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VPP control cannot stabilize the posture during walking for high VPP location

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1 Introduction

In human walking, stabilization of the upright posture of the body is important. To stabilize the posture, a control method, which keeps the direction of the ground reaction force toward a certain point above the center of mass (CoM) called Virtual Pivot Point (VPP), has been proposed [1]. Because the dynamics with this control is similar to that of a roly-poly toy, we expect that this control can stabilize the posture when the VPP is above the CoM. However, the VPP location that guarantees the stability and dynamical mechanism of the stabilization by the VPP control remain unclear. In this study, we first investigated the postural stability using the previous model [1] by changing the VPP location and found that the VPP control cannot stabilize the posture during walking for high VPP location. Based on the obtained result, we reduced the model to clarify the stability and found that the dynamics of the reduced model is equivalent to Mathieu's equation, which explains the reason why the VPP control cannot stabilize the posture during walking for high VPP location.

2 Stability of the previous model

The previous model [1] consists of a rigid body and two massless spring legs in a sagittal plane (Fig. 1a). The VPP is fixed on the body axis above the CoM. θ is the angle of the body. M (= 80.0 kg) and J (= 5.0 kgm^2) are the mass and moment of inertia around the CoM, respectively, of the body. r (= 0.1 m) is the length between the hip and CoM. η is the length between the CoM and VPP. g (= 9.8 m/s^2) is the gravitational constant.

We first investigated the postural stability of the model during walking by changing η from 0.17 to 0.40 m. Specifically, we took the Poincaré section when the leg tip reached right under the hip and searched fixed points on the section to find periodic solutions. We determined the stability of the obtained periodic solutions based on the maximum magnitude $|\lambda_{\max}|$ of the eigenvalues of the Jacobian matrix of the Poincaré map around the fixed points (Fig. 2). As a result, we found that the periodic solutions are stable for $0.17 \le \eta \le 0.32$ m and unstable for $0.32 < \eta \le 0.40$ m. This implies that the VPP control cannot stabilize the posture during walking for high VPP location. Furthermore, the dominant unstable mode of the unstable periodic solutions is the angular velocity of the posture and its period is as twice as the gait cycle. To clarify this mechanism, we reduced the previous model by focusing on the body rotation as described below.

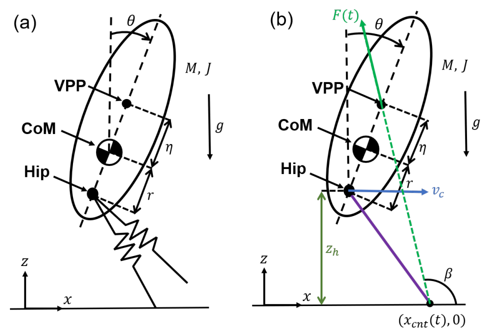

Figure 1: (a) Previous model (modified from [1]) and (b) Reduced model.

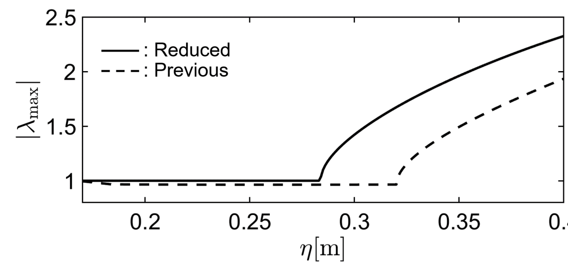

Figure 2: Maximum magnitude $|\lambda_{\max}|$ of previous and reduced models for VPP location η.

3 Reduction of the previous model

During the stable walking of the previous model, the body hardly moved vertically and moved forward at an almost constant speed. Therefore, we reduced the model so that the hip has a constant height z_h (= 0.934 m) and moves forward at a constant speed v_c (Fig. 1b). In the reduced model, we combined the ground reaction forces from the two legs using one virtual leg whose ground reaction force and leg tip position are $F(t)$ and $x_{cnt}(t)$, respectively. β is the angle of the ground reaction force with respect to the horizontal line.

