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# Observation of Dynamic Characteristics of TIG Arc†

Alber A. SADEK\*, Manabu TANAKA\*\*, Wenjie MAO\*\*\* and Masao USHIO\*\*\*\*\*

## Abstract

*An experimental study of pulsed current GTAW was carried out to determine the dynamic characteristics of the TIG arc. The effect of frequency and arc current on the phase difference and arc gain (equivalent to arc resistivity) were studied, over the range of frequency from 0.2 Hz to 1 KHz. The effect of the other welding parameters such as, arc length, electrode diameter and electrode tip angle were also investigated.*

*The results revealed the great influence of arcing current on the phase difference change criterion with increasing frequency, while the other welding parameters have almost no effect. However, the arc gain is influenced by the changes in welding parameters.*

*An equivalent LRC electrical circuit connected in series was introduced to represent the characteristics of the dynamic TIG arc. Also, the results obtained for weld pool depth showed that the current pulsation increase the weld pool depth, and this can be considered as an effective scheme for GTAW pool geometry control.*

**KEY WORDS:** (Arc Welding) (Pulsed GTAW) (Dynamic Characteristics) (Pulsed Current) (Current Frequency) (Arc Characteristics) (Dynamic Arc) (TIG Welding)

## 1. Introduction

The development of better welding methods, especially in the field of automatic welding, requires a more accurate qualitative and/or quantitative knowledge of the relationship between various welding parameters. To achieve this, an improved physical understanding of the welding process is needed. In particular, it is necessary to understand the effects of manipulation of input variables and the process output variable.

Different process modification schemes are needed to bring about such control. Most process modifications require some kind of change in the forces within the arc and the melt. The nature of these forces depends on the characteristics of the arc, such as current, temperature and the resulting heat distribution within the arc. Pulsed current is a promising device for the desired process modification that can be used in an active control arrangement.

The pulsed current is an independent variable governed essentially by the power supply. On the otherhand, the arc voltage is a dependent variable governed by the arc

itself. Arc voltage is a measure of the arc response to the effect of current through it. Therefore, a knowledge of the variation in dynamic characteristics has a fundamental role in clarifying the properties of the welding arc itself as well as their influence on welding process.

In this paper, a systematic investigation is presented which shows the results of experimental studies on the dynamic characteristics of the GTAW, supplied by full sinusoidal pulsed current over a range of frequency from 0.2 Hz to 1 KHz.

## 2. Experimental Procedure

The arc current was supplied from an analogue transistor power source by full sinusoidal pulsed current wave-forms. The pulse frequency was changed within the range from 0.2 Hz up to 1 KHz. The current was changed between the background level and the pulse level in the range of  $\pm 25$  A for an average current ranging from 100 A up to 300 A.

Throughout the entire experimental program, the

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cathode consisted of 2% La<sub>2</sub>O<sub>3</sub> -W electrodes of 3.2 mm and 5 mm in diameter. The electrodes had ground tips with solid angles of 45° and 90° to investigate the effect of electrode diameter and tip angle on the dynamic characteristics of the arc.

The anode was a water-cooled copper plate and pure Ar gas was used as a shielding gas. The arc length was varied within the range 2, 3, 4 mm during the course of the experiments to observe its effect on the dynamic characteristics of the arc.

The experimental setup for measuring and observing the dynamic characteristics of the TIG arc is shown schematically in Fig.1. An electrical shunt was used to measure the current delivered to the cathode. The welding torch was especially designed to allow the measurements of the arc voltage between the cathode and anode. These signals were recorded using multi-channel digital recorder and PC data processor.

Two thermocouples in the inlet and outlet were used to determine temperature rise of water flowing through the cooling passage. By this means the total heat delivered to the anode ( $Q_{\text{anode}}$ ) was determined by calculating:

$$Q_{\text{anode}} = 4.18 \times (T_{\text{out}} - T_{\text{in}}) \times v_{\text{water}}$$

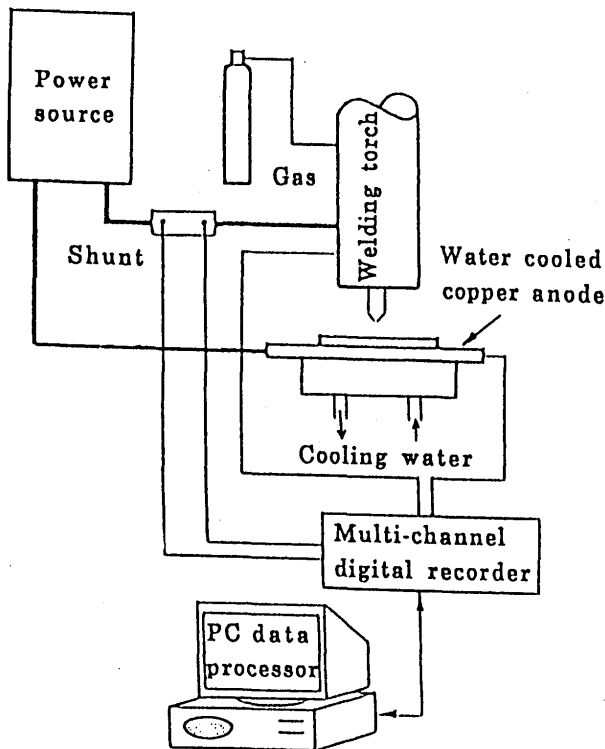


Fig.1 Schematic diagram of experimental setup

Where

- $v_{\text{water}}$  = The flow rate of the water flowing in the anode
- $T_{\text{out}}$  = Temperature of the water at the outlet
- $T_{\text{in}}$  = Temperature of the water at the inlet.

## 3. Results and Discussions

### 3.1 Arc dynamic characteristics

Figures 2 and 3 show the current and voltage wave-forms and their voltage - current hysteresis within the desired range of current pulsation frequencies at two average current levels of 150 A and 250 A respectively.

Generally, at low frequencies less than 10 Hz, the arc voltage changes between two levels corresponding to both peak current and base current. The change of current from background up to the pulse level results in a decrease in voltage, while the converse change of the arc voltage is observed with background level.

However, in the case of 150 A, with increasing frequency the average level of the arc voltage corresponding to both peak and base periods is almost equal. A further increase in frequency results in a further decrease of background period arc voltage and small increase in arc voltage during the pulse period (1 KHz).

On the other hand, in the case of 250 A, with increasing frequency, the change in arc voltage exactly coincides with that at low frequency. With a further rise in frequency the average level of the arc voltage is almost equal with a small increase in arc voltage during the pulse period (1 KHz).

