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EQUIVARIANT ALGEBRAIC VECTOR BUNDLES OVER ADJOINT REPRESENTATIONS

Dedicated to Professor Seiya Sasao on his 60th birthday

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0. Introduction

Let G be a reductive complex algebraic group and let B, F be G -modules over C . Let $\text{Vec}_G(B, F)$ denote the set of complex algebraic G -vector bundles over B whose fiber at $0 \in B$ is F , and let $\text{VEC}_G(B, F)$ denote the set of the G -isomorphism classes in $\text{Vec}_G(B, F)$. The set $\text{VEC}_G(B, F)$ has the trivial class represented by the product bundle $B \times F \rightarrow B$.

The solution of the Serre conjecture by QUILLEN [9] and SUSLIN [11] says that $\text{VEC}_G(B, F)$ is trivial for any B and F when G is trivial. In contrast to this SCHWARZ [10] discovered that $\text{VEC}_G(B, F)$ is nontrivial for some B and F when G belongs to a class of noncommutative groups that includes all classical groups (see also [5]); this depends upon an analysis of $\text{VEC}_G(B, F)$ when the ring $\mathcal{O}(B)^G$ of invariants on B is a polynomial ring in one variable. Subsequently KNOP [6] used the result of Schwarz for $G = \text{SL}_2$ to show that $\text{VEC}_G(\mathfrak{g}, F)$ is nontrivial for many irreducible G -modules F if G is connected and noncommutative, where \mathfrak{g} denotes the adjoint representation of G . Note that $\mathcal{O}(\mathfrak{g})^G$ is a polynomial ring in n variables where n is the rank of G . We refer the reader to [7] and [8] for further results, where $\text{VEC}_G(B, F)$ is studied from a different point of view.

In this paper we closely look at the result of Schwarz on the SL_2 case together with the argument of Knop to prove

Theorem A. *If G is semisimple, then $\text{VEC}_G(\mathfrak{g}, F)$ is nontrivial for all but finitely many isomorphism classes of irreducible G -modules F .*

REMARK. If G is commutative, then $\text{VEC}_G(\mathfrak{g}, F)$ is trivial for any G -module F because the action of G on \mathfrak{g} is trivial ([3, §2]).

Theorem A is a corollary of Theorem B stated below. Let R_n be the

SL_2 -module of homogeneous polynomials of degree n in two variables. According to [10] $VEC_{SL_2}(R_2, R_m)$ forms an abelian group isomorphic to C^p where $p = [(m - 1)^2/4]$. Suppose G is connected and noncommutative. We fix a system Σ of simple roots of G . Associated to a simple root $\alpha \in \Sigma$, Knop defined a map

$$\Phi^\alpha : VEC_G(\mathfrak{g}, F) \rightarrow VEC_{SL_2}(R_2, R_m)$$

where $m = \langle \chi, \alpha \rangle$ and χ is the highest weight of the irreducible G -module F . He proved that Φ^α is surjective if χ is regular, i.e. unless χ is contained in a reflecting hyperplane P_β for some $\beta \in \Sigma$.

DEFINITION. We call the α -string $(\chi, \chi - \alpha, \dots, \chi - m\alpha)$ of χ *singular* if it is contained in some P_β and *regular* otherwise.

Clearly if χ is regular, then the α -string of χ is regular for any $\alpha \in \Sigma$. But an α -string happens to be regular even if χ is singular, e.g. if G is semisimple and of rank two, then any dominant weight has a regular α -string. Hence the following theorem extends the result of Knop mentioned above.

Theorem B. *Suppose G is connected and noncommutative. Then*

- (1) Φ^α is surjective if the α -string of χ is regular,
- (2) the image of Φ^α contains a subspace of dimension $[m/2]([m/2] - 1)/2$ if the α -string of χ is singular.

Theorem B implies that $VEC_G(\mathfrak{g}, F)$ is nontrivial provided $m \geq 4$. If G is semisimple, then there are only finitely many irreducible G -modules F such that $\langle \chi, \alpha \rangle \leq 3$ for all $\alpha \in \Sigma$. Therefore Theorem A follows from Theorem B.

