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A NOTE ON THE BOUNDARY BEHAVIOR FOR NON-NEGATIVE FINELY SUPERHARMONIC FUNCTIONS IN THE UPPER HALF SPACE

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1. Introduction

In the previous paper [3], we studied the boundary behavior for non-negative finely superharmonic functions in a Green space at the Martin boundary. In this paper, as an application of Main Theorem in [3] we give a theorem of a Littlewood type (cf. [2, Theorem]) for non-negative finely superharmonic functions in the upper half space $\boldsymbol{H}$ of the d-dimensional Euclidean space ($d\geq 2$).

2. Theorem of a Littlewood type

Theorem. *Let U be a finely open subset of $\boldsymbol{H}$ and let $\Delta_1(U)=\{\zeta\in\partial\boldsymbol{H}$: $\boldsymbol{H}\setminus U$ is minimally thin at $\zeta\}$. Suppose that u is a non-negative finely superharmonic in U. Then, at almost every point ζ of $\Delta_1(U)$ with respect to the $(d-1)$-dimensional Lebesgue measure, there exists a polar subset $N(\zeta)$ of the unit sphere with center ζ satisfying the following property: $u(z)$ converges to $f\text{-}\lim_{x(\in U)\to\zeta} u(x)$ as a point z in U approaches ζ along every ray issued from ζ which does not meet $N(\zeta)$.*

For the definitions of notations and terminologies used in Main Theorem, we refer to [3] and the word "almost every" will be later used to mean "except for a null set with respect to the $(d-1)$-dimensional Lebesgue measure".

Proof of Theorem. Let u be a non-negative finely superharmonic function in a finely open subset U in $\boldsymbol{H}$. Then, Main Theorem in [3] states that u has a fine limit at almost every point ζ of $\Delta_1(U)$. From Naïm's result [4, Théorème 12], we see that, at almost every point ζ of $\Delta_1(U)$, there exists a finely open subset $V(\zeta)$ of U such that $\boldsymbol{H}\setminus V(\zeta)$ is minimally thin at ζ and that $u(z)$ converges to $f\text{-}\lim_{x(\in U)\to\zeta} u(x)$ as a point z in $V(\zeta)$ approaches ζ in the sense of the Euclidean topology in $\boldsymbol{R}^d$. On the other hand, by Lelong-Ferrand [1, Théorème in §8], there exists a polar subset N of the unit sphere with center ζ in $\boldsymbol{R}^d$ satifying the following property: for every ray l in $\boldsymbol{H}$ issued from ζ which dose not meet N, there exists a positive ρ such that $(l \cap B(\zeta, \rho)) \setminus \{\zeta\} \subset U$, where $B(\zeta, \rho)$ is the ball with center ζ and radius ρ in $\boldsymbol{R}^d$. Therefore, we obtained the desired result.

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