It was observed also that, with current pulsation the leading edge of the arc voltage showed overshoot and undershoot corresponding to a quick rise and fall of current. These instantaneous peaks were observed within the low and medium frequencies. The occurrence of such voltage peaks will be discussed in the following sections.

The shape of voltage current dynamic hysteresis changes over the whole range of frequency. It appears from the figure similar to a rhomboid at low frequencies to a figure similar to an ellipse at high frequencies. With a further rise in frequency the smaller diameter of the ellipse become shorter. In other words, a higher voltage is occurring with the rise in current than with current fall.

These phenomena may be caused to the number of ionized species in the arc column which is less than that required for sustaining the arc at higher current pulsating level, so that a higher voltage gradient (overshoot) is necessary to increase the ionization. Conversely, with the fall in current, the ionization lags behind the current and

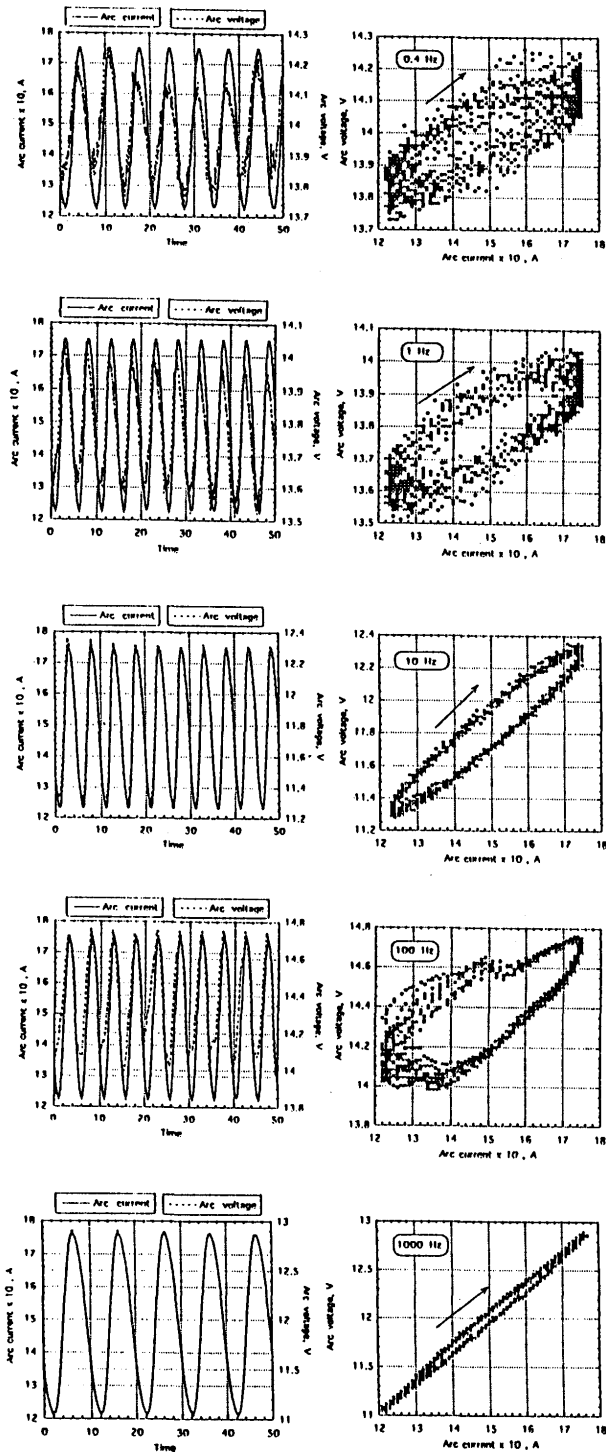


Fig.2 Dynamic characteristics changes as a function of frequency at 150 A

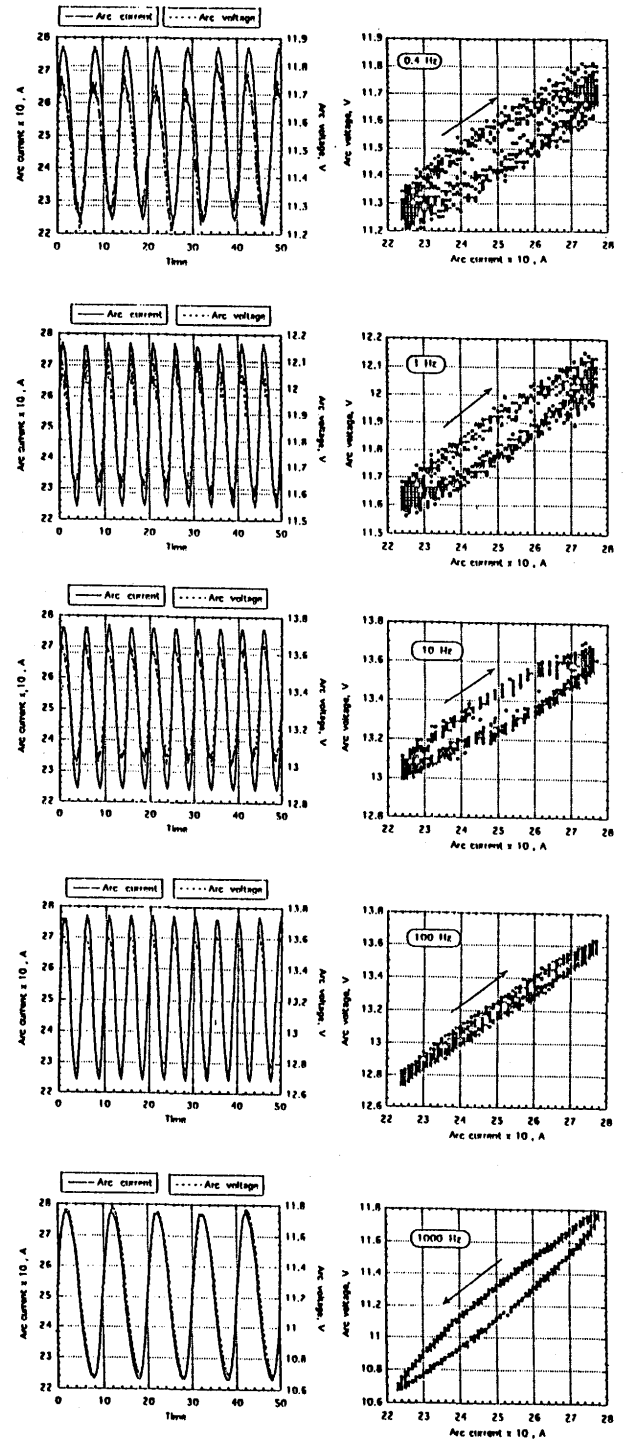


Fig.3 Dynamic characteristics changes as a function of frequency at 250 A

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the arc has a greater ionization than is required so that the current flows with a lower voltage gradient (undershoot). It follows that, the ionization changing rate is relatively slow, especially after the current base period when a recombination of ionized particle plays the main role in the relaxation process of the arc plasma. This may explain why, with a rising frequency, a remarkable decrease in voltage corresponding to the base current takes place.