1. The SL_2 case

In this section we translate the result of Schwarz on the SL_2 case into an explicit form. Let $G = SL_2$ and T be its maximal torus consisting of diagonal matrices. Remember that R_n is the G -module of homogeneous polynomials of degree n in two variables, say x and y . Since the G -orbit of $R_2^T = \{bxy \mid b \in C\}$ is dense in R_2 , the inclusion map $i : R_2^T \rightarrow R_2$ induces an injective homomorphism

$$i^* : \text{Mor}(R_2, \text{End}(R_m))^G \rightarrow \text{Mor}(R_2^T, (\text{End}(R_m))^T)^W$$

where W denotes the Weyl group $N_G(T)/T$, which is of order two. Note that the one dimensional subspaces of R_m spanned by $x^{m-n}y^n$ are mutually non-isomorphic T -modules.

Lemma 1.1. *Any element $\sigma \in \text{Mor}(R_2^T, (\text{End}(R_m))^T)^W$ is of the form*

$$(\sigma(bxy))(x^{m-n}y^n) = f_n(b)x^{m-n}y^n$$

with polynomials $f_n(b)$ such that $f_n(-b) = f_{m-n}(b)$ for $n = 0, 1, \dots, m$.

Proof. It follows from Schur's lemma that σ is of the form

$$(\sigma(bxy))(x^{m-n}y^n) = f_n(b)x^{m-n}y^n$$

with polynomials $f_n(b)$ for any n . The element of G mapping x to y and y to $-x$ is a representative of the nontrivial element of W . It acts on R_2^T as multiplication by -1 and on $(\text{End}(R_m))^T$ by conjugation of the element of $\text{End}(R_m)$ mapping $x^{m-n}y^n$ to $(-1)^n x^n y^{m-n}$. Hence it follows from the equivariance with respect to the action of W that $f_n(-b) = f_{m-n}(b)$. This proves the lemma. \square

It is well-known (and easy to prove) that $\mathcal{O}(R_2)^G$ is a polynomial ring $C[\Delta]$ where Δ is the discriminant defined by $\Delta(ax^2 + bxy + cy^2) = b^2 - 4ac$. We note that $\text{Mor}(R_2, \text{End}(R_m))^G$ is an algebra over $\mathcal{O}(R_2)^G = C[\Delta]$. The following lemma describes the algebra structure.

Lemma 1.2. $\text{Mor}(R_2, \text{End}(R_m))^G = (C[\Delta][\gamma] / \prod_{n=0}^m (\gamma - (m-2n)\sqrt{\Delta}))$ where γ is homogeneous of degree one with respect to the coordinates of R_2 and expressed on R_2^T as

$$(*) \quad (\gamma(bxy))(x^{m-n}y^n) = (m-2n)bx^{m-n}y^n.$$

REMARK. Since $(\gamma - (m-2k)\sqrt{\Delta})(\gamma - (m-2(m-k))\sqrt{\Delta}) = \gamma^2 - (m-2k)^2\Delta$, the product $\prod_{n=0}^m (\gamma - (m-2n)\sqrt{\Delta})$ is actually a polynomial of γ and Δ .

Proof. This may be known, but for the sake of completeness we shall give the proof.

First we claim that $\text{Mor}(R_2, \text{End}(R_m))^G$ is free and of rank $m+1$ as a $C[\Delta]$ -module; more precisely, the degrees of the generators are $0, 1, 2, \dots, m$. This can be seen as follows. By the self-duality of R_m and the Clebsch-Gordan formula ([4, p. 170]) we have

$$\text{End}(R_m) \cong R_m \otimes R_m \cong \bigoplus_{k=0}^m R_{2k}.$$

Hence $\text{Mor}(R_2, \text{End}(R_m))^G \cong \bigoplus_{k=0}^m \text{Mor}(R_2, R_{2k})^G$. Here it is easy to see that $\text{Mor}(R_2, R_{2k})^G$ is free and of rank one as a $C[\Delta]$ -module, in fact, the generator is given by the k th power map. This implies the claim.

