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The effect of inter vivos gift taxation on wealth inequality and economic growth

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Abstract

This study examines the effect of inter vivos gift taxation on wealth inequality and economic growth. We develop a simple model with inter vivos gifts, which are generated by altruism and gift taxation in an overlapping generation setting. The analysis shows that an increase in the gift tax rate reduces inequality, and a positive tax rate maximizes the economic growth rate. From a policy perspective, rather than exempting gifts from taxation, raising the gift tax rate to some extent reduces inequality and promotes human capital accumulation and, therefore, economic growth.

KEYWORDS
economic growth, gift taxation, human capital accumulation, intergenerational transfer, wealth inequality

JEL CLASSIFICATION
O11, O40, I24

1 | INTRODUCTION

Life-cycle savings is a popular explanation for wealth differences, but intergenerational transfer is widely recognized as a factor that plays a major role, especially at the upper end of the wealth range (Davies & Shorrocks, 2000). Introducing intergenerational transfer, some theoretical literature succeeds in showing wealth concentration (De Nardi, 2004; Laitner, 2001). However, other recent empirical studies suggest that intergenerational transfers are commonly found to make the distribution of wealth more equal (Horioka, 2009; Karagiannaki, 2017; Klevmarken, 2004; Wolff, 2002). However, Niimi and Horioka (2018) show that the receipt of intergenerational transfers increases the probability of respondents' leaving bequests to their children in Japan and the USA. The observed similarity in bequest behavior between parents and children suggests the possibility that wealth disparities are passed on from generation to generation, contributing to the persistence or widening of wealth.

In practice, 24 of 38 Organization for Economic Co-operation and Development (OECD) countries levy wealth transfer taxes, including a gift tax on inter vivos transfers (lifetime wealth transfers).[1] Although wealth transfer taxation is considered an appropriate policy to redistribute wealth and reduce inequality of opportunities, there is room for further discussion on the effect of taxation on inequality and growth. Heer (2001) finds that inheritance taxes increase both wealth inequality and welfare. Bossmann et al. (2007) show that using the coefficient of variation as the measure of inequality, bequests per se diminish wealth inequality. From a policy perspective, by levying a wealth transfer tax and redistributing revenue among the younger generation, the government can further reduce the concentration of wealth.

While taxation of gifts has been introduced in many countries, in some countries, this does not apply to gifts that are for educational purposes. As a part of the Comprehensive Reform of Social Security and Tax in 2013, a tax incentive for lump-sum gifts of educational funds was introduced in Japan, in addition to a reduction of the basic deduction of inheritance tax by 40% and an increase in the highest tax rate of both inheritance tax and gift tax.[2] Because of the tax incentive, a one-time transfer for each grandchild up to 15 million yen for educational funds is tax-exempted for their grandparents. This policy aims to encourage the transfer of assets held by older generations to younger generations and to promote economic growth.[3] However, the tax incentive may be harmful to wealth inequality and economic growth. Gift-giving itself can create disparities in assets when gifts are used to fund education; at the same time, it generates differences in educational opportunities, as not all households can afford to fund the complete education of their offspring. Thus, differences in educational opportunities lead to income inequality, which contributes to further wealth inequality in a snowballing effect.[4]

To examine the effect of gift taxation on inequality and growth, we develop a simple model with inter vivos gifts and gifts taxation in an overlapping generation setting. Moreover, we introduce human capital accumulation into the model to consider the differences in educational opportunities. The analysis shows that an increase in the gift tax rate reduces inequality, and a positive tax rate maximizes the economic growth rate in the range of plausible parameters. Thus, rather than exempting gifts from taxation, raising the gift tax rate to some extent reduces inequality and promotes human capital accumulation and, therefore, economic growth.

In a related work, Ihori (2001) examines the effect of bequest tax on economic growth with altruistic bequest motive.[5] The author shows that an increase in taxes on savings will reduce the intragenerational growth difference, while the effect of bequest tax on the growth rates is ambiguous. In contrast to Ihori (2001), we show that a positive gift tax rate maximizes the growth rate in the baseline model with different bequest motives. The mechanism behind this result is as follows. An increase in gift tax reduces necessarily private education. However, it leads to an increase in public education due to the rise in tax revenue. If the gift tax rate before a change is sufficiently low, the effect of an increase in public education on growth outweighs. Therefore, the relationship between the growth rate and the gift tax rate is non-monotonic. This study provides new qualitative insights in terms of economic growth.

This study contributes to the literature that highlights the relationship between public education and economic growth. Blankenau and Simpson (2004) examine the relationship between public education expenditures

[1]See 'Inheritance taxation in OECD Countries'.

[2]Gifts of educational funds that mainly cover tuition are exempted from taxation up to a certain amount. Unlike the U.S., in addition to tuition, this system applies to other expenditures related to education, such as fees for extracurricular activities that accumulate human capital.

[3]Based on information provided by the Ministry of Education, Culture, Sports, Science and Technology (https://www.mext.go.jp/a_menu/kaikei/zeisei/1332772.htm).

[4]See Saez and Zucman (2016).

[5]In contrast to De Nardi and Yang (2016), Ihori (2001) and Kopczuk (2013) examine the effect of bequest tax mainly on inequality in similar settings.

and growth through human capital accumulation, showing that the relation is non-monotonic. As raising the gift tax rate leads to an increase in public education expenditures, the result of this study is consistent with their findings. Azarnert (2010) analyzes the effect of free public education on human capital accumulation at different stages of economic development. The author shows that at advanced stages of development, the availability of free education crowds out private educational investments and may impede growth. Although some studies have identified income tax as a source of funding for public education, we use the gift tax as a source of funding for public education. In Section 4, we discuss the assumption of funding for public education.

In addition, this study relates to other studies that investigate the relationship between human capital accumulation and inequality with the overlapping generations model. Since the seminal papers of De la Croix and Doepke (2004) and Galor and Moav (2004), Prettner and Schaefer (2021) explain the U-shaped evolution of income inequality employing the overlapping generations model with intergenerational transfer. In contrast to these studies, we introduce wealth transfer taxation to examine the effect of gift tax on wealth inequality and growth. This paper complements existing literature by introducing gift tax into the model.

The remainder of this paper is organized as follows. Section 2 explains the model. Section 3 analyzes the effect of gift tax. Section 4 discusses how the results obtained in the baseline model change when different assumptions are applied. Finally, Section 5 concludes the paper.

2 | MODEL

Consider a three-period overlapping generations model. Time is discrete, and the economy consists of households, government, and firms under perfect competition. In this economy, an individual in the last period donates educational funds to his/her grandchild. The government imposes a tax on gifts and funds for public education. We assume a small open economy, which implies that the interest rate is equal to the world interest rate $\bar{r}$.

