Multidimensional Regularity Processing in Music:

An Examination Using Redundant Signals Effect

Short title: REDUNDANT SIGNALS EFFECTS IN MUSIC

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Abstract

Music is based on various regularities, ranging from the repetition of physical sounds to theoretically organized harmony and counterpoint. How are multidimensional regularities processed when we listen to music? The present study focuses on the redundant signals effect (RSE) as a novel approach to untangling the relationship between these regularities in music. The RSE refers to the occurrence of a shorter reaction time (RT) when two or three signals are presented simultaneously than when only one of these signals is presented, and provides evidence that these signals are processed concurrently. In two experiments, chords that deviated from tonal (harmonic) and acoustic (intensity and timbre) regularities were presented occasionally in the final position of short chord sequences. The participants were asked to detect all deviant chords while withholding their responses to non-deviant chords (i.e., the Go/NoGo task). RSEs were observed in all double- and triple-deviant combinations, reflecting processing of multidimensional regularities. Further analyses suggested evidence of coactivation by separate perceptual modules in the combination of tonal and acoustic deviants, but not in the combination of two acoustic deviants. These results imply that tonal and acoustic regularities are different enough to be processed as two discrete pieces of information. Examining the underlying process of RSE may elucidate the relationship between multidimensional regularity processing in music.

Keywords: Redundant signals effect (RSE), Race model inequality (RMI), Music perception, Harmony, Auditory perception,

Introduction

Music contains various types of sounds, and the sounds are organized based on multiple regularities, ranging from the repetition of physical aspects of sound to theoretically organized harmony and counterpoint. When we listen to music, multiple regularities should be processed simultaneously. How, then, does the brain process these regularities simultaneously? Previous studies have examined how multiple physical features of sounds are processed simultaneously. It has been suggested that the dimensions of pitch and sound intensity are processed in an integrative manner (Grau & Nelson, 1988) and that pitch and timbre information can be integrated (Hall et al., 2000). For example, Krumhansl and Iverson (1992) showed that reaction times (RTs) for categorizing the pitch dimension were longer when the timbre of the sound varied than when the timbre was fixed, and vice versa. They interpreted that the pitch and timbre dimensions are likely to interact although each dimension can be manipulated independently. However, previous studies have mainly used simple auditory stimuli, such as a sequence with a small number of pitches. The relationship between the acoustic dimensions and the tonal regularities that organize tones and chords based on musical keys and tonal hierarchies remains to be explored.

The present study aimed to examine the relationship between the multidimensional processing of the tonal and acoustic regularities in music using a redundant signals effect (RSE), which is broadly used to investigate multisensory information processing (Brandwein et al., 2011; Gibney et al., 2017; Gondan et al., 2011; Maravita et al., 2008). The RSE is a phenomenon in which reaction times (RTs) to target signals are shorter when two or three signals are presented simultaneously than when only one of the signals is presented in a task for which the same response is required by a target signal on each channel (Miller, 1982): visual signals (Mordkoff & Yantis, 1991, 1993), auditory signals (Schröter et al., 2007, 2009), and multimodal signals (Diederich & Colonius, 2004; Miller, 1986). In the auditory domain, a prerequisite for RSE is the prevention of fusion into a single percept (Schröter et al., 2007).

Raab (1962) explained this redundancy gain as the result of statistical facilitation via the so-called race model. The race model assumes that each signal of a redundant signal is processed in parallel and that a response is triggered by the *fastest* signal processing among them (Miller, 1982). When each response is determined by the winner of the race, the average RT of the redundant signals condition should be shorter than the average RT of any single signal condition (statistical facilitation). At the same time, the race model assumes that on individual trials, the RT in the redundant signals condition cannot be shorter than the fastest RT of the single signal conditions because the fastest RT determines the lower bound of the RT in the redundant signals condition. Therefore, the predicted redundancy gain follows race model inequality (RMI: Miller, 1982):

$$F_R(t) \leq F_A(t) + F_B(t)$$

for every value of RTs t, in which F_A and F_B are the cumulative distribution functions (CDF) of the RTs in the two single signal conditions, and F_R is the CDF of the RTs in the redundant signals condition. This is derived from Boole's inequality:

$$P(A \cup B) \leq P(A) + P(B)$$

where A and B are events. It states that the probability of any one event occurring is no greater than the sum of the probabilities of the individual events. The RMI analysis examines whether the cumulative probability of the redundant signals condition never exceeds the sum of the cumulative probabilities of the two single signal conditions at any RT *t*. Here, it is assumed that the processing of one signal does not affect the processing of another signal (context invariance; Gondan & Minakata, 2016; Innes & Otto, 2019; Luce, 1986; Miller, 2016).

In the RMI analysis, the inequality is sometimes violated in that the left side of the RMI is greater than the right side of the RMI: the CDF of the redundant signals condition is greater than the sum of the CDFs of the single signal conditions at a given RT t. This indicates that the probability of observing an RT shorter than t among all trials is greater in the redundant signal condition than in the sum of the single signal conditions, thereby violating the assumptions of the race model. In this case, the redundancy gain is explained by coactivation models, which assume greater activation than statistical facilitation. Coactivation models propose that activations from different channels are combined to initiate a faster response and that RTs are shorter than those predicted by statistical facilitation in the race model (Miller, 1982, 1986, 2004; Miller & Ulrich, 2003). Thus, violations of the RMI have been interpreted as a kind of integrated processing of information from individual signals (Miller, 1982, 2016; Schröger & Widmann, 1998).

