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Yasuo NAKAYAMA

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The aim of this paper is to propose a formal theory for pictorial representations. In our daily life we talk about points, lines, and areas in a map. To deal with these objects, it is convenient to use a mereological language; this language is called *Natural Representation Language* (NRL). Pictorial representations are also objects in the world; they differ from usual objects only through their representing character. We can denote, therefore, a pictorial representation by using a name and define a function from it in a part of the world. This function, called a projective function, describes how a pictorial representation corresponds to a part of the world. By giving certain constraints on projective functions we can express the intended use of a pictorial representation. Furthermore, combinations of projective functions are defined. This theory can be used to analyze aspect maps. In NRL, both representing and represented layers can be described. This theory clarifies semantic relations of pictorial representations and helps to analyze the indeterminacy problem.

1. Introduction

Today, there is no satisfactory theory for pictorial representations. Pratt (1993) attempts to define semantics for maps, but his undertaking is overly influenced by the traditional semantics. Pratt understands a formal semantics for a language as a theory that gives the truth conditions for expressions in that language. He proposes a semantics for maps according to this concept of semantics. However, it is odd to speak of the truth of maps. Normally, we ask only whether a map is appropriate for certain use. There are also different methods for projection of the reality. The answer for the question, which projection should be used for a map, depends on its intended use. Exact projection is not always needed and in some cases not desirable.

In this paper, I would like to propose a new theory for pictorial representations. This theory is based on *Natural Representation Language* (NRL) proposed by Nakayama (1999). NRL is a mereological language and can treat not only points, but also lines and areas as objects. We consider parts of a pictorial representation as objects that represent a part of the reality. In this paper, an interpretation of a pictorial representation A for B is explicated as an injection from parts of A in parts of B, where this injection preserves the part–whole relation. This injection is called a projective function. Usual maps and sketches also contain symbolic expressions that can be easily interpreted. Interpretation of symbols are considered as constraints on proper projective functions that are intended as correct readings by the designer of the given pictorial representation.

Projective functions are described as functions within the object language. Therefore, characterizations about them can be also given within it; we can talk about representing objects, represented objects, and arts of representation within this object language.

2. Pictorial Objects and Mereological Objects

What are elements of pictorial representations? We talk about points, lines, and areas in a map. This means, not only points but also lines and areas are referred to as objects. To deal with this ontology, it is convenient to use a mereological system. Recently, mereological ontology has been studied in context of representation of plural and mass objects (cf. Lønning (1997)). Link (1998) describes a comprehensive picture of a mereological system based on an algebraic system. Nakayama (1999) criticizes Link's notion of absolute atomic objects and proposes an extensional mereological theory in which individual objects are individuated by use of sortal predicates. When pictorial objects on a map are given, we can construct mereological sums of them. Lines and areas can be constructed in this way as mereological sums.

Nakayama (1999) defines a mereological theory based on Boolean algebra (cf. Appendix). Boolean algebra entails functions $\cap$, $\cup$, NON, and objects ϕ and U. Inclusion $\subset$ can be defined through the stipulation $x \subset y \equiv x \cap y = x$. The part–whole relation $\subset_p$ is defined through $x \subset_p y \equiv (x \subset y \wedge x \neq \phi)$. The name $sum(u)[\psi(u)]$ is defined as a name for the maximal object u that satisfies the condition $\psi(u)$; $sum(u)[\psi(u)]$ is called *mereological sum that satisfies* ψ.

Sortal predicates are predicates that can be used for individuation of objects; "building", "station", "lake", and "city" are examples of sortal predicates. By using a sortal predicate F, relation symbols "being a F–part of" ($\subset_F$), "being a F–atomic object" ($atom_F$), and "being a F–atomic–part of" (ε_F) can be defined as follows:

$x \subset_F y \equiv (F(x) \wedge F(y) \wedge x \subset y)$.

(x is a F–part of y *iff* x is F, y is F, and x is included in y.)

$atom_F(x) \equiv (F(x) \wedge \forall u\,(u \subset_F x \rightarrow u = x))$.

(x is a F-atomic object *iff* x is F and contains no smaller F-part in it.)
$x \, \varepsilon_F \, y \equiv (atom_F(x) \wedge x \subset_F y)$.
(x is a F-atomic-part of y *iff* x is a F-atomic object and x is a F-part of y.)