### 3.2 Effect of arc current on the phase difference angle and arc gain

In the previous section the dynamic characteristics of the arc were examined, and it was found that the values of the arc voltage corresponding to the current pulsation are different from both background and pulse levels. These differences may also depend on the values of the welding system parameters. This seems to be caused by the difference of phase angles between current (input) and voltage (output), as well as the arc gain (resistivity). All of these parameters were measured and calculated during the data processing by means of the Digital Fourier Transform (DFT) process and shown in Fig. 4.

From the experimental results it can be said that:

- 1) At constant current, with increasing frequency, arc gain increases. This may be related to the arc column temperature which is lower at low frequency due to the heat losses during pulsating periods. With increasing frequency the pulsating period becomes very short and the arc column temperature increases, resulting in an increase of the arc gain. From another view point, with increasing frequency the arc become more stable and the arc stiffness is also improved. With respect to the Kaufman stability criterion<sup>1)</sup> which states that for a stable point of arc operation

$$dv/di \ R > 0 \text{ or } R > |dv/di|$$

This means the arc gain will be increased to achieve good stability of the arc. On the other hand, with increasing current at same frequency, the values of gain decrease. This can be clearly understood due to the increasing number of charge carriers with increasing current.

- 2) At relatively low current, 100 A, the phase difference angle increases with increasing frequency, i.e. the voltage leads the current. While at moderate current, 150 A, the phase difference angle increases and then decreases with further increase of the frequency. On the other hand, at high currents over 200 A, the phase difference angle decreases with increasing frequency and reaches (negative) values, i.e. the current leads the voltage. These phenomena can be explained as follows:

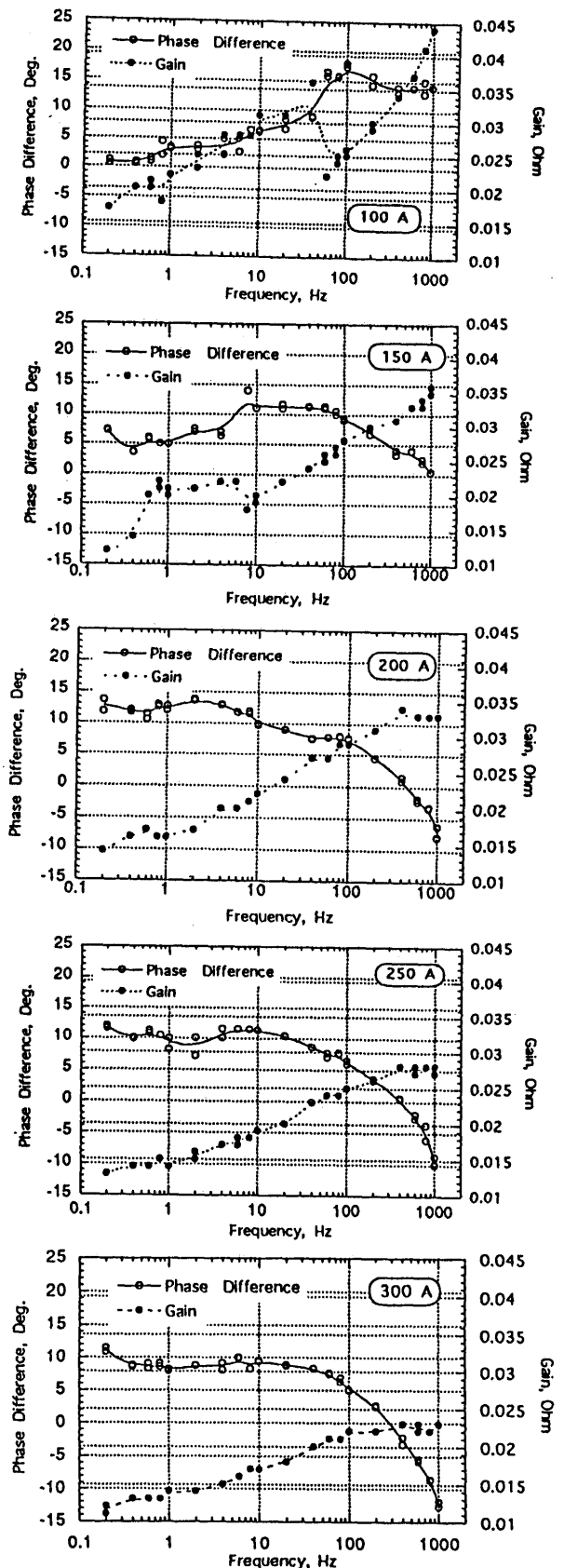


Fig.4 Phase difference angle and arc gain change criterion as a function of frequency at different average arc currents.

At low frequency, and according to the movement of the arc column from background pulse (small arc) to peak pulse (large arc), the emitting area changes. In other words, the number of emitted electrons changes as well as the current density as a function of current pulsation. Increasing current from background level to pulse level, leads to the overshoot of voltage occurring because of an increase in the number of current carriers. With a converse change of current, the arc voltage momentarily drops to a very low level which may be caused by sufficient ionization being achieved in the previous peak duration, or due to the increased energy dissipation from the arc column. Thus, the voltage always leads the current and the phase angle almost has positive values.

At high frequency the time of the background current period is shorter. Thus the above described instantaneous overshoot of voltage is no longer observed and the wave-form of arc voltage follows the current wave-form. The frequency at which the current wave-form leads the voltage wave-form is a function of average arc current, or in other words, the number of charge carriers, as shown in Fig. 4.