When the signals are different enough to be processed as two discrete pieces of information, RTs in the redundant signals condition are shorter than RTs predicted by statistical facilitation, and this RMI violation is interpreted as the coactivation of separate perceptual modules occurring and causing an RSE (Mordkoff & Danek, 2011). This has been demonstrated in both visual (Mordkoff & Yantis, 1993) and auditory (Fiedler et al., 2011; Schröter et al., 2007) experiments in which two signal dimensions of one perceptual object were manipulated. For example, Mordkoff and Yantis (1993) showed that redundant signals in different perceptual dimensions, such as shape (e.g., X) and color (e.g., green), caused RMI violation and suggested coactivation

(Experiments 1–3), whereas redundant signals in the same perceptual dimension, such as two colors (e.g., green and red), did not cause RMI violation and suggested statistical facilitation rather than coactivation (Experiment 5). Consistent with this pattern, Fiedler et al. (2011) reported shorter RTs in the redundant signals condition than those predicted by the race model and interpreted this result as the presence of coactivation when participants detected tones at specific frequencies and locations, which were different perceptual dimensions in the auditory domain. Based on these observations, Fiedler et al. (2011) and Mordkoff and Yantis (1993) have proposed that coactivation is caused by activations in separate perceptual modules, while statistical facilitation is caused by activations within a perceptual module. Therefore, the RMI analysis would provide further insight into the underlying processing of the RSE beyond simple RT analysis.

In the present study, two experiments were conducted to investigate the relationship between multidimensional regularity processing in music, ranging from acoustic to tonal regularities by comparing the redundancy gains produced by different combinations of deviants in the regularities. In Experiment 1, the targets were two types of deviants, and the redundant signal was a double deviant. In Experiment 2, the targets were three types of deviants, and the redundant signals were double and triple deviants.

Experiment 1

Experiment 1 examined an RSE elicited by the detection of tonal (harmonic) and acoustic (intensity) deviants that occurred independently or simultaneously. In the tonal deviant, the dominant–tonic progression, which is the authentic motion in Western harmony, was violated by replacing the final tonic chord with a harmonically irregular supertonic chord (i.e., harmonic deviant). In the acoustic deviant, the intensity of the last chord was attenuated relative to that of the preceding chords in the sequence (i.e., intensity deviant). The RT in the double-deviant condition should be shorter than the shortest RT in the single-deviant condition (i.e., RSE). The detection of harmonic deviants requires the schematic representation of the harmonic regularity, while the detection of intensity deviants requires the regularities extracted from the current auditory context (Ishida & Nittono, 2022; Koelsch, 2009). Because the double deviant consisted of qualitatively different deviant dimensions (i.e., deviance in the tonal and acoustic regularities), the RMI would be violated and coactivation of separate perceptual modules would be suggested.

Methods

Participants

An a priori analysis using G*Power (Faul et al., 2007) indicated that 40 participants would be needed to detect the effect dz = 0.476, which was calculated using data from Schröter et al. (2009) (Experiment 1 in the tone offset condition) with power $1-\beta = .90$ to test the presence of RSE. However, 60 participants were sampled in this experiment because the experiment was conducted online, and a larger quantity of data exclusion was predicted than in an offline experiment. Participants were recruited from Lancers, an online crowdsourcing service in Japan. Participants were excluded who had at least one condition with a hit rate lower than 80%, as well as those with pre- and post-experiment mismatches in gender and age data. Data on the remaining 53 participants (13 women and 40 men, M = 41.8 years old, range 20–61 years) were used to test the hypotheses. None of the participants reported hearing impairments. The participants had various types of musical experience, with a mean of 4.4 years of extracurricular musical lessons (range 0–

39 years). The protocol was approved by the Behavioral Research Ethics Committee of the Osaka University School of Human Sciences, Japan (HB021-076), and informed consent was obtained from all participants. All participants received 600 Japanese yen as an honorarium.

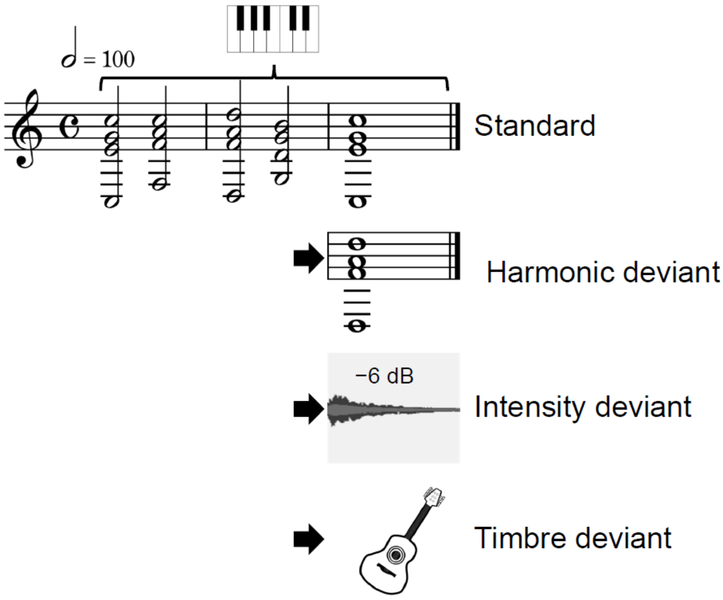

Fig. 1 Schematic illustration of the stimuli. The chord sequence was played by the piano timbre. For the harmonic deviant, the final chord was altered to the harmonically irregular chord. For the intensity deviant, the intensity was decreased by 6 dB. For the timbre deviant, the piano timbre was altered to the guitar timbre. The timbre deviant was only presented in Experiment 2. Each deviant was combined to create double- and triple-deviant conditions.

Stimuli and Procedure

Fig. 1 shows a schematic illustration of the stimuli used in the present study. Chord sequences that followed the rules of Western harmony (I→IV→II→V→I) were composed and played with a piano timbre. The duration of each of the first four chords was 600 ms, and the final chord was 1,200 ms, such that the overall duration of each sequence was 3,600 ms. All chord sequences were transposed into seven major keys (C major, C# major, D major, D# major, E major, F major, and F# major). The stimuli were generated using Studio One Prime (Version 4.6.2; PreSonus) and edited using Adobe Audition (version 13.0.12; Adobe Systems Incorporated). At the final chord of the original sequence (standard), two types of deviance were presented: a harmonically irregular supertonic chord as the *harmonic deviant* (H) and a chord decreased by 6 dB from the standard as the *intensity deviant* (I). The two types of deviance were presented either independently (H and I) or simultaneously (HI). All stimuli were the same as those used by Ishida and Nittono (2022), who examined event-related potential responses to harmonic and intensity deviants.