2.1 | Households

There are two types of individuals: L and H. Assume that the initial human capital stock of type H is larger than that of type L ($h_0^H > h_0^L$). The difference between these two types is the initial human capital. The population of each type in each generation is one, and there is no population growth. There are two families or dynasties, consisting of one grandparent, one parent, and one child, and each lives for three periods. In the first period of life, individuals born in period t−1 receive both public and private education and make no economic decisions. In the second period of life, they supply human capital h_t to the labor market and allocate their income to consumption $c_{1,t}$ and saving s_t. In the third period of life, they retire and allocate their savings to consumption $c_{2,t+1}$ and gifts b_{t+1} for their grandchild.[6] Unlike Ihori (2001), McDonald and Zhang (2012) and Zilcha (2003), and some related studies, we do not consider leaving a bequest to their offspring as a form of physical capital. Our main attention here is the role of bequest as a form of human capital investment. We assume that a nuclear family (parents and children) receives a transfer from living grandparents with the use of transfer limited to education. The budget constraint for type i individuals born in period t−1 is given by

$$w_t^i h_t^i = c_{1,t}^i + s_t^i, \tag{1}$$

$$(1+\bar{r})s_t^i = c_{2,t+1}^i + b_{t+1}^i, \tag{2}$$

[6]Although most literature considers the transfer of goods, Cardia and Michel (2004) consider time transfers and show that it may occur when intergenerational altruism is insufficient to generate bequests.

where w^i denotes the wage rate of type *i*. Each individual does not consider the effect of taxation when making decisions about giving educational funds. Individuals draw utility from consumption and the amount of donations. Since they cannot observe their offspring's educational outcome in this model, we assume that the motive of giving is the joy of giving.[7] The utility function of individuals born in period $t-1$ is given by

$$U^i_{t-1} = lnc^i_{1,t} + \beta lnc^i_{2,t+1} + \gamma lnb^i_{t+1}. \tag{3}$$

The utilities from future consumption and donation are discounted by β and γ, respectively. From (1), (2), and (3), solving the maximization problem of households, the optimal choices are

$$c^i_{1,t} = \frac{1}{1+\beta+\gamma} w^i_t h^i_t, \tag{4}$$

$$c^i_{2,t+1} = \frac{\beta(1+\bar{r})}{1+\beta+\gamma} w^i_t h^i_t, \tag{5}$$

$$b^i_{t+1} = \frac{\gamma(1+\bar{r})}{1+\beta+\gamma} w^i_t h^i_t. \tag{6}$$

2.2 | Production

Firms produce goods by employing the efficiency unit of labor h_t and physical capital K_t from household savings, and the capital depreciates fully. The production function is given by

$$y_t = K_t^{\alpha} \left(a_1 h^H_t + a_2 h^L_t\right)^{1-\alpha}. \tag{7}$$

where y_t denotes output at time t while a_1 and a_2 are productivity parameters for each type of worker.[8] Assume that $a_1 \geq a_2$.[9] Under perfect competition, wages and interest rates are equal to the marginal products of each input in equilibrium. According to the assumption of a small open economy, $r_t = r_{t+1} = \bar{r}$ is satisfied. Thus, the wage rates are

$$w^H_t = \frac{\partial y_t}{\partial h^H_t} = (1-\alpha)\left(\frac{\bar{r}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} a_1 \equiv a_H, \tag{8}$$

$$w^L_t = \frac{\partial y_t}{\partial h^L_t} = (1-\alpha)\left(\frac{\bar{r}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} a_2 \equiv a_L. \tag{9}$$

2.3 | Government

The government taxes gifts at the tax rate $\tau_b \in (0,1)$. All revenue is used for public education, which both types can receive as school education. Tax revenue is generally used for various purposes. The objective of taxing

[7]See Andreoni (1989), Galor and Zeira (1993) and Glomm and Ravikumar (1992).

[8]Generally, the relationship high-skilled and low-skilled labor could be either substitutive or complementary. For analytical tractability, we adopt the substitutive case. Bräuninger and Vidal (2000) employ a similar production function in the OLG framework and analyze how public and private education affects inequality and growth.

[9]When human capital *h* is sufficiently accumulated, the differences between each type of human capital are small, and the existence of a wage difference may be unnatural. However, if we consider two types of non-mobile regions, we can simulate a situation wherein a difference in wages exists even if the difference in the amount of human capital is small, such as between urban and rural areas.

intergenerational transfer is a distributional purpose. Public education also serves as a redistributive policy. In order to focus on distributional purposes, we limit the use of tax revenue. In each period, the government's budget constraint is balanced as follows:

$$e_t = \tau_b b_t^H + \tau_b b_t^L. \tag{10}$$

2.4 | Education

There are many theoretical specifications for the function of human capital production. Most studies assume that it depends on parental human capital or educational investment from their parents. Following Arcalean and Schiopu (2010) and Bearse et al. (2005), we assume the constant elasticity of substitution function forms of public and private education.[10] The expenditures for private education d_t^i is financed by after-tax gifts $(1-\tau_b)b_t^i$. Public education is provided by the government. Under the utility function defined as (3), individuals care about the amount of educational gift rather than the breakdown. When individuals make decisions, they overlook the fact that public education is funded by taxation of gifts. The amount of human capital for type i individuals born in period t is determined by

$$h_{t+1}^i = \left[\varepsilon\left(d_t^i\right)^q + (1-\varepsilon)\left(e_t\right)^q\right]^{\frac{1}{q}}. \tag{11}$$

Here, ε represents the weight of each type of education. The larger the value of ε, the greater the effect of private education on human capital accumulation. Few studies investigate the elasticity of substitution between public education and private education, except Houtenville and Conway (2008) and Vinson (2022). Houtenville and Conway (2008) suggest that the elasticity of substitution between private and public inputs in education is larger than what the Cobb–Douglas specification suggests.[11] It seems that private and public education are substitutable rather than complementary, especially when we consider private education as a private tutoring school. In a recent work, Vinson (2022) finds that the elasticity of substitution between public and private expenditures in education is about 2.4. This value indicates that q is approximately 0.58 in the present model. Hereafter, we focus on the case where $q > 0$. We will discuss this assumption in Section 4 and provide the results in the case of $q = 0$ and $q < 0$ cases.

