Conjoined NPs in PTQ

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§1. This paper is an attempt to incorporate conjoined NPs (or terms) into Montague's fragment of English as presented in Montague (1973),[1] with which general familiarity is presupposed here. In the appendix I shall give formal proofs of two theorems in Montague grammar.

§2. Montague (1973; hereinafter PTQ) does not include a rule for conjoining terms (or, to use a linguist's jargon, a rule for conjoining NPs); this is to limit the verb forms in PTQ to 3rd person singulars. His rule S13 reads:

(1) If $\alpha, \beta \in P_T$, then $F_9(\alpha, \beta) \in P_T$

where F_9 is a structural operation of disjunction, i.e., $F_9(\alpha, \beta) = \alpha$ **or** β. In order to incorporate conjoined NPs into PTQ fragment, we replace S13 by S13′:

(2) If $\alpha, \beta \in P_T$, then $F_8(\alpha, \beta), F_9(\alpha, \beta) \in P_T$

F_8 and F_9 are as defined in PTQ; particularly, $F_8(\alpha, \beta) = \alpha$ **and** β. Consequent upon the adoption of S13′ we will need changes/additions in other parts of the PTQ grammar — in syntax because of the resultant necessity of subject-verb agreement, and in translation rules because of the newly produced combination of conjoined terms and verbs, each of which I will take up in the following two sections.

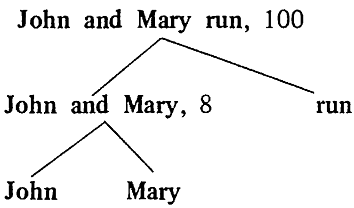
§3. We have basically two alternatives in approaching a satisfactory treatment of subject-verb agreement. First we could follow the kind of treatment in PTQ. S4 for instance could be revised as S4′ (:recall that T is a category of t/IV):

(3) 1. If $\alpha \in P_{t/IV}$ and $\alpha \neq F_8(\beta, \gamma)$ where $\beta, \gamma \in P_T$ and $\delta \in P_{IV}$, then $F_4(\alpha, \delta) \in P_t$, where $F_4(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb (i.e., member of B_{IV}, B_{TV}, $B_{IV/t}$ or $B_{IV//IV}$) in δ by its third person singular present.

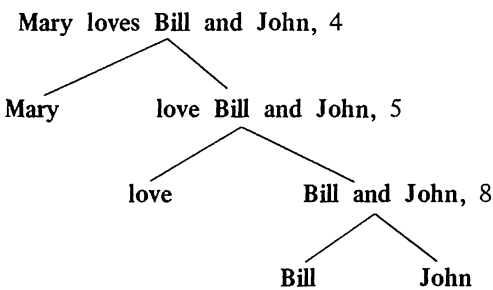
2. If $\alpha \in P_{t/IV}$ and $\alpha = F_8(\beta, \gamma)$ where $\beta, \gamma \in P_T$ and $\delta \in P_{IV}$, then $F_{100}(\alpha, \delta) \in P_t$, where $F_{100}(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb in δ by its plural present.[2]

Sample analysis trees illustrating the rules proposed are:

(4) a.



b.



S4′ alone is not of course enough to cover the entire subject-verb agreement phenomena that crop up in PTQ. We have to change S17 along the same line in order to account for negatives, present perfects, etc., which is left to the reader, the revision being a trivial task, given time and patience.

The second approach to the number agreement is a transformational one. Although a syntactic transformation (in the technical sense of the term as employed in generative grammars) as such is nowhere to be found in PTQ, it is an obvious extension of it that fits the system fairly comfortably, as shown in Partee (1975, 1976). As a generative grammarian I find the transformational approach very attractive and revealing. The only catch one might feel some discomfort at is the proviso that the agreement be obligatory. This puts it in a class with the reflexive transformation (cf. Partee (1975)). Negatively viewed, this is another loss in generalization (that all transformations be optional (in Partee's extension of PTQ)). Positively viewed, we have another piece of evidence that supports the necessity of an obligatory transformation in the extended PTQ fragment (: See in this connection Cooper and Parsons (1976)). In either case there is no doubt that we can simplify the rules S4 and S17, which is reminiscent of the countervailing effects of phrase structure rules and syntactic transformations upon each other with respect to the rule simplification in a transformational generative grammar although Montague's syntactic rules S1-17 come nowhere close to phrase structure rules.[3] S4 for instance would be a simple concatenation of terms and intransitive verbs. Thus:

(5) S4′. If $\alpha \in P_{t/IV}$ and $\delta \in P_{IV}$, then $F_6(\alpha, \delta) \in P_t$.