This experiment was conducted online using Inquisit Web (Version 6.5.2; Millisecond Software, LLC). The participants first provided written informed consent and information regarding their age, gender, and musical experience. They then adjusted their own acoustic devices (e.g., headphones or speakers) to an optimal sound volume. The experimental task was then explained, followed by a practice session. In the Go/NoGo task, the participants were asked to respond by pressing a key as quickly and accurately as possible in response to any deviants that occurred and to withhold the response if the standard chord occurred. The trial began with the presentation of a fixation cross. A chord sequence was presented after 600 ms. The presentation of the fixation cross was terminated by a response or 1,200 ms after the onset of the final chord. Following the offset of the fixation cross, when the response was a hit or a correct rejection, “correct” was displayed as feedback for 1,500 ms. Error responses were followed by an explanation of the deviant that was displayed as error feedback until the participants pressed the space key. The intertrial interval was 1,100 ms. Sixty trials were presented in one block (i.e., 10 trials for each deviant condition and 30 trials for the standard condition) in random order. Three blocks were presented such that a total of 30 trials were presented in each deviant condition and 90 trials in the standard condition. After each block, the participants were allowed to take a short break and were given feedback on their performance in the preceding block (i.e., hit rate and number of false responses). In the practice block, all three deviant conditions (three trials each) and the standard condition (nine trials) were randomly presented. To verify whether the stimuli presented were perceived as deviants following the Go/NoGo task, the participants were asked to rate how well the final chord fit the preceding musical context. These ratings suggested that all deviants were perceived as deviance (see Supplementary Material). The duration of the experiment was approximately 35 minutes.

Statistical Analysis

For each deviant condition, the RTs for all trials except the no-response trials were averaged to obtain the mean RTs of each participant. The presence of RSE was examined using paired t-tests that compared the mean RT in the double-deviant condition with the shorter mean RT in the two single-deviant conditions (Hecht et al., 2008a). Then, possible violation of RMI, which was defined as $F_{HI}(t) + F_C(t) \leq F_H(t) + F_I(t)$, was examined using CDFs F_H, F_I, F_{HI}, and $F_C(t)$. H, I, and HI indicate the harmonic, intensity, and double deviants, respectively. Here, $F_C(t)$ was introduced to control for the effects of guess responses by including false alarm RTs in the NoGo trials as a control condition C (i.e., kill-the-twin correction: Eriksen, 1988;

Ineq. 8: Gondan & Minakata, 2016). For each participant, F_H, F_I, F_{HI}, and $F_C(t)$ were calculated by applying the CDF of the normal distribution to the RTs, and the difference between $F_{HI}(t) + F_C(t)$ and $F_H(t) + F_I(t)$ was evaluated using a permutation test (Gondan & Minakata, 2016). Note that, the shorter the RT in a condition, the greater the cumulative probability at a given RT t. The CDFs of each participant were divided into 10 deciles, and the first 5 decile points were submitted to the permutation test (Gondan, 2010). The significance level was set at .05.

Results and Discussion

Mean RTs and CDFs for each deviant condition in Experiment 1

Fig. 2 shows the RTs for each deviant condition, the CDFs for each deviant condition, and the sum of the single-deviant CDFs. In the CDF plot, the horizontal axis indicates the bins of RTs generated when the RTs for each condition of each participant were arranged in decreasing time order and separated into 10 deciles. Table 1 shows the mean RTs, standard deviations, and hit rates for each deviant condition. The false alarm rate in the standard condition was 1.1%.

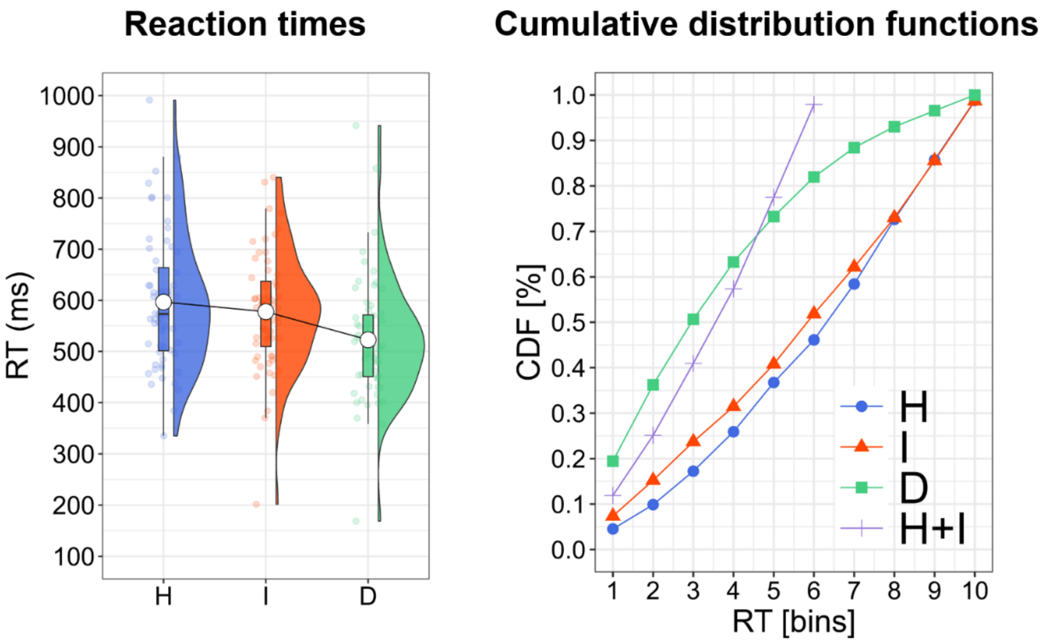

Fig. 2 Mean RTs and CDFs for each deviant condition in Experiment 1. The left panel shows the RTs for each deviant condition in Experiment 1. White dots indicate the mean RTs. The right panel shows the CDFs of the RTs for each deviant condition. The horizontal axis indicates bins of RTs, with RTs arranged in order of decreasing time and separated into 10 deciles. The vertical axis indicates the cumulative probability. The purple CDF, calculated as H + I, exceeds 1.0 and is thus truncated before reaching 1.0. H indicates the harmonic deviant, I indicates the intensity deviant, HI indicates the double deviant, and H + I indicates the sum of the harmonic and intensity deviant.