### 3.3 Effect of electrode diameter and tip angle

**Figure 5** shows the phase difference and arc gain versus the frequency as a function of electrode diameter and electrode tip angle, measured at 250 A with an arc gap of 3 mm. The electrode with 5 mm diameter and tip angle of  $90^\circ$  showed lower gain values, followed by the 5 mm electrode and tip angle of  $45^\circ$  and 3.2 mm electrode diameter with a tip angle of  $45^\circ$ , in that order. This may be attributed to the following two main causes:

- 1) The arc gain increases with increasing temperature as showed earlier. In this respect, the electrode with a 5 mm diameter and tip angle of  $90^\circ$  is running at lower temperatures than the other investigated electrodes due to its lower current density
- 2) At constant current, when a small diameter electrode is used, its resistance will be higher than the electrode with large diameter. Thus, the measured total gain values of the arc, in the case of small diameter electrodes, will be higher than in the case of large diameter electrodes.

On the other hand, changes in phase difference angle were observed at relatively low frequencies ( $<10$  Hz). Electrodes with 3.2 mm diameters and tip angles of  $45^\circ$  showed the lowest changes in phase difference angle followed by the electrodes with 5 mm diameters and tip angles of  $45^\circ$  and  $90^\circ$  in that order. However, at high frequencies ( $> 10$  Hz) these changes disappeared. These may be related to arc fluctuation during arc pulsation and the amount of energy dissipation. At low frequencies the

effect of energy dissipation represents the main cause of phase difference due to the relatively long period of pulsating time. However, at high frequencies this effect is not maintained because of the shorter period of pulsating time.

### 3.4 Effect of arc gap

**Figures 6 and 7** show the measured phase difference angles and arc gains as a function of arc gap at 150 A and 250 A respectively. Naturally, the gain increases with increasing arc gap and decreases with increasing arc current. As the changes in phase difference angle, at both arc currents are very small they can be considered to be within the experimental errors.

### 3.5 Electrical stability of the dynamic arc

Direct contact experiments (no arc) were carried out at 150 A and 250 A to explain qualitatively the electrical stability of the dynamic arc and in addition to have a clearer understanding of the dynamic characteristics of the arc and to clarify more precisely the reasons for changes in phase difference angles from positive to negative values. In other words to examine why the voltage leads current at low frequencies while at high frequencies current leads the voltage.

The same experimental setup, as shown in Fig.1, was used and the phase difference angle and gain were measured as a function of current frequencies. The results obtained are shown in **Fig.8**

Generally, the phase difference angle increases with increasing current pulsation frequency. The increase in phase difference angle for 150 A is smaller than that for 250 A. On the other hand, the gain values started to increase with frequency and then decreased after 100 Hz in case of 150 A and after 1 Hz in case of 250 A. These changes in gain values may be related to the change in the number of free electrons per cubic meter of the conductor (in this case the electrode is considered as the conductor), as a function of current and its pulsating period. With an increase in the number of free electrons the conductivity increases and gain decreases. Thus, it can be said that at 150 A, a frequency of 100 Hz is needed to reach this state, while in case of 250 A, frequency of 1 Hz is enough.

To explain the criterion of changes in phase difference angle, let the electrode be considered as a conductor carrying an electric current. Thus a magnetic field will be set up in the space surrounding the electric current. According to Coulomb's Law the interaction of this field with the second current is said to produce force between the two electric currents. In this respect, inductance will

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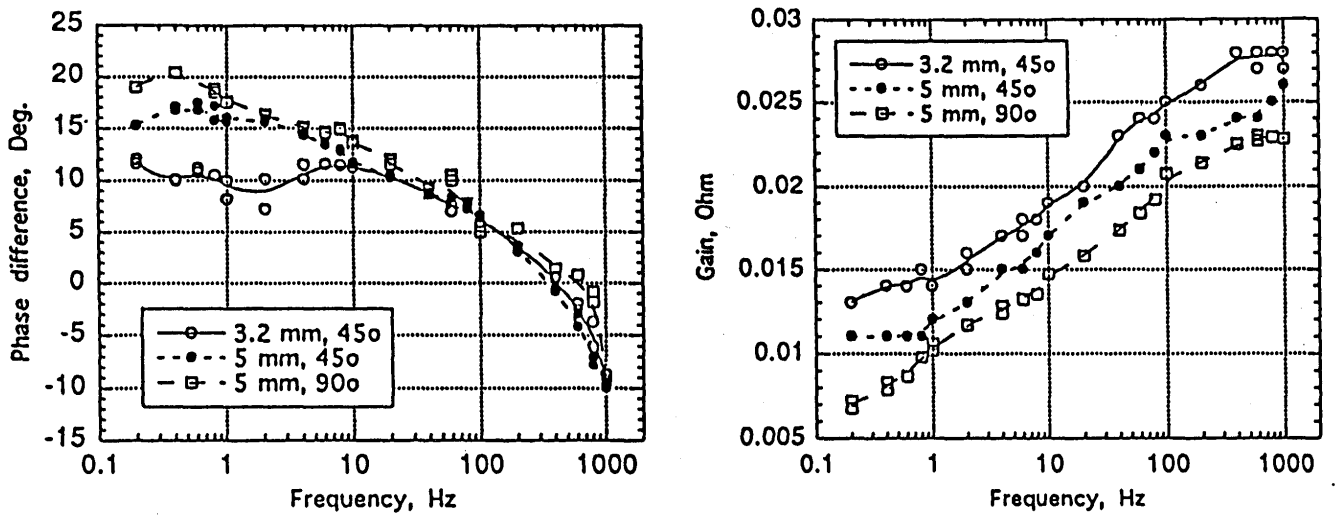


Fig.5 Effect of electrode diameter and tip angle on the phase difference angle and arc gain