2.5 | Dynamics

In this economy, we define the steady state as a situation in which both types of human capital grow at the same rate. By substituting (6) and (10) into (11), each type of human capital for $i, j \in \{H, L\}, i \neq j$ is

$$h_{t+1}^i = \Theta\left[\varepsilon\left(1-\tau_b\right)^q\left(a_i h_{t-1}^i\right)^q + (1-\varepsilon)\tau_b^q\left(a_i h_{t-1}^i + a_j h_{t-1}^j\right)^q\right]^{\frac{1}{q}}, \tag{12}$$

where $\Theta = \frac{\gamma(1+\bar{r})}{1+\beta+\gamma}$. While the difference in savings among households seems to be appropriate as wealth inequality, we define inequality of this model as $\phi_t \equiv \frac{h_t^L}{h_t^H}$ (the ratio of human capital) because saving in this model is finally determined by the human capital of each type. Since we assume that $h_0^H > h_0^L$ and h^L never outweighs h^H from the definition, ϕ is

[10] Gamlath and Lahiri (2018) set the variable elasticity of substitution function form to examine how the degree of substitutability between public and private educational expenditures affects an economy's transitional and long-run economic performance.

[11] Concerning private and public education being substitutable in this model, in Yakita (2010), learning at home and in school is assumed to be fully substitutable. Moreover, Azarnert (2010) and Glomm and Kaganovich (2003) assume that private input in education is a perfect substitute for public input.

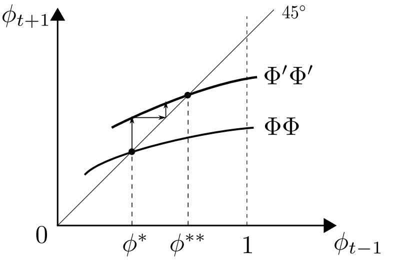

FIGURE 1 Transition of ϕ. In this economy, we define inequality as $\phi \in (0,1)$. The closer ϕ is to 1, the lower the inequality is. The law of motion for inequality ϕ is illustrated by curve $\Phi\Phi$. The curve $\Phi\Phi$ intersects the 45-degree line at a point less than 1. Therefore, a unique stable steady state exists in this economy. An increase in the gift tax rate is illustrated as an upward shift of curve $\Phi\Phi$ to $\Phi'\Phi'$. This implies that an increase in the gift tax rate reduces inequality.

between 0 and 1. When ϕ approaches 1, the inequality is implied to have declined. From (12), the dynamic system of the economy is given by the following equation:

$$\phi_{t+1} = \phi_{t-1}\left[\frac{\varepsilon\left(1-\tau_b\right)^q a_L^q + (1-\varepsilon)\tau_b^q\left(a_L + \frac{a_H}{\phi_{t-1}}\right)^q}{\varepsilon\left(1-\tau_b\right)^q a_H^q + (1-\varepsilon)\tau_b^q\left(a_H + \phi_{t-1}a_L\right)^q}\right]^{\frac{1}{q}}. \tag{13}$$

In this model, the human capital of generation $t+1$ does not depend on the human capital of their parent (generation t) but that of the grandparent (generation $t-1$). Therefore, the dynamics also depend only on the state before any two periods. Combinations $\left(\phi_{t+1}, \phi_{t-1}\right)$ satisfying (13) are illustrated by the curve $\Phi\Phi$ in Figure 1, which is upward-sloping and intersects the 45-degree line at a point less than 1 when $\tau_b \in (0,1)$.[12] We can confirm that ϕ^* exists such that it satisfies $\phi^* < 1$. Therefore, a unique stable steady state exists in this economy.

3 | EFFECTS OF GIFT TAX

We examine how the gift tax rate affects growth rate and inequality. To begin with, we examine the effect on the steady state.

3.1 | Steady state

Proposition 1. *An increase in the gift tax rate reduces inequality.*

Proof. See Appendix A.2.

The intuition is as follows. An increase in the gift tax rate reduces the amount of private education but increases the amount of public education. When private and public education are substitutable, a larger total amount of education implies greater human capital accumulation. Type L, with less private education, will relatively accumulate

[12]The proof is relegated to the Appendix A.1. The dynamics of ϕ is different from the other cases when τ_b equals 0 or 1.

more human capital than type H when public education increases due to higher taxes. Thus, inequality will be reduced. As the amount of gifts is proportional to the income, the burden on wealthy individuals increases when taxes are raised. Meanwhile, an increase in tax revenue is directly reflected in an increase in public education; hence the benefits they receive are equal regardless of their type. Therefore, the relatively poor type L is more likely to promote the accumulation of human capital, which reduces inequality.

Next, we analyze the effect on the growth rate in the steady state. We define the growth rate as $g^i_{t+1} \equiv \frac{h^i_{t+1}}{h^i_t}$. In the steady state, as both types of human capital grow at the same rate, we omit subscript i. From (12), we derive the growth rate for two periods:

$$g = \Theta\left[\varepsilon\left(1-\tau_b\right)^q a_H^q + (1-\varepsilon)\tau_b^q\left(a_H+\phi a_L\right)^q\right]^{\frac{1}{q}}. \tag{14}$$

As mentioned, human capital accumulation at time t+1 depends only on human capital stock at time t−1. That is, no connection of human capital stock exists between odd and even periods. This is why we focus on the growth rate for two periods.

> **Proposition 2.** The relationship between the growth rate and the gift tax rate is non-monotonic, and a positive gift tax rate maximizes the growth rate.

Proof. See Appendix A.3.

The intuition is as follows. A higher gift tax rate reduces private education and increases public education in the same way as it affects inequality. As an increase in both types of education has a positive effect on human capital accumulation, it has a positive effect on the growth rate by increasing public education and a negative effect on the growth rate by decreasing private education. In addition, since a rise in gift tax rate impacts inequality ϕ, there is another effect through ϕ. According to Proposition 1, ϕ increases as the gift tax rate rises. A higher ϕ leads to a relatively higher level of public education. Consequently, it improves the growth rate.

To summarize the effects, there are two positive effects and one negative effect of an increase in the tax rate on growth rate. Whether an increase in the gift tax rate positively affects the growth rate depends on the value of the gift tax rate. When it is sufficiently small, even if both education types are substitutable, the marginal effect of an increase in public education overcomes the marginal effect of a decrease in private education. As the tax rate becomes higher, the negative effect becomes larger and eventually exceeds the sum of positive effects. Therefore, a tax rate that maximizes the economic growth rate exists between 0 and 1. We also find that we cannot simultaneously minimize inequality and maximize the growth rate to operate the gift tax rate. The results suggest that, rather than exempting gifts from taxation, raising the gift tax rate to some extent reduces inequality and promotes human capital accumulation and, therefore, economic growth.[13]

3.2 | Transition

In this section, we provide a brief discussion on the transition path. To determine the short-run effect of gift tax, consider a scenario where initially at a steady state, the gift tax rate rises from sufficiently small positive τ_b to τ_b' in period t'. First, consider the transition of inequality ϕ. The increase in the gift tax rate is illustrated as an upward shift of curve $\Phi\Phi$ to $\Phi'\Phi'$ in Figure 1. As shown in Figure 1, ϕ increases monotonically and converges to a new steady state ϕ^{**}.