F_6 is the structural operation as in S7, i.e., $F_6(\alpha, \delta) = \alpha\delta$. S17 would also be simplified along the obvious line. The first clause of S17, for instance, would be:

(6) $F_{11}(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb in δ by its negative.

All the necessary morphological adjustments will be now carried over to and taken care of by the following subject-verb agreement transformation[4]:

(7) S25: *Subject-Verb Agreement.*

If $\phi \in P_t$ and ϕ has the form:

$_t(_T(\alpha)\ _{IV}(\delta))$

then if $\alpha = F_8(\beta, \gamma)$ where $\beta, \gamma \epsilon P_T$

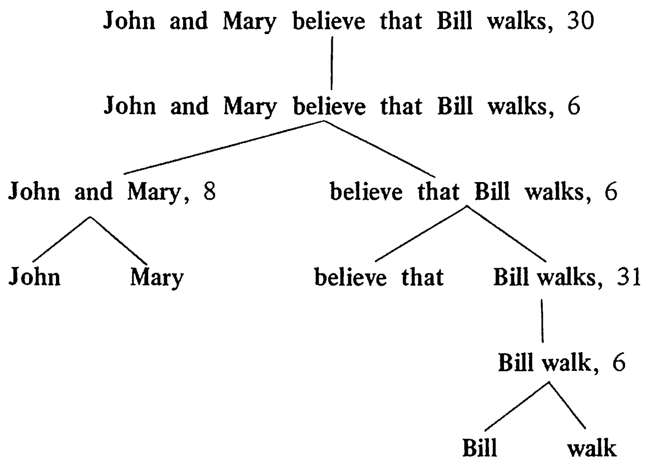
then $F_{30}(\alpha, \delta) \in P_t$ where $F_{30}(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb by its plural present, and if $\alpha \neq F_8(\beta, \gamma)$ where $\beta, \gamma \epsilon P_T$

then $F_{31}(\alpha, \delta)\ \epsilon P_t$ where $F_{31}(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb by its third person singular present,

where a verb is a member of $\cup_{A\ \epsilon\ \{IV, TV, IV/t, IV//IV\}} B_A \cup$ {**have, will**}.

Have and **will** will be each introduced by F_{14} and F_{12} in S17. Although F_{30} and F_{31} are complex structural operations, nonetheless they are specifiable, as demonstrated in the transformational literature. A sample analysis tree illustrating both S13′ and S25 would be:

(8)



Whether we should elaborate upon S4 along the suggested line or adopt a transformational approach in dealing with the subject-verb agreement pursuant to the adoption of S13′ is a problem that cannot be easily settled. Particularly no argument based on simplicity bears any relevance on the issue in the PTQ framework.

§4. The addition of new syntactic rules S13′ and S25 (here we adopt the transformational version of number agreement) necessitates corresponding translation rules. Our translation rule corresponding to the number agreement transformation is an identity mapping, which I will designate as T25:

(9) T25. If $\phi \epsilon P_t$ and ϕ translates ϕ', $F_{30}(\phi)$ and $F_{31}(\phi)$ translate into ϕ'.

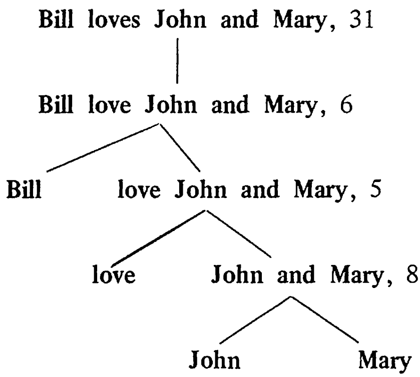
Corresponding to S13′, we will have:

(10) T13. If $\alpha, \beta \epsilon P_T$ and α, β translate into α', β' respectively, then α **and** β translates into $\hat{P}\ [\alpha'(P)\ \&\ \beta'(P)]$ and α **or** β translates into $\hat{P}\ [\alpha'(P) \vee \beta'(P)]$.

To illustrate T25 and T13 (particularly the translation of α **and** β), we will take up the following sentence and the corresponding analysis tree:

(11) a. **Bill loves John and Mary.**

b.



Using the proposed translation rules as well as those given in PTQ, we get the following translation. (The arrow "⟶" reads "translates as.")