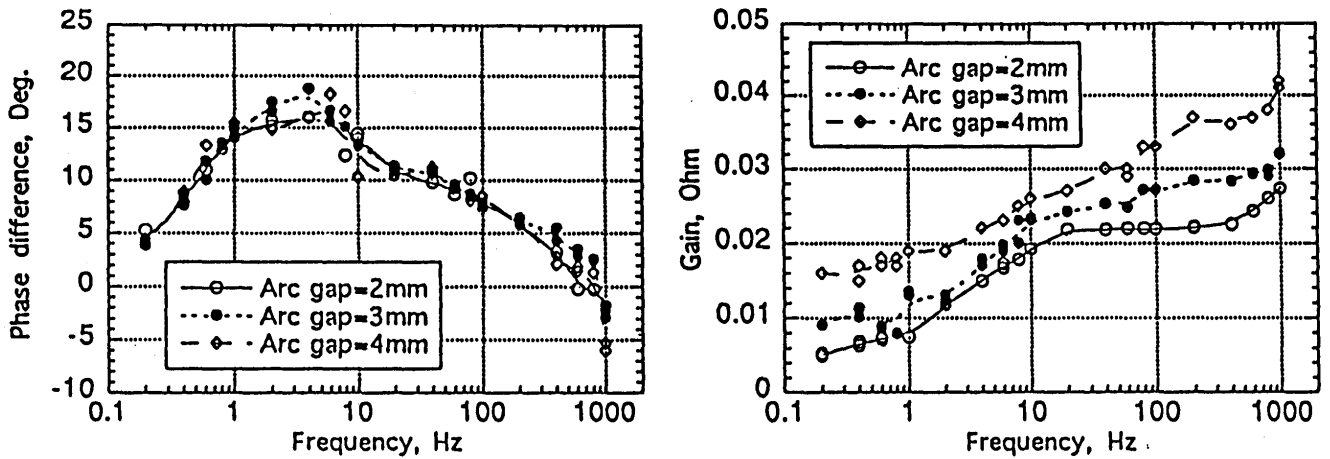


Fig.6 Effect of arc gap changes on the phase difference angle and arc gain at 150 A.

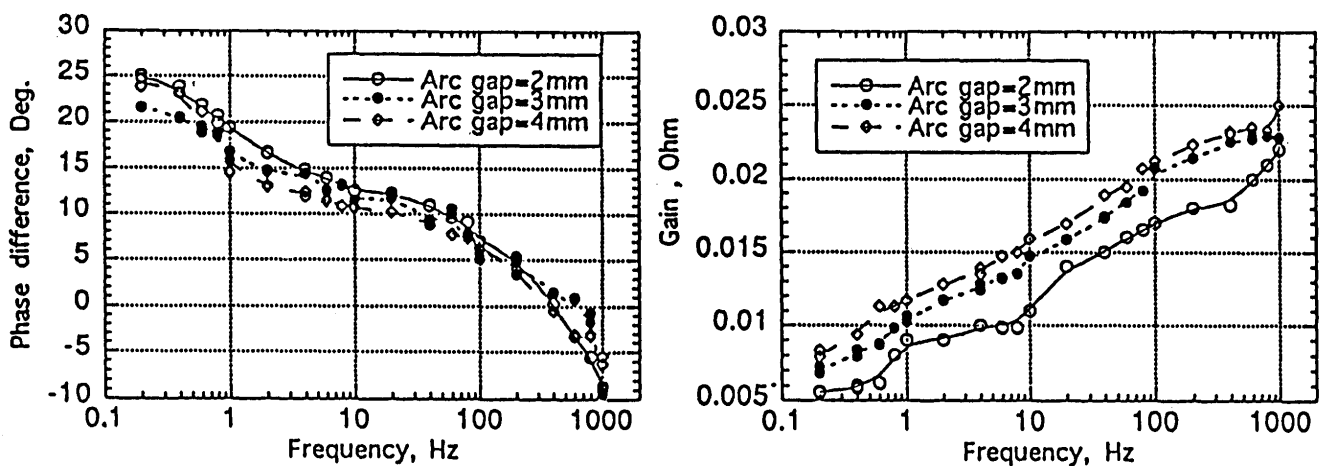


Fig.7 Effect of arc gap changes on the phase difference angle and arc gain at 250 A.

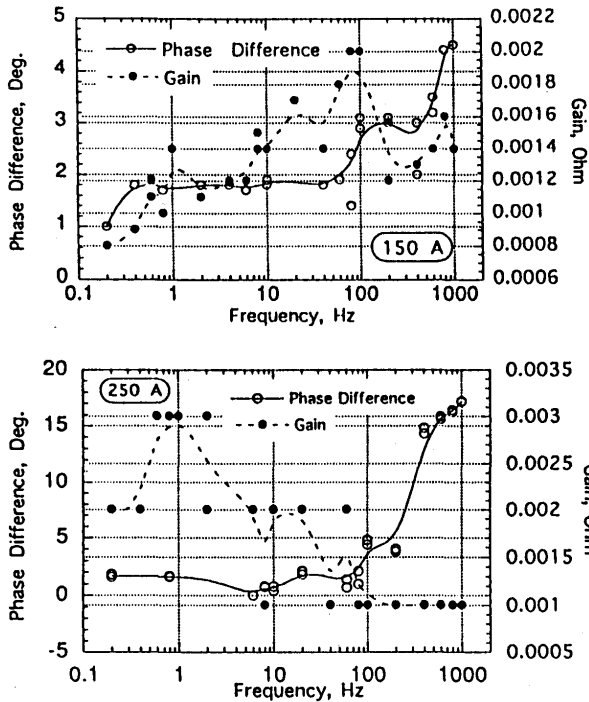


Fig.8 Phase difference and arc gain change behaviour as a function of frequency, measured at direct contact (no arc) circuit.

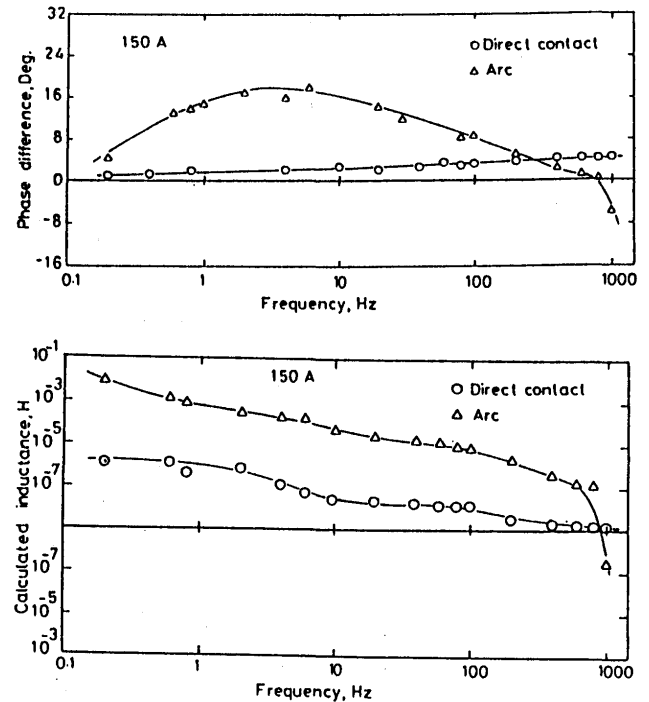


Fig.9 Calculated inductance and phase angle for direct contact circuit and arc circuit, as a function of frequency at 150 A.

be introduced in the circuit. From the experimental measurements, the inductance of the tungsten electrode ( $L_E$ ) can be calculated by using the following equation

$$L_E = R \tan \phi / \omega$$

Where

$R$  = measured gain

$\phi$  = measured phase difference angle

$\omega$  = radiant frequency

It is possible to run the same calculations under the arc conditions and calculate the total inductance ( $L_T$ ). The value  $L_T$  is a summation of both electrode inductance ( $L_E$ ) and arc inductance ( $L_A$ ). By using vector rules it become possible to calculate the arc inductance ( $L_A$ ). Figures 9 and 10 shows the calculation results for both inductance and the related phase difference angles as a function of frequency at 150 A and 250 A respectively.

