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In [2], we have proved that a right non-singular ring \( R \) is right FPF (=every finitely generated faithful right \( R \)-module generates the category of right \( R \)-modules) if and only if (1) \( R \) is right bounded, (2) The multiplication map \( Q \otimes Q \rightarrow Q \) is an isomorphism and \( Q \) is flat as a right \( R \)-module, where \( Q \) means the maximal right quotient ring of \( R \), (3) For any finitely generated right ideal \( I \) of \( R \), \( Tr_R(I) \oplus r_R(I) = R \) (as ideals), where \( Tr_R(I) \) means the trace ideal of \( I \) and \( r_R(I) \) means the right annihilator ideal of \( I \). This characterization implies a following result of S. Page. "Let \( R \) be a right non-singular right FPF-ring and \( Q \) be the maximal right quotient ring of \( R \). Then \( Q \) is also right FPF and is isomorphic to a finite direct product of full matrix rings over abelian regular self-injective rings." However, as we can see from an example in section 1, not all non-singular right FPF-rings arise in this fashion.

Therefore, in this paper, we shall give a necessary and sufficient condition for a non-singular right FPF-ring to split into a finite direct product of full matrix rings over FPF-rings whose maximal right quotient rings are abelian regular self-injective rings. More precisely, we shall prove the following theorem.

**Theorem 1.** Let \( R \) be a non-singular right FPF-ring. Then the following conditions are equivalent.

1. \( R \cong \prod_{i=1}^{l} M_{n_i}(S_i) \), where each \( S_i \) is a non-singular right FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring.
2. \( R \) contains a faithful and reduced FPF idempotent and \( R \) satisfies general comparability.

By Y. Utumi [5], non-singular (right) continuous rings are shown to be (Von Neumann) regular, and S. Page has determined the structure of regular (right) FPF-rings. Therefore we are interested in the structure on non-singular right quasi-continuous, right FPF-rings. In section 2, as an application of Theorem 1, we shall determine the structure of non-singular right quasi-continuous right FPF-rings.
0. Preliminaries

Throughout of this paper, we assume that a ring $R$ has identity and all modules are unitary.

Let $R$ be a ring. Then we say that $R$ is right FPF if every finitely generated faithful right $R$-module is a generator in the category of right $R$-modules. If $R$ is a right non-singular right FPF-ring, then $R$ is also left non-singular by Theorem 3 of [4]. Therefore, we simply call $R$ a non-singular right FPF-ring.

Let $e$ be an element of $\text{id}(R)$ (=the set of all idempotents of $R$), where $R$ is a non-singular right FPF-ring. Then we say that $e$ is a faithful and reduced FPF idempotent if the right $R$-module $eR$ is faithful and the ring $eRe$ is a non-singular right FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring, where a regular ring $R$ is said to be abelian if every idempotent of $R$ is central.

A ring $R$ satisfies general comparability provided that for any $e,f$ in $\text{id}(R)$, there exists $h \in B(R)$ (=the set of all central idempotents of $R$) such that $h(eR) \leq h(fR)$ and $(1-h)(fR) \leq (1-h)(eR)$, where $h(eR) \leq h(fR)$ means that $h(eR)$ is isomorphic to a direct summand of $h(fR)$.

Let $M$ be a right $R$-module. Then we use $r_R(M)$ to denote the right annihilator ideal of $M$, and we use $L_r(M)$ to denote the lattice of all submodules of $M$. $M$ is said to have the extending property of modules for $L_r(M)$, provided that for any $A$ in $L_r(M)$, there exists a direct summand $A^*$ of $M$ such that $A \subseteq A^*$, where $A \subseteq A^*$ means that $A$ is an essential submodule of $A^*$.

We say a ring $R$ is right quasi-continuous if (1) $R$ has the extending property of modules for $L_r(R)$, and (2) for $A$ and $B$ in $L_r(R)$, which are direct summand of $R$ with $A \cap B = 0$, $A \oplus B$ is also a direct summand of $R$.

1. Proof of Theorem 1

In this section, we shall prove Theorem 1. First we show the following lemma.