$\subset_F$ and ε_F express two well known notions in AI-research. $\subset_F$ corresponds to IS-A relation and ε_F corresponds to INSTANCE-OF relation, where these two notions are relativized by a sortal predicate. This relativized IS-A relation might be useful for combination of different domain knowledge in order to avoid inconsistency that might be generated through a combination. Furthermore, the part-whole relation $\subset_p$ corresponds to the PART-OF relation in semantic networks.

The notion of cardinality of objects can be recursively defined (cf. Appendix). When a sortal predicate is applicable to objects, they become countable with respect to the predicate. $cd_F(x) = n$ means, the cardinality of x with respect to F is n, where F is a sortal predicate. By using this notion, we can express a statement like "there are five schools represented on this map" (cf. Section 3).

Based on these theories, Nakayama (1999) develops a language called *Natural Representation Language* (NRL) in order to formally express meanings of sentences with plural and mass terms and in order to deal with plural and mass anaphora. In NRL, quantifiers, such as *all, most, more than n% of*, can be directly expressed and Nakayama (1998a) proposes to extend NRL to a system of hypothetical reasoning.

It is easy to show that $\subset_p$ is a partial ordering except ϕ (cf. Nakayama (1999)). Because $\subset_p$ is transitive, we can derive "Hamburg is a city in Europe" from "Hamburg is a city in Germany", provided that we know that Germany is a part of Europe:

$$Germany \subset_p Europe \rightarrow (Hamburg \ \varepsilon_{city} \ \mathrm{CITY} \wedge Hamburg \subset_p Germany \rightarrow Hamburg \ \varepsilon_{city} \ \mathrm{CITY} \wedge Hamburg \subset_p Europe).$$

CITY is used here as a name for the sum of all cities.

In this paper, it will be shown that NRL is useful not only for semantic description of sentences in natural languages (cf. Nakayama (1999)) but also for semantic description of pictorial representations. As we will see later, NRL can also be used to combine visual and verbal information (cf. section 5).

3. Projective Functions for Pictorial Representations

Typical pictorial representations are maps and sketches. Usual maps and sketches can be seen as objects in the world. Their elements can be described like other objects in the world; they differ from usual objects only by their representing features. I denote, therefore, maps and sketches by using terms and use functions to describe their representing character.

I use a function to describe a relation between a map and the reality; this function from a map in the world is called *projective function*. I define, at first, relation symbols *function* and *projective-function*:

DEFINITION 1

$graph(G, m) \equiv (\forall x\ (x \subset_p m \rightarrow \exists!\ y\ G(x, y)) \wedge \forall x\ (\neg x \subset_p m \rightarrow \neg \exists y\ G(x, y)))$,

where $\exists!\ y\ \psi(y)$ means there is exactly one y such that $\psi(y)$.

(G is a graph for m *iff* G correlates any parts of m to objects in the reality.)

$function(f, m)$, if there is a graph G for m such that $\forall x \forall y\ (x \subset_p m \rightarrow (y = f(x) \equiv G(x, y)))$, where values of f are undefined for objects outside of m. This function f is called the *function generated by the graph G*.

(Instead of a graph, we will use a partial function that exactly corresponds to the graph.)

$projective\text{-}function(f, m) \equiv$

$(function(f, m) \wedge \forall x_1 \forall x_2\ (x_1 \subset_p m \ \wedge\ x_2 \subset_p m \rightarrow (x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2))) \wedge \forall x\ (x \subset_p m \rightarrow f(x) \neq \phi) \wedge f(x_1 \cap x_2) = f(x_1) \cap f(x_2) \ \wedge\ f(x_1 \cup x_2) = f(x_1) \cup f(x_2))$.

(f is a projective function for m *iff* f is an injection from parts of m in parts of the world and f preserves the operations $\cap$ and $\cup$.)

As we will see later, a projective function is an injection that preserves the part–whole relation. Because of this property of maps, we can freely combine information from a map and real experience; we obtain an inverse function that projects objects of a part of the world onto a map and preserves the part–whole relation in the reality. To localize a current place in a map, we usually try to map information from the reality onto the map. For example, when I see a building of a large bank in front of me, I try to find the name of this bank on a city map to orient myself.

We can describe the world by constructing a map, but we can also describe contents of a map. To do the latter, we write names of objects onto a map. For example, when an area c_1 on a map m_1 has the name "*Hamburg*", this name can be written onto the map. The intended association between an area and the name is expressed by the formula *associated* $(c_1, Hamburg, m_1)$. We can describe properties of a map by describing constraints on this function. In our example, we should have $f_1(c_1) = Hamburg$, where f_1 is a function from the map m_1 into the real world. This example shows a method to give constraints on projective functions.