From the experimental results and calculation, the following remarks became obvious:

- 1) The investigated circuit (direct contact circuit) became a highly inductive circuit with increasing frequency where gain became small and the phase angle increased (voltage leads the current). This result is in good agreement with the physical properties of inductors.
- 2) Also, the inductance values tend to decrease at high frequencies because of additional magnetic loss effects<sup>2)</sup>.
- 3) In case of the arc circuit, the situation is different, where, with increasing frequency the phase difference decreases and reaches a negative sign, indicating that the

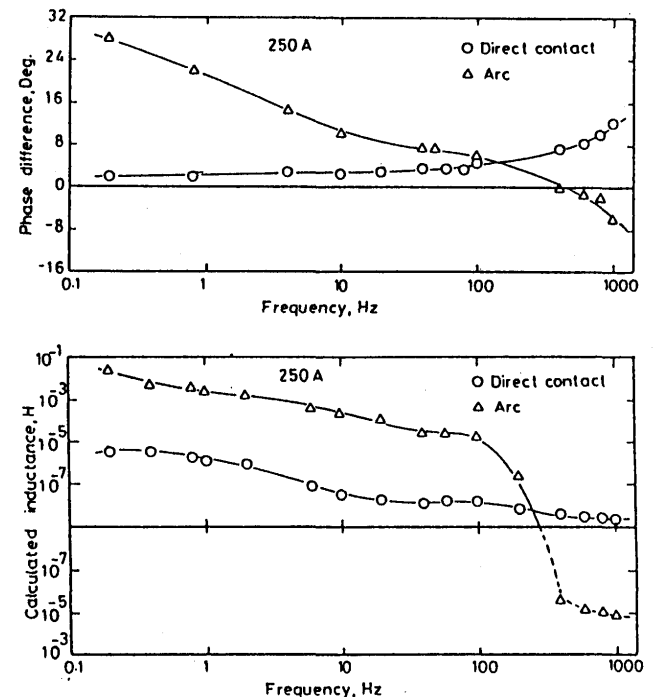


Fig.10 Calculated inductance and phase angle for direct contact circuit and arc circuit, as a function of frequency at 250 A.



current leads the voltage at high frequencies. Moreover, the calculated inductance decreases with increasing frequencies and changes its sign to negative as a function of phase angle, while the absolute value becomes higher (as shown as dotted line in the figures). These changes suggest that there is another effect occurring and appearing during arcing with increasing frequency. This may be related to the anode effect as will be discussed below.

These changes can be discussed clearly if the arc is considered as an electronic circuit. In this case the analysis of electronic circuits is facilitated by replacing all or part of a network by an equivalent circuit which for certain purposes, has the same characteristics as the original. According to the phase difference angle change criterion, it can be assumed that the arc circuit is equivalent to a circuit consists of inductors, resistors and capacitors. For simplicity, and by definition, the arc circuit can be represented on a macro scale level as inductors at the cathode and its voltage fall region at which the current is flowing, resistors at the arc column, and capacitors at the anode and its fall voltage region at which electric field is generated due to a point charge.

Now by considering the physical and electrical properties of each component <sup>(2-4)</sup> and comparing these with the experimental results of phase angle changes, the following suggested equivalent circuit can be introduced as a function of welding current:

## 1) At relatively low current (100 A)

According to the lower number of charge carriers, and the smallest anode region, the effect of capacitors (anode zone effect) is very small and can be neglected. The equivalent circuit in this case can be assumed to be a series connection of inductors and resistors. The frequency response is determined by letting  $\omega$  assume several values, three of which are shown schematically in Fig. 11. Note that the schematic illustration in Fig. 11 is for a pure LR circuit.

From Figs. 4 and 11, it can be clearly observed that this assumption is in good agreement with the experimental results from the view point of the phase angle change criterion.

## 2) At moderate current (250 A)

In this case the number of charge carriers is relatively high and their density increases with increasing frequency. Thus the anode effects (capacitors) will be greater with increasing frequency. The equivalent circuit in this case can be assumed to be a series connection LRC.

The phase difference angle of this circuit is the net result of the double actions of both inductance and capacitors as a function of frequency. At low frequency,

the inductance has the main effect, with increasing frequency, the inductance effect decreases and the capacitance will have the main effect on phase difference changes as shown schematically in Fig. 12.

From Figs. 4 and 12, it can be said that this assumption is in good agreement with the experimental results.

## 3) At high current (>200 A)

Here, the number of charge carriers is very high and also their density increases greatly with increasing frequency. Moreover, the arc root is much wider which means that the anode effect (capacitors effect) is effectively increasing with increasing frequency. Assuming a series LRC circuit as equivalent to the arc circuit and the phase plot method is used to determine the frequency response at four assumed values of frequencies. Taking into consideration that the effect of resistance is very small due to the higher conductivity of the arc, the natural frequency ( $\omega_n = 1/\sqrt{LC}$ ) is used instead of  $\omega$ .

The result is shown schematically in Fig. 13, which is also in good agreement with the experimental results from the view point of the phase angle change criterion.

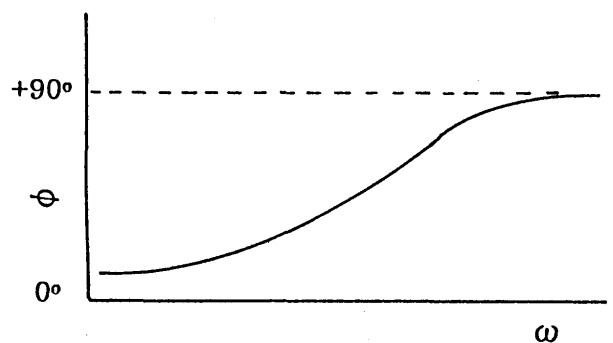


Fig. 11 Schematic plot for the corresponding phase angle changes as a function of frequency in the case of pure LR series circuit.