Suppose $\gamma \in \text{Mor}(R_2, \text{End}(R_m))^G$ is homogeneous and of degree one. Then it follows from Lemma 1.1 that

$$(\gamma(bxy))(x^{m-n}y^n) = c_n bx^{m-n}y^n$$

with constants c_n such that $c_n = -c_{m-n}$. Let $g \in G$ be the unipotent matrix with 1 in the upper right hand corner. Since $gx = x$ and $gy = x + y$ (hence $g(xy) = x^2$

+ xy), it follows from equivariance that

$$\gamma(bx^2 + bxy) = g\gamma(bxy)g^{-1}.$$

We view elements in $\text{End}(R_m)$ as matrices by taking a basis $\{x^m, x^{m-1}y, \dots, y^m\}$ of R_m . Since γ is homogeneous and of degree one, the entries of the matrix $\gamma(ax^2 + bxy + cy^2)$ are linear combinations of a, b and c . The equivariance of γ with respect to the action of T implies that the (i, j) entries of $\gamma(ax^2 + bxy + cy^2)$ vanish whenever $|i - j| \geq 2$. (In fact, the diagonal entries are scalar multiples of b , the $(i, i + 1)$ entries are those of a , and the $(i + 1, i)$ entries are those of c .) In particular, the $(1, j)$ entries of $\gamma(bx^2 + bxy)$ are zero for $j \geq 3$. The vanishing of the $(1, j)$ entries ($3 \leq j \leq m + 1$) of the matrix at the right hand side of the identity above yields $m - 1$ equations among the constants c_n . An elementary computation shows that

$$c_n = (1 - n)c_0 + nc_1.$$

This together with the relation $c_n = -c_{m-n}$ shows $c_n = (m - 2n)c_0/m$. The identities (*) are then obtained by setting $c_0 = m$.

The identities (*) imply that $\gamma^j (0 \leq j \leq m)$ are linearly independent over $C[\mathcal{A}]$ when restricted to R_2^T . Since the G -orbit of R_2^T is dense in R_2 , the $\gamma^j (0 \leq j \leq m)$ are linearly independent over $C[\mathcal{A}]$ as elements of $\text{Mor}(R_2, \text{End}(R_m))^G$. Moreover the identities (*) show that the element $\prod_{n=0}^m (\gamma - (m - 2n)\sqrt{\mathcal{A}})$ is zero when restricted to R_2^T , and hence zero actually as an element of $\text{Mor}(R_2, \text{End}(R_m))^G$. As claimed above $\text{Mor}(R_2, \text{End}(R_m))^G$ is free and of rank $m + 1$ as a $C[\mathcal{A}]$ -module. This shows that the identity $\prod_{n=0}^m (\gamma - (m - 2n)\sqrt{\mathcal{A}}) = 0$ is the only relation in $\text{Mor}(R_2, \text{End}(R_m))^G$. This completes the proof. \square

Denote by M_k^m (resp., N_k^m) the linear space consisting of homogeneous elements of degree k in $i^* \text{Mor}(R_2, \text{End}(R_m))^G$ (resp., $\text{Mor}(R_2^T, (\text{End}(R_m))^T)^W$) and set $M^m = \prod_{k \geq 1} M_k^m$, $N^m = \prod_{k \geq 1} N_k^m$. An elementary calculation together with Lemmas 1.1 and 1.2 shows that

$$\dim N_k^m = \begin{cases} (m + 1)/2, & \text{if } m \text{ is odd} \\ (m + 1 + (-1)^k)/2, & \text{if } m \text{ is even,} \end{cases}$$

$$\dim M_k^m = \begin{cases} [k/2] + 1, & \text{if } k \leq m - 2 \\ \dim N_k^m, & \text{if } k \geq m - 1, \end{cases}$$

and

$$\dim N^m/M^m = [(m - 1)^2/4].$$

Remember that $\mathcal{A} : R_2 \rightarrow C$ is an invariant polynomial. It is known that any element of $\text{Vec}_{\text{SL}_2}(R_2, R_m)$ is trivial over $\mathcal{A}^{-1}(C - (0))$ ([5, VII.2.6]). Moreover, given $E \in \text{Vec}_{\text{SL}_2}(R_2, R_m)$, there is a finite subset S of $C - (0)$ such that E is trivial

