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A CHARACTERIZATION OF CERTAIN DOMAINS WITH GOOD BOUNDARY POINTS IN THE SENSE OF GREENE-KRANTZ, III

Dedicated to Professor Masaru Takeuchi on his 60th birthday

Akio KODAMA*

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Introduction

This is a continuation of our previous papers [9, 10, 12]. For a domain $D$ in $\mathbb{C}^n$, we denote by $\text{Aut}(D)$ the group of all biholomorphic automorphisms of $D$ and write $\partial D$ (resp. $\bar{D}$) for the boundary (resp. closure) of $D$.

Let $D$ be a bounded domain in $\mathbb{C}^n$ and $x \in \partial D$. Assume that $x$ is an accumulation point of an $\text{Aut}(D)$-orbit. Can we then determine the global structure of $D$ from the local shape of $\partial D$ near $x$? Of course, this is impossible without any further assumptions, as one may see in the examples such as the direct product of the open unit disk in $\mathbb{C}$ and an arbitrary bounded domain in $\mathbb{C}^{n-1}$. In the previous papers [2,8,9,10,12], this was exclusively studied in the case where $\partial D$ near $x$ coincides with the boundary of a generalized complex ellipsoid

$E(n;n_1,\cdots,n_s;p_1,\cdots,p_s)$

$=\{(z_1,\cdots,z_s)\in \mathbb{C}^{n_1} \times \cdots \times \mathbb{C}^{n_s}; \sum_{i=1}^s \|z_i\|^{2p_i} < 1\}$

in $\mathbb{C}^n=\mathbb{C}^{n_1} \times \cdots \times \mathbb{C}^{n_s}$, where $p_1,\cdots,p_s$ are positive real numbers and $n_1,\cdots,n_s$ are positive integers with $n=n_1+\cdots+n_s$.

The purpose of this paper is to establish the following extension of some results obtained in [2, 9, 10, 12]:

**Theorem.** Let $D$ be a bounded domain in $\mathbb{C}^n$ and $E=E(n;n_1,\cdots,n_s;p_1,\cdots,p_s)$ a generalized complex ellipsoid in $\mathbb{C}^n$. Let $x \in \partial D$. Assume that the following three conditions are satisfied:

1. $p_1,\cdots,p_s$ are all positive integers;
2. $x \in \partial E$ and there exists an open neighborhood $Q$ of $x$ in $\mathbb{C}^n$ such that

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$D \cap Q = E \cap Q$; and

(3) $x$ is a good boundary point of $D$ in the sense of Greene and Krantz [6], that is, there exist a point $b \in D$ and a sequence $\{\varphi_v\} \subset \text{Aut}(D)$ such that $\varphi_v(b) \to x$ as $v \to \infty$.

Then we have $D = E$ as sets. In particular, at least one of the $p_i$'s must be equal to 1.

Note that the existences of a point $b \in E$ and a sequence $\{\varphi_v\} \subset \text{Aut}(E)$ such that $\varphi_v(b) \to x$ as $v \to \infty$ are not assumed in the theorem. Hence, this does not follow directly from the results obtained in [7 or 12]; and also this gives an affirmative answer to Problem 1 in [11, p.62] in the case where $\partial D$ near $x$ is $C^\omega$-smooth. In the special case $n_i=1$ for all $i=1,\cdots,s$, we know by [9, 10] that our theorem holds even for arbitrary $0 < p_1,\cdots,p_s \in \mathbb{R}$ (not necessarily integers). And, in its proof, Rudin's extension theorem [16] of holomorphic mappings defined near boundary points of the unit ball $B^n$ in $\mathbb{C}^n$ played a crucial role. Notice that this theorem of Rudin can be applied no longer to the case $n_i \geq 1$ in general. However, employing a recent result due to Dini and Selvaggi Primicerio [3] instead of that due to Rudin and using the same scaling technique as in [12], we can prove the theorem above.

As an immediate consequence of our theorem, we now obtain the following:

**Corollary.** For arbitrary integers $p_1,\cdots,p_s \geq 2$, any bounded domain $D$ in $\mathbb{C}^n$ with a point $x \in \partial D \cap \partial E (n_1,\cdots,n_s; p_1,\cdots,p_s)$ near which $\partial D$ coincides with $\partial E (n_1,\cdots,n_s; p_1,\cdots,p_s)$ cannot have any $\text{Aut}(D)$-orbits accumulating at $x$.

