ\Pi_2\circ F(z)=\Pi_1(z)$ for all $z\in D$ by the principle of analytic continuation. This combined with the fact that $\Phi^{-1}\circ\Pi_2\circ F: D\to E_1$ is a proper mapping yields at once that $D=E$ as sets.

Finally, since $\mathrm{Aut}(E)=\mathrm{Aut}(D)$ is now non-compact by the hypothesis (3) of the theorem, one concludes that $n_1>0$, i.e., at least one of the p_i's must be equal to 1. (Recall the understanding made after (1.2).) This completes the proof of the theorem. Q.E.D.

REMARK 1. In the proof above, one can assume that the continuous extension $H: U^{**}\cap\bar{D}\to W^{**}\cap\bar{E}^*$ of F is the restriction of a biholomorphic mapping from U^{**} onto W^{**} (after shrinkign U^{**} and W^{**} if necessary). In fact, this immediately follows from [4,14] or [1], because by the construction both $U^{**}\cap\partial D$ and $W^{**}\cap\partial E^*$ are C^ω-smooth strictly pseudoconvex real hypersurfaces in C^n and $H: U^{**}\cap\partial D\to W^{**}\cap\partial E^*$ is a CR-homeomorphism.

REMARK 2. In the theorem, assume the following (2)* instead of (2):

(2)* *There exist a point $\tilde{x}\in\partial E$, open neighborhoods Q of x, $\tilde{Q}$ of $\tilde{x}$, and a biholomorphic mappiong $\Gamma: Q\to\tilde{Q}$ such that $\Gamma(x)=\tilde{x}$ and $\Gamma(D\cap Q)=E\cap\tilde{Q}$.*

Then, a glance at our proof of the theorem tells us that *D is biholomorphically equivalent to E and that at least one of the p_i's must be equal to* 1.

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Department of Mathematics
Faculty of Science
Kanazawa University
Kanazawa 920–11
Japan