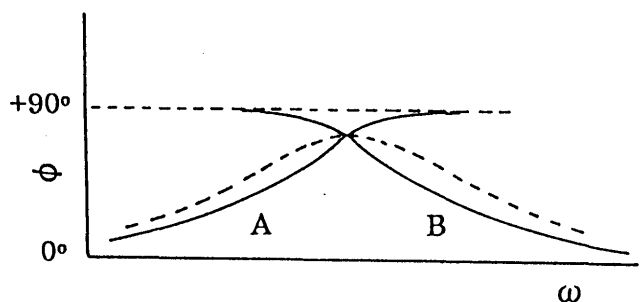


Fig. 12 Schematic plot for the corresponding phase angle changes as a function of frequency in the case of pure LRC series circuit.

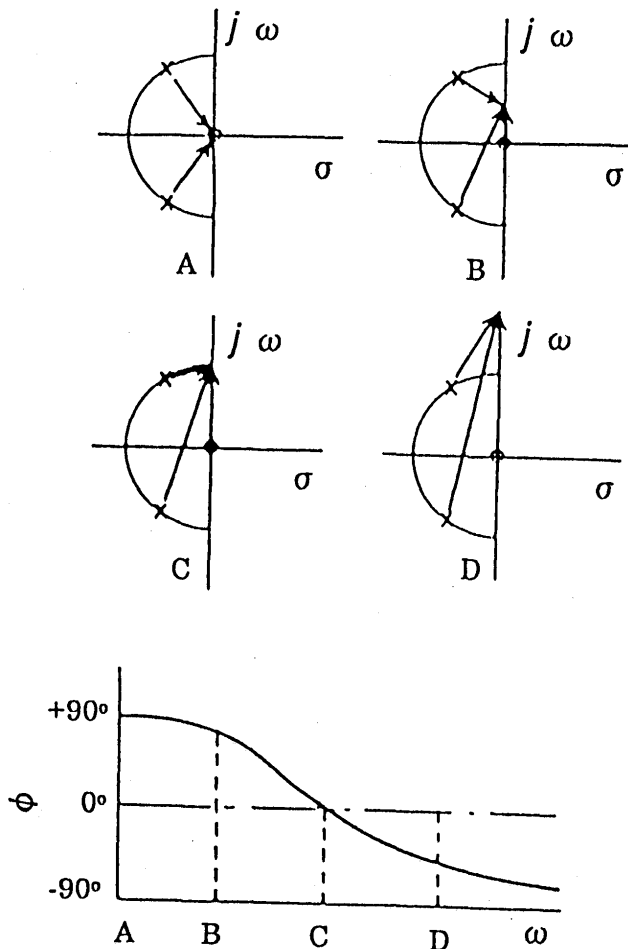


Fig.13 Phasors which determine the frequency response shown for different values of frequency.

### 3.6 Anode power measurements

Figure 14 shows the measured power at the anode as a function of current frequency at 159 A and 250 A. Generally, the increase in anode power (heat) with increasing current at the same frequency is attributed to the greater area of highly ionized gas in contact with the anode. On the other hand, at constant current, with increasing frequency the anode power increases. This can be explained by considering the energy supplied to the anode by the incident electrons which consists of :

- 1) Kinetic energy which, on the assumption of electrons in free fall through the anode fall voltage  $V_a$ , gives the power component  $IV_a$ , plus a small amount due to the thermal energy of the electrons; and
- 2) Potential energy, with each electron giving an energy of  $e\Phi$  to the anode.

It can be shown that for any pulse shape superimposed on the average DC value, the value of  $I_{pulse}$  is greater than the one for a steady value (DC) case. In fact,

$$I_{pulsing} \sim n \cdot I_{DC}$$

where  $n$  is the shape factor of the pulse and is 1.5 for full sinusoidal pulses<sup>3)</sup>. From this simple order of magnitude

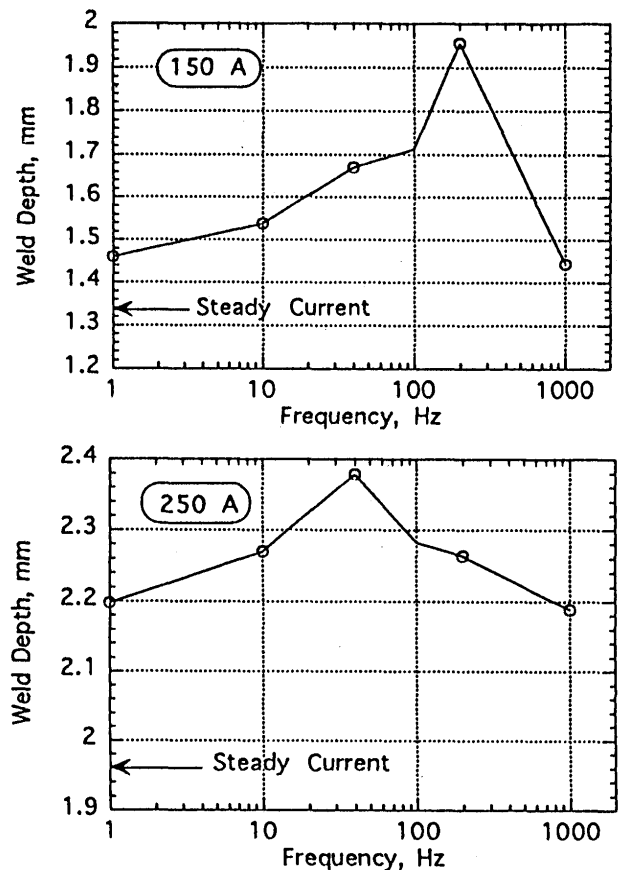


Fig.14 Measured power at copper cooled anode as a function of frequency at 150 A and 250 A.

argument, one can deduce that current pulsing increases the power component ( $IV_a$ ) at the anode.

However, with further increases of frequency the anode power (heat) decreases after about 200 Hz in the case of 150 A and about 40 Hz in the case of 250 A. This reflects the effect of the anode drop voltage zone on the total measured power at the anode. This phenomenon can be explained from two points of view.