As the mean RT was shorter for intensity deviants than for harmonic deviants, the mean RTs were compared between intensity deviants and double deviants. The double-deviant RTs were significantly shorter than the intensity-deviant RTs, $t(52) = 9.81$, $p < .001$, $dz = 1.35$. The results of the permutation test showed that F_{HI} was significantly larger than the sum of the single-deviant CDFs within the first to fifth deciles: t_{max} =

5.43, t_{crit} = 2.17, p < .001. This violation of the RMI suggests the presence of a coactivation process.

Table 1

Mean RTs (ms), standard deviations (SD), and hit rates (%) for each deviant condition

	Experiment 1 (*N* = 53)			Experiment 2 (*N* = 68)						
Conditions	H	I	HI	H	I	T	HI	HT	IT	HIT
Mean	583	572	514	642	641	517	559	502	495	487
SD	118	98	99	120	112	101	100	96	94	91
HIT rate	98.8	98.7	100	97.3	93.6	99.9	99.4	99.9	99.8	99.9

Note: H indicates harmonic deviant, I indicates intensity deviant, and T indicates timbre deviant.

Coactivation results have been reported when signals are sufficiently differentiable (Fiedler et al., 2011; Mordkoff & Danek, 2011; Mordkoff & Yantis, 1993). Separate activation by tonal and acoustic deviants in the perceptual stage was suggested by Ishida and Nittono (2022), who showed independent detection processes of harmonic and intensity deviants, which are the same stimuli as those used in this study. Therefore, the current results can be interpreted as the tonal and acoustic deviants producing activation in separate perceptual modules, and their activations were summed, as suggested by Fiedler et al. (2011).

It is also possible that coactivation commonly occurs when several perceptual dimensions are integrated into a single auditory object. Fiedler et al. (2011) observed coactivation by a single tone that had two target-defining dimensions (i.e., specific frequency and location), but this coactivation may have been attributable to the fact that the percept was a single auditory object, as suggested by Mordkoff & Danek (2011) in the visual domain. Therefore, coactivation may be common in a musical context in which multidimensional information is integrated into one musical object regardless of the perceptual dimensions of the signals. To examine this possibility, an additional acoustic deviant, a timbre deviant, was introduced in Experiment 2.

Although all deviants were perceived as deviance (see Supplementary Material), because only one type of chord sequence was used in Experiment 1, the harmonic deviant may have been detected as a change in the melodic contour. This possibility was examined in Experiment 2 using various melodic contours.

Experiment 2

Experiment 2 was designed to replicate the results of Experiment 1, with the following improvements. First, different types of chord sequences with different melodies were used to avoid the possibility that harmonic deviance would be detected as a melodic contour change. This use of various types of chord sequences has been applied in previous electrophysiological studies to examine the processing of deviance from functional harmony (Koelsch et al., 2000, 2003).

Second, a third type of deviance, a change in timbre, was added to simultaneously examine the perceptual processes of multidimensional regularities in music. In Experiment 1, although a coactivation process between harmonic and intensity deviants was observed, this finding was limited to two deviant dimensions. Therefore, timbre (i.e., choice of musical instrument), an important acoustic dimension in music, was selected as the third deviant. We expected that the double deviants generated by combinations of harmonic, intensity, and timbre deviants would elicit redundancy gains. If coactivation was common in a musical context in which multidimensional information was integrated into one musical object, regardless of the types of perceptual dimensions of the signals, all types of RMI would be violated.

Experiment 2 also examined whether a triple deviant would elicit an RSE. Although RSEs elicited by three signals have been examined in tri-modalities (i.e., visual, auditory, and tactile; Couth et al., 2018; Diederich & Colonius, 2004; Gondan & Röder, 2006; Hagmann & Russo, 2016; Hecht et al., 2008a, 2008b; Pomper et al., 2014) and within the visual modality (Engmann & Cousineau, 2013), it has remained unclear whether triple signals within the auditory modality elicit RSE. Thus, the present study aimed to explore the RSEs elicited by three signals in the context of music processing. Similar to the visual modality, we expected that RSEs would occur for the triple deviant in the auditory modality. Moreover, if coactivation reflecting integrated processing is common in a musical context, the triple deviant consisting of different dimensions of auditory deviants would cause the violation of the RMI and suggest coactivation.

