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# Low-dimensional feedback control model that utilizes redundant degrees of freedom

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## 1 Introduction

When we reach our hand to a target position, the joint trajectories and muscle activities are not uniquely determined. This problem is called a redundancy problem, and many studies have proposed criteria under which the central nervous system (CNS) determines the trajectory. On the other hand, the uncontrolled manifold (UCM) hypothesis explains that the CNS performs movements allowing diverse solutions due to the redundancy [1]. For example, the combination of shoulder, elbow, and wrist joint angles that realize a specific hand position forms a manifold in the joint angle space. Based on the UCM hypothesis, reaching movements are represented not as point-to-point movements but as movements between manifolds that allow a variety of solutions in the joint angle space. We propose a lowdimensional feedback control model using a sandglass-type neural network and Hebbian learning rule, which realizes such movements between manifolds.

#### 2 Methods

#### 2.1 Control model

Fig. 1 shows the overview of our low-dimensional feedback control model. Here, we assume that the controlled object is an arm. The control model consists of a sandglasstype neural network (SNN) and feedback control neurons fully coupled to neurons in the middle (third) layer, and receives the sensory signals representing arm posture. If the SNN learns the identity map, neural activities of the middle layer represent compressed input signals as internal states, and the output layer outputs signals representing the same posture as the input signals. By controlling the activities of neurons in the middle layer by the control neurons that outputs error signals, the network outputs the arm posture at the next time step to reach to a target.

#### 2.2 Experimental Method

Simulation experiments were conducted to verify the performance of the proposed model. The controlled object was a three-joint arm. In this experiment, the dynamics of the arm was ignored: the output of the network was immediately reflected in the arm posture. The input/output of the neural network is the joint angle  $\theta_i$  (i = 1, 2, 3) and  $\theta'_i = 2\pi - \theta_i$  which represent the redundancy of sensory in-



Figure 1: Overview of low-dimensional feedback control model.

formation. The hand position (x, y) relative to the first (base) joint of the arm is expressed in a Cartesian coordinate system.

SNN was trained to learn the identity map using the back-propagation method with a randomly generated dataset of joint angles. Then, the coupling weights from control neurons to the neurons in the middle layer were learned by a modified version of Hebbian learning rule proposed by Klopf [2]:

$$\Delta w_{ij} = \alpha \left( -w_{ij} + c \Delta v_i \Delta u_j \right) \tag{1}$$

where  $w_{ij}$  is the weight from the *i*-th (i = 1, 2) control neuron to the *j*-th (j = 1, 2, 3) middle layer neuron,  $\Delta v_i$  and  $\Delta u_j$  show the changes in the output  $v_i$  of the *i*-th control neuron, and the output  $u_j$  of the *j*-th neuron in the middle layer, respectively.  $\alpha$  and *c* are constants. The output  $\mathbf{v} = (v_1, v_2)$  of the control neuron during reaching is the error  $\mathbf{v} = \mathbf{x} - \mathbf{x}^d$  of the current hand position  $\mathbf{x} = (x, y)$  to the desired position  $\mathbf{x}^d = (x^d, y^d)$ .

### 3 Results and Discussion

Fig. 2 shows the initial and final arm postures when the target positions are  $x^d = -4, -3, \dots, 5$  (red line). The blue

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**Figure 2:** Reaching results to targets  $x^d = -4, -3, \dots, 5$ .



Figure 3: Arm postures when a disturbance is given during reaching to a target  $y^d = 1$ .

and black lines show the initial and the final arm postures for successful reaching. In this simulation, successful reaching movements were achieved except  $x^d = 5$ .

The success rate of reaching to specified points from 50 randomly generated initial postures was examined. The success rate of the targets near the boundaries of the range of motion was low. However, the rate was higher when the range of motion of the joints was limited than the rate when no limitation. This suggests that the range of motion of joints in our body contributes to the improvement of control ability. On the other hand, successful reaching was observed to the area outside of the training data set when the range of motion was not limited, showing the generalization capability of the model.

Fig. 3 shows the posture change of the arm when a disturbance was given in the middle of reaching. The red dashed line is the target position  $(y^d = 1)$ , the blue and red lines are the initial and the final posture, respectively, the black lines are the posture during reaching, and the black dashed line is the hand trajectory. The hand trajectory was newly generated in response to the disturbance, and the hand reached to the target position. The model succeeded in reaching even when some of the joint angles were fixed. Such flexibility of motor control indicates that the model can generate trajectories to a target manifold utilizing redundant



Figure 4: Internal representation of hand position. The dark blue line represents the reaching trajectory.

degrees of freedom. The above results were obtained not only when the target position was given as a line but also when it was given as a point (x, y).

Fig. 4 shows a representation of the hand position x in the space of the outputs of the middle layer cells. Each colored manifold represents the combination of various joint angles for specific hand positions  $x = -4, -3, \dots, 5$ , respectively. Most of the manifolds for the x-coordinate (Fig. 4) and *y*-coordinate (not shown) were curved like an egg shape. Hence, the inner representation was highly non-linear to the hand position (x, y), and relatively close to a Cartesian coordinate system to the joint angles. The dark blue line represents the reaching trajectory from the initial posture to  $x^d = 0$  in Fig. 2. The hand moved along the approximately normal direction to manifolds representing x coordinate of the hand and reached to the target manifold. When reaching failed, the manifold representing the target position was small or not parallel to the manifold involving the start position.

Although the hand trajectory during reaching generated by this model resulted in a curved trajectory (Fig. 3), human hand trajectories are almost straight [3]. This curvature of the trajectory is attributed to the fact that the movement is controlled in a space similar to the joint angle space (Fig. 4). The hand trajectory would become straight if it is controlled in a space similar to the task space. Thus, the CNS may learn spatial representations that could facilitate task execution or improve performance.

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#### References

[1] J. P. Scholz, and G. Schöner, "The uncontrolled manifold concept: identifying control variables for a functional task," *Experimental Brain Research.*, Vol. 126, No. 3, pp. 289–306, 1999

[2] H. Klopf, "A neuronal model of classical conditioning," *Psychobiology*, Vol. 16, No. 2, pp. 85–125, 1988

[3] P. Morasso, "Spatial control of arm movements," *Experimental Brain Research.*, Vol. 42, No. 2, pp. 223–227, 1981