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Effects of Murphy number on quadrupedal running gait based on a simple model

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1 Introduction

Different quadrupedal mammals use different gaits in high-speed running [1]. For example, cheetahs use a galloping gait, which shows flight phase twice during one stride. In contrast, horses use a galloping gait, which shows once [2]. It remains unclear why running gaits of quadrupedal mammals vary between species.

The dimensionless moment of inertia, called Murphy number, is important to determine the stability of quadrupedal gaits [3–5]. In this research, we investigated the effect of Murphy number on different quadrupedal running gaits using a simple model in our previous research [6].

2 Model

We used a 2D model composed of a rigid body and two massless springs (Fig. 1). The springs represent the fore and hind legs (Legs F and H) and are connected to the body by smoothly rotating joints. X and Z are the horizontal and vertical positions, respectively, of the center of mass (CoM). θ is the angle of the body. M and J are the mass and moment of inertia around the CoM of the body, respectively. The distance between the leg joints is $2D$. The position of the CoM of the body is shifted forward by αD from the center of the leg roots as in our previous study [6]. K is the nominal length of the legs. When Leg $i\,(i = \mathrm{F},\mathrm{H})$ is in the air, its length remains L_0 and its angle also maintains the touchdown angle γ_i^{td}. For convenience, all of the following expressions are formulated in dimensionless quantities using M, D, and a time scale $\sqrt{D/g}$ (g is the gravitational constant).

The equations of motion are given by

$$\ddot{x} + \sum_{i=\mathrm{F,H}} f_i \sin\gamma_i = 0 \tag{1}$$

$$\ddot{z} - \sum_{i=\mathrm{F,H}} f_i \cos\gamma_i + 1 = 0 \tag{2}$$

$$\mu\ddot{\theta} + \sum_{i=\mathrm{F,H}} d_i f_i \cos(\gamma_i - \theta) = 0, \tag{3}$$

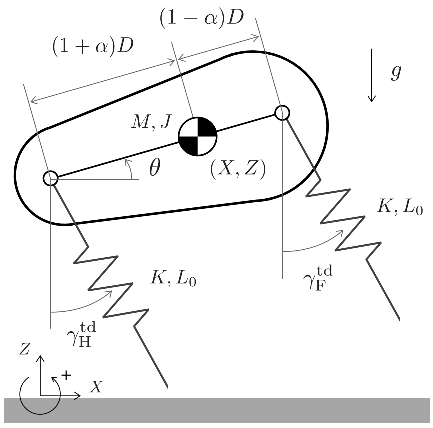

Figure 1: Simple model composed of a rigid body with center of mass offset and two massless springs.

where $x = X/D$, $z = Z/D$,

$$f_i = \begin{cases} 0 & \text{swing phase} \\ k(l_0 - l_i) & \text{stance phase} \end{cases} \qquad i = \mathrm{F},\mathrm{H},$$

$d_F = -(1-\alpha)$, $d_H = (1+\alpha)$, $\mu = J/(MD^2)$, $k = KD/(Mg)$, $l_0 = L_0/D$, and l_i and γ_i are the length and angle, respectively, of Leg i ($i = \mathrm{F},\mathrm{H}$), and from now on, $\dot{*}$ indicates the derivative of variable $*$ with respect to τ. The dimensionless moment of inertia μ corresponds to the Murphy number.

The conditions of touchdown ($r_i^{\mathrm{td}} = 0$) and liftoff ($r_i^{\mathrm{lo}} = 0$) of Leg i are given by

$$\begin{aligned} r_i^{\mathrm{td}} &= z + d_i \sin\theta - l_0 \cos\gamma_i^{\mathrm{td}} = 0 \\ r_i^{\mathrm{lo}} &= l_0 - l_i = 0 \end{aligned} \qquad i = \mathrm{F},\mathrm{H}, \tag{4}$$

where γ_i^{td} is the touchdown angle of Leg i ($i = \mathrm{F},\mathrm{H}$).

We defined the Poincaré section at the apex ($\dot{z} = 0$) and numerically searched for periodic solutions and classified them into four types of sequences corresponding to different quadrupedal running gaits (Seqs. A–D in Fig. 2). In particular, Seqs. A, B, and D correspond to cheetah's galloping, horse's galloping, and pronking, respectively.

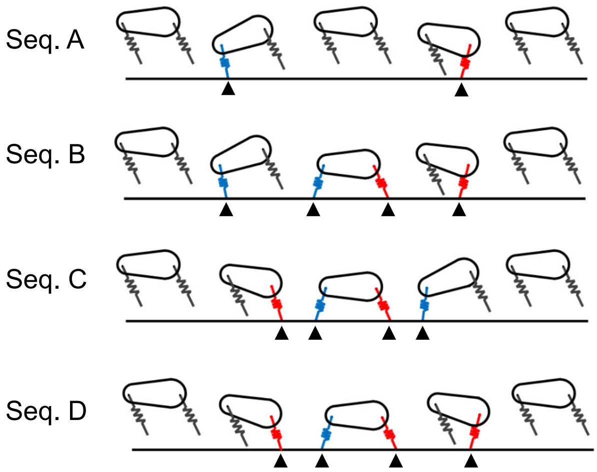

Figure 2: Four sequences of periodic solutions.

3 Result & discussion

We searched periodic solutions for various values of μ for $\alpha = 0.2$, $k = 6$, $l_0 = 2$, and the forward speed at the apex $\dot{x}_a = 4$. The obtained solutions are classified by the Murphy number μ and the angular velocity at the apex $\dot{\theta}_a$ (Fig. 3). Specifically, Seq. A was found in large $|\dot{\theta}_a|$ and stable in small μ. Seq. D was found in small $|\dot{\theta}_a|$ and stable in middle μ. Seq. B was stable over a wide range of μ.

The Murphy numbers of cheetahs and horses were estimated as 0.9 [7] and 1.5 [6], respectively. Seq. A, which corresponds to the galloping in cheetahs, has stable solutions around $\mu = 0.9$. In addition, Seq. B, which corresponds to the galloping in horses, has stable solutions around $\mu = 1.5$. At the poster presentation, we also would like to show the relationship between the Murphy number and stable gait for other quadrupedal mammals to clarify the effects of the Murphy number on the dynamics of high-speed running in quadrupedal mammals.

Acknowledgement

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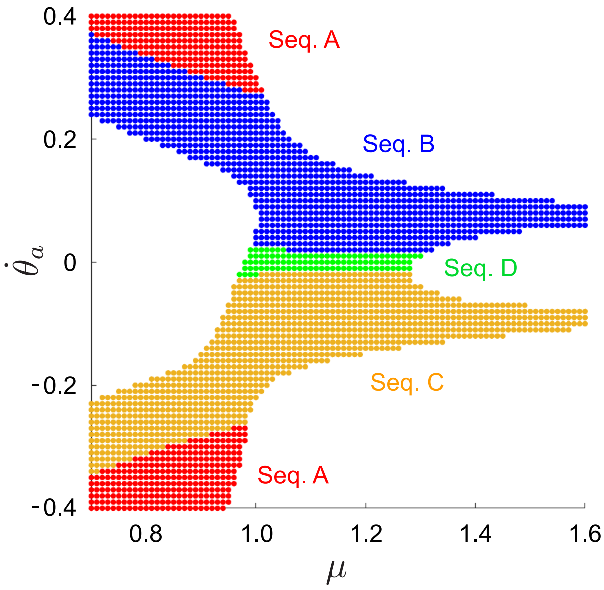

Figure 3: Gait classification of obtained stable periodic solutions for μ and $\dot{\theta}_a$.

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