Supplementary Material for "Industrial Technology Boundary, Product Quality Choice, and Market Segmentation"

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This supplementary material is organized as follows. Section 1 summarizes the best responses of firm 0 and firm 1. Section 2 provides supplementary details for Lemma 1, and a complete proof for the existence of a unique pure-strategy Nash equilibrium in Lemma 1 (1) when $\Delta^s \in (0, \underline{\Delta})$ and a unique pure-strategy Nash equilibrium in Lemma 1 (3) when $\Delta^s \in (\overline{\Delta}, 2t)$.

Note that the demand system is:

$$D_0(p_0,p_1)=\begin{cases} D_0^{00}(p_0,p_1)=1 & if\ p_1-p_0\in[t,\infty), \\ D_0^{0B}(p_0,p_1)=\frac{1}{2}\hat{x}^L(p_0,p_1)+\frac{1}{2} & if\ p_1-p_0\in[t-\Delta^s,t), \quad (1) \\ D_0^{BB}(p_0,p_1)=\frac{1}{2}\hat{x}^H(p_0,p_1)+\frac{1}{2}\hat{x}^L(p_0,p_1) & if\ p_1-p_0\in(-t,t-\Delta^s), \quad (2) \\ D_0^{B1}(p_0,p_1)=\frac{1}{2}\hat{x}^H(p_0,p_1) & if\ p_1-p_0\in(-t-\Delta^s,-t], \quad (3) \\ D_0^{11}(p_0,p_1)=0 & if\ p_1-p_0\in(-\infty,-t-\Delta^s], \end{cases}$$

and $D_1(p_0,p_1)=1-D_0(p_0,p_1)$.

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The locations of indifferent consumer for type-L and type-H consumers are:

$$\hat{x}^L(p_0, p_1) = \frac{p_1 - p_0 + t}{2t}; \quad \hat{x}^H(p_0, p_1) = \frac{p_1 - p_0 + t + \Delta^s}{2t}$$

1 Best Response

1.1 Best Response of Firm 0

In the first case when $D_0^{00}(p_0, p_1) = 1$, firm 0's profit $D_0^{00}(p_0, p_1) \cdot p_0 = p_0$ is strictly increasing in p_0, so it is optimal for firm 0 to set $p_0(p_1) = p_1 - t$. The last case when $D_0^{11}(p_0, p_1) = 0$ is never optimal for firm 0 since it always gets 0 profit. Next I focus on the three cases in between denoted as (1), (2) and (3).
To solve for the best response of firm 0, I consider the following profit-maximization problems of firm 0 in each of the three cases.

1. Case (1):

$$\begin{aligned} \max_{p_0} \quad & D_0^{0B}(p_0, p_1) \cdot p_0 = \frac{1}{2}(\hat{x}^L(p_0, p_1) + 1) \cdot p_0 = \frac{1}{2} p_0 (1 + \frac{p_1 - p_0 + t}{2t}) \\ \text{subject to} \quad & p_0 \in (p_1 - t, p_1 - t + \Delta^s], \\ & p_0 \geq 0. \end{aligned}$$

   First, I derive the interior solution by solving the first-order condition and obtain $p_0^{0B}(p_1) = \frac{1}{2}(p_1 + 3t)$, which exists when $p_1 \in (5t - 2\Delta^s, 5t)$. To derive the corner solutions when $p_1 \notin (5t - 2\Delta^s, 5t)$, I re-write the condition of case (1), $p_1 - p_0 \in [t - \Delta^s, t)$, as the condition on p_0, which is, $p_0 \in [p_1 - t, p_1 - t + \Delta^s)$.

Thus, by $p_1 \geq 0$, I summarize the candidates of best response under case (1),

$$p_0(p_1) = \begin{cases} p_1 - t + \Delta^s & if\ p_1 \in [0, 5t - 2\Delta^s], \\ p_0^{0B}(p_1) = \frac{1}{2}(p_1 + 3t) & if\ p_1 \in (5t - 2\Delta^s, 5t), \\ p_1 - t & if\ p_1 \in [5t, +\infty). \end{cases}$$

2. Case (2):

$$\max_{p_0} D_0^{BB}(p_0, p_1) \cdot p_0 = \frac{1}{2}(\hat{x}^L(p_0, p_1) + \hat{x}^H(p_0, p_1)) \cdot p_0 = \frac{1}{2}p_0(\frac{p_1 - p_0 + t}{2t} + \frac{p_1 - p_0 + t + \Delta^s}{2t})$$

$$\text{subject to} \quad p_0 \in (p_1 - t + \Delta^s, p_1 + t),$$
$$p_0 \geq 0.$$

Similarly, I can obtain interior solution $p_0^{BB}(p_1) = \frac{1}{4}(2p_1 + 2t + \Delta^s)$ solving the first-order condition. This interior solution exists when $p_1 \in (-t + \frac{1}{2}\Delta^s, 3t - \frac{3}{2}\Delta^s)$. Then I derive the corner solutions by the condition on p_0, $p_0 \in (p_1 - t + \Delta^s, p_1 + t)$. Thus, I summarize the candidates of best response under case (2),

$$p_0(p_1) = \begin{cases} p_1 + t & if\ p_1 \in [0, -t + \frac{1}{2}\Delta^s], \\ p_0^{BB}(p_1) = \frac{1}{2}(p_1 + t + \frac{1}{2}\Delta^s) & if\ p_1 \in (-t + \frac{1}{2}\Delta^s, 3t - \frac{3}{2}\Delta^s), \\ p_1 - t + \Delta^s & if\ p_1 \in [3t - \frac{3}{2}\Delta^s, +\infty). \end{cases}$$

By $p_1 \geq 0$ and my assumption that $0 < \Delta^s < 2t$, the candidates reduce to:

$$p_0(p_1) = \begin{cases} p_0^{BB}(p_1) = \frac{1}{2}(p_1 + t + \frac{1}{2}\Delta^s) & if\ p_1 \in [0, 3t - \frac{3}{2}\Delta^s), \\ p_1 - t + \Delta^s & if\ p_1 \in [3t - \frac{3}{2}\Delta^s, +\infty). \end{cases}$$

3. Case (3):

$$\max_{p_0} \quad D_0^{B1}(p_0,p_1)\cdot p_0 = \frac{1}{2}\hat{x}^H(p_0,p_1)\cdot p_0 = \frac{1}{2}p_0\cdot\frac{p_1-p_0+t}{2t}$$

$$\text{subject to} \quad p_0 \in [p_1+t, p_1+t+\Delta^s),$$

$$p_0 \geq 0.$$

Again, I first obtain the interior solution $p_0^{B1}(p_1) = \frac{1}{2}(p_1+t+\Delta^s)$ by solving the first-order condition, and it exists when $p_1 \in (-t-\Delta^s, -t+\Delta^s]$.