By giving constraints on projective functions, they can be specified. Some of these constraints are formally definable.

DEFINITION 2

We presuppose here an axiomatic system that defines distance of two points in space.

standard-name (f, m), if for all names N in the map m, it holds: $\forall x$ (*associated* $(x, N, m) \rightarrow f(x) = N)$.

(*standard-name* (f, m) means a person who follows the instruction given by f will use all names in the map m in the same way as in natural language.)

standard-attribution (f, m), if for all class names C in the map m, it holds:

$\forall x$ (*associated* $(x, C, m) \rightarrow f(x) \varepsilon_F C)$.

(*standard-attribution* (f, m) means a person who follows the instruction given by f will use all class names in the map m in the same way as in natural language.)

standard-icon (f, m), if for all icons δ in the map m that are associated with the class C, it holds: $\forall x$ (*associated* $(x, \delta, m) \wedge$ *icon-for* $(\delta, C) \rightarrow f(x) \varepsilon_F C)$.

(*standard-icon* (f, m) means a person who follows the instruction given by f will use all icons in the map m as usual.)

$1/n$*-contaction* $(f, m) \equiv \forall x_1 \forall x_2 (x_1 \subset_p m \wedge x_2 \subset_p m \rightarrow dis(f(x_1), f(x_2)) = n \times dis(x_1, x_2)$, where *dis* is a function of the distance between centers of two regions.

($1/n$*-contaction* (f, m) means a person who follows the instruction given by f will multiply the distance between areas on m by n to get their right distance in the reality.)

proportion-preserving $(f, m) \equiv \exists u$ ($1/u$*-contaction* $(f, m))$.

(*proportion-preserving* (f, m) means there is a real number u such that $1/u$-*contaction* (f, m). When this condition holds, the proportion of the reality is preserved in m.)

length-preserving $(f, m) \equiv 1/1$*-contaction* (f, m).

$[r\%]$*-fuzzy-proportion* $(f, m) \equiv \exists u \forall x_1 \forall x_2 (x_1 \subset_p m \wedge x_2 \subset_p m \rightarrow u \times (1 - r/100) \times dis(x_1, x_2) < dis(f(x_1), f(x_2)) \wedge dis(f(x_1), f(x_2)) < u \times (1 + r/100) \times dis(x_1, x_2))$.

($[r\%]$*-fuzzy-proportion* (f, m) means the distortion of proportion on m is maintained within the limit of $r\%$. This characterization becomes important for schematic maps. In schematic maps, proportion is slightly skewed but this distortion should be limited so that it is still possible to recognize the shapes of original objects.)

Sometimes we want to characterize partial features of a pictorial representation. This partial characterization is especially important for schematic maps.

DEFINITION 3

complete $(f,\ m,\ C) \equiv \forall x\ (x \subset_p m \wedge f(x)\ \varepsilon_F\ C \rightarrow associated\,(x,\ C,\ m))$.

(*complete* $(f,\ m,\ C)$ means, in m, all objects of the class C in the m-part of the reality are associated with the name of C, where m-part of the reality denotes the part of the reality described by m.)

*proportion-preserving** $(f,\ m,\ C) \equiv \exists u \forall x_1 \forall x_2\ (associated\,(x_1,\ C,\ m) \wedge associated\,(x_2,\ C,\ m) \rightarrow dis\,(f(x_1),\ f(x_2)) = u \times dis\,(x_1,\ x_2))$.

(*proportion-preserving** $(f,\ m,\ C)$ means, in m, proportion between objects associated with the name of C is preserved.)

orientation-preserving $(f,\ m,\ C) \equiv$ *proportion-preserving** $(f,\ m,\ C)$.

$[r\%]$*-fuzzy-proportion** $(f,\ m,\ C) \equiv \exists u \forall x_1 \forall x_2\ (associated\,(x_1,\ C,\ m) \wedge associated\,(x_2,\ C,\ m) \rightarrow (u \times (1 - r/100) \times dis\,(x_1,\ x_2) < dis\,(f(x_1),\ f(x_2)) \wedge dis\,(f(x_1),\ f(x_2)) < u \times (1 + r/100) \times dis\,(x_1,\ x_2))$.

It is easy to prove the following proposition :

PROPOSITION 1

The following statements straightforwardly follow from definition 1 and 2.

a) For any name N and any class C on the map m, *standard-name* $(f,\ m) \wedge$ *standard-attribution* $(f,\ m) \rightarrow (\exists x\ (associated\,(x,\ N,\ m) \wedge associated\,(x,\ C,\ m)) \rightarrow N\ \varepsilon_F\ C)$.