[13]The welfare implications are essential when we consider the implementation for public policies. However, the welfare ratio defined by U^L / U^H approaches 1 irrespective of the gift tax rate in the long run.

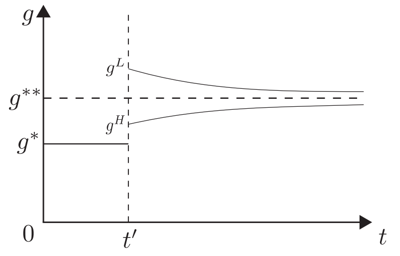

FIGURE 2 Transition of growth rate. The horizontal axis represents time. g^* represents the growth rate at the initial steady state, while g^{**} represents the growth rate at a new steady state. At time t', both of the growth rates jump differently. After the initial jump, both of the growth rates converge to a new steady state.

Second, consider the transition of growth rate. From the definition, we derive the growth rate for two periods of type H and L:

$$g^H_{t+1}=\frac{h^H_{t+1}}{h^H_{t-1}}=\Theta\left[\varepsilon\left(1-\tau_b\right)^q a_H^q+(1-\varepsilon)\tau_b^q\left(a_H+\phi_{t-1}a_L\right)^q\right]^{\frac{1}{q}} \tag{15}$$

$$g^L_{t+1}=\frac{h^L_{t+1}}{h^L_{t-1}}=\Theta\left[\varepsilon\left(1-\tau_b\right)^q a_L^q+(1-\varepsilon)\tau_b^q\left(a_L+a_H/\phi_{t-1}\right)^q\right]^{\frac{1}{q}} \tag{16}$$

The change in the gift tax rate has both direct and indirect effects on the growth rate. As $\phi_{t'-1}$ is already determined at changed period t', the change in the gift tax rate does not affect the growth rate through inequality in the changed period. The direct effect can be positive or negative. If τ_b' is sufficiently small, the growth rate of both types will initially jump. On the one hand, as shown in Figure 2, the growth rate of type L initially jumps up and decreases over time. On the other hand, the growth rate of type H also jumps down but increases over time. Note that after the increase in the tax rate, only ϕ affects the growth rate. Therefore, the growth rate of type H increases over time, but the growth rate of type L decreases over time. Finally, the growth rate of both types converges to a new steady state growth rate g^{**}. In summary, if the tax rate is sufficiently small, it converges to the new steady state without dropping below the growth rate in the previous steady state.

4 | DISCUSSION

For analytical tractability, we put some restrictions on parameters and assumptions in the baseline model. This section discusses how the results obtained above change when different assumptions are applied.

4.1 | Other tax revenues

In reality, the government collects money for public expenditures in many ways. If the gift tax rate is zero, there is no public education in the previous settings. The issue with the assumption that public education is financed only by gift taxation is that if there exists sufficient other funding for public education, the gift tax rate being zero might be optimal for growth. In other words, the result that a positive gift tax rate maximizes the growth rate might depend crucially on the assumption. This subsection examines the case in which another funding of public education exists. For tractability, we introduce interest income tax as another funding source for public education. Therefore, the budget constraint for the second period of life is rewritten as

$$\left[1+(1-\tau_r)\bar{r}\right]s_t^i = c_{2,t+1}^i + b_{t+1}^i, \tag{17}$$

where τ_r denotes the interest income tax rate. The government's budget constraint is also rewritten as follows:

$$e_t = \tau_b b_t^H + \tau_b b_t^L + \tau_r \bar{r} s_{t-1}^H + \tau_r \bar{r} s_{t-1}^L. \tag{18}$$

Note that the savings in period t−1 appear to be in the constraint of government in period t because the interest income generated by the savings in period t−1 is subject to taxation of period t. Then, each type of human capital for $i, j \in \{H, L\}, i \neq j$ is

$$h_{t+1}^i = \frac{\left\{\varepsilon\left[(1-\tau_b)\psi a_i h_{t-1}^i\right]^q + (1-\varepsilon)\left[(\tau_b\psi+\eta)\left(a_i h_{t-1}^i + a_j h_{t-1}^j\right)\right]^q\right\}^{\frac{1}{q}}}{1+\beta+\gamma} \tag{19}$$

where $\psi = \gamma\left[1+(1-\tau_r)\bar{r}\right]$ and $\eta = \tau_r \bar{r}(\beta+\gamma)$. In the same procedure, the dynamic system of this economy is described by the following equation:

$$\phi_{t+1} = \phi_{t-1}\left\{\frac{\varepsilon\left[(1-\tau_b)\psi a_L\right]^q + (1-\varepsilon)(\tau_b\psi+\eta)^q\left(a_L + \frac{a_H}{\phi_{t-1}}\right)^q}{\varepsilon\left[(1-\tau_b)\psi a_H\right]^q + (1-\varepsilon)(\tau_b\psi+\eta)^q\left(a_H + \phi_{t-1}a_L\right)^q}\right\}^{\frac{1}{q}}. \tag{20}$$

In the steady state, as ϕ is constant over time, we obtain the implicit function of ϕ and τ:

$$\varepsilon(1-\tau_b)^q\psi^q\left(a_H^q - a_L^q\right) + (1-\varepsilon)(\tau_b\psi+\eta)^q\left[(a_H+\phi a_L)^q - \left(a_L + \frac{a_H}{\phi}\right)^q\right] = 0. \tag{21}$$

Applying the same procedure as in the proof of Proposition 1, we obtain the following result.

Proposition 3. Even in the presence of interest income taxation, an increase in the gift tax rate reduces inequality.

Regardless of the presence of interest income taxation, an increase in the gift tax rate reduces inequality. The benefit from an increase in public education is always higher for the poor than the rich. Taxation of gifts fills in the gap of human capital between type L and type H, similar to the previous settings.