Since $-t+\Delta^s > -t+\frac{1}{2}\Delta^s$, the ranges of conditions on p_1 in cases (2) and (3) overlap. Therefore, when $p_1 \in (-t+\frac{1}{2}, -t+\Delta^s)$, firm 0 has to choose between the two functions in case (2) and case (3). To determine which function is firm 0's best response when $p_1 \in (-t+\frac{1}{2}, -t+\Delta^s)$, I compare firm 0's profits under case (2) and case (3): $\pi_0^{(2)}|_{p_0=\frac{1}{4}(2p_1+2t+\Delta^s)} = \frac{(2p_1+t+\Delta^s)^2}{32t}$ and $\pi_0^{(3)}|_{p_0=\frac{1}{2}(p_0+t+\Delta^s)} = \frac{(p_1+t+\Delta^s)^2}{16t}$.

By comparing the profits $\pi_0^{(2)}$ and $\pi_0^{(3)}$, I know that when $\Delta^s \in (0, \sqrt{2}t]$, $\pi_0^{(2)} \geq \pi_0^{(3)}$ for any $p_1 \geq 0$; when $\Delta^s \in (\sqrt{2}t, 2t]$, $\pi_0^{(2)} \geq \pi_0^{(3)}$ if $p_1 \geq -t+\frac{1}{\sqrt{2}}\Delta^s$. Note that $-t+\frac{1}{\sqrt{2}}\Delta^s > -t+\frac{1}{2}\Delta^s$, where $-t+\frac{1}{2}\Delta^s$ is the lower bound of case (2). Hence, firm 0 always obtains higher profit in case (2) than in case (3).

By $p_1 \geq 0$ and my assumption that $0 < \Delta^s < 2t$, $-t+\frac{1}{2}\Delta^s < 0$, and hence, firm 0's best response is $\frac{1}{4}(2p_1+2t+\Delta^s)$ if $p_1 \in (0, 3t-\frac{3}{2}\Delta^s]$. Therefore, I obtain the best response functions of firm 0:

$$BR_0(p_1) = \begin{cases} p_0^{BB}(p_1) = \frac{1}{2}(p_1+t+\frac{1}{2}\Delta^s) & if\ p_1 \in [0, 3t-\frac{3}{2}\Delta^s], \\ p_1 - t + \Delta^s, & if\ p_1 \in (3t-\frac{3}{2}\Delta^s, 5t-2\Delta^s] \\ p_0^{0B}(p_1) = \frac{1}{2}(p_1+3t) & if\ p_1 \in (5t-2\Delta^s, 5t], \\ p_1 - t & if\ p_1 \in (5t, +\infty). \end{cases}$$

1.2 Best Response of Firm 1

The first case when $D_1(p_0, p_1) = 1 - D_0^{00}(p_0, p_1) = 0$ is never optimal for firm 0 since it always gets 0 profit. In the last case when $D_1(p_0, p_1) = 1 - D_0^{11}(p_0, p_1) = 1$ firm 0's profit $1 \cdot p_1 = p_1$ is strictly increasing in p_1, so it is optimal for firm 1 to set $p_1(p_0) = p_0 - t - \Delta^s$.

Now, I solve the following profit-maximization problems of firm 1 under the three cases (1), (2) and (3).

1. Case (1):

$$\begin{aligned} \max_{p_1} \quad & D_1^{0B}(p_0, p_1) \cdot p_1 = \frac{1}{2}(1 - \hat{x}^L(p_0, p_1)) \cdot p_1 = \frac{1}{2}p_1(1 - \frac{p_1 - p_0 + t}{2t}) \\ \text{subject to} \quad & p_1 \in (p_0 + t - \Delta^s, p_0 + t], \\ & p_1 \geq 0. \end{aligned}$$

   First, I derive the interior solution by solving the first-order condition and obtain $p_1^{0B}(p_0) = \frac{1}{2}(p_0 + t)$, which exists when $(-t, -t + 2\Delta^s]$. To derive the corner solutions when $p_1 \notin (-t, -t + 2\Delta^s]$, I re-write the condition of case (1), $p_1 - p_0 \in [t - \Delta^s, t)$, as the condition on p_1, which is, $p_1 \in [p_0 + t - \Delta^s, p_0 + t)$. By $p_1 \geq 0$ and my assumption that $t \in (0, 1)$, I summarize the candidates of best response under case (1),

$$p_0(p_1) = \begin{cases} p_1^{0B}(p_0) = \frac{1}{2}(p_0 + t) & if \ p_0 \in [0, -t + 2\Delta^s] \\ p_0 + t - \Delta^s & if \ p_1 \in [-t + 2\Delta^s, +\infty). \end{cases}$$

2. Case (2):

$$\max_{p_1} \quad D_1^{BB}(p_0,p_1)\cdot p_1 = [1-\frac{1}{2}(\hat{x}^L(p_0,p_1)-\frac{1}{2}\hat{x}^H(p_0,p_1)]\cdot p_1 = p_1(1+\frac{1}{2}\frac{\Delta^s}{2t})$$
$$\text{subject to} \quad p_1 \in (p_0-t, p_0+t-\Delta^s),$$
$$p_1 \geq 0.$$

Next, I derive the interior solution by solving the first-order condition and obtain $p_1^{0B}(p_0) = \frac{1}{4}(2p_0+2t-\Delta^s)$, which exists when $(-t+\frac{3}{2}\Delta^s, 3t-\frac{1}{2}\Delta^s]$. To derive the corner solutions when $p_1 \notin (-t+\frac{3}{2}\Delta^s, 3t-\frac{1}{2}\Delta^s]$, I re-write the condition of case (2), $p_1-p_0 \in (-t, t-\Delta^s)$, as the condition on p_1, which is, $p_1 \in [p_0-t, p_0+t-\Delta^s)$. By $p_1 \geq 0$ and my assumption that $t>0$, I summarize the candidates of best response under case (2),

$$p_0(p_1) = \begin{cases} p_0+t-\Delta^s & if\ p_1 \in [0, -t+\frac{3}{2}\Delta^s] \\ p_1^{BB}(p_0) = \frac{1}{2}(p_0+t-\frac{1}{2}\Delta^s) & if\ p_1 \in (-t+\frac{3}{2}\Delta^s, 3t-\frac{1}{2}\Delta^s] \\ p_0-t & if\ p_1 \in (3t-\frac{1}{2}\Delta^s, +\infty). \end{cases}$$