Lemma 1. Let $R$ be a non-singular right FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring. Then for any positive integer $n$, $M_n(R)$ satisfies general comparability.

Proof. First we assume that $n=1$. Since $Q$, the maximal right quotient ring of $R$, is an abelian regular ring, $B(Q) = B(R)$, hence $\text{id}(Q) = \text{id}(R)$. Let $e$ and $f$ are idempotents of $R$. Then $eR \cap fR = efR$ and $ef$ is idempotent since $\text{id}(R) = B(R)$. So $fR \cap eR \leq fR = f(fR)$ and $(1-f)(fR)$ (= 0) $\leq (1-f)(eR)$.

Therefore $R$ has general comparability. Next let $n>1$, and assume that the Theorem holds for $n-1$. Let $e$ and $f$ are idempotent of $M_n(R)$. Then we note
that $eR$ and $fR$ are isomorphic to $e_1 \oplus \cdots \oplus e_n R$ and $f_1 R \oplus \cdots \oplus f_n R$ as right $R$-modules, where $e_i$ and $f_i$ ($i=1, 2, \ldots, n$) are idempotents of $R$. We set $A = e_1 R \oplus \cdots \oplus e_n R$ and $B = f_1 R \oplus \cdots \oplus f_n R$. By induction hypothesis, there exist central idempotents $h_1, h_2$ such that $h_1(e_1 R) \leq h_1(f_1 R), (1-h_1)(f_1 R) \leq (1-h_1)(e_1 R)$, and $h_2 A' \leq h_2 B', (1-h_2)B' \leq (1-h_2)A'$. Now we set that $t_1 = h_1 h_2$ and $t_2 = (1-h_1)(1-h_2)$. Then $t_1$ and $t_2$ are in $B(R)$, and $t_1 t_2 = 0$. Moreover we see that $t_1(e_1 R) \leq t_1(f_1 R)$ and $t_1 A' \leq t_1 B'$. Hence $t_1 A \leq t_1 B$. Similarly, $t_2 B \leq t_2 A$. Further we set $g_1 = h_1(1-h_2), g_2 = h_2(1-h_1)$ and $g = g_1 + g_2$. Then since $h_1(e_1 R) \leq h_1(f_1 R)$, we have $g_1(e_1 R) \leq g_1(f_1 R)$ and $g_1(f_1 R) \approx g_1(e_1 R) \oplus D_1$ for some $D_1$. Similarly, we obtain that $g_2(e_1 R) \approx g_2(f_1 R) \oplus C_1$ for some $C_1$. Furthermore we have that $g_1 B' = g_1 A' \oplus D_2$ and $g_2 A' = g_1 B' \oplus C_2$ for $D_2, C_2$. We see that $gA \approx (g_1 e_1 R) \oplus g_1 f_1 R \oplus g_2 B' \oplus g_2 A') \oplus (C_1 \oplus C_2)$ and $gB \approx (g_1 e_1 R) \oplus g_2 f_1 R \oplus C_1 \oplus C_2 \oplus (D_1 \oplus D_2)$. On the other hand, since $C_1 \leq g_1(e_1 R) \leq g_1 R$ and $C_1 \leq g_2 A' \leq (n-1)(g_1 R), C_1 \oplus C_2 \leq (n-1)(g_1 + g_2)R \leq (n-1)gR$. Similarly, $D_1 \oplus D_2 \leq (n-1)gR$. Hence by induction hypothesis, there exists a central idempotent $k$ of $R$ such that $k(C_1 \oplus C_2) \leq k(D_1 \oplus D_2)$ and $(1-k)(D_1 \oplus D_2) \leq (1-k)(C_1 \oplus C_2)$. Set $t_3 = k$ and $t_4 = g(1-k)$. Then $t_3, t_4$ are in $B(R)$ and $t_3 t_4 = 0$ and $t_3 + t_4 = g$. We see that $t_3 A \leq t_2 B$ and $t_4 B \leq t_2 A$. Now we set $e = t_3 + t_4$. Then $eA \leq eB$ and $(1-e)B \leq (1-e)A$. Therefore $M_n(R)$ has general comparability.