Clearly this gives an affirmative answer to the following conjecture of Greene and Krantz [6, p. 200]: Let $x$ be a boundary point of the domain $E = \{(z_1, z_2) \in \mathbb{C}^2; |z_1|^4 + |z_2|^4 < 1\}$. Then any weakly pseudoconvex bounded domain $D$ in $\mathbb{C}^2$ with $x \in \partial D$ near which $\partial D$ coincides with $\partial E$ cannot have any $\text{Aut}(D)$-orbits accumulating at $x$.

The author would like to express his sincere gratitude to Professors Gilberto Dini and Angela Selvaggi Primicerio for informing him of their recent work [3]; and also for their stimulating conversations on the structure of generalized complex ellipsoids, which were done during his stay at the University of Florence.

1. **Preliminaries**

For later purpose, in this section we shall recall a recent result on localization principle of holomorphic automorphisms of generalized complex ellipsoids due to Dini and Selvaggi Primicerio [3], which plays an essential role in our proof.

For convenience and without loss of generality, in the following we will always assume
and write a generalized complex ellipsoid $E$ in the form
\[(1.2)\quad E = E(n; n_1, n_2, \ldots, n_s; 1, p_2, \ldots, p_s).\]

Here it is understood that 1 does not appear if $n_1 = 0$, and also this domain is the unit ball $B^*$ in $\mathbb{C}^n$ if $s = 1$.

For a generalized complex ellipsoid $E$ as in (1.2), we denote by $\mathcal{W}(E)$ the set consisting of all weakly, but not strictly, pseudoconvex boundary points of $E$. Then it can be seen that
\[(1.3)\quad \mathcal{W}(E) = \{(z_1, z_2, \ldots, z_s) \in \partial E; \|z_2\| \cdots \|z_s\| = 0\}
\subset \bigcup_{i=2}^{s} \{(z_1, \ldots, z_i, \ldots, z_s) \in \mathbb{C}^{n_1} \times \cdots \times \mathbb{C}^{n_i} \times \cdots \times \mathbb{C}^{n_s}; z_i = 0\}.

We can now state the result due to Dini and Selvaggi Primicerio [3] in the following form:

**Theorem D-S.** Let $E_1, E_2$ be generalized complex ellipsoids in $\mathbb{C}^n$ with $C^\omega$-smooth boundaries and $\mathcal{W}(E_1), \mathcal{W}(E_2)$ the sets of weakly pseudoconvex boundary points of $E_1, E_2$ respectively, as in (1.3). Let $x_1 \in \partial E_1, x_2 \in \partial E_2$ and $U_1, U_2$ open neighborhoods of $x_1, x_2$ in $\mathbb{C}^n$, respectively. Assume that:

1. $\mathcal{W}(E_1)$ and $\mathcal{W}(E_2)$ are contained in the union of finitely many complex linear subspaces of $\mathbb{C}^n$ of codimension at least 2;
2. $U_1 \cap \partial E_1$ is a connected open subset of $\partial E_1$;
3. $\Psi: U_1 \cap E_1 \to U_2 \cap E_2$ is a biholomorphic mapping that can be extended to a continuous mapping $\overline{\Psi}: U_1 \to E_1 \to E_2$ with $\overline{\Psi}(x_1) = x_2$ and $\overline{\Psi}(U_1 \cap \partial E_1) \subset \partial E_2$.

Then $\Psi$ extends to a biholomorphic mapping $\Phi$ from $E_1$ onto $E_2$.

As noted by themselves in [3], the assumption (1) cannot be dropped in general; and also, after shrinking $U_1$ if necessary, one may further assume that $\Psi$ is defined on all of $U_1$.

We finish this section by the following:

**Definition.** Let $E_1 = E(n; n_1, n_2, \ldots, n_s; 1, p_2, \ldots, p_s)$ and $E_2 = E(n; m_1, m_2, \ldots, m_t; 1, q_2, \ldots, q_t)$ be two generalized complex ellipsoids in $\mathbb{C}^n$. Then we say that $E_1$ precedes $E_2$ if $s \leq t$ and there exists a permutation $\sigma$ of the set $\{2, \ldots, t\}$ such that $(p_i, n_i) = (q_{\sigma(i)}, m_{\sigma(i)})$ for $i = 2, \ldots, s$.