3. Case (3):

$$\max_{p_1} \quad D_1^{B1}(p_0,p_1)\cdot p_1 = [1-\frac{1}{2}\hat{x}^H(p_0,p_1)]\cdot p_1 = p_1\cdot(1-\frac{1}{2}\cdot\frac{p_1-p_0+t+\Delta^s}{2t})$$
$$\text{subject to} \quad p_1 \in (p_0-t-\Delta^s, p_0-t],$$
$$p_1 \geq 0.$$

Now I derive the interior solution by solving the first-order condition and obtain $p_1^{B1}(p_0) = \frac{1}{2}(p_0+3t-\Delta^s)$, which exists when $(5t-\Delta^s, 5t+\Delta^s]$. To derive the corner solutions when $p_1 \notin (5t-\Delta^s, 5t+\Delta^s]$, I re-write the condition of case (3), $p_1-p_0 \in (-t-\Delta^s, -t]$, as the condition on p_1, which is, $p_1 \in (p_0-t-\Delta^s, p_0-t]$.

By $p_1 \geq 0$, I summarize the candidates of best response under case (1),

$$p_0(p_1) = \begin{cases} p_0 - t & if\ p_1 \in [0, 5t - \Delta^s] \\ p_1^{B1}(p_0) = \frac{1}{2}(p_0 + 3t - \Delta^s) & if\ p_1 \in (5t - \Delta^s, 5t + \Delta^s] \\ p_0 - t - \Delta^s & if\ p_1 \in (5t + \Delta^s, +\infty). \end{cases}$$

Since $-t + 2\Delta^s > -t + \frac{3}{2}\Delta^s$, conditions on p_0 in case (1) and case (2) overlap. When $p_0 \in (-t + \frac{3}{2}\Delta^s, -t + 2\Delta^s)$, it is optimal for firm 1 to set the price in case (1) because $\pi_1^{(1)}|_{p_1 = \frac{1}{2}(p_0+t)} = \frac{(p_0+t)^2}{16t} > \frac{(\Delta^s - 2p_0 - 2t)^2}{32t} = \pi_1^{(2)}|_{p_1 = \frac{1}{4}(2p_0 + 2t - \Delta^s)}$ for any $t \in (0, 1)$, $\Delta^s \in (0, 2t)$ and $p_0 \geq 0$. Also note that, by my assumption that $0 < \Delta^s < 2t$, I must have $3t - \frac{1}{2}\Delta^s < 5t - \Delta^s$. Thus, I summarize the following best response of firm 1:

$$BR_1(p_0) = \begin{cases} p_1^{0B}(p_0) = \frac{1}{2}(p_0 + t) & if\ p_0 \in [0, -t + 2\Delta^s], \\ p_1^{BB}(p_0) = \frac{1}{2}(p_0 + t - \frac{1}{2}\Delta^s) & if\ p_0 \in (-t + 2\Delta^s, 3t - \frac{1}{2}\Delta^s], \\ p_0 - t & if\ p_0 \in (3t - \frac{1}{2}\Delta^s, 5t - \Delta^s], \\ p_1^{B1}(p_0) = \frac{1}{2}(p_0 + 3t - \Delta^s) & if\ p_0 \in (5t - \Delta^s, 5t + \Delta^s], \\ p_0 - t - \Delta^s & if\ p_0 \in (5t + \Delta^s, +\infty). \end{cases}$$

2 Supplementary Details for Lemma 1

Based on indicators h, l, which denote whether group H, L is supplied by firm 0 and firm 1, there are 16 equilibrium candidates: NN, $0N$, $N0$, $1N$, $N1$, NB, BN, $1B$, $B0$, 10, $B1$, 00, 11, 01, BB, and $0B$. Candidates NN, $0N$, $N0$, $1N$, $N1$, NB and BN do not exist because I assumed both consumer groups are always fully covered. Now I show that candidates $1B$, $B0$, 10, $B1$, 00, 11, 01 do not exist either.

[1B]: The $1B$ equilibrium requires that $\hat{x}^H(p_0,p_1) \leq 0$ and $\hat{x}^L(p_0,p_1) \in (0,1)$, which respectively implies that $p_1 - p_0 \in (-\infty, -t-\Delta^s]$ and $p_1 - p_0 \in (-t,t)$. However, $p_1 - p_0 \in (-\infty, -t-\Delta^s] \cap (-t,t) = \emptyset$.

[B0]: The $B0$ equilibrium requires that $\hat{x}^H(p_0,p_1) \in (0,1)$ and $\hat{x}^L(p_0,p_1) \geq 1$, which respectively implies that $p_1 - p_0 \in (-t-\Delta^s, t-\Delta^s)$ and $p_1 - p_0 \in [t,\infty)$. However, $p_1 - p_0 \in (-t-\Delta^s, t-\Delta^s) \cap [t,\infty) = \emptyset$.

[10]: The 10 equilibrium requires that $\hat{x}^H(p_0,p_1) \leq 0$ and $\hat{x}^L(p_0,p_1) \geq 1$, which respectively implies that $p_1 - p_0 \in (-\infty, -t-\Delta^s]$ and $p_1 - p_0 \in [t,\infty) = \emptyset$. However, $p_1 - p_0 \in (-\infty, -t-\Delta^s] \cap [t,\infty) = \emptyset$.

[B1]: The $B1$ equilibrium requires that $\hat{x}^H(p_0,p_1) \in (0,1)$ and $\hat{x}^L(p_0,p_1) \leq 0$, which respectively implies that $p_1 - p_0 \in (-t-\Delta^s, t-\Delta^s)$ and $p_1 - p_0 \in (-\infty, -t]$. Then I have $p_1 - p_0 \in (-t-\Delta^s, t-\Delta^s) \cap (-\infty, -t]$. Therefore, the firms solve

$$\max\nolimits_{p_0} \pi_0^{B1}(p_0,p_1) = \frac{1}{2}\hat{x}^H(p_0,p_1)p_0, \ \max_{p_1} \pi_1^{B1}(p_0,p_1) = \left(\frac{1}{2}(1-\hat{x}^H(p_0,p_1)) + \frac{1}{2}\right)p_1$$

subject to $p_1 - p_0 \in (-t-\Delta^s, t-\Delta^s) \cap (-\infty, -t]$.