(If f for m interprets names and class names in conformity with the standard, then it holds : if there is a graphic object in m that is associated with both the object N and the class C, then N is a C.)

b) For any name N and any class C on the map m, *standard-name* $(f, m) \wedge$ *standard-icon* $(f,\ m) \rightarrow (\exists x\ (associated\,(x,\ N,\ m) \wedge associated\,(x,\ \delta,\ m) \wedge icon\text{-}for\,(\delta,\ C)) \rightarrow N\ \varepsilon_F\ C)$.

(If f for m interprets names and icons in conformity with the standard, then it holds : if there is a graphic object in m that is associated with both the object N and the icon δ for the class C, then N is a C.)

c) *projective-function* $(f,\ m) \rightarrow \forall x_1 \forall x_2\ (x_2 \subset_p m \rightarrow (x_1 \subset_p x_2 \rightarrow f(x_1) \subset_p f(x_2)))$.

(If f is a projective function for m, then, for all parts of m, f preserves the part-whole relation.)

d) *projective-function* $(f,\ m) \rightarrow \forall x_1 \forall x_2\ (x_1 = x_2 \equiv f(x_1) = f(x_2))$.

(If f is a projective function for m, then f is an injection from m in a part of the world.)

Proof (a), (b), and (d) are obvious. I will only prove the proposition (c). Suppose *projective-function* $(f, m) \wedge x_2 \subset_p m \wedge x_1 \subset_p x_2$. Then, it suffices to show $f(x_1) \subset_p f(x_2)$. From the definition of $\subset_p$, $(x_1 \subset_p x_2 \equiv (x_1 \neq \phi \wedge x_1 \cap x_2 = x_1)) \wedge (f(x_1) \subset_p f(x_2) \equiv (f(x_1) \neq \phi \wedge f(x_1) \cap f(x_2) = f(x_1)))$. From the definition of *projective-function* (f, m), $f(x_1) \neq \phi \wedge f(x_1) \cap f(x_2) = f(x_1 \cap x_2)$. From $x_1 \cap x_2 = x_1$, $f(x_1) \cap f(x_2) = f(x_1)$. Hence, $f(x_1) \subset_p f(x_2)$. $\dashv$

The first two statements show that simple subject–predicate statements are expressible on a map. For example, by using an icon for school and by writing a name of a school near the icon, we can express that the referred building is a school. Now, a statement such as "there are five schools represented on this map m" can be expressed in NRL :

$\exists x\ (x \subset_p m \wedge associated(x, \delta, m) \wedge icon\text{-}for(\delta, \text{SCHOOL}) \wedge \text{cd}_{building}(f(x)) = 5)$.
(There is a part of m that represent a sum of five schools. SCHOOL denotes here the sum of all schools in the world. $f(x)$ denotes a mereological object in the reality that is represented in m by the area x).

In this formula, the statement *associated* (x, δ, m) has a collective reading. See Nakayama (1999) for a detailed treatment of collective and distributive reading.

The last two statements in proposition 1 state that a projective function is an injection that preserves the part–whole–relation.

Particular properties of the projective function can be easily defined in case by case. Pratt (1993) used an example of a map consisting of dark and light parts, where dark parts represent water and light parts represent land. Let m be the map shown in fig.1 and f be a proper projective function for m. It is, then, easy to characterize f, so that f expresses the intended interpretation of m ; we can require the following property from f :

$\forall x\ (x \subset_p m \wedge dark(x) \rightarrow f(x) \subset_p \text{WATER}) \wedge \forall x\ (x \subset_p m \wedge light(x) \rightarrow f(x) \subset_p \text{LAND})$.
(Any parts of m that have a dark color represent water and any parts of m that have a light color represent land.)

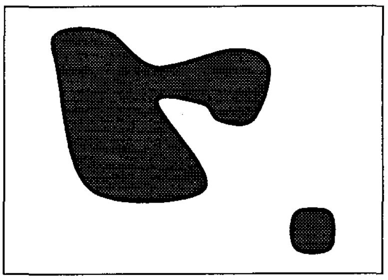

Fig.1 A map with two lakes

Actually, to interpret this map, Pratt (1993) uses no mereological concept and uses a function from points on a map into the reality without giving any special constraint on the function. However, parks and lakes on a map are normally considered as individual objects, as this can be observed in the fact that some parks and lakes have a name. We need, therefore, mereological ontology with relativization by sortal predicates in order to properly describe a map. Furthermore, the intended function is a projective function that preserves the part–whole relation.