Next, we examine the effect of the presence of interest income taxation on growth rate. From (19), we derive the growth rate for two periods:

$$g = \frac{1}{1+\beta+\gamma}\left\{\varepsilon\left[(1-\tau_b)\psi a_H\right]^q + (1-\varepsilon)(\tau_b\psi+\eta)^q(a_H+\phi a_L)^q\right\}^{\frac{1}{q}}. \tag{22}$$

The derivative of the growth rate for two periods g with respect to τ_b at $\tau_b = 1$ is always negative. However, the derivative at $\tau_b = 0$ is not always positive. Thus, in some parameters, the optimal gift tax rate for maximizing growth is zero. Even in the presence of interest income taxation, however, we can show that the gift tax rate being zero is not optimal for growth in the range of plausible parameter values. To confirm the above result, we conduct some numerical experiments. We shall emphasize that, despite using some reasonable parameter values, our goal is to check the properties of the growth rate qualitatively rather than quantitatively. We choose the parameters of the model such that the growth rate fits the empirical observations of advanced countries. Considering the

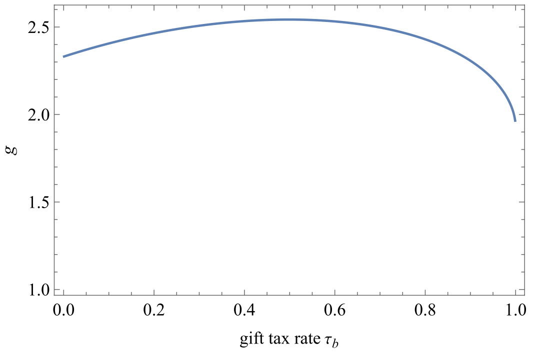

FIGURE 3 A numerical example of a discussion model. The horizontal axis represents the gift tax rate $\tau_b \in (0, 1)$. The vertical axis represents the growth rate for two periods g. When $\tau_b \fallingdotseq 0.5$, the growth rate for two periods is maximized in this example.

three-period overlapping generations model, one period has a length of 25 years. The literature on real business cycle suggests a discount factor of future consumption of around 0.99 per quarter, that is, $\beta = 0.99^{100}$. The real interest rate is set to 4% per year, that is, $\bar{r} = 1.67$. There are few studies on the relative weight of private education ϵ. We choose the intermediate case, that is, $\epsilon = 0.5$. According to Houtenville and Conway (2008), the elasticity of substitution between private and public inputs in education is larger than zero. In a recent study, Vinson (2022) finds that the elasticity of substitution between public and private inputs in education is about 2.4. This value indicates that q is approximately 0.58 in the present model, so we choose $q = 0.58$. Following Bossmann et al. (2007), the utility weight of the donation is set to $\gamma = 0.09$. In Japan, the tax rate of interest income is about 15%, that is, $\tau_r = 0.15$. The remaining values are $a_H = 19$ and $a_L = 9$.

According to Figure 3, we confirm the properties of the growth rate in a certain range of parameters. As mentioned in Section 3.1, an increase in the gift tax rate has one negative and two positive effects on growth. When the gift tax rate is sufficiently low, positive effects overcome the negative effect, similar to the previous settings. This is why the gift tax rate being zero is not optimal. However, public education is provided to some extent without taxation of gifts because of the presence of interest income taxation. If the relative weight of private education ϵ is sufficiently large because of the substitutability of education (recall that $q > 0$), the negative effects always overweigh the positive effects. Therefore, the taxation of gifts is not efficient. All donations from grandparents should be used for private education. In such a case, the gift tax rate being zero is optimal for growth.

4.2 | The elasticity of substitution between public and private education

For analytical tractability, the main analyses focus on the case in which public and private education are relatively substitutable. Few studies estimate the elasticity of substitution except Houtenville and Conway (2008) and Vinson (2022). Both studies support our assumption in the main analyses that q is positive. Furthermore, there is the growing literature that examines the relationship between public education spending and private inputs in education. Kim (2001), using the Panel Study of Income Dynamics, shows that higher school expenditures do not lead to changes in child care time among the higher-educated mothers, while the lower-educated mothers reduce their child care time. Pop-Eleches and Urquiola (2013) find that Romanian parents consider school quality as a substitute for parental involvement, such as the willingness to help with homework or pay for tutoring services. Yuan and Zhang (2015) show that increases in public education spending are associated with significant decreases in household spending on private tutoring in urban China. According to these studies, the relationship between public expenditures and private inputs in education are more likely to be substitutable than complementary.[14]

However, private inputs in education are multidimensional. In the aforementioned studies, the private inputs include time inputs such as helping child with homework and mother's child care time. Private education in the present model should be interpreted as inputs of goods, and thus the result obtained from the aforementioned studies may not be appropriate for the present model. Other factors such as gender, country, and parental background would affect whether private inputs serve as substitutes or complements to public expenditures. It would be dependent on how the government intervenes, especially on whether the intervention is for primary or higher education. From a theoretical perspective, Blankenau and Simpson (2004) employ the human capital production function where private and public expenditures are imperfect substitutes. The authors' interpretation is that primary and secondary school, where students acquire general skills, are basically funded by public expenditures, but private expenditures mainly finance a college education, on-the-job training and continued education, which develop specific skills. Human capital can be interpreted as the combination of those skills.

Therefore, the value of the elasticity of substitution between public and private education is inconclusive. Because the gifts for educational purposes should be used in various forms, it is worthwhile analyzing the strong complementary case in which q is not positive. This subsection examines the strong complementary case where q is not positive. We provide analytical results, followed by a numerical experiment to evaluate the impact of changing q on the growth rate.

First, we provide the results of $q = 0$ case. In this case, the human capital production function (12) turns into the Cobb–Douglas function, $h^i_{t+1} = \left(d^i_t\right)^{\varepsilon}\left(e_t\right)^{1-\varepsilon}$. The same procedure gives us the following dynamics of ϕ[15]:

$$\phi_{t+1} = \left(\frac{a_L}{a_H}\right)^{\varepsilon} \phi^{\varepsilon}_{t-1}. \tag{23}$$

The dynamics does not depend on the gift tax rate τ_b. Since $0 < \varepsilon < 1$, the dynamics has a unique stable steady state. In the steady state, any changes in the gift tax rate do not affect inequality. Regardless of the gift tax rate, inequality ϕ converges towards $\phi^* = \left(a_L/a_H\right)^{\varepsilon/(1-\varepsilon)}$.

The growth rate at the steady state in the case of $q = 0$ is given by

$$g = \left(1-\tau_b\right)^{1-\varepsilon} \tau^{\varepsilon}_b \Theta a^{\varepsilon}_H \left(a_H + \phi a_L\right)^{1-\varepsilon}. \tag{24}$$

Since the gift tax rate does not affect inequality in the steady state, from (24), the derivative of the growth rate with respect to the gift tax rate is

$$\frac{\partial g}{\partial \tau_b} = \frac{1-\varepsilon-\tau_b}{\tau^{\varepsilon}_b \left(1-\tau_b\right)^{1-\varepsilon}} \Theta a^{\varepsilon}_H \left(a_h + \phi a_L\right)^{1-\varepsilon}. \tag{25}$$

With the inspection of (25), we confirm that the growth maximizing tax rate τ^*_b is equal to the weight of public education $1 - \varepsilon$.