By solving the maximization problem, I have $(p_0^{*B1}, p_1^{*B1}) = (\frac{5t+\Delta^s}{3}, \frac{7t-\Delta^s}{3})$, and $(\pi_0^{*B1}, \pi_1^{*B1}) = (\frac{(5t+\Delta^s)^2}{36t}, \frac{(7t-\Delta^s)^2}{36t})$. Substituting p_1^{*B1} and p_0^{*B1} into the constraint $p_1 - p_0 \in (-t-\Delta^s, t-\Delta^s) \cap (-\infty, -t]$, I have $\Delta^s \in (-5t, t) \cap [\frac{5t}{2}, \infty) = \emptyset$.

[00]: The 00 equilibrium requires that $\hat{x}^H(p_0,p_1) \geq 1$ and $\hat{x}^L(p_0,p_1) \geq 1$, from which I have $p_1 - p_0 \in [t-\Delta^s, \infty) \cap [t,\infty) = [t,\infty)$. Suppose on the contrary there exist equilibrium prices $\{p_0^{*00}, p_1^{*00}\}$ constituting the 00 equilibrium. Then, firm 0 must have set the highest possible price under the constraint $p_1 - p_0 \in [t,\infty)$ given the rival's equilibrium price, i.e., $p_0^{*00} = p_1^{*00} - t$. At this equilibrium, $\pi_1^{*00} = 0$. However, given $p_0^{*00} = p_1^{*00} - t$, firm 1 can always obtain a positive profit by deviating to set price $p_1'^{00} = p_1^{*00} - \epsilon$, where ϵ is an infinitesimally small positive integer[1], which contradicts to p_1^{*00} being the equilibrium price.

[11]: The 11 equilibrium requires that $\hat{x}^H(p_0, p_1) \leq 0$ and $\hat{x}^L(p_0, p_1) \leq 0$, from which I have $p_1 - p_0 \in (-\infty, -t - \Delta^s] \cap (-\infty, -t] = (-\infty, -t - \Delta^s]$. Suppose there exist prices $\{p_0^{*11}, p_1^{*11}\}$ constituting the 00 equilibrium. Then, firm 1 must have set the highest possible price under the constraint $p_1 - p_0 \in (-\infty, -t - \Delta^s]$ given the rival's equilibrium price, i.e., $p_1^{*11} = p_0^{*11} - t - \Delta^s$. At this equilibrium, $\pi_0^{*11} = 0$. However, given $p_1^{*11} = p_0^{*11} - t - \Delta^s$, firm 0 can always obtain a positive profit by deviating to set price $p_0'^{11} = p_0^{*11} - \epsilon$, where ϵ is an infinitesimally small positive integer[2], which is a contradiction.

[01]: The 01 equilibrium requires that $\hat{x}^H(p_0, p_1) \geq 1$ and $\hat{x}^L(p_0, p_1) \leq 0$, from which I have $p_1 - p_0 \in [t - \Delta^s, \infty) \cap (-\infty, -t] = [t - \Delta^s, -t]$. Suppose there exist $\{p_0^{*01}, p_1^{*01}\}$ constituting the 01 equilibrium. Then, firm 1 must have set the highest possible price under the constraint $p_1 - p_0 \in [t - \Delta^s, -t]$ given the rival's equilibrium price, i.e., $p_1^{*01} = p_0^{*01} - t$. However, given $p_1^{*01} = p_0^{*01} - t$, firm 0 can always obtain a higher profit by deviating to set $p_0'^{01} = p_0^{*01} + \epsilon$, where ϵ is an infinitesimally small positive integer[3], which is a contradiction.

[1]In this deviation, firm 1's demand becomes positive because $\hat{x}^L(p_0^{*00}, p_1'^{00}) = \frac{1}{2} + \frac{t-\epsilon}{2t} < 1$

[2]In this deviation, firm 0's demand becomes positive because $\hat{x}^H(p_0'^{11}, p_1^{*11}) = \frac{1}{2} - \frac{t-\epsilon}{2t} > 0$.

[3]Since $p_1^{*01} - p_0'^{01} = -t - \epsilon \in [t - \Delta^s, -t]$, this deviation does not change the demand faced by firm 0. Therefore, firm 0's profit increases by ϵ upon deviation.

2.1 Supplementary Details for Lemma 1 (1)

In the BB equilibrium, firms solve

$$\max_{p_0} \pi_0^{BB}(p_0, p_1) = \left(\frac{1}{2}\hat{x}^H(p_0, p_1) + \frac{1}{2}\hat{x}^L(p_0, p_1)\right) p_0,$$
$$\max_{p_1} \pi_1^{BB}(p_0, p_1) = \left(\frac{1}{2}(1 - \hat{x}^H(p_0, p_1)) + \frac{1}{2}(1 - \hat{x}^L(p_0, p_1))\right) p_1$$

subject to $p_1^{*BB} - p_0^{*BB} \in (-t, t - \Delta^s)$. Solving the above two maximization problems, I have $(p_0^{*BB}, p_1^{*BB}) = (\frac{6t+\Delta^s}{6}, \frac{6t-\Delta^s}{6})$ and $(\pi_0^{*BB}, \pi_1^{*BB}) = (\frac{(6t+\Delta^s)^2}{72t}, \frac{(6t-\Delta^s)^2}{72t})$. Substituting (p_0^{*BB}, p_1^{*BB}) into the constraint yields

$$0 \leq \Delta^s < \frac{3}{2}t \quad \text{(existence condition).}$$

2.1.1 Firm 0's deviation incentives:

[**11**]: Firm 0 will never deviate by inducing 11; otherwise, its profit becomes zero.

[**0B**]: Given $p_1 = p_1^{*BB}$, if firm 0 deviates by inducing 0B, it chooses the deviation price $p_0'^{0B}$ by solving

$$\max_{p_0} \pi_0^{0B}(p_0, p_1^{*BB}) = \left(\frac{1}{2} + \frac{1}{2}\hat{x}^L(p_0, p_1^{*BB})\right) p_0$$
$$s.t.\ t - \Delta^s \leq p_1^{*BB} - p_0 < t \Longleftrightarrow -\frac{\Delta^s}{6} < p_0 \leq \frac{5\Delta^s}{6}.$$

From the first-order condition, I have the deviation price and profit

$$p_0'^{0B} = \frac{24t - \Delta^s}{12},\ \pi_0'^{0B} = \frac{(24t - \Delta^s)^2}{576t}.$$

I confirm that $p_0'^{0B} > \frac{5\Delta^s}{6}$, meaning that $\pi_0^{0B}(p_0, p_1^{*BB})$ increases in p_0 for $p_0 \in (\frac{\Delta^s}{6}, \frac{5\Delta^s}{6}]$. Therefore, firm 0's optimal deviation profit is $\pi_0^{0B}(\frac{5\Delta^s}{6}, p_1^{*BB}) = \frac{5\Delta^s(4t-\Delta^s)}{24t}$, which is always weakly less than π_0^{*BB}. Therefore, firm 0 never deviates to induce 0B.