Note that every generalized complex ellipsoid precedes itself and that the unit ball $B^*$ in $\mathbb{C}^n$ precedes any generalized complex ellipsoid in $\mathbb{C}^n$. 

\[(1.1)\quad p_1 = 1 < p_2, \ldots, p_s \in \mathbb{Z}, \quad 0 < n_2, \ldots, n_s \in \mathbb{Z}\]
2. Proof of the Theorem

With the same assumption and notation as in (1.1) and (1.2), we write the given $E$ and $\bar{E}$ in the form $E = E(n; n_1, n_2, \ldots, n_s, 1, p_2, \ldots, p_s)$ and $x = (x_1, x_2, \ldots, x_s) \in C^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}$.

In order to prove the theorem, we prepare the following:

Lemma. The domain $D$ is biholomorphically equivalent to a generalized complex ellipsoid $\tilde{E}$ that precedes $E$.

Proof. The following proof will be presented in outline, since the details of the steps can be filled in by consulting the corresponding passages in the proof of [12; Theorem I].

If $s = 1$, i.e., $E = B^n$, then $x$ is a $C^\omega$-smooth strictly pseudoconvex boundary point of $D$, and hence, $D$ is biholomorphically equivalent to $B^n$ by Rosay [15].

Assume that $s > 1$. According to the form of $x$, we shall divide the proof into two cases as follows:

Case A. $x = (x_1, 0, \ldots, 0)$.

In this case, there exists a sequence $\{\phi_v\} \subset \text{Aut}(E)$ such that $\phi_v(o) \to x$ as $v \to \infty$, where $o \in E$ denotes the origin of $C^n$. Hence, $D$ is biholomorphically equivalent to $E$ by Kodama, Krantz and Ma [12].

Case B. $x = (x_1, \ldots, x_i, \ldots, x_s)$ with some $x^O_i (2 < i < s)$.

First of all, passing to a subsequence if necessary, one may assume that $\phi_v(b) \in D \cap Q = E \cap Q \subset E$ for all $v$. So there exists a sequence $\{\psi_v\} \subset \text{Aut}(E)$ such that

(2.1) $\psi_v(\phi_v(b)) = (0, z_2, \ldots, z_s)$ for $v = 1, 2, \ldots$;

(2.2) each $\psi_v$ can be written in the form

$$\psi_v(z) = (A^v z_1 + b^v)/(c^v z_1 + d^v), \quad z_2/(c^v z_1 + d^v)^{1/p_2}, \ldots, z_s/(c^v z_1 + d^v)^{1/p_s}$$

for $z = (z_1, z_2, \ldots, z_s) \in E \subset C^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}$.

Moreover, if we define the holomorphic mappings $\psi^*_v : B^{n_1} \to C^{n_1}$ by

(2.3) $\psi^*_v(z_1) = (A^v z_1 + b^v)/(c^v z_1 + d^v)$ for $z_1 \in B^{n_1}$,

then $\psi^*_v \in \text{Aut}(B^{n_1})$ for all $v = 1, 2, \ldots$. (For the structure of Aut($E$), see [12].) Setting $y^v = \phi_v(b) = (y^v_1, y^v_2, \ldots, y^v_s)$ for $v = 1, 2, \ldots$, we have now

(2.4) $\psi^*_v(y^v_1) = 0$ for all $v = 1, 2, \ldots$. 

On the other hand, since \( \|x_1\|^2 + \sum_{i=2}^{s} \|x_i\|^{2p_i} = 1 \) and \( x_i \neq 0 \) for some \( 2 \leq i \leq s \), we see that

\[
(2.5) \quad \text{the point } x_1 = \lim_{v \to \infty} y_1^v \text{ is contained in } B^{s_1},
\]

which implies that \( \{y_1^v\} \) lies in a compact subset of \( B^{s_1} \). This combined with (2.4) guarantees that \( \{y_1^v\} \) has a convergent subsequence in \( \text{Aut}(B^{n_1}) \) [13; p.82]. Here we assert that, after taking a subsequence if necessary, \( \{\psi_\nu\} \) converges to some \( \psi \in \text{Aut}(E) \). In fact, this can be seen as follows. With the same notation as in section 1 of [12], we can express \( \text{Aut}(B^{n_1}) = U(n_1,1)/S^1 \), where \( U(n_1,1) \) is a special kind of linear Lie group and \( S^1 \) is closed normal subgroup of \( U(n_1,1) \). Hence \( \text{Aut}(B^{n_1}) \) is the base space of the principal fiber bundle \( \pi : U(n_1,1) \to U(n_1,1)/S^1 \). Let us assume that \( \lim_{v \to \infty} \psi_\nu^v = \psi \in \text{Aut}(B^{n_1}) \). Then there exists a \( C^\omega \)-smooth local cross section \( \gamma \) of \( \pi : U(n_1,1) \to \text{Aut}(B^{n_1}) \) defined on an open neighborhood \( O \) of \( \psi_\nu \). Without loss of generality, we may assume that