Proof of Theorem 1.

(1) $\Rightarrow$ (2): By Lemma 1, $R$ has general comparability, and it is easily seen that $R$ contains a faithful and reduced FPF idempotent.

(2) $\Rightarrow$ (1): Let $g$ be a a faithful and reduced FPF idempotent of $R$. We claim that $g$ is a faithful abelian idempotent of $Q$. Set $H = r_0(gQ)$, then since $Q$ is right self-injective regular, $H = eQ$ for some central idempotent $e$ of $R$. Further by Theorem 1 of [3], $B(R) = B(Q)$, so $e$ must be zero since $gR$ is faithful. Hence $gQ$ is a faithful right $Q$-module. We note that $gQg$ is a maximal right quotient ring of $gRg$. Thus $g$ is a faithful abelian idempotent of $Q$. Since $Q$ is also right FPF by [4, Theorem 2], $Q \leq n(gQ)$ for some positive integer $n$. For each $i=1, 2, \ldots, n$ let $e_i$ be the supremum of all $e \in B(R)$ for which $(eQ)g \leq t(eQ)g$. By [2, Theorem 10, 15], $e_i Q \leq t(e_i gQ)$. Then $e_i \leq e_1 \cdots \leq e_n = 1$, and we define that $f_1 = e_1$, and $f_i = e_i - e_{i-1}$ for all $i=2, \ldots, n$. Note that the $f_i$s are pairwise orthogonal, and $\bigvee f_i = \bigvee e_i = 1$. Therefore $Q = \prod_{i=1}^n f_i Q$, and since $B(R) = B(Q)$, we see that $R = \prod_{i=1}^n f_i R$. Thus it only remains to show that each of the rings $f_i R$ is isomorphic to a $t \times t$-matrix ring over a FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring. Since $f_i Q \leq f_i gQ \leq gQ$ and $gQg$ is abelian, we see that the ring $f_i Q$ is abelian. Furthermore, $f_i Q$ is a maximal right quotient ring of $f_i R$. Now consider any integer $t \in \{2, \ldots, n\}$. Note that $f_i Q$ is a regular ring of index $t$. We have that $f_i \leq e_i$.
and \( e_tQ \lesssim t(e_tgQ) \), whence \( f_tQ \lesssim t(f_tgQ) \). Since \( R \) has general comparability, there exists a central idempotent \( e \) of \( R \) such that \((t-1)(egR) \lesssim eR \) and \((1-e)R \lesssim (t-1)(1-e)gR \). Then \((1-e)Q \lesssim (t-1)(1-e)gQ \), so \( 1-e \lesssim e_{t-1} \). Thus \( f_t \lesssim 1-e_{t-1} \lesssim e \). Therefore \((t-1)(f_tgR) \lesssim f_tR \). We have that \( f_tR = (t-1)(f_tgR) \oplus A \) for some \( A \). Next by general comparability of \( R \), there exists a central idempotent \( e \) of \( R \) such that \((t-1)(egR) \lesssim eR \) and \((1-e)R \lesssim (1-e)gR \). Then \( hA = h(f_tgR) \oplus B \) and \((1-h)(f_tgR) \lesssim (1-h)A \oplus C \) for some \( B \) and \( C \). Now \( f_tR = t(hf_tgR) \oplus (t-1)C \). Hence there exist idempotents \( k_i \) (\( i = 1, 2, 3, 4 \)) of \( f_tR \) such that \( k_iR \approx (hf_tgR) \), \( k_2R \approx B \), \( k_3 \approx t((1-h)A) \), and \( k_4 \approx (t-1)C \). We claim that each \( k_i \) is central. If \( k_1 \) is not central, then \( \text{Hom}_Q(k_1Q, (f_t-k_1)Qk_1 \neq 0) \), whence \( \text{Hom}_Q(hf_tgQ, (f_t-k_1)Qk_1 \neq 0) \). Let \( \varphi \) be a nonzero element of \( \text{Hom}_Q(hf_tgQ, (f_t-k_1)Q) \). Then \( \varphi(hf_tgQ) \) is a direct summand of \( (f_t-k_1)Q \). In particular, \( \varphi(hf_tgQ) \) is a projective and hence we have that \( hf_tgQ = E \oplus \ker \varphi \) for some \( E \). Since \( E \approx \varphi(hf_tgQ) \), we have that \( E \lesssim hf_tgQ \). But then \( hf_tgQ \) contains a direct sums of \( t+1 \) nonzero pairwise isomorphic ideals, which contradicts the index of \( f_tQ \). Therefore \( k_1 \) is central. Likewise for \( k_2, k_3, k_4 \). Next we show that \( k_2 \) and \( k_3 \) are zero. It is easily seen that \( k_2 \) is zero since \( f_tgR \) is a faithful right \( f_tR \)-module. Since \( C \lesssim (1-h)(f_tgR) \), \( R \approx (t-1)C \lesssim (t-1)(1-h)f_tgR \). Hence \( (1-h)f_tk_2 \leq (t-1) \cdot (1-h)f_tk_4 \), so \( (1-h)f_tk_4 \leq e_{t-1} \). But since \( e_{t-1} = e_t-f_t, e_{t-1} \) is orthogonal to \( f_t \). Thus \( k_4 = (1-h)f_tk_4 = 0 \). Consequently, \( f_tR = k_tR \oplus k_2R \approx t(hf_tgR) \oplus t((1-h)f_tgR) \approx t(f_tgR) \). Thus \( f_tR = M_t(f_tgRf_tg) \) as rings, since \( f_tgRf_tg \) is a maximal right quotient ring of \( f_tgRf_tg \) and is an abelian regular right self-injective ring. Thus the proof is complete.