[B1]: Given $p_1 = p_1^{*BB}$, if firm 0 deviates by inducing B1, it chooses the deviation price $p_0'^{B1}$ by solving

$$\max_{p_0} \pi_0^{B1}(p_0, p_1^{*BB}) = \frac{1}{2}\hat{x}^H(p_0, p_1^{*BB})p_0$$
$$s.t. \; -t-\Delta^s < p_1^{*BB} - p_0 \leq -t \Longleftrightarrow \frac{12t-\Delta^s}{6} \leq p_0 < \frac{12t+5\Delta^s}{6}.$$

From the first-order condition, I have the deviation price and profit

$$p_0'^{B1} = \frac{12t+5\Delta^s}{12}, \; \pi_0'^{B1} = \frac{(12t+5\Delta^s)^2}{576t}.$$

I confirm that $p_0'^{B1} < \frac{12t+5\Delta^s}{6}$ always holds. Moreover, $p_0'^{B1} \geq \frac{12t-\Delta^s}{6}$ if $\Delta^s \geq \frac{12t}{7}$. Then, the optimization system has an interior solution $p_0 = p_0'^{B1}$ when $\Delta^s \in [\frac{12t}{7}, 2t)$, which leads to a deviation profit $\pi_0'^{B1}$ strictly less than π_0^{*BB}. Moreover, when $\Delta^s \in [0, \frac{12t}{7})$, since $p_0'^{B1} < \frac{12t-\Delta^s}{6}$, $\pi_0^{B1}(p_0, p_1^{*BB})$ decreases in p_0 when $p_0 \in [\frac{12t-\Delta^s}{6}, \frac{12t+5\Delta^s}{6})$. Then, firm 0's optimal deviation profit is $\pi_0^{B1}(\frac{12t-\Delta^s}{6}, p_1^{*BB}) = \frac{\Delta^s(12t-\Delta^s)}{24t}$, which is strictly less than π_0^{*BB}. Therefore, firm 0 never deviates to induce B1.

[00]: Given $p_1 = p_1^{*BB}$, if firm 0 deviates by inducing 00, it chooses the deviation price $p_0'^{00}$ such that the following condition is satisfied:

$$p_1^{*BB} - p_0'^{00} \in [t, \infty) \Longleftrightarrow p_0'^{00} \leq -\frac{\Delta^s}{6}.$$

Since the price must be nonnegative, firm 0 never deviates to induce 00.

2.1.2 Firm 1's deviation incentives:

[00]: Firm 1 will never deviate by inducing 00; otherwise, its profit becomes zero.

[**0B**]: Given $p_0 = p_0^{*BB}$, if firm 1 deviates by inducing 0B, it chooses the deviation price $p_1'^{0B}$ by solving

$$\max_{p_1} \pi_1^{0B}(p_0^{*BB}, p_1) = \frac{1}{2}\left(1 - \hat{x}^L(p_0^{*BB}, p_1)\right) p_1$$
$$s.t.\ t - \Delta^s \le p_1 - p_0^{*BB} < t \Longleftrightarrow \frac{12t - 5\Delta^s}{6} \le p_1 < \frac{12t + \Delta^s}{6}.$$

From the first-order condition, I have the deviation price and profit

$$p_1'^{0B} = \frac{12t + \Delta^s}{12},\ \pi_1'^{0B} = \frac{(12t + \Delta^s)^2}{576t}.$$

I confirm that $p_1'^{0B} < \frac{12t+\Delta^s}{6}$ always holds. Moreover, $p_1'^{0B} \ge \frac{12t-5\Delta^s}{6}$ if $\Delta^s \ge \frac{12t}{11}$. Then, the optimization system has an interior solution $p_1 = p_1'^{0B}$ when $\Delta^s \in [\frac{12t}{11}, 2t)$, which leads to a deviation profit $\pi_1'^{0B}$ strictly less than π_1^{*BB}. Moreover, when $\Delta^s \in [0, \frac{12t}{11})$, since $p_1'^{0B} < \frac{12t-5\Delta^s}{6}$, $\pi_1^{0B}(p_0^{*BB}, p_1)$ decreases in p_1 for $p_1 \in [\frac{12t-5\Delta^s}{6}, \frac{12t+\Delta^s}{6})$. Then, firm 1's optimal deviation profit is $\pi_1^{0B}(p_0^{*0B}, \frac{12t-5\Delta^s}{6}) = \frac{\Delta^s(12t-5\Delta^s)}{24t}$, which is Iakly less than π_1^{*BB} if

$$0 \le \Delta^s \le \underline{\Delta}. \quad \text{(no-deviation condition)}$$

[**B1**]: Given $p_0 = p_0^{*BB}$, if firm 1 deviates by inducing B1, it chooses the deviation price $p_1'^{B1}$ by solving

$$\max_{p_1} \pi_1^{B1}(p_0^{*BB}, p_1) = \left(\frac{1}{2}(1 - \hat{x}^H(p_0, p_1^{*BB})) + \frac{1}{2}\right) p_1$$
$$s.t.\ -t - \Delta^s < p_1 - p_0^{*BB} \le -t \Longleftrightarrow -\frac{5\Delta^s}{6} < p_1 \le \frac{\Delta^s}{6}.$$

From the first-order condition, I have the deviation price and profit

$$p_1'^{B1} = \frac{24t - 5\Delta^s}{12},\ \pi_1'^{B1} = \frac{(24t - 5\Delta^s)^2}{576t}.$$

I can confirm that $p_1'^{B1} > \frac{\Delta^s}{6}$ always holds. Therefore, $\pi_1^{B1}(p_0^{*BB}, p_1)$ increases in p_1 for $p_1 \in (\frac{-5\Delta^s}{6}, \frac{\Delta^s}{6}]$. Then, firm 1's optimal deviation profit is $\pi_1^{B1}(p_0^{*BB}, \frac{\Delta^s}{6}) = \frac{\Delta^s(4t-\Delta^s)}{24t}$, which is strictly less than π_1^{*BB}. Therefore, firm 1 never deviates to induce B1.

