Title	Spatially distributed force measurement for small terrestrial animals
Author(s)	Guan, Yue; Venkadesan, Madhusudhan
Citation	The 11th International Symposium on Adaptive Motion of Animals and Machines (AMAM2023). 2023, p. 192-193
Version Type	VoR
URL	https://doi.org/10.18910/92333
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Spatially distributed force measurement for small terrestrial animals

Yue Guan[1,2], Madhusudhan Venkadesan[2]
[1]Department of Mechanical Engineering, University of Memphis, USA
yguan1@memphis.edu
[2]Department of Mechanical Engineering and Material Science, Yale University, USA
m.venkadesan@yale.edu

1 Introduction

We present here a sensor that is constructed using off-the-shelf components and can measure all three components of the surface traction that is of the order of 10 Pa (e.g., 0.1 g over 1 cm^2), with millimeter spatial and millisecond temporal resolution. This paper is concerned with the measurement of all three components of spatially distributed surface forces (called traction), such as would be needed to study and understand the locomotion biomechanics of terrestrial animals. Although sensors such as pressure mats are routinely used in human biomechanics, we presently lack the technology to measure pressure distributions of lighter animals of the order of 10–100 g. In addition to the sensor, we present a rational design methodology to adapt the sensor for specific size and force scales.

Although pressure mats are not sensitive enough for light animals, such as mudskippers, multiple load cells may be used like in [6]. But that significantly limits the spatial resolution. Alternatively, sensors have been developed based on photoelasticity, which couples optical properties such as transmissivity or birefringence with elastic strains so that the tractions may be inferred by imaging the substrate's strain field [1, 2, 3]. However, photoelasticity has not found widespread adoption on account of challenging fabrication problems and additional issues because the animal often occludes light transmission in precisely the regions where force needs to be measured. Traction force microscopy (TFM) is another promising technology that measures elastic strains in the substrate using scanning microscopy [4]. It is widely adopted in cellular-scale measurements but is not suitable for animals where 1–100 ms temporal resolution is important. Our technique is inspired by these past approaches. Instead of bulk strain fields, we image the deformation of a thin elastic substrate and back-calculate the applied tractions by inverting an appropriate asymptotic version of the elasticity equations (Figure 1).

2 Theoretical foundation

Using a tensed thin sheet as the substrate, which behaves like a membrane rather than an elastic shell, the normal traction component q is directly proportional to the deformed curvature when pre-stress dominates the behavior [5],

$$q\,(x,y) \approx -T \cdot \nabla^2 w \approx -T\,(w_{,xx} + w_{,yy}) \tag{1}$$

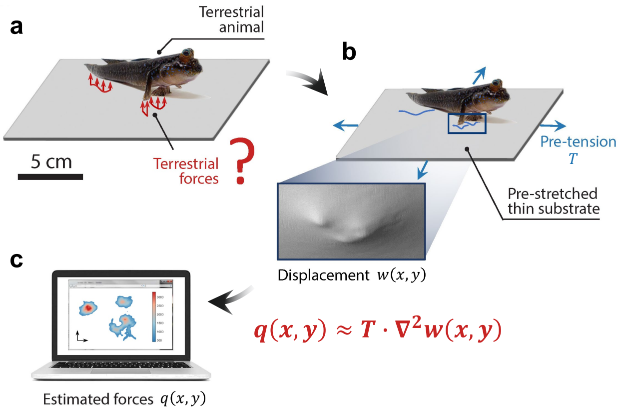

Figure 1: Proposed force measurement technique. **a**, The approach aims at measuring the in-vivo force distribution $q(x,y)$ on terrestrial small animals. **b**, A pre-stretched thin substrate deforms to yield normal displacements $w(x,y)$. **c**, The normal forces $q(x,y)$ are estimated using the pre-tension T and measured $w(x,y)$.

where q is the normal traction, x, y are in-plane coordinates, T is the pre-tension, w is the normal membrane deflection, ∇^2 denotes the second covariant derivative, and the portion after the comma denotes partial derivatives.

The tangential tractions p_x and p_y can also be inferred from the measured deformation using membrane elasticity:

$$\begin{aligned} p_x\,(x,y) \approx &-(C_1+C_2)\,u_{x,xx} - C_1 u_{x,yy} + C_2 u_{y,xy} \\ &-(C_1+C_2)\,w_{,x}w_{,xx} - C_1 w_{,x}w_{,yy} - C_2 w_{,y}w_{,xy}, \end{aligned} \tag{2}$$

$$\begin{aligned} p_y\,(x,y) \approx &-(C_1+C_2)\,u_{y,yy} - C_1 u_{y,xx} - C_2 u_{x,xy} \\ &-(C_1+C_2)\,w_{,y}w_{,yy} - C_1 w_{,y}w_{,xx} - C_2 w_{,x}w_{,xy}, \end{aligned} \tag{3}$$

where u_x and u_y are in-plane deformation components. C_1 and C_2 are material properties. The three constants, T, C_1, and C_2, could either be specified by material and tension choice or estimated through a calibration using known loads.