(12) **John** $\longrightarrow j^*$: T1(d)

Mary $\longrightarrow m^*$: T1(d)

John and Mary $\longrightarrow \hat{P}\ [j^*(P)\ \&\ m^*(P)]$: T13

love $\longrightarrow$ **love′** : T1(a)

love John and Mary $\longrightarrow$ love′ $[{}^\wedge\hat{P}\ [j^*(P)\ \&\ m^*(P)]$: T5

Bill $\longrightarrow$ b^* : T1(d)

Bill love John and Mary $\longrightarrow$

$b^*(^\wedge \mathbf{love'}[^\wedge \hat{P}\, [j^*(P) \,\&\, m^*(P)]])$: T4

Bill loves John and Mary $\longrightarrow$

$b^*(^\wedge \mathbf{love'}[^\wedge \hat{P}\, [j^*(P) \,\&\, m^*(P)]])$: T25

To make the proposed translation rules more perspicuous, I shall below carry out the usual reduction of the last formula above (: read the dotted arrow "$- - \rightarrow$" as "converts to"; conversion here is of course based on definitions, notational conventions, meaning postulates, and rules of intensional logic, as in PTQ[5]).

(13)	$b^*(^\wedge \mathbf{love'}[^\wedge \hat{P}\, j^*(P) \,\&\, m^*(P)]])$	
$- - \rightarrow$	$\hat{P}P\{^\wedge b\}(^\wedge \mathbf{love'}[^\wedge \hat{P}[j^*(P) \,\&\, m^*(P)]])$	: Superstar Definition
$- - \rightarrow$	$^\wedge \mathbf{love'}[^\wedge \hat{P}[j^*(P) \,\&\, m^*(P)]]\ \{^\wedge b\}$	: Abstraction Application
$- - \rightarrow$	$^{\vee\wedge} \mathbf{love'}[^\wedge \hat{P}[j^*(P) \,\&\, m^*(P)]]\ (^\wedge b)$	: Brace Convention
$- - \rightarrow$	$\mathbf{love'}\ [^\wedge \hat{P}[j^*(P) \,\&\, m^*(P)]]\ (^\wedge b)$	: Down-Up Cancellation
$- - \rightarrow$	$\mathbf{love'}(^\wedge b, {}^\wedge \hat{P}[j^*(P) \,\&\, m^*(P)])$	: Relation Notation
$- - \rightarrow$	$^\wedge \hat{P}[j^*(P) \,\&\, m^*(P)]\ \{\hat{y}\mathbf{love'}_*(^{\vee\wedge} b, {}^\vee y)\}$	: Extensional TV Theorem
$- - \rightarrow$	$^{\vee\wedge} \hat{P}[j^*(P) \,\&\, m^*(P)]\ (\hat{y}\mathbf{love'}_*(^{\vee\wedge} b, {}^\vee y))$	: Brace Convention
$- - \rightarrow$	$\hat{P}[j^*(P) \,\&\, m^*(P)]\ (\hat{y}\mathbf{love'}_*(b, {}^\vee y))$	: Down-Up Cancellation
$- - \rightarrow$	$j^*(\hat{y}\mathbf{love'}_*(b, {}^\vee y)) \,\&\, m^*(\hat{y}\mathbf{love'}_*(b, {}^\vee y))$	: Abstraction Application

(Since the two conjuncts above are identical save the occurrences of "j" and "m," I will further reduce only the left conjunct, the right conjunct being similarly reducible.)

$j^*(\hat{y}\mathbf{love'}_*(b, {}^\vee y))$

$\dashrightarrow$ $\hat{P}P\{{}^\wedge j\}(\hat{y}\mathbf{love}'_*(b, {}^\vee y))$: Superstar Definition

$\dashrightarrow$ $\hat{y}\mathbf{love}'_*(b, {}^\vee y)\ \{{}^\wedge j\}$: Abstraction Application

$\dashrightarrow$ ${}^\wedge\hat{y}\mathbf{love}'_*(b, {}^\vee y)\ \{{}^\wedge j\}$: Intensional Abstraction Convention

$\dashrightarrow$ ${}^{\vee\wedge}\hat{y}\mathbf{love}'_*(b, {}^\vee y)\ ({}^\wedge j)$: Brace Convention

$\dashrightarrow$ $\hat{y}\mathbf{love}'_*(b, {}^\vee y)\ ({}^\wedge j)$: Down-Up Cancellation

$\dashrightarrow$ $\mathbf{love}'_*(b, {}^{\vee\wedge} j)$: Abstraction Application

$\dashrightarrow$ $\mathbf{love}'_*(b, j)$: Down-Up Cancellation

Thus the original formula reduces to:

$$\mathbf{love}'_*(b, j)\ \&\ \mathbf{love}'_*(b, m)$$

Appendix: Proofs of Weak and Strong Common Noun Theorems.