**Example.** There exists a non-singular two-sided FPF-ring \( R \), which is not isomorphic to a full matrix ring of non-singular FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring.

**Proof.** Let \( D \) be a Prüfer domain which is not a principal ideal domain and let \( I \) be a non-principal finitely generated ideal of \( D \). We set \( R = \left( \begin{array}{cc} D & I \\ I^{−1} & D \end{array} \right) \).

It is easy to see that \( R \) is non-singular FPF-ring, and \( R \) does not satisfy general comparability. Hence by Theorem 1, \( R \) is not isomorphic to a full matrix ring over a non-singular FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring.

2. **Non-singular quasi-continuous FPF-rings**

In this section, as an application of Theorem 1, we shall determine the structure of non-singular quasi-continuous FPF-rings. First we recall that a ring \( R \) is right quasi-continuous if (1) \( R \) has the extending property of modules for \( L_t(R) \), and (2) For any \( A, B \) in \( L_t(R) \), which are direct summand of \( R \) with
In [3], K. Oshiro has studied quasi-continuous modules, and proved the following.

**Proposition 1** ([3, Propositions 1.5 and 3.1]). Let $M$ be a right $R$-module. Then the following conditions are equivalent.

1. $M$ is right quasi-continuous.
2. (i) $M$ has the extending property of modules for $L_r(M)$.
   (ii) For any $A$ and $B$ in $L_r(M)$ such that $B\ltimes M$ i.e. $B$ is a direct summand of $M$, and $A \cap B = 0$, every homomorphism of $A$ to $B$ is extended to a homomorphism of $M$ to $B$.
3. Every decomposition $E(M) = E_1 \oplus E_2 \oplus \cdots \oplus E_n$ implies that $M = (E_1 \cap M) \oplus (E_2 \cap M) \oplus \cdots \oplus (E_n \cap M)$, where $E(M)$ denotes the injective hull of $M$.

In order to determine the structure of non-singular quasi-continuous FPF-rings, we shall need the following lemma.