over $\mathcal{A}^{-1}(C - S)$ ([3, 6.2]). Hence one can find a transition function ϕ_E of E in $\text{Mor}(\mathcal{A}^{-1}(C - (S \cup (0))), \text{End}(R_m))^G$. The choice of ϕ_E is not unique and one can always arrange ϕ_E such that the restriction $\phi_E|_{R_2^T}$ is defined at 0 with value the identity. By the T -equivariance $\phi_E|_{R_2^T}$ is a diagonal matrix with rational functions as entries with respect to the basis $\{x^m, x^{m-1}y, \dots, y^m\}$ of R_m . We expand those rational functions into formal power series of the coordinate b of R_2^T . This gives a correspondence

$$\phi : \text{Vec}_{\text{SL}_2}(R_2, R_m) \rightarrow 1 + N^m$$

defined by $\phi(E) = \phi_E|_{R_2^T}$. A general result of Schwarz [10] or Kraft-Schwarz [5, VII.3.4] applied to the SL_2 case implies

Theorem 1.3 ([5], [10]). *The map ϕ induces a bijection*

$$\Psi : \text{VEC}_{\text{SL}_2}(R_2, R_m) \cong 1 + N^m/M^m.$$

2. The map Φ^α

In this section G is connected and noncommutative. We recall the definition of the map $\Phi^\alpha : \text{VEC}_G(\mathfrak{g}, F) \rightarrow \text{VEC}_{\text{SL}_2}(R_2, R_m)$ mentioned in the introduction. Let T be a maximal torus of G . Denote the Lie algebra of T by \mathfrak{t} . Let L be the subgroup of G generated by T and the root groups U_α and $U_{-\alpha}$ (see [2, 26.3]). Let L' be the commutator subgroup of L and Z be the identity component of the center of L . Then L' is isomorphic to SL_2 or SO_3 , the subgroup Z is a codimension one torus in T and $L = ZL'$. We choose and fix an element $\xi_0 \in \mathfrak{t}$ whose centralizer is exactly L . This is equivalent to saying that $\xi_0 \in P_\alpha$ but $\xi_0 \notin P_\beta$ for any $\beta \neq \alpha \in \Sigma$. Denote by \mathfrak{a} the affine space $\xi_0 + \text{Lie } L' \subset \mathfrak{g}$, which is L invariant. The action of Z on \mathfrak{a} is trivial and \mathfrak{a} is isomorphic to R_2 as L' -varieties because $\text{Lie } L'$ and R_2 are isomorphic representations.

Given $E \in \text{Vec}_G(\mathfrak{g}, F)$, we restrict it to \mathfrak{a} . Since \mathfrak{a} is fixed under the action of Z , the restricted bundle $E|_{\mathfrak{a}}$ decomposes into eigenbundles according to the weights of F viewed as a Z -module. Let $(E|_{\mathfrak{a}})^\chi$ denote the eigenbundle of $E|_{\mathfrak{a}}$ corresponding to the highest weight χ restricted to Z . Since Z commutes with L' , $(E|_{\mathfrak{a}})^\chi$ is an L' -vector bundle. The correspondence $E \rightarrow (E|_{\mathfrak{a}})^\chi$ induces the desired map Φ^α .

3. Proof of Theorem B

Let $\mathcal{A} \in C[\mathfrak{g}]^G$ be the discriminant and put $\mathfrak{g}_0 = \mathcal{A}^{-1}(C - (0))$. For a finite subset $S \subset C - (0)$ we set $\mathfrak{g}_S = \mathcal{A}^{-1}(C - S)$. Similarly we set $\mathfrak{t}_0 = \mathfrak{t} \cap \mathfrak{g}_0$, $\mathfrak{t}_S = \mathfrak{t} \cap \mathfrak{g}_S$. Since \mathfrak{g}_0 is the set of regular semisimple elements, we have

$$(3.1) \quad \mathfrak{g}_0 = G \times^{N_G(T)} \mathfrak{t}_0.$$

We construct a G -vector bundle over \mathfrak{g} by glueing the product G -vector

