Title	A note on the boundary behavior for non-negative finely superharmonic functions in the upper half space
Author(s)	Masaoka, Hiroaki
Citation	Osaka Journal of Mathematics. 1991, 28(2), p. 461-462
Version Type	VoR
URL	https://doi.org/10.18910/9542
rights	
Note	

Masaoka, H.
Osaka J. Math.
28 (1991), 461–462

A NOTE ON THE BOUNDARY BEHAVIOR FOR NON-NEGATIVE FINELY SUPERHARMONIC FUNCTIONS IN THE UPPER HALF SPACE

Hiroaki MASAOKA

(Received Nobember 6, 1989)
(Revised September 20, 1990)

1. Introduction

In the previous paper [3], we studied the boundary behavior for non-negative finely superharmonic functions in a Green space at the Martin boundary. In this paper, as an application of Main Theorem in [3] we give a theorem of a Littlewood type (cf. [2, Theorem]) for non-negative finely superharmonic functions in the upper half space $\boldsymbol{H}$ of the d-dimensional Euclidean space ($d \geq 2$).

2. Theorem of a Littlewood type

Theorem. *Let U be a finely open subset of $\boldsymbol{H}$ and let $\Delta_1(U)=\{\zeta \in \partial \boldsymbol{H}$: $\boldsymbol{H} \backslash U$ is minimally thin at $\zeta\}$. Suppose that u is a non-negative finely superharmonic in U. Then, at almost every point ζ of $\Delta_1(U)$ with respect to the $(d-1)$-dimensional Lebesgue measure, there exists a polar subset $N(\zeta)$ of the unit sphere with center ζ satisfying the following property: $u(z)$ converges to $f\text{-}\lim_{x(\in U) \to \zeta} u(x)$ as a point z in U approaches ζ along every ray issued from ζ which does not meet $N(\zeta)$.*

For the definitions of notations and terminologies used in Main Theorem, we refer to [3] and the word "almost every" will be later used to mean "except for a null set with respect to the $(d-1)$-dimensional Lebesgue measure".

Proof of Theorem. Let u be a non-negative finely superharmonic function in a finely open subset U in $\boldsymbol{H}$. Then, Main Theorem in [3] states that u has a fine limit at almost every point ζ of $\Delta_1(U)$. From Naïm's result [4, Théorème 12], we see that, at almost every point ζ of $\Delta_1(U)$, there exists a finely open subset $V(\zeta)$ of U such that $\boldsymbol{H} \backslash V(\zeta)$ is minimally thin at ζ and that $u(z)$ converges to $f\text{-}\lim_{x(\in U) \to \zeta} u(x)$ as a point z in $V(\zeta)$ approaches ζ in the sense of the Euclidean topology in $\boldsymbol{R}^d$. On the other hand, by Lelong-Ferrand [1, Théorème in §8], there exists a polar subset N of the unit sphere with center ζ in $\boldsymbol{R}^d$ satifying the following property: for every ray l in $\boldsymbol{H}$ issued from ζ which dose not meet N, there exists a positive ρ such that $(l \cap B(\zeta, \rho)) \setminus \{\zeta\} \subset U$, where $B(\zeta, \rho)$ is the ball with center ζ and radius ρ in $\boldsymbol{R}^d$. Therefore, we obtained the desired result.

Acknowledgement. The author wishes to thank Professor Ikegami for valuable comments. And he also wishes to thank the referee for valuable comments and advices which give an improvement of the above Theorem.

References

[1] J. Lelong-Ferrand: *Étude au voinsinage de la frontière des fonction surharmonique positive dans un demi-espace*, Ann. Sci. École Norm. Sup., **66** (1949), 125–159.
[2] J.E. Littlewood: *On functions subharmonic in a circle* (*II*), Proc. London Math. Soc. (2), **28** (1928), 383–394.
[3] H. Masaoka: *On the behavior of non-negative finely superharmonic functions at the Martin boundary*, J. Math. Kyoto Univ., **30** (1990), 193–205.
[4] L. Naïm: *Sur le rôle de la frontière de R. S. Martin dans la théorie du potentiel*, Ann. Inst. Fourier, **7** (1957), 183–281.

Department of Mathematics
Osaka City University
Sugimoto, Sumiyoshi-ku
Osaka, 558, Japan