Methods

Participants

This experiment was preregistered before sampling (https://osf.io/m3j4c). An a priori analysis was conducted using the power contour (Baker et al., 2021). The power contour is a function of the number of trials and the sample size, given a mean difference, within-participant standard deviation, and between-participant standard deviation. Thus, we conducted this analysis to determine the optimal combination of trials and sample sizes to ensure sufficient power for the comparison of each double-deviant CDF and each summed single-deviant CDF. The results of Experiment 1 showed that the RMI was significantly violated at the 1st–5th decile points when the summed CDFs and double-deviant CDFs were compared at decile points using paired *t*-tests. Specifically, the mean difference of −12 ms and the between-participants standard deviation of 24 ms at the 5th decile point were used to estimate the power contour because the effect size of the 5th decile point ($dz = -0.476$) was the smallest among the 1st–5th decile points. The within-participant standard deviation was set at 50, which was considered sufficiently large. The results showed that a sample size greater than 59 would be needed to obtain a power of $1-\beta > .90$ in 20 trials, which was the minimum number included in the analysis.[1] Considering the possibility of outliers and missing values, 90 participants were initially recruited. However, because the quantity of incomplete data was greater than expected, an additional 30 participants were recruited before the data were analyzed. None of these participants participated in Experiment 1. After excluding participants who had at least one condition with a hit rate lower than 80% and those with pre- and post-experiment mismatches of gender and age data, data from the remaining 68 participants (30 women and 38 men, 30–67 years old, $M = 42.8$ years old) were used to test the hypotheses. None of the participants reported hearing impairments. The participants had various types of musical experience, with a mean of 5.3 years of extracurricular musical lessons (range 0–40 years). The protocol was approved by the Behavioral Research Ethics Committee of the Osaka University School of Human Sciences, Japan (HB022-062), and informed

[1]Although we changed the method of analysis from the preregistered protocol, the post-hoc simulation validated that this sample size was large enough to detect a violation of RMI (see the Supplementary Materials).

consent was obtained from all participants. All participants received 900 Japanese yen as an honorarium.

Stimuli and Procedure

Three types of chord sequences with various melodic contours were composed by manipulating the chord inversion, and all sequences followed the same harmonic progression (I→IV→II→V→I) as in Experiment 1. The stimuli were generated using Cubase (Version 12.0.50; Steinberg) and edited using Adobe Audition (Version 22.6.66; Adobe Systems Incorporated). Three types of deviants occurred in the final chord of the original sequence (standard): a harmonically irregular supertonic chord as the *harmonic deviant* (H), a chord decreased by 6 dB from the standard as the *intensity deviant* (I), and a chord played with a guitar timbre as the *timbre deviant* (T). By combining these deviance types, seven deviant conditions were created: three single-deviant conditions in which each deviant occurred independently (H, I, and T), three double-deviant conditions in which two of the three deviants occurred simultaneously (HI, HT, and IT), and one triple-deviant condition in which the three types of deviants occurred simultaneously (HIT).

The online experiment was again conducted using Inquisit Web (Version 6.6.2; Millisecond Software, LLC). The procedure was identical to that in Experiment 1, with some exceptions. In Experiment 2, the number of trials in one block was 70 (i.e., 5 trials for each deviant condition, 35 trials for the standard condition). Feedback after each trial was not presented because of the larger number of conditions and the longer experimental time. The five blocks were presented such that the total number of trials in each deviant condition and the standard condition were 25 and 175, respectively. Feedback was provided only during the break period. In the practice block, the participants were presented with all seven deviant conditions (2 trials each) and the standard condition (14 trials) in random order. The duration of the experiment was 40 minutes.

Statistical Analysis

We revised the preregistered analysis protocol in two ways according to Godon and Minakata's (2016) tutorial, although virtually the same results (i.e., coactivations in HI and HT conditions and statistical facilitation in IT and HIT conditions) were obtained before and after the change. First, we did not trim RTs below 1,200 ms or above 200 ms, because RTs are not conditioned on a specific range in Miller's RMI. Second, we included RTs in the NoGo trials (false alarms in the standard condition) in RMIs to control for the effects of guess responses (i.e., kill-the-twin correction: Eriksen, 1988; Gondan & Minakata, 2016). Data were analyzed at two levels, as in Experiment 1. First, the presence of RSE was examined using paired *t*-tests that compared the mean RT in the triple-deviant condition with the shortest mean RT in the three double-deviant conditions (Hecht et al., 2008a). Second, possible violations of RMI were evaluated using the CDFs F_H, F_I, F_T, F_{HI}, F_{HT}, F_{IT}, and F_{HIT}. HI, HT, IT, and HIT indicate the double-deviant condition of the harmonic deviant + intensity deviant, harmonic deviant + timbre deviant, and intensity deviant + timbre deviant, and the triple-deviant condition of the harmonic deviant + intensity deviant + timbre deviant, respectively. We used Gondan and Vorberg's (2021) Ineq. 5, because our goal was to examine the coactivation of triple-deviant signals in the triple-deviant condition. The RMI for the triple-deviant condition was defined as $F_{HIT}(t) \leq F_{HI}(t) + F_{HT}(t) + F_{IT}(t) - F_H(t) - F_I(t)$. To ensure that the triple-deviant RMI was as conservative as possible, the two smallest CDFs among the three single-deviant conditions, $F_H(t)$ and $F_I(t)$, were used as the last two terms of the definition. This type of triple-deviant RMI is violated only by a system with genuine trimodal coactivation, since this RMI considers all coactivations produced by combinations of two signals. In an exploratory analysis, Gondan and Vorberg's Ineq. 4, which models two coactivations by two of three signals, was also tested because we only observed two coactivations in this experiment (see Supplementary Material). The CDFs were again divided into 10 deciles, and the first 5 decile points were submitted to the permutation test (Gondan, 2010). The significance level was set at .05. For the multiple comparisons in the *t*-tests between the three double deviant-conditions, the significance level was corrected to $\alpha = .016$ using the Bonferroni correction.

Results and Discussion

Fig. 3 shows the RTs for each deviant condition, the CDFs for each deviant condition, and the synthesized CDFs. Table 1 shows the mean RTs, *SD*s, and hit rates for each deviant condition. The false alarm rate in the standard condition was 2.0%. All double- and triple-deviant conditions produced RSEs. However, different models have been suggested to explain the redundancy gains in each deviant condition.