I call the theory presented in this section *the extended* NRL or ENRL.

4. What are Representations?

Palmer (1978) characterizes a representation system as a system that has the following five aspects:

a) what the represented world is;
b) what the representing world is;
c) what aspects of the represented world are being modeled;
d) what aspects of the representing world are doing the modeling;
e) what are the correspondences between the two worlds.

In this sense, ENRL characterizes pictorial representations as representation systems. According to ENRL, a pictorial representation m is a representation whose function f satisfies the condition *projective–function* (f, m). Palmer's five aspects can now be explicated as follows:

a) The represented world is the intended model.
b) The representing world is a pictorial representation m.
c) The aspects of the represented world that are being modeled are specified by giving constraints on possible projective functions. Examples for these constraints are defined in definition 2; they are *proportion–preserving, standard–name,* etc.
d) When only some parts of a pictorial representation are used for modeling the world, these parts can be specified by using conditions in the form $R(f, m, C)$, where m stands for the pictorial representation, f denotes the intended projective function, and C is a class of objects that are described in m. Examples for R are defined in definition 3; they are *complete, proportion–preserving**, etc.
e) The correspondences between the two worlds is expressed by a projective function f.

In summary: a part of the real world is represented by a pictorial representation m, where the intended projective function f for m shows how the world is represented by m and how to use m.

5. Combining Information from Pictorial Representations with Information from Verbal Expressions

Within ENRL, it is easy to combine information given by different media. In this section, it is discussed how to combine information from a pictorial representation with information given by verbal expressions.

Within ENRL, verbal information about the world is directly expressed, whereas information contained in a pictorial representation is expressed by using a projective function. A projective function is a partial function whose domain consists of parts of a pictorial representation. Some of them are characterized by certain properties or identified by using names, and they stand in certain spatial relations to each other. A proper projective function for a pictorial representation should be characterized in order to use it correctly. Relations between pictorial representations and projective functions give the characterization for a correct use. These characterizing relations are relations defined in section 3, like *standard-attribution, proportion-preserving*, etc.

Within ENRL, information from a pictorial representation and information from verbal expressions can be straightforwardly combined. Tappe and Habel (1998) point out that there are different representation layers, i.e. the layer of graphical entities and the layer of real-world entities. By using a natural language, we can talk about both of the layers. Think about the situation in which a man points to a line l_1 on the map m and says "This long line represents a railway. There is a station on this line. You can see a church near the station. Peter's house lies next to this church." In this talk, two representation layers are mixed up. This talk can be translated into ENRL as follows, where f is a proper projective function for m :

$(l_1 \; \varepsilon_{figure}$ LINE $\wedge\; f(l_1) \subset_p$ RAILWAY $\wedge\; on\,(d_1,\; d_2,\; m) \wedge\; f(\mathrm{d}_1) \; \varepsilon_{building}$ STATION $\wedge\; associated\,(d_2,\; †,\; m) \wedge\; icon\text{-}for\,(†,$ CHURCH$) \wedge\; near\,(d_2,\; d_1) \wedge\; d_3 \; \varepsilon_{building}$ HOUSE $\wedge\; d_3$ *belong to Peter* $\wedge\; next\text{-}to\,(d_3,\; f(d_2)))$.

(l_1 is a line, l_1 represents a part of the whole railway, d_1 is on l_1 (in m), d_1 represents a station, d_2 is associated with the icon † for a church, d_2 is placed near d_1, d_3 is a house that belongs to Peter, and d_3 is located next to the church represented by d_2.)

Here, l_1, d_1, d_2, and d_3 are Skolem constant symbols. Nakayama (1999) uses Skolem symbols to express anaphoric relations. Detailed explanations can be found there.

We can combine information from verbal expressions and information from the map m. Suppose that there is a school with the name N that is 2cm away from the church mentioned before. Suppose that $1/100000$-*contaction*(f, m) holds. This means that the distance of the school and the church is 2km. This is expressed within ENRL as follows:

$1/100000\text{-}contaction(f, m) \rightarrow (d_4 \subset_p m \wedge associated(d_4, N, m) \wedge d_5 = f(d_4) \wedge d_5 = N \wedge d_5\ \varepsilon_{building}\ \text{SCHOOL} \rightarrow (dis(d_4, d_2) = 2cm \rightarrow dis(d_5, f(d_2)) = 2km))$.

(When the scale of m is 1/100000 and when d_4 represents a school with the name N, then the distance between the school and the church mentioned before is 2km, because their distance on m is 2cm.)