Although no use was made of above, we will here give proofs of two theorems in PTQ grammar, the (Weak) Common Noun Theorem and the Strong Common Noun Theorem (cf. Partee (1975), Appendix B) in view of their significance in carrying out logical reductions. The intensional logic here is based on the modal system S5 with quantifiers and equality (cf. Lewis and Langford (1932)), that is, any formalization adequate for the classical first-order predicate calculus with equality supplemented by the following axiom schemes and rules of inference (see Kripke (1959)):

A1: $\Box A \rightarrow A$

A2: $\sim\Box A \rightarrow \Box\sim\Box A$

A3: $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$

R1: If $\vdash A$ and $\vdash A \rightarrow B$, $\vdash B$.

R2: If $\vdash A$, $\vdash \Box A$.

Below I shall feel free to use derived rules of inference that may be obtainable within the system; throughout the proofs, δ translates any member of B_{CN} other than **price** or **temperature** (cf. the meaning postulate (2) in PTQ, p.263).

First I shall prove the Weak Common Noun Theorem, i.e.,
$\Box[\delta(x) \leftrightarrow \delta(x) \,\&\, (Eu)\,(x = {}^\wedge u)]$.

1.	$\Box[\delta(x) \rightarrow (Eu)\,(x = {}^\wedge u)]$	: PTQ Meaning Postulate (2)
2.	$\rightarrow \delta(x)$	
3.	$\Box[\delta(x) \rightarrow (Eu)\,(x = {}^\wedge u)] \rightarrow [\delta(x) \rightarrow (Eu)\,(x = {}^\wedge u)]$	: A1
4.	$\delta(x) \rightarrow (Eu)\,(x = {}^\wedge u)$	: 1, 3, Modus Ponens
5.	$(Eu)\,(x = {}^\wedge u)$	: 2, 4, Modus Ponens
6.	$\delta(x) \,\&\, (Eu)\,(x = {}^\wedge u)$	: 2, 5, Conjunction
7.	$\delta(x) \rightarrow \delta(x) \,\&\, (Eu)\,(x = {}^\wedge u)$	: 2–6, Conditional Proof
8.	$\rightarrow \delta(x) \,\&\, (Eu)\,(x = {}^\wedge u)$	
9.	$\delta(x)$	: 8, Simplification
10.	$\delta(x) \,\&\, (Eu)\,(x = {}^\wedge u) \rightarrow \delta(x)$	: 8-9, Conditional Proof
11.	$\delta(x) \leftrightarrow \delta(x) \,\&\, (Eu)\,(x = {}^\wedge u)$	: 7, 10, Material Equivalence
12.	$\Box[\delta(x) \leftrightarrow \delta(x) \,\&\, (Eu)\,(x = {}^\wedge u)]$	: 11, R2

Next I will prove the Strong Common Noun Theorem, i.e.,
$\Box[(Ex)[\delta(x) \,\&\, P\{x\}] \leftrightarrow (Eu)\,[\delta_*(u) \,\&\, P\{{}^\wedge u\}]]$.

$\blacktriangleright (Ex)\ [\delta(x)\ \&\ P\{x\}]$	
$(Ex)\ [\delta(x)\ \&\ (Eu)\ (x = {}^\wedge u)\ \&\ P\{x\}]$	: 1, Common Noun Theorem
$(Ex)\ (Eu)\ [\delta(x)\ \&\ (x = {}^\wedge u)\ \&\ P\{x\}]$	: 2, Quantifier Exportation
$(Ex)\ (Eu)\ [\delta({}^\wedge u)\ \&\ (x = {}^\wedge u)\ \&\ P\{{}^\wedge u\}]$	: 1, 3, Substitution of Equivalents
$(Eu)\ [\delta({}^\wedge u)\ \&\ P\{{}^\wedge u\}]$	: 4, Vacuous Parts Elimination
$(Eu)\ [\delta_*(u)\ \&\ P\{{}^\wedge u\}]$	: 5, Substar Definition
$x)\ [\delta(x)\ \&\ P\{x\}] \rightarrow (Eu)\ [\delta_*(u)\ \&\ P\{{}^\wedge u\}]$	: 1–6, Conditional Proof
$\blacktriangleright (Eu)\ [\delta_*(u)\ \&\ P\{{}^\wedge u\}]$	
$\rightarrow \delta_*(u)$	
$\Box(Au)\ (Ex)\ (x = {}^\wedge u)$	: A theorem of intensional logic
$\Box(Au)(Ex)(x = {}^\wedge u) \rightarrow (Au)(Ex)(x = {}^\wedge u)$	: 10, A1
$(Au)\ (Ex)\ (x = {}^\wedge u)$	: 10, 11, Modus Ponens
$(Ex)\ (x = {}^\wedge u)$	: 12, Universal Instantiation
$\delta_*(u)\ \&\ (Ex)\ (x = {}^\wedge u)$	: 9, 13, Conjunction
$\delta_*(u) \rightarrow \delta_*(u)\ \&\ (Ex)\ (x = {}^\wedge u)$	: 9–14, Conditional Proof
$\rightarrow \delta_*(u)\ \&\ (Ex)\ (x = {}^\wedge u)$	
$\delta_*(u)$	: 16, Simplification
$\delta_*(u)\ \&\ (Ex)\ (x = {}^\wedge u) \rightarrow \delta_*(u)$	: 16–17, Conditional Proof
$\delta_*(u) \leftrightarrow \delta_*(u)\ \&\ (Ex)\ (x = {}^\wedge u)$	: 15, 18, Material Equivalence
$(Eu)\ [\delta_*(u)\ \&\ (Ex)\ (x = {}^\wedge u)\ \&\ P\{{}^\wedge u\}]$	: 8, 19, Substitution of Equivalents