[11]: Given $p_0 = p_0^{*BB}$, if firm 1 deviates by inducing 11, it chooses the deviation price $p_1'^{11}$ such that the following condition is satisfied:

$$p_1'^{11} - p_0^{*BB} \in (-\infty, -t - \Delta^s] \Longleftrightarrow p_1'^{11} \leq -\frac{5\Delta^s}{6}.$$

Since the price must be nonnegative, firm 1 never deviates to induce 11.

Summary of Lemma 1 (1): If the existence condition and no-deviation condition are simultaneously satisfied, i.e., $0 \leq \Delta^s \leq \underline{\Delta}$, then the BB equilibrium exists.

2.2 Supplementary Details for Lemma 1 (3)

In the 0B equilibrium, firms solve

$$\max_{p_0} \pi_0^{0B}(p_0, p_1) = \left(\frac{1}{2} + \frac{1}{2}\hat{x}^L(p_0, p_1)\right) p_0,$$
$$\max_{p_1} \pi_0^{0B}(p_0, p_1) = \frac{1}{2}(1 - \hat{x}^L(p_0, p_1))p_1$$

subject to $p_1^{*0B} - p_0^{*0B} \in [t - \Delta^s, t)$. By solving the problems, I have $(p_0^{*0B}, p_1^{*0B}) = (\frac{7t}{3}, \overline{\Delta})$, and $(\pi_0^{*0B}, \pi_1^{*0B}) = (\frac{49t}{36}, \frac{25t}{36})$. Substituting (p_0^{*0B}, p_1^{*0B}) into the constraint yields

$$\overline{\Delta} \leq \Delta^s < 2t \quad \text{(existence condition)}.$$

2.2.1 Firm 0's deviation incentives:

[11]: Firm 0 will never deviate by inducing 11; otherwise, its profit becomes zero.

[BB]: Given $p_1 = p_1^{*0B}$, if firm 0 deviates by inducing BB, it chooses the deviation price $p_0'^{BB}$ by solving

$$\begin{aligned} &\max_{p_0} \pi_0^{BB}(p_0, p_1^{*0B}) = \left(\frac{1}{2}\hat{x}^H(p_0, p_1^{*0B}) + \frac{1}{2}\hat{x}^L(p_0, p_1^{*0B})\right) p_0 \\ &s.t. \; -t < p_1^{*0B} - p_0 < t - \Delta^s \Longleftrightarrow \frac{2t+3\Delta^s}{3} < p_0 < \frac{8t}{3} \; and \; \Delta^s < 2t. \end{aligned}$$

From the first-order condition, I have the deviation price and profit

$$p_0'^{BB} = \frac{16t + 3\Delta^s}{12}, \; \pi_0'^{BB} = \frac{(16t+3\Delta^s)^2}{288t}.$$

I can confirm that $p_0'^{BB} < \frac{8t}{3}$ always holds. Moreover, $p_0'^{BB} > \frac{2t+3\Delta^s}{3}$ if $\Delta^s < \frac{8t}{9}$. Then, the optimization system has an interior solution $p_0 = p_0'^{0B}$ when $\Delta^s < \frac{8t}{9}$, which leads to a deviation profit $\pi_0'^{BB}$ strictly less than π_0^{*0B}. Moreover, when $\Delta^s \geq \frac{8t}{9}$, since $p_0'^{BB} \leq \frac{2t+3\Delta^s}{3}$, $\pi_0^{BB}(p_0, p_1^{*0B})$ decreases in p_0 for $p_0 \in (\frac{2t+3\Delta^s}{3}, \frac{8t}{3})$. Then, firm 0's optimal deviation profit is strictly less than $\pi_0^{0B}(\frac{2t+3\Delta^s}{3}, p_1^{*0B})$, is strictly less than π_0^{*0B}.[4] Therefore, firm 0 never deviates to induce BB.

[01]: Notice first that this case exists if and only if $x^H(p0, p1) \geq 0$ and $x^L(p0, p1) \leq 1$, from which I have $p1 - p0 \in [t - \Delta^s, -t]$. Since I have assumed $0 \leq \Delta^s < 2t$, this deviation never happens.

[B1]: Given $p_1 = p_1^{*0B}$, if firm 0 deviates to B1, it chooses the deviation price $p_0'^{B1}$ by solving

$$\begin{aligned} &\max_{p_0} \pi_0^{B1}(p_0, p_1^{*BB}) = \frac{1}{2}\hat{x}^H(p_0, p_1^{*BB})p_0 \\ &s.t. \; -t - \Delta^s < p_1^{*0B} - p_0 \leq -t \Longleftrightarrow \frac{8t}{3} \leq p_0 < \frac{8t+3\Delta^s}{3}. \end{aligned}$$

From the first-order condition, I have the deviation price and profit

$$p_0'^{B1} = \frac{8t+3\Delta^s}{6}, \; \pi_0'^{B1} = \frac{(8t+3\Delta^s)^2}{144t}.$$

[4] At $p_0 = \frac{2t+3\Delta^s}{3}$, $\hat{x}^H = 1$. Then, this deviation case coincides with 0B.

I can confirm that $p_0'^{B1} < \frac{8t}{3}$ always holds, meaning that $\pi_0^{B1}(p_0, p_1^{*0B})$ decreases in p_0 for $p_0 \in [\frac{8t}{3}, \frac{8t+3\Delta^s}{3})$. Then, firm 0's optimal deviation profit is $\pi_0^{B1}(\frac{8t}{3}, p_1^{*0B}) = \frac{2\Delta^s}{3}$, which is strictly less than π_0^{*0B}. Therefore, firm 0 never deviates to induce B1.

[**00**]: Given $p_1 = p_1^{*0B}$, if firm 0 deviates by inducing 00, it chooses the deviation price $p_0'^{00}$ such that the following condition is satisfied:

$$p_1^{*0B} - p_0'^{00} \in [t, \infty) \Longleftrightarrow p_0'^{00} \leq \tfrac{2t}{3}.$$

Its optimal deviation profit is $\pi_0'^{00} = \frac{2t}{3}$, which is strictly less than π_0^{*0B}. Therefore, firm 0 never deviates to induce 00.

2.2.2 Firm 1's deviation incentives:

[**00**]: Firm 1 will never deviate by inducing 00; otherwise, its profit becomes zero.