**Lemma 3.** Let $R$ be a non-singular right quasi-continuous right FPF-ring. Then $R$ satisfies general comparability.

Proof. Let $e$ and $f$ are idempotents of $R$, and consider an exact sequence $0 \to eR \cap fR \to fR \to (1-e)fR \to 0$. Since $R$ is right quasi-continuous, $R$ has the extending property of modules for $L_r(R)$, $(1-e)fR$ is projective. Thus $eR \cap fR$ is a direct summand of $fR$. Similarly, $eR \cap fR \ltimes \oplus eR$. Hence $eR = (eR \cap fR) \oplus eR$ and $fR = (eR \cap fR) \oplus fR$. It follows that if there exists a central idempotent $h$ of $R$ such that $h(eR) \leq h(fR)$ and $(1-h)(fR) \leq (1-h)(eR)$, then $h(eR) \leq h(fR)$ and $(1-h)(fR) \leq (1-h)(eR)$. Hence $R$ satisfies general comparability. Therefore, we may assume that $eR \cap fR = 0$. Let $X$ denote the collection of all triples $(A, B, \varphi)$ such that $A \subseteq eR$, $B \subseteq fR$, and $\varphi: A \to B$ is an isomorphism. Define a partial order on $X$ by setting $(A', B', \varphi') \leq (A', B', \varphi')$ whenever $A' \subseteq A'$, $B' \subseteq B'$, and $\varphi'$ is an extension of $\varphi'$. By Zorn’s lemma, there exists a maximal element $(A', B', \varphi')$ in $X$. Since $R$ has the extending property of modules for $L_r(R)$, there exist direct summands $A^*, B^*$ of $R$ such that $A' \subseteq A^*$, $B' \subseteq A^*$. Then since $R$ is right quasi-continuous, by the condition (2) of Proposition 1, we have homomorphisms $\varphi: A^* \to B^*$, and $\varphi: B^* \to A^*$, which are extensions of the isomorphisms $\varphi'$ and $(\varphi')^{-1}$ respectively. We show that $\varphi$ is an isomorphism. Let $m$ be an element of $A^*$ such that $\varphi(m) = 0$. Then since $A' \subseteq A^*$, $f = \{r \in R | mr \in A'\}$ is an essential right ideal of $R$ and $\varphi'(mr) = 0$ for any $r \in f$. Thus $mf = 0$ since $\varphi'$ is an isomorphism, and $m = 0$. Hence $\varphi$ is a monomorphism. Next let $m$ be any element of $B^*$. Then for any element $r$ of $H = \{r \in R | mr \in B'\}$, $\varphi \circ \psi(mr) = \varphi'(\varphi')^{-1}(mr) = mr$, where $\varphi$ is an epimorphism. Therefore $\varphi$ is an isomorphism. On the other hand, by the maximality of $(A', B', \varphi')$, $(A', B', \varphi') = (A^*, B^*, \varphi)$, hence $A'$ and $B'$ are direct summand of...
Thus $eR = A' \oplus e_1 R$ and $fR = B' \oplus f_1 R$. Set $I = r_R(e_1 R)$. Since $R$ is right FPF, there exists a central idempotent $h$ of $R$ such that $I = hR$ by [3, Theorem 1]. In this case, we have that $h(eR) = h(A') \leq h(B') \leq h(fR)$. Next we show that $f_1 R(1 - h) = 0$. If $f_1 R(1 - h) \neq 0$, then there is a nonzero element $x \in f_1 R(1 - h)$. Since $xR$ is non-singular, $xR$ is projective, so that there exists an idempotent $t$ of $R$ such that $xR = tR$ by the extending property to $R$. We note that $tRh = xRh = 0$. Since $t \neq 0$, it follows that $t \in hR$. Then $yt \neq 0$ for some $y \in e_1 R$. We have an exact sequence $xR \rightarrow tR \rightarrow ytR \rightarrow 0$, and that $ytR$ is projective. Thus there exists a monomorphism $g : ytR \rightarrow xR$. Since $yt \in e_1 R$ and $x \in f_1 R$, we obtain that $(A' \oplus y_1 R, B' \oplus g(ytR), \varphi' \oplus g) \in X$. But this contradicts the maximality of $(A', B', \varphi')$. Therefore $f_1 R(1 - h) = 0$. Now $(1 - h)(fR) = (1 - h)(B') \simeq (1 - h)(A') \leq (1 - h)(eR)$. Therefore $R$ satisfies general comparability.