bundles $\mathfrak{g}_0 \times F \rightarrow \mathfrak{g}_0$ and $\mathfrak{g}_S \times F \rightarrow \mathfrak{g}_S$ over $\mathfrak{g}_{S_0} = \mathfrak{g}_0 \cap \mathfrak{g}_S$ using a transition function, where $S_0 = S \cup (0)$. The transition function is a G -equivariant morphism

$$\varphi : \mathfrak{g}_{S_0} \rightarrow \text{GL}(F)$$

where G acts on $\text{GL}(F)$ by conjugation. It follows from (3.1) that the restriction map

$$\text{Mor}(\mathfrak{g}_{S_0}, \text{GL}(F))^G \rightarrow \text{Mor}(t_{S_0}, \text{GL}(F)^T)^W$$

is bijective, where W is the Weyl group $N_G(T)/T$. Thus we are led to study W -equivariant morphisms from t_{S_0} to $\text{GL}(F)^T$.

Decompose

$$F = \bigoplus_{\eta \in \chi(T)} M(\eta) \quad \text{as } T\text{-modules}$$

where $\chi(T)$ denotes the set of characters of T and $M(\eta)$ is a (not necessarily one dimensional) T -module with character η . It follows from Schur's lemma that

$$\text{GL}(F)^T = \prod_{\eta \in \chi(T)} \text{GL}(M(\eta)).$$

Hence an element of $\text{Mor}(t_{S_0}, \text{GL}(F)^T)^W$ is given by a family of morphisms

$$\varphi_\eta : t_{S_0} \rightarrow \text{GL}(M(\eta))$$

satisfying

$$(3.2) \quad \varphi_{w\eta}(\xi) = \bar{w} \circ \varphi_\eta(w^{-1}\xi) \circ \bar{w}^{-1} \quad \text{for all } w \in W \text{ and } \xi \in t_{S_0}$$

where $\bar{w} \in N_G(T)$ is a representative of w . The action of \bar{w} induces an isomorphism from $M(\eta)$ to $M(w\eta)$ as T -modules.

We define $\varphi_\eta = 1$ unless η is in the W -orbit of the α -string of χ , i.e. unless $\eta = w(\chi - n\alpha)$ for some $w \in W$ and $0 \leq n \leq m$. If η is in the W -orbit of the α -string of χ , then $\dim M(\eta) = 1$ ([1, p. 125, Exercise 1]); so $\text{GL}(M(\eta)) = \mathbb{C}^*$. Hence φ_η is a rational function on \mathfrak{t} which has neither zero nor a pole on t_{S_0} . Moreover in this case (3.2) reduces to

$$(3.3) \quad \varphi_{w\eta}(\xi) = \varphi_\eta(w^{-1}\xi) \quad \text{for all } w \in W \text{ and } \xi \in t_{S_0}.$$

In order to choose a family $\{\varphi_\eta\}$ which satisfies (3.3), It suffices to choose a subfamily $\{\varphi_\eta \mid \eta \text{ is in the } \alpha\text{-string of } \chi\}$ which satisfies (3.3) whenever η and $w\eta$ are in the α -string of χ . We note that the reflection s_α relative to the reflecting hyperplane P_α reflects the α -string of χ , i.e. $s_\alpha(\chi - n\alpha) = \chi - (m-n)\alpha$ for any n

Lemma 3.4. (1) *If $w(\chi - k\alpha) = \chi - l\alpha$ for some $0 \leq k, l \leq m$, then $k = l$ or $k = m - l$.*

(2) *If $w(\chi - k\alpha) = \chi - k\alpha$ and $\chi - k\alpha$ is regular, then w is the identity.*

Proof. (1) First we recall the following general fact ([1, 10.3]). Let λ, μ be elements in the closure \bar{C} of the Weyl chamber relative to the simple root system Σ . If $w\lambda = \mu$ for some $w \in W$, then $\lambda = \mu$.