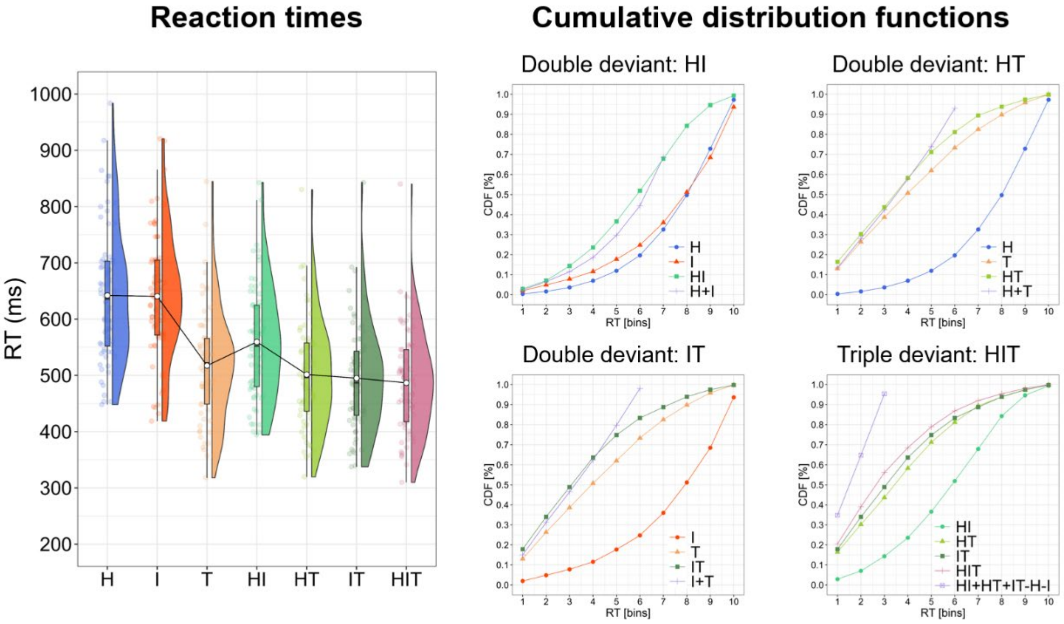

Fig. 3 Mean RTs and CDFs for each deviant condition in Experiment 2. The left panel shows the RTs for each deviant condition in Experiment 2. White dots indicate the mean RTs. The right panel shows the CDFs of the RTs for each deviant condition. The horizontal axis indicates bins of RTs, with RTs arranged in order of decreasing time and separated into 10 deciles. The vertical axis indicates the cumulative probability. The purple CDFs, calculated by summation and subtraction, exceed 1.0 and are thus truncated before reaching 1.0. HI, HT, IT, and HIT indicate the double-deviant condition of the harmonic deviant + intensity deviant, harmonic deviant + timbre deviant, and intensity deviant + timbre deviant, and the triple-deviant condition of the harmonic deviant + intensity deviant + timbre deviant, respectively.

Double-Deviant Conditions

Comparing harmonic and intensity deviants, the mean RT was shorter for intensity than for harmonic deviants. The mean RT was significantly shorter for the double (harmonic + intensity) deviant than for the intensity deviant: $t(67) = 13.83$, $p < .001$, *dz*

= 1.68. The permutation test showed that the CDF F_{HI} was significantly greater than the sum of the single-deviant CDFs $F_H + F_I$ in the 1st–5th deciles: $t_{max} = 4.70$, $t_{crit} = 2.23$, $p < .001$. The violation of the RMI was again observed in the harmonic + intensity deviant.

Comparing harmonic and timbre deviants, the mean RT was shorter for timbre than for harmonic deviants. The mean RT was significantly shorter for the double (harmonic + timbre) deviant than for the timbre deviant: $t(67) = 5.15$, $p < .001$, $dz = 0.62$. The permutation test showed that the CDF F_{HT} was significantly greater than the sum of the single-deviant CDFs $F_H + F_T$, $t_{max} = 2.73$, $t_{crit} = 1.94$, $p = .007$. Similar to the harmonic + intensity deviant, a violation of the RMI was observed.

The mean RT was shorter for timbre than for intensity deviants. The mean RT was significantly shorter for the double (intensity + timbre) deviant than for the timbre deviant: $t(67) = 6.74$, $p < .001$, $dz = 0.82$. However, the permutation test showed that the CDF F_{IT} was not significantly larger than the sum of the single-deviant CDFs $F_I + F_T$, $t_{max} = 1.72$, $t_{crit} = 2.11$, $p = .105$. When the intensity and timbre dimensions deviated, no violation of the RMI was observed.

Triple-Deviant Condition

Among the three types of double deviants, the mean RT was shortest for the intensity + timbre deviant. The mean RT was significantly shorter for the triple (harmonic + intensity + timbre) deviant than for the double (timbre + intensity) deviant: $t(67) = 3.26$, $p = .002$, $dz = 0.40$. However, the permutation test showed that the CDF F_{HIT} was not significantly greater than the quantity on the right side of the RMI definition: $t_{max} = -7.81$, $t_{crit} = 2.02$, $p = 1.000$. These results suggest that an RSE was elicited by three signals within the auditory modality. However, no violation of the RMI was observed.

General Discussion

The present study investigated the relationship between multidimensional regularities in music processing by comparing the redundancy gains produced by different combinations of harmonic, intensity, and timbre deviants. In Experiment 1, the harmonic and intensity deviants produced a redundancy gain with a violation of the RMI. In Experiment 2, all double deviants produced redundancy gains, and the triple deviant produced a further redundancy gain. Based on Schröter et al. (2007), who proposed the prevention of fusion into a single percept as one requisite for the RSE, these results suggest that multidimensional regularity information is processed at the perceptual stage. To our knowledge, this is the first study to apply an RSE for the examination of music perception, and it may also be the first to observe the triple-signal RSE within the auditory modality.