**Theorem 2.** Let $R$ be a non-singular right FPF-ring and $Q$ be the maximal right quotient ring of $R$. Then the following conditions are equivalent.

1. $R$ is right quasi-continuous.
2. $id(R) = id(Q)$.
3. $R \simeq R_1 \times \prod_{i=1}^{n(i)} M_{n(i)}(S_i)$, where $R_1$ is a non-singular right FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring, and each $S_i$ is an abelian regular self-injective ring and $n(i) \geq 2$.

**Proof.** $(1) \Rightarrow (3)$: Since $Q$ is a regular self-injective ring of bounded index, $Q$ has a faithful and abelian idempotent $e$, i.e. the right $Q$-module $eQ$ is faithful and the ring $eQe$ is abelian regular. Then $Q = eQ \oplus (1 - e)Q$. Thus by the condition (3) of Proposition 1, $R = e'R \oplus e''R$ for some idempotents $e'$, $e''$ of $R$. We show that $e'$ is a faithful and reduced FPF-idempotent. Let $I = r_R(e'R)$. Then since $R$ is right FPF, by Theorem 1 of [3], there exists a central idempotent $f$ of $R$ such that $I = fR$. Set $J = \{ r \in R | er \in R \}$. Then we obtain that $ef \subseteq eQ \cap R = e'R$ and $eef = 0$, so $ef = 0$ since $J$ is an essential right ideal of $R$. While since $eQ$ is faithful, $f$ must be zero. Hence $e'R$ is faithful. Furthermore, $eQe$ is clearly, a maximal right quotient ring of $e'Re'$. Therefore $e'$ is a faithful and reduced FPF idempotent. Moreover, by Lemma 2, $R$ satisfies general comparability. Therefore it follows from Theorem 1 that $R \simeq \prod_{i=1}^{n(i)} M_{n(i)}(S_i)$, where each $S_i$ is a non-singular right FPF-ring whose the maximal right quotient ring is an abelian regular self-injective ring. We claim that if $n(i) \geq 2$, then each $S_i$ is a self-injective regular ring. To prove this, we set $R = M_n(S)$ ($n \geq 2$), where $S$ is a non-singular right FPF-ring whose the maximal right quotient ring $Q(S)$ is abelian regular. Assume that $Q(S) \neq S$ and let $w$ be any element of $Q(S) - S$. Set $\bar{e} = (1, 0, \cdots, 0, w)$. Then clearly, $\bar{e}Q = \begin{pmatrix} Q(S), Q(S), \cdots, Q(S) \end{pmatrix}$ and
\[ (1-e)Q = \{ wx_1, wx_2, \ldots, wx_n \mid x, x, \ldots, x_n \in Q(S) \}. \]

In this case

\[ eQ \cap R = \begin{pmatrix} \mathbf{S} & \mathbf{S} & \cdots & \mathbf{S} \\ 0 & \mathbf{S} & \cdots & \mathbf{S} \end{pmatrix} \]

and \((1-e)Q \cap R = \{ wy_1, wy_2, \ldots, wy_n \mid y_1, y_2, \ldots, y_n \in J \}

\[ = \{ r \in S \mid wr \subseteq S \}. \]

Hence \((eQ \cap R) \oplus (1-e)Q \cap R = (S, \cdots, S) \neq R. \)

But this contradicts that \( R \) is right quasi-continuous. Therefore \( Q(S) = \mathbf{S}, \) so \( S \) is regular self-injective.

(3) \( \Rightarrow \) (2); Since idempotent of \( R_i \) are central, \( id(R_i) = id(Q_{\max}(R_i)). \)

Therefore \( id(R) = id(Q). \)

(2) \( \Rightarrow \) (1); By the condition (3) of Proposition 1, it suffices to show that if \( Q = e_1Q \oplus e_2Q \oplus \cdots \oplus e_nQ \) for some idempotents \( e_i \) of \( Q \), then \( R = (e_1Q \cap R) \oplus (e_2Q \cap R) \oplus \cdots (e_nQ \cap R). \)

But since \( id(R) = id(Q), \) this is clear.

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References