These examples demonstrate that combination of information from a pictorial representation and information from verbal expressions is straightforward within ENRL.

6. Combination of Pictorial Representations

In this section, we define how to combine different pictorial representations. There are two ways of combining functions; they are unification and composition.

DEFINITION 4

Let G_1 be the graph which generates the projective function f_1 for m_1. Let G_2 be the graph that generates the projective function f_2 for m_2.

a) $union(G, f_1, f_2) \equiv \forall x \forall y\, (G(x, y) \equiv (G_1(x, y) \vee G_2(x, y)))$.
(G is the union of f_1 and f_2 *iff* G is the union of G_1 and G_2.)

b) $united\text{-}graph(G, f_1, f_2) \equiv (union(G, f_1, f_2) \wedge \forall x\, (x \subset_p (m_1 \cap m_2) \rightarrow f_1(x) = f_2(x)))$.
(G is the united graph of f_1 and f_2 *iff* G is the union of f_1 and f_2, and G is a graph for $m_1 \cup m_2$.)

c) If *united-graph*(G, f_1, f_2) holds, then the function generated by G is denoted by $[f_1 \cup f_2]$ and called *the unification of* f_1 and f_2.

d) $composition(G, f_1, f_2) \equiv \forall x \forall y\, (G(x, y) \equiv (x \subset_p m_1 \wedge f_1(x) \subset_p m_2 \wedge y = f_2(f_1(x))))$.
(G is the composition of f_1 and f_2 *iff* G correlates every part x of m_1 to the object $f_2(f_1(x))$.)

e) The function generated by the graph G that is the composition of f_1 and f_2 is denoted by $[f_2 * f_1]$ and called *the composition of f_1 and f_2*.

Composition and unification of functions can be combined, if the conditions for them described in definition 4 are satisfied. Thus we can construct a complex combination like $[f_1 \cup [f_2 * f_3] \cup f_2 \cup f_4]]]$. We can use many pictorial representations to interpret the reality, if they are altogether consistent. It is also our usual praxis to use different maps and sketches in order to go to an unknown place.

In some cases, it is possible to construct a virtual map by combining different maps in a proper way. This construction is easy, when all maps that should be combined overlap with each other and their proper projective functions are proportion preserving. Let us think about such an example. Suppose that there are maps m_1 and m_2 with projective functions f_1 and f_2. Suppose that f_1 and f_2 satisfy $1/n$*-contaction* (f_1, m_1), $1/k$*-contaction* (f_2, m_2), and $f_1(m_1) \cap f_2(m_2) \neq \phi$. In this case, we can define a virtual map m_3 with $m_3 = g_1(m_1) \cup g_2(m_2)$, $f_3 = [f_1 * g_1^{-1}] \cup [f_2 * g_2^{-1}]$, and $\forall x \; (x \subset_p \; (g_1(m_1) \cap g_2(m_2)) \equiv f_3(x) \subset_p \; (f_1(x) \cap f_2(x)))$, where g_i^{-1}, means the inverse function of g_i. These conditions are needed to correctly correlate m_1 and m_2. Now, we can construct a virtual map with the scale $(1:r)$, when we give the constraints n/r*-contraction* (g_1, m_1) and k/r*-contraction* (g_2, m_2) on g_1 and g_2. It holds then: $1/r$*-contaction* (f_3, m_3), r/n*-contraction* $(g_1^{-1}, g_1(m_1))$, and r/k*-contraction* $(g_2^{-1}, g_2(m_2))$. In this way, we can construct a proportion preserving virtual map from two proportion preserving maps.

7. Aspect Maps

According to Brendt *et al* (1998), an aspect map is a spatial organization structure that represents one or more aspects of geographic entities. To represent a part of the world, different aspects of it can be extracted from the reality. An example of aspect maps is a transportation network map that reconstructs only some aspects of the original city map. In this section, aspect maps will be formally characterized by using ENRL.

There are some aspects that are crucial for transportation network maps. Their most important feature is the completeness of lines and stations. It is formally expressed as *complete* $(f, m, \text{LINE}) \wedge$ *complete* $(f, m, \text{STATION})$. The formula *complete* $(f, m, \text{LINE}) \wedge$ *complete* $(f, m, \text{STATION})$ means, all lines and stations in the m-part of the reality are represented in m. The completeness of stations is a nice property. We would not trust a transportation network map that misses several stations.