The redundant gains can be explained by two possible processes. First, deviations from tonal regularity and deviations from acoustic regularity can be perceptually dissociable. Previous studies have suggested that coactivation occurs when signals are different enough to be processed as discrete pieces of information (Fiedler et al., 2011; Mordkoff & Danek, 2011; Mordkoff & Yantis, 1993). Among them, Fiedler et al. (2011) explained that when the processing of two signals with discrete dimensions occurs in two separate perceptual modules, two activations occur to produce RSEs with a coactivation model. In contrast, when the processing of two signals with the same stimulus dimension occurs in the same perceptual module, the RSEs in this case can be explained by a race model (statistical facilitation). Similarly, the combination of the tonal and acoustic deviants may produce two distinct activations, resulting in coactivation. The combined activation then initiates a response with greater redundancy gain than mere statistical facilitation. In contrast, the combination of the acoustic deviants may activate a single perceptual module. In this case, because activation is limited to a single perceptual module, the RSE can be explained by a race model (statistical facilitation), which assumes that the activation of one signal that wins the race among the two signals initiates the response.

In the detection of harmonic deviants (i.e., tonal deviants), actual input is compared to the schematic representation of musical regularities (Bigand et al., 2003; Koelsch, 2009). To detect deviants in the intensity and timbre dimensions, actual input is compared to the acoustic regularities extracted from the current auditory context (Bonetti et al., 2018; Näätänen et al., 2005; Vuust et al., 2012). Ishida and Nittono (2022) showed that event-related brain potentials elicited by harmonic and intensity deviants were additively enhanced when the two deviants occurred simultaneously, suggesting that the deviance detection process may operate separately for tonal regularity and acoustic regularity. Taken together, the detection processes of tonal and acoustic deviants (i.e., intensity and timbre) may be distinct, while the detection processes of intensity and timbre deviants may be similar at an early perceptual stage.

Second, as discussed in Fiedler et al. (2011), the coactivation could be accounted for by the parallel grains model. Grains correspond to information or activations that are processed in parallel. According to this model, different grains are activated in random delays after stimulus onset, and this activation is transmitted to a decision center (Miller & Ulrich, 2003). As soon as the criterion is attained, a response is initiated. The redundant signal, which consists of two signals, activates a larger number of grains than a single signal. The redundant signals that are processed in a common perceptual dimension activate grains within a common grain pool and cause statistical facilitation. However, redundant signals, which are processed in separate perceptual dimensions, activate grains between distinct grain pools, thus causing coactivation, as more grains are activated than in the activation of the common grain pool (Fiedler et al., 2011). The HI and HT may have activated distinct grain pools in the present study, thus causing coactivation. In contrast, IT and HIT may have activated a common grain pool, which led to statistical facilitation.

It is also possible that the RSE may have been affected by response competition (Eriksen & Eriksen, 1974; Grice et al., 1984). The coexistence of non-deviant (NoGo) channels may have inhibited the response to a deviant stimulus. For instance, shorter RTs in the triple-deviant condition can be due to the absence of response competitions, because none of the channels inhibited the response in that condition. However, the double-deviant results in the present study could not be fully explained by the response competition effect, because the combination of deviant channels affected RT results differently.

In conclusion, the present study demonstrated that multidimensional regularities in music are processed. The violation of RMI was observed only in the combination of the tonal and acoustic deviants, the detection processes of which have been shown to be separate in the brain (Ishida & Nittono, 2022; Koelsch, 2009). According to the suggestion of previous studies that coactivation occurs when signals are different enough to be processed in discrete pieces of information (Fiedler et al., 2011; Mordkoff & Danek, 2011; Mordkoff & Yantis, 1993), the deviants in the tonal and acoustic regularities suggested coactivation caused by activations from separate perceptual modules. However, the deviants in the two acoustic regularities suggest statistical facilitation caused by activations within a perceptual module. These results imply that the processing of tonal and acoustic regularities functions distinctly at the perceptual stage. The RSE was observed even in three targets defined within the auditory modality; thus, high applicability for the examination of concurrent signal processing in music was demonstrated. Finally, an examination of the underlying process of RSE using RMI may elucidate the relationship between multidimensional regularity processing in music.

Declaration

Ethics approval and consent to participate: This study was performed in line with the principles of the Declaration of Helsinki. Approval was granted by the Behavioral Research Ethics Committee of the Osaka University School of Human Sciences (Experiment 1: HB021-076; Experiment 2: HB022-062). Written informed consent for participation was obtained from all participants included in the study.

Consent for publication: All participants provided written informed consent for publication of obtained data under anonymity.

Availability of data and materials: The sound materials used and datasets analyzed for the present paper are available at https://osf.io/6txub/.

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Author's contributions:

Kai Ishida: Conceptualization, Methodology, Investigation, Data curation, Formal analysis, Visualization, Writing - original draft, and Funding acquisition. **Hiroshi Nittono:** Conceptualization, Methodology, Writing - review & editing, and Project administration.

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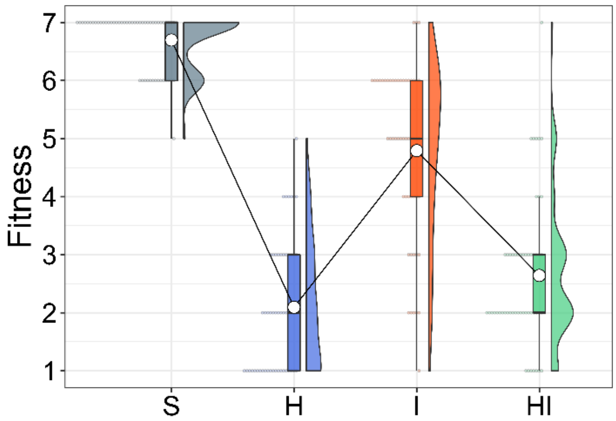
Supplementary Material

Subjective Fitness Ratings for Deviants

After completing the Go/Nogo task in Experiment 1, the participants were asked to rate how well the final chord of a sequence fit the preceding musical context on a seven-point scale (1 = *not fit at all*, 7 = *fit very well*). In four trials, one for each of the four conditions (one standard and three deviants) were presented in random keys. The fitness ratings were submitted to a one-way repeated measures analysis of variance (ANOVA) with a factor of condition (standard, harmonic deviant, intensity deviant, and double deviant). Greenhouse-Geisser ε correction was applied to compensate for the violation of sphericity. The Bonferroni correction was applied to multiple comparisons in post hoc testing. Supplementary Fig. S1 shows the participants' fitness ratings for each condition. Supplementary Table S1 shows the mean ratings and *SD*s of the four conditions. A significant main effect of the condition was obtained: $F(3, 156) = 197.31$, $p < .001$, $\varepsilon = .738$, $\eta_p^2 = .791$. The post hoc *t*-tests revealed that fitness ratings were significantly lower for the harmonic, intensity, and double deviants than for the standard (all *p*s <.001). Moreover, the ratings were significantly lower for the harmonic deviant than for the intensity and double deviants (*ps* < .001), respectively. These results suggest that the participants recognized all types of deviants and that harmonic deviance had the largest effect among them.