[**BB**]: Given $p_0 = p_0^{*0B}$, if firm 1 deviates by inducing BB, it chooses the deviation price $p_1'^{BB}$ by solving

$$\begin{aligned}
&\max_{p_1} \pi_1^{BB}(p_0^{*0B}, p_1) = \left(\frac{1}{2}(1 - \hat{x}^H(p_0^{*0B}, p_1)) + \frac{1}{2}(1 - \hat{x}^L(p_0^{*0B}, p_1))\right) p_1 \\
&s.t. \ -t < p_1 - p_0^{*0B} < t - \Delta^s \Longleftrightarrow \frac{4t}{3} < p_1 < \frac{10t - 3\Delta^s}{3}.
\end{aligned}$$

From the first-order condition, I have the deviation price and profit

$$p_1'^{BB} = \frac{20t - 3\Delta^s}{12}, \ \pi_1'^{BB} = \frac{(20t - 3\Delta^s)^2}{288t}.$$

Here, $-t < p_1'^{BB} - p_0^{*0B} < t - \Delta^s \Leftrightarrow -t < \frac{20t-3\Delta^s}{12} - \frac{7t}{3} < t - \Delta^s \Leftrightarrow \Delta^s < \frac{4t}{3}$, which does not satisfies the existence condition $\overline{\Delta} \leq \Delta^s < 2t$. Therefore, firm 1 never deviates to induce BB.

[01]: Notice first that this case exists if and only if $x^H(p0, p1) \geq 0$ and $x^L(p0, p1) \leq 1$, from which I have $p1 - p0 \in [t - \Delta^s, -t]$. Since I have assumed $0 \leq \Delta^s < 2t$, this deviation never happens.

[B1]: Given $p_0 = p_0^{*0B}$, if firm 1 deviates by inducing B1, it chooses the deviation price $p_1'^{B1}$ by solving

$$\max_{p_1} \pi_1^{B1}(p_0^{*0B}, p_1) = \left(\frac{1}{2}(1 - \hat{x}^H(p_0^{*0B}, p_1)) + \frac{1}{2}\right) p_1$$
$$s.t. \; -t - \Delta^s < p_1 - p_0^{*0B} \leq -t \iff \frac{4t - 3\Delta^s}{3} < p_1 \leq \frac{4t}{3}.$$

From the first-order condition, I have the deviation price and profit are

$$p_1'^{B1} = \frac{16t - 3\Delta^s}{6}, \; \pi_1'^{B1} = \frac{(16t - 3\Delta^s)^2}{144t}.$$

Here, since $-t-\Delta^s < p_1'^{B1} - p_0^{*0B} < -t \Leftrightarrow -t-\Delta^s < \frac{1}{3}t - \frac{1}{2}\Delta^s - \frac{7}{3}t < -t \Leftrightarrow < \Delta^s > 2t$, which does not satisfies my assumption that $0 < \Delta^s < 2t$. Therefore, firm 1 never deviates to induce B1.

[11]: Given $p_0 = p_0^{*0B}$, if firm 1 deviates by inducing 11, it chooses the deviation price $p_1'^{11}$ such that the following condition is satisfied:

$$p_1'^{11} - p_0^{*0B} \in (-\infty, -t - \Delta^s] \iff p_1'^{11} \leq \frac{4t - 3\Delta^s}{3}.$$

Its optimal deviation profit is $\pi_o'^{11} = \frac{4t-3\Delta^s}{3}$, which is weakly less than π_1^{*0B} if

$$\frac{23t}{36} \leq \Delta^s < 2t,$$

which always satisfies under the existence condition $\overline{\Delta} < \Delta^s < 2t$. Therefore, firm 1 never deviates to induce 11.

Summary of Lemma 1 (3): If the existence condition $\Delta^s \in [\overline{\Delta}, 2t)$ is satisfied, then the 0B equilibrium exists.

2.3 Supplementary Details for Section 4 Discussion

2.3.1 Marginal Costs

Let $c_0 - c_1 = \epsilon$.

By solving for the stage-2 game, the results in Lemma 1 still hold and now become:

(1) When $\Delta^s \in (0, \frac{4(\sqrt{2}-1)(c_0-c_1+3t)}{1+2\sqrt{2}})$, there exists a unique pure-strategy Nash equilibrium in which the status BB prevails. In this equilibrium, $(p_0^{*BB}, p_1^{*BB}) = (\frac{6t+\Delta^s}{6} + \frac{2}{3}c_0 + \frac{1}{3}c_1, \frac{6t-\Delta^s}{6} + \frac{1}{3}c_0 + \frac{2}{3}c_1)$.

(2) When $\Delta^s \in \left(\frac{4(\sqrt{2}-1)(3t+c_0-c_1)}{1+2\sqrt{2}}, \frac{1}{3}(5t+c_0-c_1)\right)$, there exists a mixed-strategy Nash equilibrium in which both the status BB and 0B could prevail with a positive probability. In this equilibrium, firm 0 chooses $p_0^{mix} = \frac{t(6+\beta)+(1-\beta)\Delta^s}{3(2-\beta)} + \frac{2}{3}c_0 + \frac{1}{3}c_1$, and firm 1 chooses $\overline{p}_1^{mix} = \frac{2t(6-\beta)+(1-\beta)\Delta^s}{6(2-\beta)} + \frac{1}{3}c_0 + \frac{2}{3}c_1$ with probability β and $\underline{p}_1^{mix} = \frac{4t(6-\beta)-(4-\beta)\Delta^s}{12(2-\beta)} + \frac{1}{3}c_0 + \frac{2}{3}c_1$ with probability $1-\beta$, where

$\beta = \frac{2(-3+3\sqrt{2}-c_0+\sqrt{2}c_0-c_1+\sqrt{2}c_1+6t-6\sqrt{2}t+\Delta^s)}{3-3\sqrt{2}+c_0-\sqrt{2}c_0+c_1-\sqrt{2}c_1-2t+2\sqrt{2}t+\sqrt{2}\Delta^s}$.

(3) When $\Delta^s \in \left[\frac{1}{3}(5t+c_0-c_1), 2t\right)$, there exists a unique pure-strategy Nash equilibrium in which firm 0 monopolizes type-H consumers, and both firms supply to type-L consumers (0B). In this equilibrium, $(p_0^{*0B}, p_1^{*0B}) = \left(\frac{7t}{3} + \frac{2}{3}c_0 + \frac{1}{3}c_1, \overline{\Delta} + \frac{1}{3}c_0 + \frac{2}{3}c_1\right)$.