Next, we show how the gift tax rate affects inequality and the growth rate in the steady state when q is negative. In contrast to the case in which q is positive, inequality ϕ in the steady state is a decreasing function of the gift tax rate τ_b (See Appendix A.2). Again, a rise in the gift tax rate increases public education but decreases private

[14] Gelber and Isen (2013) find that in response to a preschool program, parents tend to be significantly involved with their children, which suggest that the relationship is complementary.

[15] The derivation of dynamics is relegated to Appendix A.4.

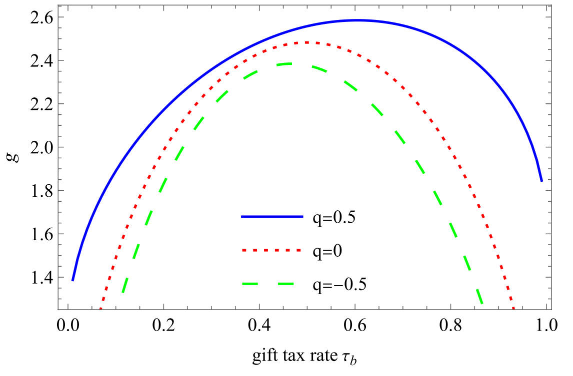

FIGURE 4 Numerical examples of different q. We set the same parameters as before, except for a_H and a_L. The gift tax rates which maximize the growth rate are approximately 0.61, 0.5, and 0.47, respectively.

TABLE 1 Parameters employed in the numerical examples.

Definition	Parameter	Value
Discount factor	β	0.37
Interest rate	$\bar{r}$	1.67
The relative weight of private education	ε	0.5
The utility weight of the gift	γ	0.09
Productivity of type *H*	a_H	28
Productivity of type *L*	a_L	11

education. The marginal benefit of increasing public education for type H individuals is larger than for type L individuals because of strong complementarity ($q < 0$). An increase in the gift tax rate benefits type H individuals. Thus, it is harmful in terms of inequality. In terms of the growth rate, the intuition is similar to the case in which q is positive. However, the effect on growth rate through ϕ is opposite, so there are one positive and two negative effects. When the gift tax rate is sufficiently small, the positive effect overcomes two negative effects. As the gift tax rate rises, two negative effects become large and, at some point, begin to dominate to the positive effect. In line with the case in which q is positive, a gift tax rate maximizes the growth rate.

For the quantitative evaluation of the impact of changing q on the growth rate, we additionally conduct a numerical experiment. Figure 4 shows the growth rate in the steady state at different values of q. We set the same parameters as in Section 4.1, except for a_H and a_L. Table 1 summarizes the parameters used in this numerical experiment. The gift tax rates which maximize the growth rate are 0.61, 0.5, and 0.47, respectively. From those figures, we can deduce the fact that the larger the value of q, the larger the gift tax rate, which maximizes the growth rate.

4.3 | Decision of the gift

Within the preceding analysis, each individual only cares about the amount of the gifts rather than the breakdown. They do not take the effect of taxation into consideration when they make decisions about giving educational funds due to the utility function specification. It seems to be natural in the present paper that individuals choose the amount of educational gifts, taking into account that public education is financed by a part of the gift through taxation. It is important to confirm how individuals' behavior, growth rate, and inequality change when they are aware of the interaction between gifts and the provision of public education.

However, this issue is beyond the scope of the present paper, so we propose a natural modification and note conjectures. It could be addressed by changing the specification of the utility function. Following Azarnert (2010) and Zilcha (2003), the introduction of descendants' human capital could be a natural modification. Considering the human capital of their grandchildren, each individual pays attention to how much her gift contributes to public education. They simultaneously also consider the amount of the gift which the other type of individual decides, because the provision of public education depends on the amount of the gift by the other type as well. The total amount of the gift is essential for individuals especially when public and private education are relatively substitutable. Therefore, this modification may lead to a strategic interaction among individuals. If the interaction between individuals is strategic substitutes, both types of individuals would reduce both public and private expenditures in education, thereby impairing economic growth. In contrast, if the interaction between individuals is strategic complements, both types of individuals would increase educational expenditures. In this case, the strategic interaction may lead to promote economic growth. Although this modification would enrich the present model, the derivation and analysis of the equilibrium are expected to be complex. While this extension exceeds the scope of the present paper, it would be worthwhile to pursue it for future work.

5 | CONCLUSION

In response to the introduction of the tax incentive in Japan, we developed a three-period overlapping generations model with inter vivos gifts and human capital accumulation. Then, we analyzed how the gift tax rate affects wealth inequality and economic growth. The analysis showed that an increase in the gift tax rate reduces inequality, and a positive tax rate maximizes the economic growth rate in the baseline model. Even in the presence of other funding for public education, a positive gift tax rate maximizes the growth rate in the range of plausible parameters. If the gift tax rate is sufficiently small, the growth rate never falls below the previous steady state in the transition path. The results suggest that, rather than exempting gifts from taxation, raising the gift tax rate, to some extent, reduces inequality and promotes human capital accumulation and economic growth. Even in the presence of other funding for public education, the result does not change. In recent decades, the share of inherited wealth in total private wealth has increased in some countries. Therefore, it is crucial to investigate the effects of wealth transfer taxation, which, in turn, is expected to be influential in future.

For analytical simplification, some factors that might have an effect have been ignored. In overlapping generations models with human capital accumulation, the analysis often includes educational-related issues, such as school loans, subsidies, and borrowing constraints. These factors might affect human capital accumulation through education, and thus, economic growth rate and inequality. Yakita (2004) examines the policy effects on the balanced growth rate and welfare with the introduction of school loans. Kitaura and Yakita (2010) introduce the constraint on educational choice and analyze the effects on economic development and fertility. In contrast to this study, Grossmann and Poutvaara (2009) distinguish intentional bequest from educational investment. They show that a small bequest tax may improve efficiency in an overlapping generations framework with only the intended bequest, and it may mitigate the distortion of educational investment. Although we assumed that the use of gifts is restricted only to education, essentially, individuals face the borrowing constraint for education in childhood. The gift may relax the constraint to attain education. We also assume that the fertility rate is exogenous. The quality–quantity trade-off of children is often observed in the literature. As the number of children might affect the amount of gifts per child and the total amount of gifts, fertility decisions are also crucial in this field. Consideration of the effects of these factors is a subject of future work.