\[
\{\gamma_\nu^v\} \subset O \quad \text{and in (2.3)} \quad \gamma(\psi_\nu^v) = \begin{pmatrix} A^v & b^v \\ c^v & d^v \end{pmatrix} \quad \text{for } v = 1,2,\ldots.
\]

Then we have

\[
\lim_{v \to \infty} \begin{pmatrix} A^v & b^v \\ c^v & d^v \end{pmatrix} = \gamma(\psi_1) \in U(n_1,1).
\]

This combined with (2.2) assures that \( \{\psi_\nu\} \) converges to some \( \psi \in \text{Aut}(E) \), as desired. Now, notice here that each \( \psi_\nu \) as well as \( \psi \) are defined on \( B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s} \) and, in fact,

\[
(2.6) \quad \{\psi_\nu; v = 1,2,\ldots\} \subset \text{Aut}(B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s});
\]

\[
(2.7) \quad \psi_\nu(z) \to \psi(z) \quad \text{(resp. } \psi_\nu^{-1}(z) \to \psi^{-1}(z)\text{)} \quad \text{uniformly on compact subsets of } \text{Aut}(B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}).
\]

Hence we have \( z^\circ := \lim_{v \to \infty} (y^v) = \psi(x) \in \partial E \), because the set \( \{x,y^v; v = 1,2,\ldots\} \) is now compact in \( B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s} \) by (2.5). Therefore, Case I in the proof of [12; Theorem I] does not occur in our Case B. Once it is shown that there exists a small open neighborhood \( U \) of \( z^\circ \) such that \( \psi_\nu^{-1}(E \cap U) \subset E \cap Q = D \cap Q \) for all sufficiently large \( v \), the rest of our proof can be done with exactly the same arguments as in the proof (Case II, pp.181–190) of [12; Theorem I] only by setting \( \Gamma = \text{id}_{C^n} \) throughout. Therefore, it is enough to prove the existence of such a neighborhood \( U \) of \( z^\circ \). To this end, taking (2.5) into account, we choose an open neighborhood \( V \) of \( x \) with compact closure in \( (B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}) \cap Q \). Then, by (2.6) and (2.7) we see that \( \psi(V) = \lim_{v \to \infty} \psi_\nu(V) \) is an open neighborhood of \( z^\circ \) and
\[ \psi^{-1}_{\nu}(|(V)|) \subset (B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}) \cap Q \] for all sufficiently large \( v \). Hence, every open neighborhood \( U \) of \( z^* \) with \( U \subset \psi(V) \) satisfies the requirement above. This completes the proof of the lemma in the case \( n_1 > 0 \).

Finally, consider the case \( n_1 = 0 \). Then, setting \( \Gamma = \text{id}_{C^n} \) and also \( \psi_{\nu} = \text{id}_{C^n} \) for all \( \nu \), and proceeding along exactly the same line as in the proof (Case II, pp. 181–190) of [12; Theorem I], we can check that \( D \) is biholomorphically equivalent to some generalized complex ellipsoid \( \bar{E} \) that precedes \( E \); thereby completing the proof of the lemma.

Q.E.D.

Proof of the Theorem. After relabeling the indices, one may assume that

\[ n_2 = \cdots = n_k = 1 < n_{k+1}, \ldots, n_s \text{ for some integer } k (1 \leq k \leq s). \]

Here it is understood that all \( n_2, \ldots, n_s \geq 2 \) if \( k = 1 \), and also \( n_2 = \cdots = n_s = 1 \) if \( k = s \).