The equation of motion of the reduced model is given by

$$(J + Mr^2)\ddot{\theta} = Mgr\sin\theta - (r+\eta)F(t)\cos(\beta - \theta) \qquad (1)$$

where

$$\beta = \arctan \frac{z_h + (r+\eta)\cos\theta}{v_c nt + (r+\eta)\sin\theta - x_{cnt}(t)} \tag{2}$$

$$F(t) = \frac{F_m}{2}\left(1 - \cos\frac{2\pi}{T}t\right) + F_0 \tag{3}$$

$$x_{cnt}(t) = \begin{cases} \left\{v_c - \frac{L}{t_2}\right\}t & (t_1 \le t \le t_2) \\ \left\{v_c + \frac{2L}{t_3 - t_2}\right\}t - \frac{nTL}{t_3 - t_2} & (t_2 \le t \le t_3) \\ \left\{v_c - \frac{L}{t_4 - t_3}\right\}t + \frac{nTL}{t_4 - t_3} & (t_3 \le t \le t_4) \end{cases} \tag{4}$$

$$t_1 = (n-1)T,\ t_2 = \frac{(1-d)nT}{2},\ t_3 = \frac{(1+d)nT}{2},\ t_4 = nT \tag{5}$$

T is the gait cycle duration, F_{m} and F_0 are the amplitude and offset, respectively, of the ground reaction force, d is the ratio of the double support phase per T, L is the stride length, n is the number of steps. $F(t)$ and $x_{cnt}(t)$ are determined based on the previous work [1].

4 Stability analysis

In the reduced model, we defined a Poincaré section in the same way as the previous model, which is given by $t = nT$, and investigated the periodic solutions and their stability by calculating $|\lambda_{\max}|$ for $0.17 \le \eta \le 0.4$ m, as shown in Fig. 2. The obtained solutions were stable for $0.17 \le \eta \le 0.283$ m and unstable for $0.283 < \eta \le 0.4$ m. The reduced model also had the upper bound of the VPP location as the previous model. Furthermore, the upper bound of the reduced model was close to that of the previous model.

We linearized (1) around $\theta = 0$ under the assumption that the stride length is small ($L \ll 1$) and obtained the following linearized equation:

$$\xi'' + (A + B\cos\tau)\,\xi = f(\tau) \tag{6}$$

where $\xi = (z_h + r + \eta)\theta/(r+\eta)$, $\tau = 2\pi t/T$, $f(\tau)$ is the periodic function with the period of 2π, A and B are determined by the parameters of the reduced model, such as η, and $*'$ indicate the derivative of variable $*$ with respect to τ. (6) is Mathieu's equation with a periodic force and its stability is identical to that of the following homogeneous equation [2]:

$$\xi'' + (A + B\cos\tau)\,\xi = 0 \tag{7}$$

This means that the stability of the reduced model is explained by that of Mathieu's equation (7) around the origin $[\xi,\ \xi']^T = [0,\ 0]^T$, which is determined by A and B [3]. We took a Poincaré section at $\tau = 2n\pi$ and compared the stability of the periodic solutions of the reduced model with the stability diagram of Mathieu's equation (7) for A and B by changing η, as shown in Fig. 3. We also compared $|\lambda_{\max}|$ and found that they have almost the same value, as shown in Fig. 4.

In Mathieu's equation, a parametric resonance occurs around $(A,\ B) \approx (0.25,\ 0)$, as shown in Fig. 2, and the perturbation around the origin grows exponentially with the period, twice as much as the period of the external force. The periodic solutions of the reduced model also became unstable with the period twice, as much as the gait cycle in the same way as the previous model. This means that the parametric resonance caused the instability of not only the reduced model but also the previous model, which generated the upper bound of the VPP location.

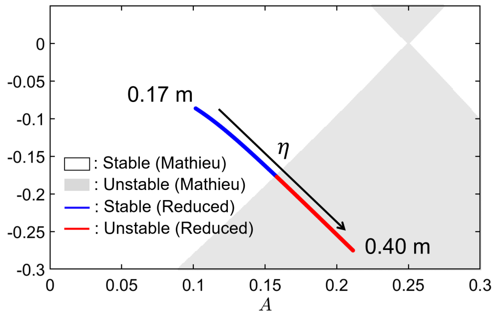

Figure 3: Comparison of stability between reduced model and Mathieu's equation.

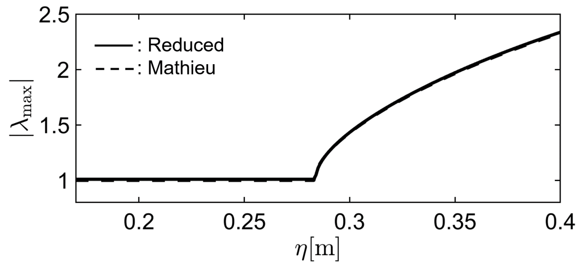

Figure 4: Maximum magnitude $|\lambda_{\max}|$ of reduced model and Mathieu's equation for VPP location η.

5 Conclusion

In this study, we found that the VPP control cannot stabilize the posture during walking for high VPP location and that the stability of Mathieu's equation explains the existence of the upper bound. In the future, we would like to investigate the reason of the upper bound based on the characteristic of Mathieu's equation.

Acknowledgement

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