21. $(Eu)(Ex)[\delta_*(u) \,\&\, (x = {}^\wedge u) \,\&\, P\{{}^\wedge u\}]$: 20, Quantifier Exportation

22. $(Eu)(Ex)[\delta({}^\wedge u) \,\&\, (x = {}^\wedge u) \,\&\, P\{{}^\wedge u\}]$: 21, Substar Definition

23. $(Eu)(Ex)[\delta(x) \,\&\, (x = {}^\wedge u) \,\&\, P\{x\}]$: 22, Substitution of Equivalents

24. $(Ex)[\delta(x) \,\&\, P\{x\}]$: 23, Vacuous Parts Elimination

25. $(Eu)[\delta_*(u) \,\&\, P\{{}^\wedge u\}] \rightarrow (Ex)[\delta(x) \,\&\, P\{x\}]$: 8–24, Conditional Proof

26. $(Ex)[\delta(x) \,\&\, P\{x\}] \leftrightarrow (Eu)[\delta_*(u) \,\&\, P\{{}^\wedge u\}]$: 7, 25, Material Equivalence

27. $\Box[\,[(Ex)\,\delta(x) \,\&\, P\{x\}] \leftrightarrow (Eu)[\delta_*(u) \,\&\, P\{{}^\wedge u\}]\,]$: 26, R2

Notes.

1. But I would like to exclude the discussion of phrasal conjunction (see Lakoff and Peters (1966)); hence the conjunction in this paper is to be understood as the sentential conjunction. For obvious typographical reasons, I shall use A, E, & for $\wedge$, $\vee$, $\wedge$, respectively. Cooper (1977) and Parsons (1975) each contains a useful list of misprints in PTQ as it appears in Thomason (ed.) (1974).

2. Another version may read:

> If $\alpha \in P_{t/IV}$ and $\alpha = F_8(\beta, \gamma)$ where $\beta, \gamma \in P_T$ and $\delta \in P_{IV}$, then $F_{100}(\alpha, \delta) \in P_t$, where $F_{100}(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb by **are** if it is **be**, otherwise $\delta' = \delta$.

This is based on the fact that the plural forms of English verbs are the same as those of infinitives save for **be**, which takes on the form **are**. Either treatment is consistent with the facts of English.

As they stand, both versions have at least two drawbacks.

1) They produce sentences like **John walks and run, Mary runs and walk**, etc., which is also true of PTQ as pointed out in Partee (1975). To overcome this undesirable consequence we will have to consider the structure of δ above, but I will leave the matter at that; 2) They both produce sentences like **John and Bill are a man**, which is not grammatical. It seems we can easily code the information to avoid them into either version above – we can for instance specify that the sequence **are** $_{T}$(**a** $_{CN}(\alpha)$) is to be changed to **are** $_{T}(_{CN}(\alpha'))$, where α' is the result of replacing the first common noun in α by its plural form.

3. Needless to say this does not preclude the possibility of a metamorphosis from PTQ grammar into a version of transformational grammar. In fact one such proposal or demonstration, whichever may be the case, is given in Cooper and Parsons (1976) in a fairly rigorous fashion.

4. The numbering is from Partee (1975), this paper being at least in spirit an attempt at extending her grammar. Re minor inadequacies of S25, see the last paragraph of the footnote 2 above. Cooper and Parsons (1976) propose an affix hopping rule in a *Syntactic Structures* fashion, which I feel is far better than S25. Since I do not wish to tinker with other rules in PTQ, I will stick to S25 in this paper.

5. The names of justification used in each step of reduction, except for fairly standard ones, are from Partee (1975).

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