Supplementary Fig. S1 Fitness ratings for the final chords in the four conditions. White dots indicate mean ratings. Colored dots indicate an individual's ratings. S indicates standard, H indicates harmonic deviant, and I indicates intensity deviant.

Supplementary Table S1

Means and SDs of the fitness rating for the final chord in each condition

	Conditions (*N* = 53)			
	S	H	I	HI
Mean	6.7	2.1	4.8	2.7
SD	0.5	1.5	1.5	1.2

Note. S indicates the standard, H indicates the harmonic deviant, and I indicates the intensity deviant.

Simulation of null hypothesis rejection rates in Experiment 2

To validate the planned sample size (59) in Experiment 2, we conducted a post hoc simulation of null hypothesis rejection rates under a coactivation model. Based on the mean RTs and SDs of Experiment 1 (H: $M = 583$, $SD = 118$; I: $M = 572$, $SD = 98$; HI: M

= 514; *SD* = 99), we randomly generated RTs of each participant following the inverse Gaussian distribution using “statmod” (Smyth et al., 2017, version 1.5.0), which is an R package. After the calculation of the CDFs from the generated RTs, we submitted the first five decile points were submitted to the permutation test. When the sample size was 59, the number of deviant trials was 25, and the iteration was 10,000, the null hypothesis rejection rate was 99.93%. Thus, these results indicate that the sample size of Experiment 2 was large enough to detect a violation of RMI.

Test of a different type of RMI: Ineq. 4 of Gondan and Vorberg (2021)

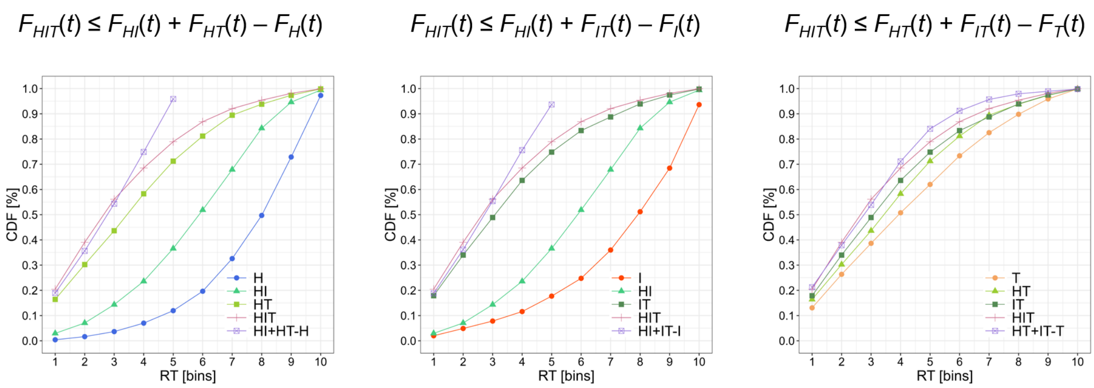
We conducted an exploratory analysis of the triple-deviant RMI of Ineq. 4 (Gondan & Vorberg, 2021), which considers two coactivations by combinations of two of three signals. This analysis was conducted to examine whether Gondan and Vorberg’s Ineq. 4 could model the present results, where coactivation was selectively observed in HI and HT, but not in IT. Ineq. 4 of Gondan and Vorberg (2021) is defined as follows:

$$F_{HIT}(t) \leq F_{HI}(t) + F_{HT}(t) - F_{H}(t) \tag{1}$$

$$F_{HIT}(t) \leq F_{HI}(t) + F_{IT}(t) - F_{I}(t) \tag{2}$$

$$F_{HIT}(t) \leq F_{HT}(t) + F_{IT}(t) - F_{T}(t) \tag{3}$$

This definition can be formed in three different ways, and each version considers two coactivations: (1) coactivation of HI and HT; (2) coactivations of HI and IT; and (3) coactivation of HT and IT. Based on the distinct signal coactivation model (Ulrich & Miller, 1997, Table 1), in which coactivation is represented by an additional racer, it was expected that (2) and (3) would be violated because the coactivation of HI and HT is suggested in the double-deviant RMI. The CDFs calculated based on (1) to (3) are illustrated in Supplementary Fig. S2. The CDFs were divided into 10 deciles, and the first five decile points were submitted to the permutation test for each form. However, none of the forms of RMI were violated: t_{max} = 1,73, t_{crit} = 2.30, p = .148 for (1), t_{max} = 1.56, t_{crit} = 2.27, p = .192 for (2), and t_{max} = 1.00, t_{crit} = 2.28, p = .389 for (3). The faster response to timbre deviance and the contamination of response competition might have caused these unclear results. Future research should examine selective violations of Ineq. 4, such as by controlling for the competition or salience of responses.



Supplementary Fig. S2 CDFs for each deviant condition in Experiment 2. The horizontal axis indicates bins of RTs arranged in order of decreasing time and separated into 10 deciles. The vertical axis indicates the cumulative probability. The purple CDFs, calculated by summation and subtraction, exceeded 1.0 and were truncated before reaching 1.0.