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APPENDIX A

A.1 | Proof of the properties of the curve ΦΦ

We prove that ϕ_{t+1}-locus is upward sloping and intersects the 45-degree line at a point less than 1 when $\tau_b \in (0,1)$. From (13), we obtain

$$\frac{\partial \phi_{t+1}}{\partial \phi_{t-1}} = P(\phi_{t-1})Q(\phi_{t-1}), \tag{26}$$

where

$$P(\phi_{t-1}) \equiv \left[\frac{\varepsilon(1-\tau_b)^q a_L^q + (1-\varepsilon)\tau_b^q \left(a_L + \frac{a_H}{\phi_{t-1}}\right)^q}{\varepsilon(1-\tau_b)^q a_H^q + (1-\varepsilon)\tau_b^q (a_H + \phi_{t-1} a_L)^q} \right]^{\frac{1}{q}},$$

$$Q(\phi_{t-1}) \equiv \frac{\varepsilon^2(1-\tau_b)^{2q} a_H^q a_L^q + \varepsilon(1-\varepsilon)\tau_b^q (1-\tau_b)^q (a_H + \phi_{t-1} a_L)^{q-1} \left(\frac{a_H^q a_L}{\phi_{t-1}^{q-1}} + a_H a_L^q \right)}{\left[\varepsilon(1-\tau_b)^q a_H^q + (1-\varepsilon)\tau_b^q (a_H + \phi_{t-1} a_L)^q\right] \left[\varepsilon(1-\tau_b)^q a_L^q + (1-\varepsilon)\tau_b^q \left(a_L + \frac{a_H}{\phi_{t-1}}\right)^q\right]}.$$

All variables take positive values, then we find $\partial\phi_{t+1}/\partial\phi_{t-1} > 0$. This means that the locus is upward-sloping. When ϕ_{t-1} takes 1, $\partial\phi_{t+1}/\partial\phi_{t-1} < 1$ and $\phi_{t+1} < 1$ hold. It is because $P(\phi_{t-1})$ is less than 1 due to $a_H > a_L$ and the numerator of $Q(\phi_{t-1})$ is smaller than the denominator. We can decompose (26) into three parts as follows:

$$\frac{\partial\phi_{t+1}}{\partial\phi_{t-1}} = R(\phi_{t-1})S(\phi_{t-1})T(\phi_{t-1}) \tag{27}$$

where

$$R(\phi_{t-1}) \equiv \left[\varepsilon(1-\tau_b)^q a_H^q + (1-\varepsilon)\tau_b^q(a_H+\phi_{t-1}a_L)^q\right]^{-\frac{1}{q}-1},$$
$$S(\phi_{t-1}) \equiv \left[\varepsilon(1-\tau_b)^q a_L^q + (1-\varepsilon)\tau_b^q\left(a_L+\frac{a_H}{\phi_{t-1}}\right)^q\right]^{\frac{1}{q}-1},$$
$$T(\phi_{t-1}) \equiv \varepsilon^2(1-\tau_b)^{2q}a_H^q a_L^q + \varepsilon(1-\varepsilon)\tau_b^q(1-\tau_b)^q(a_H+\phi_{t-1}a_L)^{q-1}\left(\frac{a_H^q a_L}{\phi_{t-1}^{q-1}} + a_H a_L^q\right).$$

As ϕ_{t-1} approaches 0, $R(\phi_{t-1})$ and $T(\phi_{t-1})$ approach constant, however, $S(\phi_{t-1})$ approaches infinity. Thus, we obtain $lim_{\phi_{t-1}\to 0}\partial\phi_{t+1}/\partial\phi_{t-1} = \infty$. Until now, we have confirmed that the curve crosses 45-degree line at points less than 1 at least once.

To ensure that the curve crosses 45-degree line only once and the steady state is stable, we prove that the curve is concave. Using (27), we obtain second-order derivative of (13) with respect to ϕ_{t-1}:

$$\frac{\partial^2\phi_{t+1}}{\partial\phi_{t-1}^2} = R(\phi_{t-1})S(\phi_{t-1})T(\phi_{t-1})\left[\frac{R'(\phi_{t-1})}{R(\phi_{t-1})} + \frac{S'(\phi_{t-1})}{S(\phi_{t-1})} + \frac{T'(\phi_{t-1})}{T(\phi_{t-1})}\right]. \tag{28}$$

Because $R(\phi_{t-1}), S(\phi_{t-1})$ and $T(\phi_{t-1})$ are positive, if the sum in the square brackets of (28) is negative, the curve is concave. Upon some inspections, we obtain the followings:

$$\frac{R'(\phi_{t-1})}{R(\phi_{t-1})} = -\frac{(1+q)(1-\varepsilon)\tau_b^q a_L(a_H+\phi_{t-1}a_L)^{q-1}}{R(\phi_{t-1})},$$
$$\frac{S'(\phi_{t-1})}{S(\phi_{t-1})} + \frac{T'(\phi_{t-1})}{T(\phi_{t-1})} = \frac{(1-q)\varepsilon(1-\varepsilon)\tau_b^q(1-\tau_b)^q a_H a_L}{(a_H+\phi_{t-1}a_L)^{2-q}S(\phi_{t-1})T(\phi_{t-1})} \times\left[\varepsilon(1-\tau_b)^q a_L^{q-1}\left(\frac{a_H^{q+1}}{\phi_{t-1}^{q+1}} + a_L^{q+1}\right) + (1-\varepsilon)\tau_b^q\left(a_L+\frac{a_H}{\phi_{t-1}}\right)^{q+1}\right].$$

We find that the sum in the square brackets of (28) is negative given that $q \geq -1$. Therefore, the curve $\Phi\Phi$ intersects the 45-degree line at a point less than 1 when $\tau_b \in (0,1)$. In the case of $\tau_b = 0$, the slope of (13) is $a_L/a_H < 1$. Thus, $\phi^* = 0$ is a unique stable steady state. In the case of $\tau_b = 1$, (13) does not depend on ϕ_{t-1} anymore. ϕ immediately reaches 1.