3 Numerical validation using truth tests

We assessed the effectiveness of our approach using simulation studies. In a truth test (Figure 2), a circular elastic sheet under tension was loaded on a region resembling a curved fin to simulate normal and shear components ob-

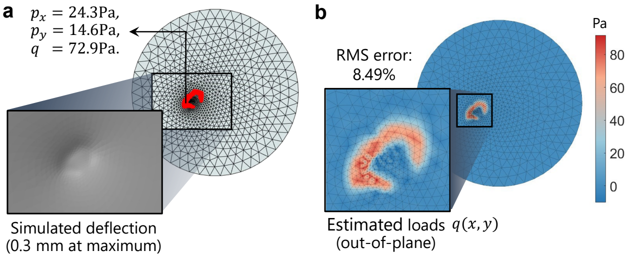

Figure 2: Numerical validation. a, Millinewton level forces, p_x, p_y, and q, were uniformly applied in an irregular 'C' shaped region on a pre-stretched PTFE sheet, causing a 0.3 mm indentation. **b**, The proposed technique accurately identified the loading area and magnitudes.

served in mudskipper fins during locomotion [6]. The deflection profile was simulated using Finite Element Analysis (Abaqus, S3R elements) and perturbed with 5% synthetic uniform noise to capture imaging uncertainty. Force estimation employed Eqs. (1) to (3) and numerical differentiation to approximate displacement derivatives. The contact region was accurately found and the RMS estimation error was always less than 10%.

4 Design criteria and boundary effects

There are two aspects to the design. The first is to factor in operational requirements for the sensor such as sensitivity, material strength, and so on. The second is to identify criteria where the boundary effects may be neglected. The design criteria are (Figure 3): (**1**) the pre-tension T should be below a threshold to not tear the material, (**2**) the elastic wave should be much faster than the fastest animal movement, (**3**) the membrane's elastic stress should be negligible compared to the pre-tension induced stress, (**4**) the deflection due to the lightest detectable point-load $P_{\min}$ should exceed the imaging capabilities $w_{\min}$, (**5**) the spatial resolution should exceed $d_{\min}$ for discriminating two point-loads $P_{\min}$, and (**6**) the maximum deflection under a specified load $P_{\max}$ should be below some threshold $w_{\max}$ to not affect the animal's behavior. For a membrane of radius R, thickness h, and made of material with Young's modulus E and Poisson's ratio ν, the three dimensionless design parameters are the membrane radius $\hat{R} = R/h$, thickness $\hat{h} = h/\frac{12(1-\nu^2)T}{E}$, and pre-tension $\hat{T} = T/\frac{P_{\min}}{2\pi w_{\min}}$. The boundary effects are defined in terms of the length scale $\ell = \sqrt{D/T}$ for pre-tension T, and membrane bending stiffness $D = Eh^3/12\left(1-\nu^2\right)$. The accuracy is above 95% if the measurement region is at least 3ℓ away from the boundary. For our chosen design (Figure 3), $\ell \approx 0.3$ mm, for a sensor that is 330 mm in diameter.

5 Experimental realization

We built a custom optical force apparatus to measure the spatial force distribution with millimeter spatial resolution, millisecond time resolution, and millinewton point-load detection ability by applying the design methodology. The experimental realization uses a store bought PTFE membrane (star in Figure 3), a supporting frame for pre-tension control, and a non-contact 3D Digital Image Correlation system (Correlated Solutions, Inc.) with two high-speed cameras (AX100, Photron, Inc.) to capture the dynamic 3D deformation profile of the membrane during animal locomotion.

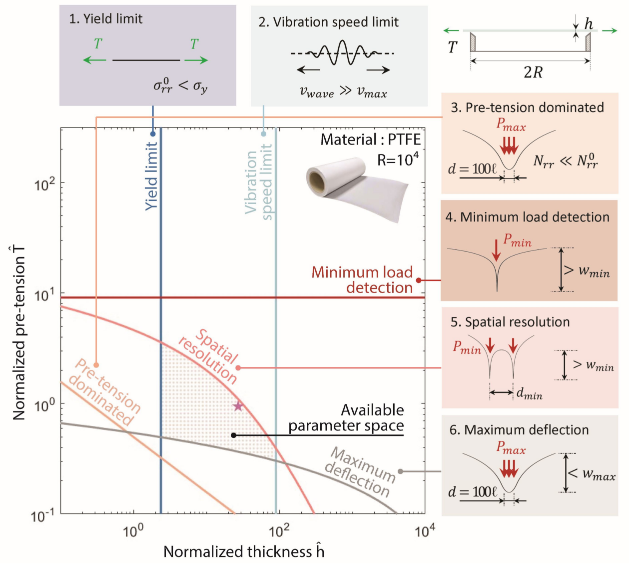

Figure 3: Design criteria for user needs. Six criteria define the available normalized parameter space of thickness ($\hat{h}$) and pre-tension ($\hat{T}$) as the membrane radius exceeds 10^4. Insets, analytical models for each design criterion.

A 25.4-micrometers-thick PTFE shim stock (McMaster-Carr: catalog #1192N35) fits the design criteria for a membrane that is 330 mm in diameter. When pre-stretched at 15 N/m, the setup satisfies all the design criteria, with mudskipper locomotion in mind, and achieves sub-millimeter spatial resolution with millisecond time capability while still satisfying the quasi-static assumption of the equations.

Although our specific realization targets mudskippers, the proposed force apparatus applied more broadly through the use of the design tool that we have developed.

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