Equilibrium profits are:

(1) $(\pi_0^{*BB}, \pi_1^{*BB}) = \left(\frac{(6t+\Delta^s-2c_0+2c_1)^2}{72t}, \frac{(6t+\Delta^s-2c_1+2c_0)^2}{72t}\right)$;

(2) $\pi_0^{*mix} = \frac{(8t-\Delta^s)(-2c_0+2\sqrt{2}c_0+2c_1-2\sqrt{2}c_1-2t+2\sqrt{2}t-\sqrt{2}\Delta^s)^2}{8(1-\sqrt{2})t(-4c_0+4\sqrt{2}c_0+4c_1-4\sqrt{2}c_1-4t+4\sqrt{2}t-2\Delta^s-\sqrt{2}\Delta^s)}$,

$\pi_1^{*mix} = \frac{(3+\sqrt{2})(\Delta^s)^2}{32t}$;

(3) $(\pi_0^{*0B}, \pi_1^{*0B}) = \left(\frac{(7t-c_0+c_1)^2}{36t}, \frac{(5t-c_1+c_0)^2}{36t}\right)$.

Solving $\pi_1^{*BB}\big|_{\Delta^s=0} = \pi_1^{*mix}\big|_{\Delta^s=\Delta}$, I have $\Delta = 4\sqrt{3-2\sqrt{2}}$. Note that this is identical to the threshold in Proposition 2.

When marginal costs of the two firms are the same, i.e., $c_0 = c_1 \equiv c$,

(1) $(p_0^{*BB}, p_1^{*BB}) = \left(\frac{6t+\Delta^s}{6} + c, \frac{6t-\Delta^s}{6} + c\right)$;

(2) $p_0^{mix} = \frac{t(6+\beta)+(1-\beta)\Delta^s}{6-3\beta} + c$,

$\overline{p}_1^{mix} = \frac{2t(6-\beta)+(1-\beta)\Delta^s}{6(2-\beta)} + c$,

$\underline{p}_1^{mix} = \frac{4t(6-\beta)-(4-\beta)\Delta^s}{12(2-\beta)} + c$;

(3) $(p_0^{*0B}, p_1^{*0B}) = \left(\frac{7t}{3} + c, \overline{\Delta} + c\right)$.

2.3.2 Fixed Costs

I prove that status 0B ($\overline{\Delta} \leq \Delta^s < 2t$) can be achieved as an equilibrium outcome.

Since both π_0^{*0B} and π_1^{*0B} is independent of s_0 and s_1, both Π_0^{*0B} and Π_1^{*0B} decreases in s_0 and s_1, respectively. Then, firm 1 would choose $s_1^{*0B} = 0$ and firm 0 would choose $s_1^{*0B} = \overline{\Delta}$ such that status 0B is possible. We, therefore, obtain firms' equilibrium profits as follows:

$$\Pi_0^{*0B} = \frac{t(49-50t)}{36}, \ \Pi_1^{*0B} = \frac{25t}{36}.$$

Next, I confirm whether firms would deviate by triggering the mix of statuses BB and 0B. If such deviation happens, Δ^s must satisfy $\underline{\Delta} < \Delta^s < \overline{\Delta}$. Since π_1^{mix} increases in Δ^s, Π_1^{mix} decreases in s_1. Then, given s_1^{*0B}, firm 1 would never deviate from $s_1^{*0B} = 0$. Next, given $s_1^{*0B} = 0$, suppose firm 0 deviates by letting $\underline{\Delta} < \Delta^s < \overline{\Delta}$, its optimal deviation quality is obtained by $s_0' = \arg\max_{s_0} \Pi_0^{mix}$, and the optimal profit is Π_0'. Since π_0^{mix} increases in $\Delta^s \in (\underline{\Delta}, \overline{\Delta})$, we have

$$\begin{aligned}\Pi_0' &< \pi_0^{mix}\big|_{\Delta^s=\overline{\Delta}} - \frac{s_0^2}{2}\bigg|_{s_0=\underline{\Delta}} \\ &= \frac{t(-17689 + 5586\sqrt{2} - 17044992t + 12063168\sqrt{2}t)}{1764(-1+\sqrt{2})(-22+7\sqrt{2})},\end{aligned}$$

which is strictly less than Π_0^{*0B} for any $t > 0$.

Next, I confirm whether firms would deviate by triggering status BB. If such deviation happens, Δ^s must satisfy $0 \leq \Delta^s \leq \underline{\Delta}$. Given $s_1 = 0$, suppose firm 0 deviates by letting $0 \leq \Delta^s \leq \underline{\Delta}$, its optimal deviation quality is obtained by solving

$$\max_{s_0} \Pi_0^{*BB}\big|_{s_1=0}, \; s.t. \; 0 \leq s_0 \leq \underline{\Delta},$$

from which I obtain an interior solution $s_0 = \frac{6t}{35}$ with a profit $\frac{18t^2}{35}$. Notice that the restriction condition is always satisfied. The profit under the interior solution is weakly less than Π_0^{*0B} if

$$0 < t < \frac{1715}{2398} \approx 0.715. \tag{1}$$

Suppose now firm 1 deviates. Then, given $s_0 = \overline{\Delta}$, its optimal deviation quality is obtained by solving

$$\max_{s_1} \Pi_1^{*BB}\big|_{s_0=\overline{\Delta}}, \; s.t. \; \frac{108\sqrt{2}-145)t}{21} \leq s_1 \leq \overline{\Delta},$$

from which I obtain the corner solution $s_1 = \frac{108\sqrt{2}-145)t}{21}$ with a profit $\frac{2}{63}(11 - 9\sqrt{2})(68-45\sqrt{2})t^2$. The profit under the corner solution is weakly less than Π_1^{*0B} if

$$0 < t < \frac{175}{-12464 + 8856\sqrt{2}} \approx 2.903.$$

To summarize, the 0B status constitutes an SPNE when Condition (1) holds.

2.3.3 Spillovers

With spillovers, results that are parallel to Proposition 2 are as follows:

(1) when $\Delta \in \left[0, \frac{\left(4\sqrt{3-2\sqrt{2}}\right)t}{(1-\theta)}\right]$, there exists an SPNE outcome in which both firms choose the highest quality $\underline{s} + \Delta$;

(2) when $\Delta \in \left(\frac{\left(4\sqrt{3-2\sqrt{2}}\right)t}{(1-\theta)}, \frac{2t}{(1-\theta)}\right)$, there exists an SPNE outcome in which firm 0 chooses the highest quality $\underline{s} + \Delta$ whereas firm 1 chooses the lowest quality $\underline{s}$.