A.2 | Proof of Proposition 1

In the steady state, as ϕ is constant over time, we obtain

$$1 = \left[\frac{\varepsilon(1-\tau_b)^q a_L^q + (1-\varepsilon)\tau_b^q\left(a_L+\frac{a_H}{\phi}\right)^q}{\varepsilon(1-\tau_b)^q a_H^q + (1-\varepsilon)\tau_b^q(a_H+\phi a_L)^q}\right] \equiv F(\phi). \tag{29}$$

From (29), ϕ in the steady state satisfies

$$\left(a_L+\frac{a_H}{\phi}\right)^q - (a_H+\phi a_L)^q = \frac{\varepsilon(a_H^q - a_L^q)(1-\tau_b)^q}{(1-\varepsilon)\tau_b^q}. \tag{30}$$

Define the following function:

$$M(\phi,\tau_b) \equiv \left(a_L + \frac{a_H}{\phi}\right)^q - (a_H + \phi a_L)^q - \frac{\varepsilon(a_H^q - a_L^q)(1-\tau_b)^q}{(1-\varepsilon)\tau_b^q}. \tag{31}$$

Applying the implicit function theorem, we obtain

$$\frac{\partial \phi}{\partial \tau_b} = -\frac{M_{\tau_b}(\phi,\tau_b)}{M_\phi(\phi,\tau_b)} = \frac{\varepsilon \phi^{q+1}(a_H^q - a_L^q)(1-\tau_b)^{q-1}}{(1-\varepsilon)\tau_b^{q+1}(a_H+\phi a_L)^{q-1}(a_H+\phi^{q+1}a_L)}. \tag{32}$$

Since we assume that $q > 0$, $\frac{\partial \phi}{\partial \tau_b}$ is positive.

A.3 | Proof of Proposition 2

To confirm that a gift tax rate maximizes the growth rate, we demonstrate that $lim_{\tau_b \to 0} \partial g / \partial \tau_b = \infty$ and $lim_{\tau_b \to 1} \partial g / \partial \tau_b < 0$. Differentiating (14) with respect to τ_b and substituting (32), we derive

$$\frac{\partial g}{\partial \tau_b} = \Theta\left[\varepsilon(1-\tau_b)^q a_H^q + (1-\varepsilon)\tau_b^q (a_H+\phi a_L)^q\right]^{\frac{1-q}{q}} \times \left[-\frac{\varepsilon a_H^q}{(1-\tau_b)^{1-q}} + \frac{(1-\varepsilon)(a_H+\phi a_L)^q}{\tau_b^{1-q}} + \frac{\varepsilon a_L \phi^{q+1}(a_H^q - a_L^q)}{(1-\tau_b)^{1-q}(a_H+\phi^{q+1}a_L)}\right]. \tag{33}$$

If $q > 0$, we obtain

$$\lim_{\tau_b \to 0} \frac{\partial g}{\partial \tau_b} = \Theta\left[\varepsilon a_H^q\right]^{\frac{1-q}{q}} \left\{-\varepsilon a_H^q + \lim_{\tau_b \to 0}\left[\frac{(1-\varepsilon)(a_H+\phi a_L)^q}{\tau_b^{1-q}} + \frac{\varepsilon a_L \phi^{q+1}(a_H^q - a_L^q)}{\tau_b(1-\tau_b)^{1-q}(a_H+\phi^{q+1}a_L)}\right]\right\} = \infty. \tag{34}$$

We can rewrite (33) as follows:

$$\frac{\partial g}{\partial \tau_b} = \Theta\left[\varepsilon(1-\tau_b)^q a_H^q + (1-\varepsilon)\tau_b^q (a_H+\phi a_L)^q\right]^{\frac{1-q}{q}} \times \left\{\frac{(1-\varepsilon)(a_H+\phi a_L)^q}{\tau_b^{1-q}} + \frac{-\varepsilon(\tau_b a_H + \phi^{q+1}a_L) + \varepsilon(1-\tau_b)\phi^{q+1}a_H^q a_L}{\tau_b(1-\tau_b)^{1-q}(a_H+\phi^{q+1}a_L)}\right\}. \tag{35}$$

The second term in the curly brackets goes to negative infinity as τ_b approaches 1. That is, $lim_{\tau_b \to 1} \frac{\partial g}{\partial \tau_b} < 0$. Therefore, there is at least one τ_b between 0 and 1, which maximizes the growth rate.

A.4 | Dynamics when $q = 0$

When $q = 0$, the human capital accumulation function can be Cobb–Douglas form:

$$h_{t+1}^i = (d_t^i)^\varepsilon (e_t)^{1-\varepsilon} = \tau_b^{1-\varepsilon}(1-\tau_b)^\varepsilon \Theta a_i^\varepsilon (h_{t-1}^i)^\varepsilon \left[a_i h_{t-1}^i + a_j h_{t-1}^j\right]^{1-\varepsilon}. \tag{36}$$

The same procedure yields

$$\phi_{t+1} = \frac{h_{t+1}^L}{h_{t+1}^H} = \frac{\tau_b^{1-\varepsilon}(1-\tau_b)^\varepsilon \Theta a_H^\varepsilon (h_{t-1}^H)^\varepsilon [a_H h_{t-1}^H + a_L h_{t-1}^L]^{1-\varepsilon}}{\tau_b^{1-\varepsilon}(1-\tau_b)^\varepsilon \Theta a_L^\varepsilon (h_{t-1}^L)^\varepsilon [a_H h_{t-1}^H + a_L h_{t-1}^L]^{1-\varepsilon}} = \left(\frac{a_L}{a_H}\right)^\varepsilon \phi_{t-1}^\varepsilon. \tag{37}$$

Because of $a_H > a_L$ and $0 < \varepsilon < 1$, it is straightforward that there is a unique stable steady state which is less than 1. The value of ϕ at the steady state is $\phi^* = (a_L/a_H)^{\frac{\varepsilon}{1-\varepsilon}}$. The gift tax rate does not affect the value of ϕ at the steady state.

The growth rate is

$$g = \tau_b^{1-\varepsilon}\left(1-\tau_b\right)^{\varepsilon}\Theta a_H\left[1+\left(\frac{a_L}{a_H}\right)^{\frac{1}{1-\varepsilon}}\right]^{1-\varepsilon}. \tag{38}$$

The derivative of the growth with respect to the gift tax rate is

$$\begin{aligned}\frac{\partial g}{\partial \tau_b} &= \left[(1-\varepsilon)\tau_b^{-\varepsilon}\left(1-\tau_b\right)^{\varepsilon}-\varepsilon\tau_b^{1-\varepsilon}\left(1-\tau_b\right)^{\varepsilon-1}\right]\Theta a_H\left[1+\left(\frac{a_L}{a_H}\right)^{\frac{1}{1-\varepsilon}}\right]^{1-\varepsilon}\\ &= \left(\frac{1-\tau_b}{\tau_b}\right)^{\varepsilon}\left(1-\frac{\varepsilon}{1-\tau_b}\right)\Theta a_H\left[1+\left(\frac{a_L}{a_H}\right)^{\frac{1}{1-\varepsilon}}\right]^{1-\varepsilon}.\end{aligned} \tag{39}$$

Thus, the gift tax rate, which maximizes the growth rate, is equal to $\tau_b^* = 1-\varepsilon$.