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A NOTE ON THE BOUNDARY BEHAVIOR FOR
NON-NEGATIVE FINELY SUPERHARMONIC
FUNCTIONS IN THE UPPER HALF SPACE

HIROAKI MASAOKA

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1. Introduction

In the previous paper [3], we studied the boundary behavior for non-negative finely superharmonic functions in a Green space at the Martin boundary. In this paper, as an application of Main Theorem in [3] we give a theorem of a Littlewood type (cf. [2, Theorem]) for non-negative finely superharmonic functions in the upper half space $H$ of the $d$-dimensional Euclidean space $(d \geq 2)$.

2. Theorem of a Littlewood type

Theorem. Let $U$ be a finely open subset of $H$ and let $\Delta(U) = \{z \in \partial H : H \setminus U \text{ is minimally thin at } z\}$. Suppose that $u$ is a non-negative finely superharmonic in $U$. Then, at almost every point $z$ of $\Delta(U)$ with respect to the $(d-1)$-dimensional Lebesgue measure, there exists a polar subset $N(z)$ of the unit sphere with center $z$ satisfying the following property: $u(z)$ converges to $f\lim u(x)$ as a point $z$ in $U$ approaches $z$ along every ray issued from $z$ which does not meet $N(z)$.

For the definitions of notations and terminologies used in Main Theorem, we refer to [3] and the word "almost every" will be later used to mean "except for a null set with respect to the $(d-1)$-dimensional Lebesgue measure".

Proof of Theorem. Let $u$ be a non-negative finely superharmonic function in a finely open subset $U$ in $H$. Then, Main Theorem in [3] states that $u$ has a fine limit at almost every point $\zeta$ of $\Delta(U)$. From Naïm’s result [4, Théorème 12], we see that, at almost every point $\zeta$ of $\Delta(U)$, there exists a finely open subset $V(\zeta)$ of $U$ such that $H \setminus V(\zeta)$ is minimally thin at $\zeta$ and that $u(z)$ converges to $f\lim u(x)$ as a point $z$ in $V(\zeta)$ approaches $\zeta$ in the sense of the Euclidean topology in $\mathbb{R}^d$. On the other hand, by Lelong-Ferrand [1, Théorème in §8], there exists a polar subset $N$ of the unit sphere with center $\zeta$ in $\mathbb{R}^d$ satifying the following property: for every ray $l$ in $H$ issued from $\zeta$ which
dose not meet $N$, there exists a positive $p$ such that $(I \cap B(\xi, p)) \setminus \{\xi\} \subset U$, where $B(\xi, p)$ is the ball with center $\xi$ and radius $p$ in $\mathbb{R}^d$. Therefore, we obtained the desired result.

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References


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