Pratt (1993) proposes to use a *default representation*, because he thinks the absence of symbols can convey information. This problem is closely related with our usual expectation from a map that it represents all objects of some classes in the described area. By using ENRL, we can express when we should use a default reasoning. If it holds *complete* (f, m, C), then we can say that there is no C, where no symbol for C

stands. If *complete* (f, m, C) does not hold but we know that there are more places with *non*$-C$ than with C in the m-part of the reality, then it is appropriate to infer by default that there is probably no C-object where no symbol for C stands. However, this deals with a practical decision and it is not a genuine problem of semantics.

For transportation network maps, it is crucial that stations are located on a line according to the original order. These maps should be so constructed that there are projective functions that preserve all station orderings of all lines described in them. As Brendt *et al* correctly pointed out, the exact locations of stations and courses shall be omitted for reasons of map readability. However, locations should be roughly right, so that correlation between the map and the reality remains readable. We can express this condition of rough correctness, for example, by the formula [10%]-*fuzzy-proportion**$(f, m,$ LINE$)$ $\wedge$ [10%]-*fuzzy-proportion**$(f, m,$ STATION$)$.

In the last section, we have already discussed how to combine two projective functions, when they are proportion preserving. However, many aspect maps are not proportion preserving. Let us discuss the combination of aspect maps more closely. When two aspect maps, m_1 and m_2, represent the same part of the reality, then it holds: $f_1(m_1) = f_2(m_2)$. By using the method explained in section 7, a virtual map m_3 can be constructed from m_1 and m_2, and it holds: $m_3 = g_1(m_1) \cup g_2(m_2)$, $f_3 = [f_1 * g_1^{-1}] \cup [f_2 * g_2^{-1}]$. Hence, $m_3 = g_1(m_1) = g_2(m_2)$, $f_3 = [f_1 * g_1^{-1}] = [f_2 * g_2^{-1}]$. In many cases, to preserve f_1's property, g_1^{-1} has to have the same property. For example, to preserve f_1's property of orientation preserving, g_1^{-1} has to be orientation preserving, i.e. g_1 has to be orientation preserving. However, when we accept slight distortion of the preservation, we can often construct a virtual map with nice properties. For example, when we accept properties [10%]-*fuzzy-proportion*(g_1, m_1) and [10%]-*fuzzy-proportion*(g_2, m_2), it might be possible that f_3 newly obtains the property of proportion preserving without loss of any properties of f_1 and f_2. The construction problem of a virtual map from existing aspect maps can be seen as a problem of *constraint satisfaction*. That is a problem to find a projective function from a virtual map to the reality and projective functions from the existing aspect maps to the virtual map such that they satisfy all relevant properties of the existing aspect maps and all desirable properties for the virtual map.

8. The Indeterminacy Problem

To properly use a pictorial representation m, it is crucial to know what are presupposed as proper projective functions for m. Ignorance of this knowledge can cause a misuse of this representation system. Let m be a pictorial representation that is not proportion preserving. If we misunderstand this pictorial representation as proportion preserving, we can fail to estimate a right distance between two places described on m. To understand how to use a pictorial representation, you have to know properties of its proper projective functions.

This consideration helps to clarify the indeterminacy problem discussed in Habel (1998). According to Habel, the indeterminacy problem deals with the claim that pictorial representations are completely determined and committed to details, whereas propositional representations can be underdetermined. As Habel points out, even if a pictorial description is completely determined, there are different interpretation possibilities of it. Habel explains these different interpretation possibilities through the possibilities of taking different axiomatized geometric systems. It is, however, more general and natural to think that these interpretation possibilities are given by the possibilities of taking different proper projective functions for the same pictorial representation.

A pictorial representation has its own structure, but usually not all of its structural properties are used for representation of the world. Maps and sketches differ in the relevance of their details. For example, a typical map is proportion preserving and it has many classes of objects that are exhaustively mentioned with respect to the described area, but most sketches are not proportion preserving and it has few classes of objects that are exhaustively mentioned with respect to the described area. When two points are connected on a sketch, this connection might represent a property in the reality but neither distance nor orientation nor length of the line will play a representational role. Normally, sketches represent only restricted parts of the reality. This consideration can be summarized as follows:

a) A proper projective function f of a typical map m fulfills the following properties: $\textit{proportion-preserving}(f, m) \wedge \textit{standard-name}(f, m) \wedge \textit{standard-attribution}(f, m) \wedge \textit{standard-icon}(f, m)$, and there are many classes C such that $\textit{complete}(f, m, C)$.

b) A proper projective function f of a typical sketch m fulfills the following properties: $\textit{standard-name}(f, m) \wedge \textit{standard-attribution}(f, m)$, and there is a part m^* of m and some classes C such that $m^* \subset_p m \wedge \textit{complete}(f, m^*, C)$.