By virtue of the Lemma, \( D \) is now biholomorphically equivalent to a generalized complex ellipsoid \( \bar{E} \) in \( C^n \) that precedes \( E \). Therefore, remembering the definition of precedence and renaming the indices if necessary, we may assume that \( D \) is biholomorphically equivalent to the generalized complex ellipsoid \( E^* \) in \( C^n \) defined by

\[ E^* = \{ z = (z_1, \ldots, z_s) \in C^{n_1} \times \cdots \times C^{n_s} = C^n; \rho(z) < 1 \}, \]

where

\[ \rho(z) = \| z_1 \|^2 + \sum_{a=2}^{j} |z_a|^2 + \sum_{a=j+1}^{k} |z_a|^{2P_a} + \sum_{b=k+1}^{l} \| z_b \|^2 + \sum_{b=l+1}^{s} \| z_b \|^{2P_b} \]

for some integers \( j, l \) (\( 1 \leq j \leq k \leq l \leq s \)), with the natural understanding that some of summands may vanish (for example, \( \sum_{a=2}^{j} |z_a|^2 = 0 \) if \( j = 1 \)). Let us fix a biholomorphic mapping \( F: D \to E^* \) and take a point

\[ z^* = (z^*_1, \ldots, z^*_s) \in Q \cap \partial D \text{ with } \| z^*_1 \| \cdots \| z^*_s \| \neq 0. \]

There is a sequence \( \{ z^i \} \) in \( D \) such that

\[ z^i \to z^* \text{ and } F(z^i) \to w^* \text{ for some point } w^* \in \partial E^*. \]

Since \( z^* \) is a \( C^0 \)-smooth strictly pseudoconvex boundary point of \( D \) and since \( w^* \) satisfies Condition \( (P) \) in the sense of Forstnerič and Rosay [5], the inverse mapping \( F^{-1}: E^* \to D \) of \( F \) has a continuous extension \( G: W \cap \bar{E}^* \to \bar{D} \) by [5], where \( W \) is a sufficiently small open neighborhood of \( w^* \) in \( C^n \). Clearly \( G(w^*) = z^* \). So there exist open neighborhoods \( U^*, W^* \) of \( z^*, w^* \) in \( C^n \), respectively, such that
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\[ U^* \subset Q \cap \{ (z_1, \ldots, z_s) \in C^n; \| z_1 \| \cdots \| z_s \| \neq 0 \}; \]

\[ W^* \subset W \text{ and } G(W^* \cap \bar{E}^*) \subset U^*. \]

Take a point
\[ w^{**} = (w_1^{**}, \ldots, w_t^{**}) \in W^* \cap \partial E^* \text{ with } \| w_1^{**} \| \cdots \| w_t^{**} \| \neq 0 \]

and set \( z^{**} = G(w^{**}) \subset U^* \cap \partial D \). Then \( z^{**} \) and \( w^{**} \) are \( C^\infty \)-smooth strictly pseudoconvex boundary points of \( D \) and \( E^* \), respectively. Applying again the extension theorem of Forstnerič and Rosay [5] to the biholomorphic mappings \( F: D \to E^* \) and \( F^{-1}: E^* \to D \), one can find open neighborhoods \( U^{**}, W^{**} \) of \( z^{**}, w^{**} \), respectively, in \( C^n \) such that

(2.9) \( U^{**} \subset U^* \), \( W^{**} \subset W^* \) and \( U^{**} \cap \partial D \) is a connected subset of \( \partial D \);

(2.10) \( F \) extends to a homeomorphism \( H: U^{**} \cap \bar{D} \to W^{**} \cap \bar{E}^* \) with \( H^{-1} = G \) on \( W^{**} \cap \bar{E}^* \).

Now, define the mappings \( \Pi_1, \Pi_2: C^n \to C^n \) by setting
\[
\Pi_1(z) = (z_1, (z_2)^{p_2}, \ldots, (z_k)^{p_k}, z_{k+1}, \ldots, z_s),
\]
\[
\Pi_2(z) = (z_1, \ldots, z_j(z_{j+1})^{p_{j+1}}, \ldots, (z_k)^{p_k}, z_{k+1}, \ldots, z_s)
\]

for \( z = (z_1, \ldots, z_s) \in C^n \times \cdots \times C^n = C^n \); and consider the generalized complex ellipsoids \( E_1, E_2 \) in \( C^n \) defined by
\[
E_1 = E(n; n_1 + \cdots + n_k, n_{k+1}, \ldots, n_s; 1, p_{k+1}, \ldots, p_s),
\]
\[
E_2 = E(n; n_1 + \cdots + n_k, n_{k+1}, \ldots, n_s; 1, p_{l+1}, \ldots, p_s).
\]

Since \( n_2 = \cdots = n_k = 1 \) by (2.8) and since \( 2 \leq p_2, \ldots, p_k \in Z \) by (1.1), both \( \Pi_1 \) and \( \Pi_2 \) are proper holomorphic mappings from \( C^n \) onto \( C^n \) such that

(2.11) \( \Pi_1(E) = E_1 \) and \( \Pi_2(E^*) = E_2 \);

(2.12) \( \Pi_1 \) and \( \Pi_2 \) are injective near \( z^{**} \) and \( w^{**} \), respectively.