First, from the view point of heat balance, for a plate anode the heat input (power) is that due to condensation of electrons plus the energy gained in passing through the anode drop zone, and heat conducted and radiated from the arc. These contributions are equal to the heat loss from the anode region by conduction and radiation. Thus the power at the anode can be represented as :

$$Q_A = Q_E + Q_C + Q_R - Q_L$$

where;

$Q_A$  = net energy at the anode

$Q_E$  = electron energy

$Q_C$  = energy gained by convection and conduction

$Q_R$  = energy gained by radiation

$Q_L$  = energy lost from the anode surface by conduction and radiation.

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At constant current, with increasing frequency the changes in the values of  $Q_C$  and  $Q_R$  are very small compared with the changes in the values of  $Q_E$ . With increasing frequency the plasma jet velocity increases, leading to an increase in the surface temperature of the anode which in turn decreases the value of  $Q_E$  ( $Q_E = 3K(T_e - T_a)/2 e$ ). In addition, the energy loss term will also increase with increasing plasma jet velocity. Finally, these will result in a decrease in the energy at the anode and hence the total measured energy as shown in Fig. 14.

The second point of view, if we consider the equivalent electronic circuit properties as discussed in the previous section, with increasing frequency the effect of capacitors (anode effect) increases, and this leads to decrease in the output voltage of the circuit. The effect of capacitors appears at some frequencies depending on the current.

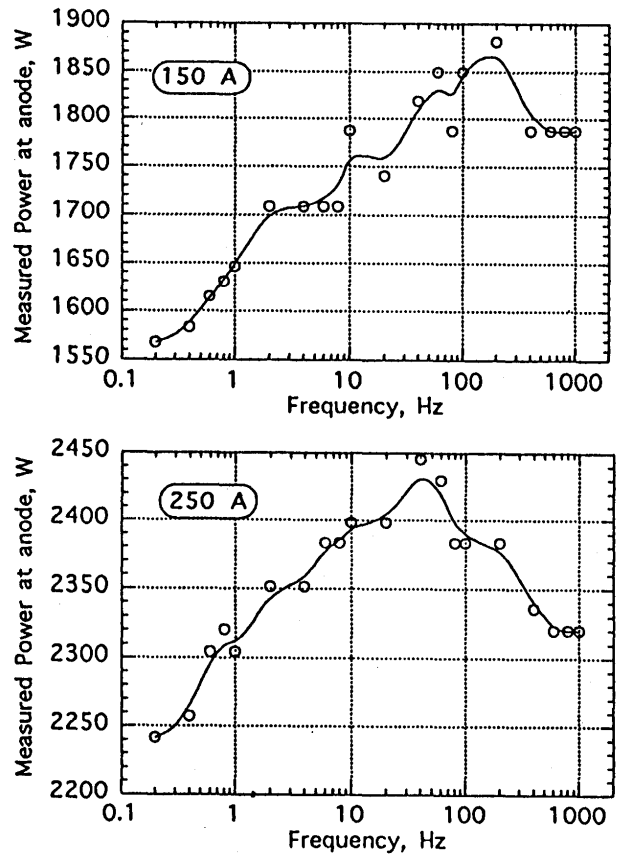
The weld pool depth was measured to achieve the above observed phenomenon of anode effect and the results are shown in the next section.

### 3.7 Weld pool depth

A series of tests was done to establish the effect of current pulsing and anode effect phenomenon on the weld pool depth. Carbon steel plates were melted by means of single bead on plate TIG welding through the desired range of frequencies. The dimensions of the carbon steel plate were 150 mm x 60 mm x 8 mm. The weld beads (100 mm) were cut transversally through the center (50 mm from the starting point) and specimens of 10 mm after the cutting line were taken. The specimens were ground, polished and macro etched by using nital 5%. The weld pool depth was measured by using a profile projector.

**Figure 15** shows the changes in weld pool depth as a function of current frequency at 150 A and 250 A and welding speed of 30 cm/min. The results are compared with those obtained at steady current. The results clearly show that the weld pool depths are higher than for those at the steady current level. Also, with increasing frequency the weld pool depth increases, due to the increase of electromagnetic body force inside the weld pool as well as the increase of the power delivered to the anode.

However, with further increases in frequency the weld pool depth reaches an optimum value and then decreases. These optimum values are coincident with the peak values of the measured power at the anode. These results clearly reveal the effect of the anode with increasing frequency.



**Fig. 15** Weld pool depth versus frequency for 3 mm arc pulsed sinusoidal at average currents of 150 A and 250 A.

## 4. Conclusions

A systematic investigation was conducted on the dynamic characteristics of the GTAW. Arc was supplied by full sinusoidal pulsed current over a range of frequencies.

The analysis of the experimental results allowed the following conclusions to be drawn;

- 1) The dynamic characteristics of voltage-current relationships are functions of current pulsating frequency. At low frequencies less than 10 Hz, the arc voltage changes between two levels corresponding to both peak current (maximum level) and base current (minimum level). The change of current from background level up to pulse level results in a decrease of voltage. However, with increasing frequency and, depending on the arc current, the average level of the arc voltage corresponding to both peak and base period is changed.
- 2) Instantaneous overshoot and undershoot peaks corresponding to the quick rise and fall of current were observed within the low and medium frequency ranges, and disappeared at high frequencies. These phenomena may be attributed to the changes in the number of ionized particles in the arc column with the current pulsating period.

3) The gain is increased with increasing frequency, at constant current due to increases in the arc temperature with frequency. However, with increasing current the gain decreases due to the increase of charge carriers.

4) The changes in phase difference angle are functions of welding current. At low current, with increasing frequency, the phase angle increases. At moderate current, the phase angle first increases and then decreases with increasing frequency. At high currents, the phase angle decreases with increasing frequency.

5) The electrode diameter and tip angle have an influence on the arc gain values. While for the phase difference angle this influence is only slightly observed at low frequencies, it disappears with increasing frequency.

6) The arc gain increases with increasing arc gap, while the phase angle criterion and values remain almost the same.

7) For GTAW arc modeling and from the analysis of the experimental results, the arc can be represented by an equivalent LRC electrical circuit connected in series. Depending on the arc current and frequency these

components react together influencing the measurement of total output voltage and phase difference. This assumption is in good agreement with the experimental results.

8) The measurements of the power delivered to the anode and weld pool reveal the great influence of anode effect with increasing frequency.

9) The results imply that with increasing frequency the electromagnetic stirring force in the weld pool grows and therefore, increases the weld pool depth independent of width. Thus, current pulsing can be used to control the weld pool geometry in GTAW.

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