When you draw a map or a sketch, you can decide which of standard constraints you take and you may introduce new symbols into it. To correctly use this pictorial representation, we need additional information about the projective function that was presupposed by the drawer. Thus these pictorial representations are underdetermined. Especially for schematic maps and sketches, certain details are often irrelevant for interpretation of the represented reality.

Sometimes, the same facts can be expressed both through a language and through a sketch. Descriptions through a sketch often have the property that the whole structure can be viewed at a glance. However, they often need verbal explanation how to read objects on them. Pictorial and verbal representations have their own advantages. Therefore, sophisticated speakers tend to use both representations in a talk.

9. Conclusions

A formal theory for pictorial representations has been presented based on a merelogical language. It was shown that a semantics of pictorial representations can be satisfactory characterized within the *Extended Natural Representation Language* (ENRL). By using this theory, combination of information from pictorial representations and information from verbal expressions could be properly described. The relation between representing and represented layers has been clarified. Furthermore, combination of pictorial representations is closely analyzed and its result is applied to analysis of aspect maps. ENRL is proposed from a theoretical interest. However, the idea proposed in this paper might be used for implementation. When ENRL is fully implemented, it can be used as an information system that can combine symbolical and spatial knowledge; it will be a system that can deal with verbal and pictorial inputs and outputs.

Appendix

In this appendix, I will present axioms of NRL, which are defined in Nakayama (1999). NRL is a theory in two-sorted logic with mereological objects and numbers.

(MA1) Axioms for Boolean algebra (minimum: ϕ, maximum: U).

(MD1) $u \subset v \equiv u \cup v = v$.

(MD2) $u \subset_p v \equiv u \subset v \wedge u \neq \phi$.

(MD3) $u \mathrm{O} v \equiv u \cap v \neq \phi$.

(MD4) $\forall u \, ((\lambda v[q(v)])(u) \equiv q(u))$.

(MA2) $\exists u \; q(u) \rightarrow \exists v (q(v) \wedge \forall u (q(u) \rightarrow u \subset_p v))$.

(MD5) $v = sum(u)[q(u)] \equiv$

$$((q(v) \wedge \forall u \; (q(u) \rightarrow u \subset_p v)) \vee (\forall u \; \neg q(u) \wedge v = \phi)).$$

(MA3) For all Skolem function symbols d_k: $d_k(u \cup v) = d_k(u) \cup d_k(v)$.

From this axiom system it follows that the part–whole relation $\subset_p$ is a partial ordering excluding the nothing:

$$\forall u (u \neq \phi \rightarrow u \subset_p u) \wedge \forall u \forall v \forall w \; (u \subset_p v \wedge v \subset_p w \rightarrow u \subset_p w) \wedge$$

$\forall u \forall v\ (u \subset_p v \wedge v \subset_p u \rightarrow u = v)$.

A ***sortal predicate*** F is characterized as follows. It provides the basis for count.

(SA1) $\neg F(\phi)$.

(SA2) $F(u) \wedge F(v) \rightarrow F(u \cup v)$.

(SA3) $F(u) \wedge F(u \cup v) \wedge v \neq \phi \wedge u \cap v = \phi \rightarrow F(v)$.

(SD1) $u \subset_F v \equiv F(u) \wedge F(v) \wedge u \subset v$.

(SD2) $atom_F(u) \equiv F(u) \wedge \forall v\ (v \subset u \wedge v \neq u \rightarrow \neg F(v))$.

(SD3) $u \varepsilon_F v \equiv atom_F(u) \wedge u \subset_F v$.

(SA4) $F(u) \rightarrow \exists v\ (v \varepsilon_F u)$.

(SD4) $v = sum_F(u)[q(u)] \equiv$
$((F(u) \wedge q(v) \wedge \forall u\ (F(u) \wedge q(u) \rightarrow u \subset_F v)) \vee$
$(\forall u\ (F(u) \rightarrow \neg q(u)) \wedge v = \phi))$.

The ***cardinality*** of physical objects and events is recursively defined, where F is a sortal predicate :

(CD1) $cd_F(u) = 1 \equiv atom_F(u)$.

(CD2) $(cd_F(v) = 1 \wedge u \cap v = \phi) \rightarrow (cd_F(u) = n \equiv cd_F(u \cup v) = n+1)$.

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