After shrinking \( U^{**} \) and \( W^{**} \) if necessary, we can therefore assume that the restrictions \( \Pi_1|U^{**}: U^{**} \to \Pi_1(U^{**}) \) and \( \Pi_2|W^{**}: W^{**} \to \Pi_2(W^{**}) \) are biholomorphic mappings. Consider here the homeomorphism
\[
\Psi := \Pi_2 \circ H \circ (\Pi_1|U^{**} \cap \bar{D})^{-1}: \Pi_1(U^{**}) \cap \bar{E}_1 \to \Pi_2(W^{**}) \cap \bar{E}_2
\]

Then, it is obvious that the hypotheses (2) and (3) of Theorem D-S hold with \( x_1 = \Pi_1(z^{**}), x_2 = \Pi_2(w^{**}), U_1 = \Pi_1(U^{**}) \) and \( U_2 = \Pi_2(W^{**}) \). Moreover, in view of (1.3), the set \( \mathcal{W}(E_1) \) (resp. \( \mathcal{W}(E_2) \)) is contained in the union of finitely many complex linear subspaces of \( C^n \) of codimension at least 2 if and only if all
\[ n_{k+1}, \ldots, n_s \geq 2 \text{ (resp. } n_{l+1}, \ldots, n_l \geq 2) \text{, which is now guaranteed by (2.8). (Note that } p_{k+1}, \ldots, p_s \geq 2 \text{ and } l \geq k \geq 1.) \text{ Therefore, Theorem D-S can be applied to obtain a biholomorphic mapping } \Phi: E_1 \to E_2 \text{ such that } \Psi(z) = \Phi(z) \text{ for all } z \in \Pi_1(U^{**}) \cap E_1, \text{ or equivalently }

\[ \Phi^{-1} \circ \Pi_2 \circ F(z) = \Pi_1(z) \text{ for all } z \in U^{**} \cap D; \]

consequently \( \Phi^{-1} \circ \Pi_2 \circ F(z) = \Pi_1(z) \) for all \( z \in D \) by the principle of analytic continuation. This combined with the fact that \( \Phi^{-1} \circ \Pi_2 \circ F: D \to E_1 \) is a proper mapping yields at once that \( D = E \) as sets.

Finally, since \( \text{Aut}(E) = \text{Aut}(D) \) is now non-compact by the hypothesis (3) of the theorem, one concludes that \( n_1 > 0 \), i.e., at least one of the \( p_i \)'s must be equal to 1. (Recall the understanding made after (1.2).) This completes the proof of the theorem.

Q.E.D.

**Remark 1.** In the proof above, one can assume that the continuous extension \( H: U^{**} \cap D \to W^{**} \cap E^* \) of \( F \) is the restriction of a biholomorphic mapping from \( U^{**} \) onto \( W^{**} \) (after shrinking \( U^{**} \) and \( W^{**} \) if necessary). In fact, this immediately follows from [4,14] or [1], because by the construction both \( U^{**} \cap \partial D \) and \( W^{**} \cap \partial E^* \) are \( C^\infty \)-smooth strictly pseudoconvex real hypersurfaces in \( C^n \) and \( H: U^{**} \cap \partial D \to W^{**} \cap \partial E^* \) is a CR-homeomorphism.

**Remark 2.** In the theorem, assume the following (2)* instead of (2):

(2)* There exist a point \( \bar{x} \in \partial E \), open neighborhoods \( Q \) of \( x \), \( \bar{Q} \) of \( \bar{x} \), and a biholomorphic mapping \( \Gamma: Q \to \bar{Q} \) such that \( \Gamma(x) = \bar{x} \) and \( \Gamma(D \cap Q) = E \cap \bar{Q} \).

Then, a glance at our proof of the theorem tells us that \( D \) is biholomorphically equivalent to \( E \) and that at least one of the \( p_i \)'s must be equal to 1.

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**References**


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