Suppose $\chi - k\alpha$ and $\chi - l\alpha$ are both in \bar{C} . Then it follows from the above fact that $k = l$. Suppose $\chi - k\alpha$ is in \bar{C} but $\chi - l\alpha$ is not in \bar{C} . Then $s_\alpha(\chi - l\alpha) = \chi - (m - l)\alpha$ is in \bar{C} . Since $s_\alpha w(\chi - k\alpha) = \chi - (m - l)\alpha$, we are in the same situation as above, hence $k = m - l$. The remaining two cases can be treated in the same way.

(2) The isotropy subgroup of W at a regular element in \mathfrak{t} is trivial ([1, 10.3]). This implies (2). \square

We denote $\varphi_{\chi - n\alpha}$ by φ_n . We shall find a family $\{\varphi_n(0 \leq n \leq m)\}$ satisfying (3.3). Let δ be the product of positive roots. It is well known that

$$(3.5) \quad \delta(s_\beta \xi) = -\delta(\xi) \quad \text{for any } \beta \in \Sigma$$

([1, 10.2]). We take a family of polynomials $\{p_n(0 \leq n \leq m) | p_n(0) = 1\}$ in one variable such that

$$(3.6) \quad p_0 \equiv p_m \equiv 1 \quad \text{and} \quad p_n(-\delta) = p_{m-n}(\delta) \quad \text{for any } n$$

and define

$$(3.7) \quad \varphi_n(\xi) = p_n(\delta(\xi)).$$

Suppose the α -string of χ is regular. Since $\varphi_0 \equiv \varphi_m \equiv 1$, it follows from (3.3) and Lemma 3.4 that the identity $\varphi_{m-n}(\xi) = \varphi_n(s_\alpha \xi)$ for each n is the only condition which the family $\{\varphi_n\}$ must satisfy. But it is satisfied by (3.5), (3.6) and (3.7).

Suppose the α -string of χ is singular. Then we require one more condition on the family $\{p_n\}$ that they be all even functions. Since $\delta(w\xi)^2 = \delta(\xi)^2$ for any $w \in W$ by (3.5), it follows from Lemma 3.4, (3.6) and (3.7) that (3.3) is satisfied.

Let $[E_p]$ denote the isomorphism class of the G -vector bundle $E_p \in \text{Vec}_G(\mathfrak{g}, F)$ defined by a family of polynomials $\{p_n\}$ satisfying the conditions mentioned above. We shall observe $\Phi^\alpha([E_p])$. As discussed in §1 elements in $\text{VEC}_{\text{SL}_2}(\mathbb{R}_2, \mathbb{R}_m)$ are detected by their transition functions restricted to \mathbb{R}_2^T . By definition $\Phi^\alpha([E_p]) = [(E_p|_a)^x]$ and a is the affine space $\xi_0 + \text{Lie } L'$ which is isomorphic to \mathbb{R}_2 as L' -varieties. Then \mathbb{R}_2^T corresponds to $t \cap a = \{\xi_0 + bh_\alpha | b \in \mathbb{C}\}$ where $h_\alpha \in t \cap \text{Lie } L'$ with $\alpha(h_\alpha) = 1$. Thus $\Phi^\alpha([E_p])$ corresponds to the family $\{p_n(\delta(\xi_0 + bh_\alpha))\}$ through the map Ψ in Theorem 1.3. Remember that ξ_0 is chosen in such a way that $\xi_0 \in P_\alpha$ but $\xi_0 \notin P_\beta$ for any $\beta \neq \alpha \in \Sigma$. Since δ is the product of positive roots, $\delta(\xi_0 + bh_\alpha)$ is a polynomial of b with zero constant term and nonzero degree one term.

In case the α -string of χ is regular, the condition we imposed on $\{p_n\}$ is only (3.6). Then it is not difficult to see that the composition $\Psi \circ \Phi^\alpha$ is surjective, hence Φ^α is surjective as Ψ is bijective.

In case the α -string of χ is singular, the conditions we imposed on $\{p_n\}$ are (3.6) and that p_n are even functions. Then it is also not difficult to see that $\Psi \circ \Phi^\alpha$ contains the image of even degree elements of $1 + N^m$ in $1 + N^m/M^m$. An elemen-

tary calculation shows that the image is a subspace of dimension $\lfloor m/2 \rfloor (\lfloor m/2 \rfloor - 1)/2$. This completes the proof of Theorem B.

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